

Introduction to GeoGebra for Agricultural Science Teachers

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GeoGebra 5

Figure 1: Launch Screen for GeoGebra

The toolbar along the top shows 12 icons or tools, each of which has a drop-down menu, or submenu, which can be activated by clicking on the down-pointing arrow at the bottom right corner of an icon.

The following section explains each of the drop-down menus in more detail.





The GeoGebra Toolbar

Figure 2: The main GeoGebra toolbar

1st Tool from left: Move

The first button here allows you to select a previously drawn object and move it around the drawing area. The 2nd button in this menu allows you to select a point as a centre of rotation and rotate any object around it. The 3rd button allows you sketch freehand shapes which will be recognised and converted to an exact shape. The 4th button allows you to draw objects using a pen.





2nd Tool from left: Point

These buttons allow you to construct different types of points.

The first button allows you to put a point anywhere in the drawing area. Using the second button you can **attach a point** to an object and the third button you can attach or detach a point.

The fourth button allows you to select two different lines or curves and find their point of **intersection**. The fifth button constructs the **midpoint** of a line segment.

3rd Tool from left: Line through two points

This tool has eight different buttons. These construct

- 1. an infinite line through two selected points
- 2. a line segment between two selected points
- 3. a line segment of a given length from a specific point
- 4. a ray through two points
- 5. a polyline linking a number of points
- 6. a vector between two points
- 7. a vector parallel to a given vector.

4th Tool from left: Perpendicular line

There are eight buttons under the perpendicular line tool. These allow you to

- 1. draw a line perpendicular to a given line
- 2. draw a line parallel to a previously constructed line
- 3. bisect a line segment
- 4. bisect a previously constructed angle
- 5. construct **tangents** to various curves, graphs or previously defined functions
- 6. the polar or diameter line
- 7. the line of best fit.

5th Tool from left: Polygons

This can be used to construct regular and irregular polygons. Be sure to complete the polygon by finally clicking on the first or starting point.











6th tool from left: Circles and arcs

The nine buttons in the this menu allow you to construct

- 1. Circle with centre through point. The point will then be linked to the circumference of the circle
- 2. Circle with specified centre and radius. Click on the required centre. A menu appears asking you for the radius; enter the value and press the Return key.
- 3. Compass: This draws a circle whose radius is the distance between two previously drawn points.
- 4. Circle through three points. Select this tool and click on three different points.
- 5. Semicircle through two points, does exactly what it says.
- 6. Circular arc with specified centre and going through two other points. Click to to fix the centre of the circle. Click again to create the first point of the arc. If you move the mouse anti-clockwise you will see an arc being constructed. When you click again the arc is completed.
- 7. Circumcircular arc through three points, draws an arc through any three points.

8th Tool: Angles

The eight buttons in this menu deal mainly with measurement.

- 1. Angle is used to construct and measure the angle between three points, moving anticlockwise
- 2. Angle with given size. Select this tool and click to create the vertex of the arm of an angle. The next click gives the point at the angle. At this point a menu appears where you will be asked to select angle size and direction.
- 3. Distance or length, has several functions. If you use this button and click on two previously constructed points it will give you the distance between them. The function of this button is to measure the length of a line segment.
- 4. Area. Use this button on a polygon, circle or conic to display the area of the figure.
- 5. Slope, will display the slope of a selected line or line segment.



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9th Tool: Reflect objects in a line or a point

These buttons are used to construct geometric transformations.

- 1. Reflect object in a line. First select the object and then click on the line of symmetry
- 2. Reflect object in a point.
- 3. Reflect object in a circle.
- 4. Rotate object around point by an angle. Select the object to be rotated and then the centre of rotation. A menu will then appear asking for the size of the angle. You can enter a number or the variable name of a previously constructed angle or slider. The default rotation is anticlockwise.
- 5. Translate object by vector performs the translation of an object
- 6. Enlarge object from point by factor magnifies or reduces the object.

10th Tool: Slider and other items

This dropdown menu has miscellaneous functions.

- 1. **Slider** allows us to construct variables whose values can be changed simply by moving the slider.
- 2. The **Text** tool
- 3. The Insert Image tool
- 4. The **Button** tool is used to create buttons that can be used to control objects by means of a **Script**.
- 5. **Check box** to show and hide objects, is another button to control what is displayed on the screen.
- 6. The **Input box** can be used to

Insert text

Insert text allows you to place text anywhere on the drawing pad /Graphics View. Select the tool and then click on the drawing pad. A window appears. The **Symbols** button helps you to enter mathematical symbols, Greek letters etc.

Insert image

The Insert image button allows you to import various types of images into your worksheet.

Once the image is imported it is treated

as an ordinary object and you can use various tools on it. In particular, all the geometric transformation tools will work. This allows you to prepare an image in your favourite drawing application and then use it in GeoGebra.





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Move Graphics View

Most of the buttons under this tool are self explanatory.

- 1. The Move Graphics View button allows you to drag the whole drawing pad in any direction you like.
- 2. Zoom In
- 3. Zoom Out
- 4. Show/ Hide Object is a little more complicated. Use this button to click on the object that you want to show or hide. Nothing appears to happen. Now select any other tool. At this point the change you wanted occurs.
- 5. Show / Hide Label
- 6. The Copy visual style button is very useful. Suppose you have drawn a diagram in which you assigned a particular colour to an object. Later you may want to use the same colour on another object, such as a descriptive piece of text. Using this tool click on the first object; its style will be applied to next the object you click on.
- 7. Delete object deletes any object you click on when using it. Be very careful before you use this tool. It will delete not only the object you select but also any objects that depended on the selected object during their construction. Unless you are sure that you want to delete an object it is usually a much safer option to hide it.

Algebra Input Commands

GeoGebra contains a set of internal commands and functions. These can be entered directly by using the box beside the word **Input**. The Input bar is by default located in the bottom of GeoGebra window.

There are other commands available also that you can examine at your leisure in the GeoGebra help file. With these commands you can draw many types of graph that are not possible by just using the Geometry toolbar

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For example if you enter sin(x) into the Input bar and press the Enter/Return key you will find that the graph of sin x is drawn on the screen. The command Function[sin(x),0,2 pi] will draw the graph of sin x from 0 to 2π . Note the use of the square brackets and the space between the number 2 and the constant pi. We are now going to look at the ways that we can use direct input, entered into the Input bar, to perform many different constructions and operations.



Input:



Exercise 1

Task 1

Select the 1st tool from the left and then select the freehand shape from the dropdown menu. Use your mouse to draw shapes on the graphics window.

Task 2

Select the 2nd tool from the left and then select a point on the graphics window.

Use your mouse to move this point and note its co-ordinates also change.

Task 3

Use the 3rd tool from the left and select the line option from the dropdown menu. Now select 2 points on the graph to draw the line. Note its equation. Use your mouse to move one of the points and note the line also changes.

Task 4

Graph the line y=2x+1 by inputting it in the bar at the bottom of the screen and hitting the return key.

Task 5

Graph the line y=x+3 by inputting in the input bar and hitting the return key. Graph the line y=2x+3 by inputting in the input bar and hitting the return key.

Use the 2nd tool from the left and select the **intersection** from the dropdown menu. Then select the 2 lines and the intersection point will appear.

Task 6

Graph the line $\mathbf{y} = \mathbf{ax} + \mathbf{b}$ by inputting in the input bar and hitting the return key. When the prompt appears accept **create sliders**. Change the sliders and observe the change on the graph.

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One variable data Example 1

- 1. Open GeoGebra
- 2. Select the *View* option from the menu bar and click on Spread sheet. Note the new toolbar.
- 3. Close the Algebra and Graphics views by clicking on the **x** in the top right-hand corners. You are now ready to input data into the Spreadsheet.
- 4. Type the data given under column A:

Α	
Exercises	
3	
1	
7	
2	
3	
2	
0	
4	
1	
3	

This is a record of the number of Agricultural Science exercises Chris completed each school day over a two-week period.

- 5. Click on column A to highlight all its contents.
- Select the 2nd tool from the left of the toolbar and click on *One Variable Analysis*.

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3		Hultiple Variable Analysis							
4 5		Probability Calculator							
6									
7									
8									
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{1,2}

- 7. Select Analyse from the pop up window.
- 8. Select *Bar Chart* from the drop down menu.





Extensions

1. Click on the icon in on the top right-hand corner of the Bar Chart and select copy to clipart. The image can now be pasted into Word.



2. Under the heading *Data Analysis* select Σx to view relevant statistics.



3. To insert a title click on the tool second from the right and select ABC text this will allow text to be included.

Note:

- GeoGebra ignored the text written in the Spreadsheet cell A1.
- Notice if the data in the Spreadsheet view is changed the graph changes accordingly provided it is the data in one of the original cells that changed.
- ABC Text ABC Text Image OK Button

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• Data tables have a specific format which include columns and rows: The start of each column is usually labelled so if recording a physical measurement, the unit of measurement is listed beside the label e.g. Mass / kg.



One variable data Histogram Example 2

- 1.Open GeoGebra
- 2. Select the *View* option from the menu bar and click on Spread sheet.
- 3. Close the Algebra and Graphics views by clicking on the x in the top right-hand corners. You are now ready to input data into the Spreadsheet.
- 4. Type the data given under columns A and B.

Α	This is a record of the length of different plants ab	ove the soil.
Length/cm		
12	5. Click on column A to highlight all its contents.	
15	6. Select the 2 nd tool from the left of the toolbar	O GeoGebra Classic 5
15	and click on One Variable Analysis.	File Edit View Options Tools Window Help
14		
18		f_x f_x One Variable Analysis
17		
22		2
15		3 Multiple Variable Analysis
17		4 5 Probability Calculator
12		6
	/. Select Analyse from the pop up window.	7

Select *Analyse* from the pop up window. 8. Select *Histogram* from the drop down menu.

9. Use the slider which is just to the right of the title Histogram to vary the Bar or Class width.





10. Click on the icon \square on the top right-hand corner of the histogram and select copy to clipart. The image can now be pasted into Word.

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Extensions

1. Under the heading *Data Analysis* select Σx to view relevant statistics.

n	10
Mean	15.7
σ	2.8302
S	2.9833
Σχ	157
Σx²	2545
Min	12
Q1	14
Median	15
Q3	17
Max	22

Note the mean is 15.7 and the standard deviation $\sigma = 2.83$ which indicates that most of the data is close to the mean.

The max=22 and the min= 12 so the range is max- min = 10.

The median is very similar to the mean when the distribution of the data is symmetrical.

2. For manual adjustments click on the icon which is on the top right-hand corner of the screen and a new box appears which allows Classes and Frequency types to be adjusted.

	Click on Set Classes Manually diagram changes to	and the toolbar at the to	op of the
Histogram Graph Classes ☐ Set Classes Manually Class Rule: () ≤ x <	Histogram Start: Then pick the starting point of the frequency table box.	Width: 5	l of 1 and tick
 ○ < x ≤ Frequency Type Cumulative © Count ○ Relative ○ Normalised Show ☑ Histogram □ Frequency Table □ Frequency Polygon ○ Normal Curve 	Without adjustments notice the minimum value in the histogram is the minimum value in the data and the maximum value is the maximum value of the data. Tick the box to show frequency table and it appears under the graph.	Histogram v Start 12	Width 1 Image: Classes Histogram Graph Classes Image: Set Classes Manually Class Rule: Image: Set Classes Manually Image: Set Classes Manually Image: Set Classes Manually Class Rule: Image: Set Classes Manually Image: Set Classes Manually Image: Set Classes Manually <t< td=""></t<>

0.6

0

Interval

12 - 13

13 - 14

14 - 15

15 - 16

16 - 17

17 - 18

18 - 19

12 14 16 18



20 22

Frequency

2

0

1

3

0

3. Under the heading *Data Analysis* select the Show 2nd plot icon to view two plots on the same pane e.g. Bar Chart and Histogram.



Univariate Data

Involves a single variable i.e. we look at one item of data at a time from each subject e.g. Mass or length. The major purpose of Univariate analysis is to describe. It does not deal with relationships between variables.

Importing Data from other Spreadsheets

GeoGebra allows you to copy and paste data from other files e.g. Word or Excel into the GeoGebra spreadsheet:

- 1. Select and copy the data you want to import (e.g., use the keyboard shortcut *Ctrl-C* in order to copy the data to your computer's clipboard).
- 2. Click on the spreadsheet cell that should contain the first data value.
- 3. Paste the data from your computer's clipboard into GeoGebra's *Spreadsheet View* using the keyboard shortcut *Ctrl-V*.

Note: If there is a lot of data copied it may take a few seconds after pasting before it appears in the spreadsheet.



Histogram Example 3

Use GeoGebra to draw a histogram for the results of students in their class test for the data in the table shown and copy to a Word document.

Mid-interval Mark	No. of students
95	4
85	6
75	12
65	4
55	2
45	1
35	0

- 1. Open GeoGebra
- 2. Select the *View* option from the menu bar and click on Spread sheet. Note the new toolbar.
- 3. Close the Algebra and Graphics views by clicking on the x in the top right-hand corners. You are now ready to input data into the Spreadsheet.
- 4. Copy the data table and paste into the Spreadsheet.
- 5. Click on column A to highlight all its contents.
 6. Select the 2nd tool from the left of the toolbar and click on One Variable Analysis.
- 7. Note the Data Source pane opens.



- 8. The *Data Source* pane has a cog wheel-an Options icon
- 9. Select the Data with Frequency Option and a second column, titled column B appears.
- 10. Go back to the Spreadsheet and highlight the Spreadsheet column B.
- 11. Select the hand in the Data Source pane and the data is transferred.

57	Frequency
Column A	Add Selection
"Mid-interval value"	//dd Ociccation

Finally select Analyse on the bottom of the pane.



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1 2	; /•	Two Variable Regression Analysis						
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			Data with Frequency					
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12. Select <i>Histogram</i> from the drop down menu and
use the slider which is just to the right of the title
Histogram to vary the Bar or Class width.

Histogram	\sim	
		Classes

13 For manual adjustments click on the icon which is on the top right-hand corner of the screen and a new box appears which allows Classes and Frequency types to be adjusted.
 Click on Set Classes Manually Then pick the starting point of 30 and width of interval of 10 and tick the frequency table box.

Tick the box to show frequency table and it appears under the graph.





- 14 Under the heading *Data Analysis* select Σx to view relevant statistics.
- 15 Click on the icon on the top right-hand corner of the Histogram pane and select copy to clipart. The image can now be pasted into Word.



Barcharts and Histograms

Bar charts and histograms can both be used to compare the sizes of different groups. A Bar chart is made up of bars plotted on a graph. Usually, there is a space between adjacent Bars. Histogram is a chart representing a frequency distribution; heights of the bars represent observed frequencies. In other words a histogram is a graphical display of data using bars of different heights. Usually, there is no space between adjacent Bars.

Barcharts are used when the independent variable involves qualitative categories, such as different species. The dependent variable is usually discrete data i.e. only a finite number of values are possible. The number of children in a family is discrete data while the age or height of a person is continuous data.

The bars proportional to the number of observations (counts) in each category. The horizontal (x) axis represents the categories; The vertical (y) axis represents a value for those categories e.g. percentages.

A bar chart is useful for looking at a set of data and making comparisons.

A bar chart describes categorical data, and has gaps, whereas a histogram describes continuous data and hence has no gaps. Histograms are used to show distributions of variables while bar charts are used to compare variables.

Why Use Histograms and Bar Charts

Like many other visuals, histograms and bar charts are gaining increasing popularity for the following benefits.

- 1. Ease data analysis.
- 2. Useful for looking at a set of data and making comparisons.
- 3. Make report presentations more compelling and comprehensible.
- 4. Enhance effective information communication. For example, it's easier to see which items are performing better by glancing at a chart rather than looking at a string of numbers.

Pie charts show classes or groups of data in proportion to the whole data set. The entire pie represents all the data, while each slice or

segment represents a different class or group within the whole. Each slice should show significant variations. The number of categories should be generally limited to between 3 and 10.

The resources link on the last page has a Dropbox link which contains a useful pie chart template.





Box Plots

Box plots (also known as box and whisker plots) are a type of chart often used in data analysis to visually show the distribution of numerical data through displaying the data quartiles (values that divide a set of data into four equal parts) and median.

A box plot—displays the five-number summary of a set of data at a glance. The five-number summary is the *minimum*, *first quartile, median, third quartile,* and *maximum*.

In a box plot, we draw a box from the first quartile to the third quartile. A vertical line goes through the box at the median. The whiskers go from each quartile to the minimum or maximum.

Note: the median is the middle value when the values are arranged in order e.g. 3 is the median for the following set of data: 1, 2, 3, 8, 12.



25% of the data lies in the 1^{st} quartile.

The interquartile range IQR contains 50% of the data and is often written $Q_3 - Q_1$. Another 25% of the data lies in the 4th quartile.

Note: Box plots may be displayed horizontally or vertically (more common in research papers).

Interpreting Box Plots

Box plots are useful for showing the dispersion of a data set.

- The difference between the maximum and minimum gives us the range.
- The interquartile range (IQR) is the box plot showing the middle 50% of scores and can be calculated by subtracting the lower quartile from the upper quartile (e.g. Q_3-Q_1).
- The longer the box the more dispersed the data.
- When the median is in the middle of the box, and the whiskers are about the same on both sides of the box, then the distribution is *symmetric*.
- When the median is closer to the bottom of the box, and if the whisker is shorter on the lower end of the box, then the distribution is *positively skewed* (skewed right).
- When the median is closer to the top of the box, and if the whisker is shorter on the upper end of the box, then the distribution is *negatively skewed* (skewed left).

Note: An outlier is a value that lies an abnormal distance from other values in a data set. The definition leaves it up to the analyst to decide what is abnormal e.g. a value outside the interval Q_1 - 1.5 IQR or Q_3 + 1.5 IQR where IQR is the inter quartile range i.e. $Q_3 - Q_1$.



Box Plot Example

Use the same data as in the above histogram exercise on Page 8 to plot a box plot of the data which should look like this:



Stem-and-Leaf Plot

The stem-and-leaf plots main purpose is to show the frequency with which certain classes of values occur while retaining the uniqueness of each data value. It is recommended for batches of data containing between 15 and 150 data points. They give a quick clear picture of a distribution and make it easy to identify clusters of data from the lengths of the branches.

Selecting the stem-and-leaf plot for the data above gives

Note: The first digit is called the stem and the second digit the leaf e.g. 1 is a stem and 2 is a leaf.

A key should be included with a stem-and-leaf plot.

1 2 high: 2	$2 \\ 2.0$	4	5	5	5	7	7	8	
Key:	3 1	me	ans	31					



Trend or Polyline graphs illustrate trends in data over a period of time. They are composed of one or more **line segments**. For example, one axis of the graph might represent a variable value, while the other axis often displays a timeline. Multiple trends can be compared by plotting lines of various colours.

Example: The table shows the domestic milk intake for the first six months of 2019.

Use the Spreadsheet in GeoGebra to graph the data.

Month	Millions of litres	
Jan	175.3	
Feb	303.6	
Mar	708.7	
Apr	953.3	
May	1072.2	
Jun	1001.0	

1. Open GeoGebra

2. Select the *View* option from the menu bar and click on *Spreadsheet*.Note the new toolbar.

А

1 Month

2 Jan

3 Feb

4 Mar

5 Apr

6 May

10 11

12

13

14

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Insert Right

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Paste

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A1:A100

2 Delete Objects

Delete Column A

Record to Spreadsheet

Object Properties

Insert.

Create

Show Object

C

- 3. Close the Algebra view by clicking on the **x** in its top right-hand corner. You are now ready to input data into the Spreadsheet.
- 4. Copy the data table and paste into the Spreadsheet.
- 5. Insert a column between columns A and B by highlighting column A, right click and *Insert Right*.
- 6. As points have two numeric co-ordinates we denote the months by numbers from 1 to 6.

Α	В	С	
Month		Milk	
Jan	1	175.3	
Feb	2	303.6	
Mar	3	708.7	
Apr	4	953.3	
Мау	5	1072.2	
Jun	6	1001	

7. Highlight columns B and C, right click and select Create Polyline





- 8. The Polyline is drawn in the Graphics view.
- 9. Note: It may be off scale and need to be rescaled by selecting the *Move Graphics view* tool then right click the Graphics window and select Zoom to Fit.





Extensions

- Give the graph a title (using the textbox tool).
- Hide the points. (e.g. Open the Algebra window, select the points and click Show Object)
- Use the cog wheel preferences to select the positive Direction only for the X and Y axis.
- Replace the numbers on the X-axis with months (using the textbox tool).
- Highlight the relevant cells in columns A, B and C, right click and select Create Table and then move it into position in the graph. Right click in the table and select Object Properties to edit the text.

The data used was sourced from https://www.cso.ie/en/releasesandpublications/er/ms /milkstatisticsdecember2019





Glossary of Mathematical Terms

- Averages allow us to compare sets of data. There are different ways of expressing the average. The three most common are the *mean*, the *mode* and the *median*.
- Mean: is the sum of the data values divided by the number of values.
- Mode: is the value that occurs most often.
- Median: is the middle value when the values are arranged in order.
- **Standard of Deviation** σ : this is a measure of the spread or how much the data in a set deviates from the mean.
- Statistic: is often used to refer to numbers that describe or summarise data. For example, finding the mass of thousands of 18 month- old shorthorn heifers and then determining an average mass i.e. single value that can be considered "normal." This single value can also be called a statistic.
- Variable: is the characteristic observed and recorded in a study.
- **Bar chart** is a graph that presents categorical data with rectangular bars with heights or lengths proportional to the category values that they represent. The bars can be plotted vertically or horizontally. The x-axis will have no scale and a space between bars.
- **Graphs** are a common method to visually illustrate relationships in the data. The purpose of a graph is to present data that are too numerous or complicated to be described adequately in the text and in less space.
- **Histogram** also called a *frequency distributions graph*, is a specialised type of bar graph that resembles a column graph, but without any gaps between the columns. It is used to represent data from the measurement of a continuous variable. Individual data points are grouped together in classes to show the frequency of data in each class.
- **Pie charts** show classes or groups of data in proportion to the whole data set. The entire pie represents all the data, while each slice or segment represents a different class or group within the whole. Each slice should show significant variations. The number of categories should be generally limited to between 3 and 10.
- **Box plot** may be either horizontal or vertical. It is used to display a statistical summary of one or more box-and- variables, such as the minimum, lower quartile, median, and maximum. It may also identify the outlier data. The spacing between the different parts of the box indicates the degree of dispersion and whether the data distribution is symmetrical or skewed.



Project Maths GeoGebra Resources

https://www.projectmaths.ie/for-teachers/professional-development/learn-to-use-geogebra

Download GeoGebra	Install GeoGebra	Set up GeoGebra
Want to download and install GeoGebra? Simply follow the instructions in this tutori	Want to download and install GeoGebra on multiple PCs? Simply follow the instruction	This tutorial shows you how to change some basic settings on GeoGebra to make it eas
Move the graphics view	Inserting a picture	Introduction to statistics
Moving and scaling the graphics view is one of the most useful things to know In Geo	When linking maths to real-life it ccan be useful to insert a picture into GeoGebra	Use GeoGebra with Census@SChool data to create charts, calculate means, standard dev

To Download GeoGebra visit the link shown and select GeoGebra Classic 5 from the bottom rhs. <u>https://www.geogebra.org/download</u>

Useful resource on statistics

https://www.projectmaths.ie/for-teachers/modular-course/modular-course-1-probability-and-statistics/

Useful YouTube video clips

Analysing data: take a look at the first 3 minutes of this video which is very relevant

https://www.youtube.com/watch?v=8A2NVgTpKgE

One variable Graphing and analysis: first 2 minutes very relevant

https://www.youtube.com/watch?v=8A2NVgTpKgE

One variable graphing and analysis first 5 minutes very relevant.

https://www.youtube.com/watch?v=lLzBzsx1lec

One variable graphing and analysis first 12 minutes very relevant.

https://www.youtube.com/watch?v=4Ucy6bVUDPs

