The following task is taken from Mathematical Modelling for Teachers by Jürgen Maaß, Niamh O'Meara, Patrick Johnson and John O'Donoghue.

4.2 Which Road Should We Take?

This idea is credited to Mrs. M. Spiegl and Mr. T. Lehner who prepared a mathematics course unit for a workshop on teaching mathematics in Linz, Austria in the summer term of 2014. We acknowledge their contribution with thanks.

Overview: When using a satellite navigation system you will often be provided with more than one possible route. Mathematics can help us determine the most suitable route. If we assume that we are not just looking for the shortest driving time, then other factors will need to be taken into consideration such as shopping on the way, paying Aunt Anna a visit, enjoying the scenic view, or avoiding tolls. When such considerations are taken in account our deliberations might become more complex.

How would you present this problem to your students?
Please jot down some ideas so that you can compare your suggestions to ours.

Implementation: We suggest inventing a story around the problem. For example:
Your geography teacher is organising a field trip from your school (Europagymnasium Auhof, Linz) via *bus to the abandoned opencast pit in Eisenerz, once the principal centre of Austrian Iron mining. The bus operator enquired which route we would like to take. Route planners (such as Google Maps or Open Street Map) provide us with three suggestions:*

1. *The first route takes us south* via *the A1 motorway passing the city of Steyr.*
2. *The second route is* via *the A9 motorway and then takes us east.*
3. *The third route takes us southeast passing the city of St Valentin.*

A rough sketch of the three suggested routes are shown in Fig. 4.2. Using Google Maps, or another web-based mapping service, construct a detailed description of the route, outlining travel time, distance travelled and roads traversed. What considerations must be taken into account when you compare these three routes?

We propose a brainstorming session with the entire class. After studying the map, what suggestions have the class come up with concerning the three routes? Here are our remarks:

- Route 1 (via A7/A1): According to Google Maps the route is 141 km long. It takes a car 2 h and 8 min to travel this route and you have to pay tolls on the highways. We have to ask the bus driver if he has an electronic pass for the tolls. According to traffic reports there is a construction site on the national road 115, north of Landl, possibly resulting in delays. If this is the route to be taken, we have to check the status of the construction site once again before departure. Approximately 64% of the route is on mountain roads with only 18% on highways.
- Route 2 (via A9): According to Google Maps the route is 163 km long. It takes a car 2 h and 8 min to travel this route. The largest proportion of this route is via motorway. In the Ennstal area we would go down national road 146, and towards

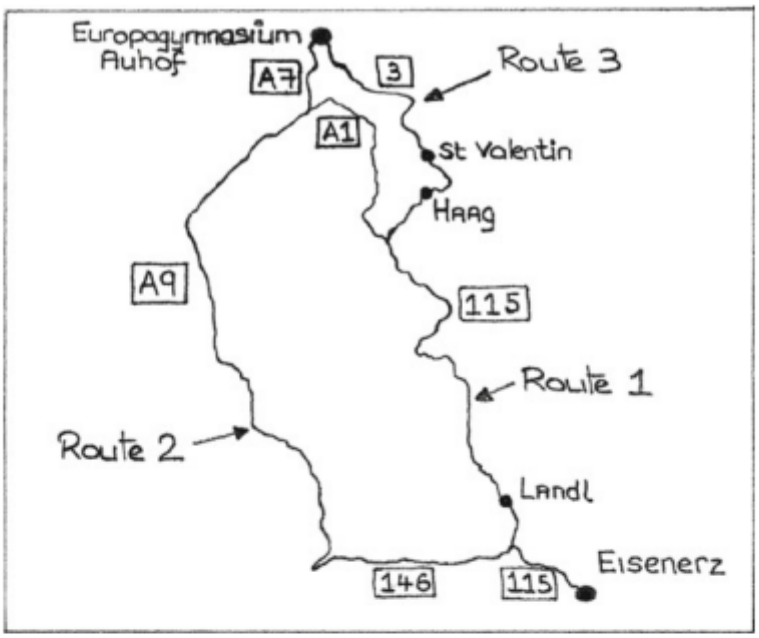

Fig. 4.2 Map of three possible routes

the end travel for a few kilometres on national road 115. On this route we have to pay a toll for using the Bosruck tunnel. We have to ask the bus operator if he has an annual pass for the tunnel or if we have to pay for using the tunnel.

- Route 3 (via Haag): According to Google Maps the route is 147 km long. It takes a car 2 h and 15 min to travel this route. Initially we proceed eastwards along the national road 3 before turning south to join the A1, heading southeast for a few kilometres. We then merge onto the national road 42 and head south again to join up with the national road 115.

Do we have enough information now to make an informed decision? We would like to point out that everyday decisions regarding what route to take are generally made without much deliberation. The level of detail outlined above is generally sufficient as the difference in time and cost between routes is minimal once no additional reasons for changing our route arise. However, if one does not simply take the information on the internet for granted you might notice that the suggested driving time is for cars. Buses on the other hand might take longer on these roads. In this case we need to put more thought into our decision.

We would like to point out that, as in most modelling scenarios, we need to decide if we believe the information provided by sources such as the internet, advertisements, companies or public authorities. Mathematics can be a means of verifying such information and checking if it makes sense. However, this requires some time and effort on our part. We consider it very important for students to realise in school that situations requiring such decisions do frequently arise in real life. In this book we

advocate the teaching of applicable mathematics in a way that students can use it to verify facts they are not entirely convinced of.

How can we verify the information provided by Google Maps? Below we have listed possible options that you could consider:

1. Use a different satellite navigation system.
2. Use a map and calculate the distance the old-fashioned way (using pins and a string; this is a good way of practicing calculations with map scales).
3. Accept the proposed distance but calculate the driving time yourself.

Here we shall only focus on option 3. Students should realise themselves that they need information on the average speed a bus travels down a motorway, or on national roads in the plains or in the mountains. To assist them in this endeavour they could call the bus operator or make their own estimates.

We will now carry out a short modelling task using estimated values (or values deemed realistic for buses by automobile associations in Austria):

- average speed on the motorway: 80 km/h
- average speed on national roads in the plains: 60 km/h
- average speed on national roads in the mountains: 40 km/h

Bear in mind, as with all modelling problems we performed, we can revisit these assumptions later on or carry out further research on them and recalculate how these changes influence our results (i.e. driving time). Depending on the model, it might be advantageous to see what effect changes made to the initial values such as average speeds have on the desired value to be calculated such as total travel time, and if this can be expressed as a function.

Using our assumed values, route 2 consists of 109 km of motorway, which takes 1 h and 22 min to travel (provided there is no traffic jam or no queue for the toll, as the bus might be able to use an electronic toll system). We will then travel down the Ennstal national road for approximately 38 km (38 min) and then travel another 16 km on national road 115 through the mountains (24 min). According to our calculations it will take a bus approximately 2 h 24 min; 16 min slower than the time it would take a car, according to Google Maps.

How accurate was your calculation of the driving time? What degree of accuracy makes sense in this case? Our calculation was kept rather inaccurate on purpose, as we only wanted to re-confirm our assumption that a bus would take longer, on average, than the times Google Maps suggested for a car.

We calculated the driving time for the other routes in a similar fashion. For route 1 the result was approximately 3 h, and for route 3 approximately 3 h 10 min. Therefore, if driving time is the decisive factor, route 2 is the best option.

As we have just seen, driving time is influenced by the approximations of the average speed that the bus can travel. If your class requires more information on this, we suggest analysing the various driving times using a spreadsheet. We have chosen to use a spreadsheet as it will allow us to perform accurate calculations whilst also allowing us to make quick changes and immediately see the revised results. Sample spreadsheet outputs are given in Table 4.1.

Table 4.1 Spreadsheet data for calculating total driving time

Driving time: Linz - Erzberg		
Assumed average speed of a bus		
Motorway	80 km/h	
National road in the plains	60 km/h	
National road in the mountains	40 km/h	
Route 1 (via A7/A1):	**Distance**	**Driving Time**
Motorway	26 km	19.5 min
National road in the plains	24 km	24 min
National road in the mountains	91 km	136.5 min
Total Distance	141 km	180 min
Route 2 (via A9):	**Distance**	**Driving Time**
Motorway	109 km	81.75 min
National road in the plains	38 km	38 min
National road in the mountains	16 km	24 min
Total Distance	163 km	143.75 min
Route 3 (via Haag):	**Distance**	**Driving Time**
Motorway	9 km	6.75 min
National road in the plains	47 km	47 min
National road in the mountains	91 km	136.5 min
Total Distance	147 km	183.50 min

Using the spreadsheet it would be relatively straightforward to calculate how long the journey would take on an express bus that can travel at increased speeds. By simply altering the average speed at which the bus can now travel our total travel time will be automatically updated. Given that the average speed of the express bus is 100 km/h on motorway, 80 km/h on national roads in the plains, and 60 km/h in the mountains, the total travel time for each route is recomputed in Table 4.2.

As you can see, these results are even faster than those indicated by Google Maps. We encourage you to discuss this point with your students. Why do they think this is the case? What factors could be affecting the travel times suggested by Google Maps? Will the travel times suggested by Google Maps always remain the same?

Finally our class will have to make a decision on which route to take. Travel time and cost will probably be the two key factors that our class will need to consider before reaching a decision. Due to relatively small differences in travel time, the decisive factor will probably be whether or not the tolls on the A1, A7 and A9 motorway will incur extra cost, and how much that cost will be.

Comment: We have located this example in Upper Austria, as these three routes are relatively close in both distance and travel time, with each having its pros and cons. It could be an interesting task for your students to find a similar example closer to home with routes that are comparable as we have done in our example. Without

Table 4.2 Data for calculating total driving time for express bus

Driving time: Linz - Erzberg		
Assumed average speed of a bus		
Motorway	100 km/h	
National road in the plains	80 km/h	
National road in the mountains	60 km/h	
Route 1 (via A7/A1):	**Distance**	**Driving Time**
Motorway	26 km	15.6 min
National road in the plains	24 km	18 min
National road in the mountains	91 km	91 min
Total Distance	141 km	124.6 min
Route 2 (via A9):	**Distance**	**Driving Time**
Motorway	109 km	65.4 min
National road in the plains	38 km	28.5 min
National road in the mountains	16 km	16 min
Total Distance	163 km	109.9 min
Route 3 (via Haag):	**Distance**	**Driving Time**
Motorway	9 km	5.4 min
National road in the plains	47 km	35.25 min
National road in the mountains	91 km	91 min
Total distance	147 km	131.65 min

localisation, the problem will lose its appeal. This insight holds true of all problems incorporating reality. If the reality to be modelled is too far removed, it is not as motivating as local problems. Therefore, it is often up to the teachers to adapt a proposed lesson based on reality to the living environment of their students and let them find up-to-date data that is meaningful to them.