



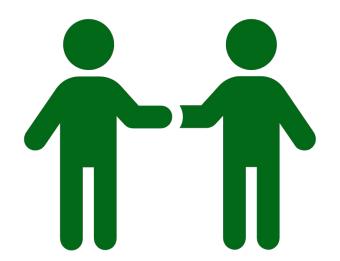


National Seminar 1

Applied Mathematics



Welcome

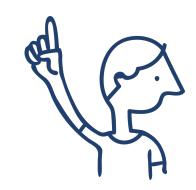


Expectations for Online CPD





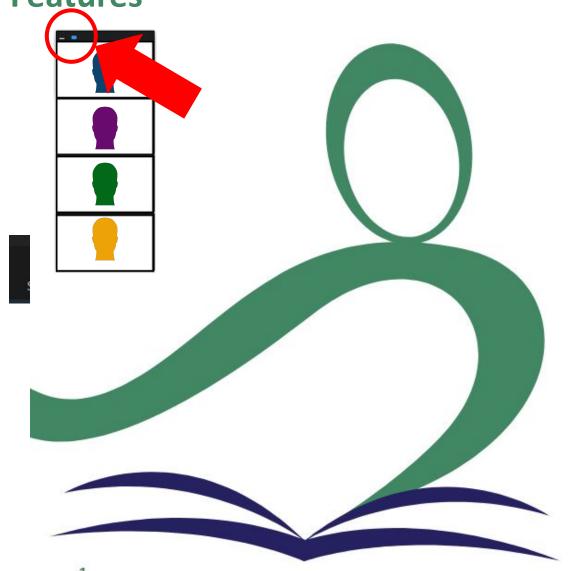






The PDST does not give permission for this CPD event to be recorded.

Zoom Features







IV

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National Seminar 1

Applied Mathematics



Role of the PDST

What we are:

- Teachers & School Leaders.
- Teacher Educators.
- Facilitators/Enablers.
- Purveyors of Lifelong Learning.

What we are not:

- Evaluators.
- Policy or Curriculum Developers.
- Exam Creators.



Keys

Resource



Reflection



Prior Knowledge



Teaching Approach



Booklet activity



Contact us



Group work



Planning





Key Messages

Core to the specification is a non-linear approach empowered by the use of rich pedagogy which promotes the making of connections between various Applied Mathematics learning outcomes.

Strand 1 of the specification is a unifying strand and emphasises the importance of utilising modelling across all learning outcomes.

Applied Mathematics is rooted in authentic problems as a context for learning about the application of Mathematics to design solutions for real-world problems and to develop problem solving skills applicable to a variety of disciplines.



Schedule for Seminar 1

09:30 – 11:15	Session 1: Overview of the PD and relevant documents. Analysis of the specification and Learning Outcomes. Introduction to Mathematical Modelling.
11:15 - 11:30	Break
11:30 - 13:00	Session 2: Development of Algorithms Introduction to Networks and Graph Theory
13:00 - 13:45	Lunch
13:45 – 15:10	Session 3: Algorithms and their Applications
15:10 – 15:30	PLCs and Q&A



By the end of this session you will have:

Been introduced to the professional development programme, its components and timelines.

Explored the Applied Maths specification in terms of content as well as teaching, learning and assessment.

An understanding of Mathematical Modelling, the process involved and its benefits to student learning.





Overview of Professional Development (PD) and Supports Available

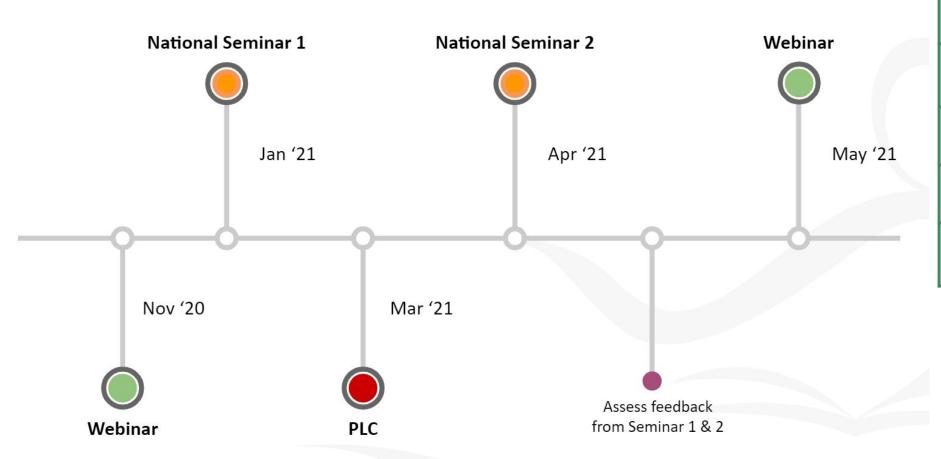
- 9 X 1 day seminars
 - Full day PD workshops
- 5 X Professional Learning Communities (PLCs)
 - Autumn and Spring each year in response to teacher PD needs and relevant to the particular community.
- 4 X Webinars
 - Live events discussing new material with Q&A
- 2 X Geogebra Workshops
 - Exploring the use of technology in teaching Applied Maths.





Overview of PD and Supports Available

Year 1 Nov 2020 - June 2021



Year 2	Year 3
3 X National Seminars	4 X National Seminars
2 X PLCs	2 X PLCs
1 X Webinar	1 X Webinar
1 X Geogebra Workshop	1 X Geogebra Workshop





PDST Sustained Support

What is it?

The PDST Sustained support model invites schools to engage with bespoke support based on individual school context.

Schools will have access to a PDST advisor for a number of school visits during a school year.

Sustained support is a collaborative process towards educational change and improvement in learner outcomes.

How do I apply?

https://pdst.ie/schoolsupport

What others say

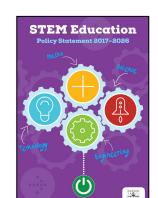
'I thought SSE was this big concept which floated in a sky of terminology it's what I and we do to bring change in our classrooms so children can reach their full potential. The SSE guidelines hold great conversation starters. I will actually use them going forward.'

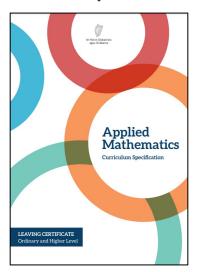




Overview of Key Documents













"Students can, of their own initiative, transfer and apply skills learned in one context to another context." Looking at Our School, p. 16



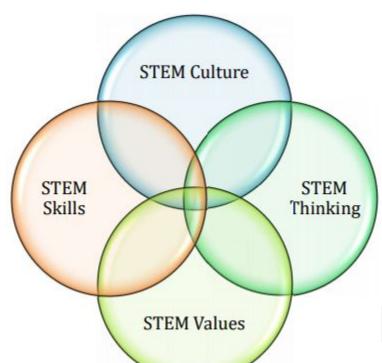
Pillars of STEM Education in Ireland

Complex problem solving.

Collaboration with others.

Creativity.

- Student-centred.
- Collaborative.
- Productive failure.



- Making connections.
- Making choices.
- Managing change.

- Mindset.
- Disposition.
- Community.





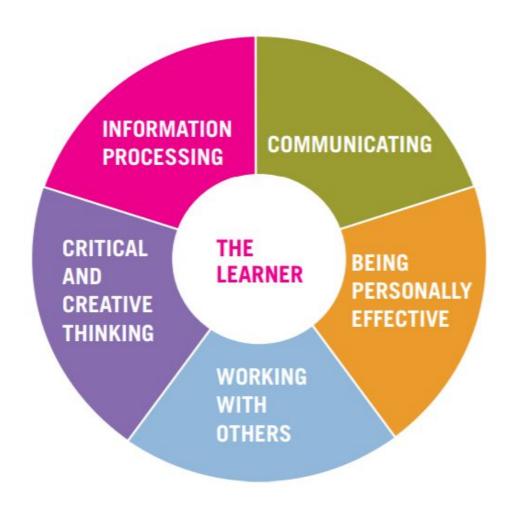


Response to Task Wall Padlet Wall/Menti



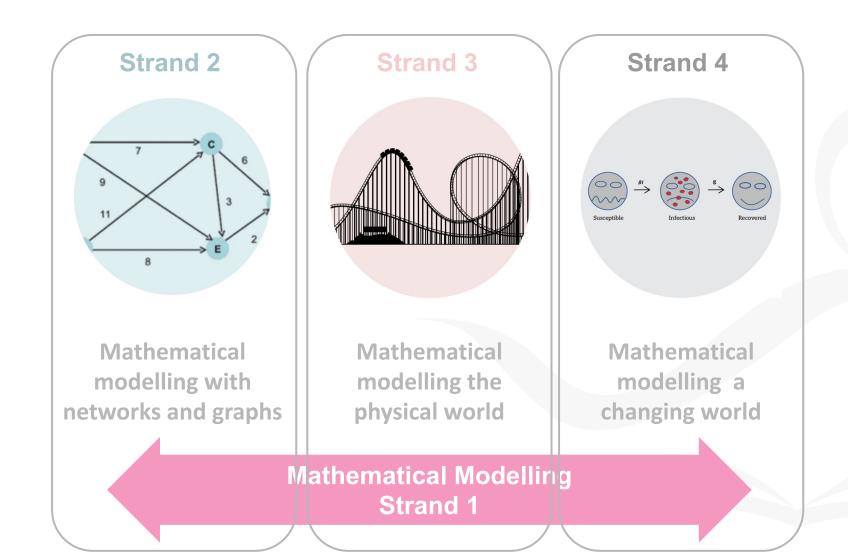
Outline of Key Skills







Structure of Curriculum



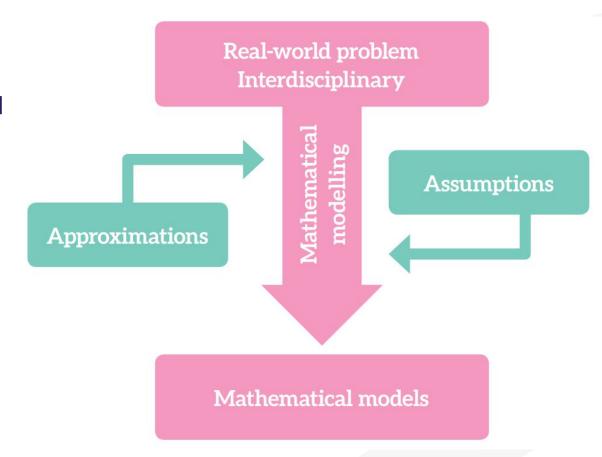


Overview of New Specification

Strand 1: Mathematical modelling

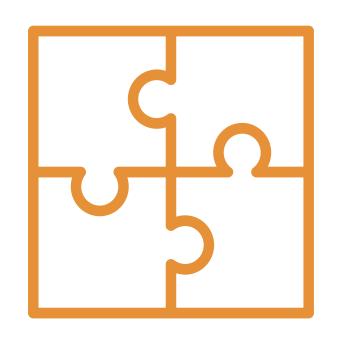
In this **unifying strand** students learn about mathematical modelling as a process that will develop skills such as:

- Formulating problems
- Translating problems into mathematics
- Computing solutions
- Evaluating solutions





Assessment and Coursework



Written Assessment 80%

Modelling Project 20%

Ordinary and Higher Level

(Applied Maths Specification, p. 22)





Task 2: Specification Analysis



Analyse your allocated topic based on the Group number you are assigned:

Group 1

Aims & Objectives of Specification

Pg 6-7

Group 2

Overview & Structure

Pg 9-10

Group 3

Teaching & Learning

Pg 13

Group 4

Strand 1
Mathematical
Modelling
Pg 16



Share the key messages from your piece and one point about what this means for your teaching of Applied Mathematics with the group.

What student skills are being developed from your reading of the specification?

Padlet link: https://tinyurl.com/y7vjtmh7



Feedback from Groups





Development of Learning Outcomes



Strand 1: Mathematical modelling

In this unifying strand students learn about mathematical modelling as a process that uses mathematics to represent, analyse, make predictions or otherwise provide insight into real-world phenomena. The process is iterative and translates between the real world and mathematics in both directions and involves a number of stages. As they model authentic problems, students learn to appreciate the importance of mathematics in understanding the world around them and realise that although mathematical models are not perfect predictors of what will happen in the real world, they can offer important insights into key elements of a problem. Students become comfortable with uncertainty; not knowing an answer immediately does not deter them and learning from their peers is a valuable part of the process. They learn the importance of assumptions to the modelling process, and how they affect the validity of a model. They recognise that mathematical models are used to inform many decisions that directly affect their lives, and that being able to critically evaluate mathematical models is a desirable skill for them to acquire.

STUDENTS LEARN ABOUT	STUDENTS SHOULD BE ABLE TO
The problem-solving cycle	 describe a systematic process for solving problems and making decisions
Formulating problems	 research the background to a problem to analyse factors or variables that affect the situation determine information relevant to the problem decompose problems into manageable parts determine what assumptions are necessary to simplify the problem situation
Translating problems into mathematics	 use abstraction to describe systems and to explain the relationship between wholes and parts abstract the knowledge needed to build a mathematical model translate the information given in the problem together with the assumptions into a mathematical model that can be solved

Modelling Project Overview

"Modelling problems require the solver to research the situation themselves, make reasonable assumptions, decide which variables will affect the solution and develop a model that provides a solution that best describes the situation Specification p. 10





Common brief issued annually by the State Examinations Commission (SEC) for both OL and HL.

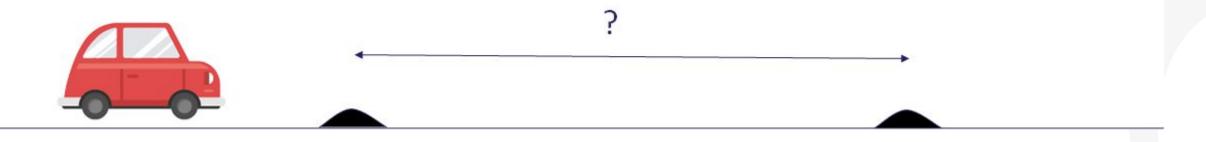
Students present a solution to an authentic modelling problem.

Allows students to demonstrate proficiency in course content and skills.



Task 3: Introduction to Mathematical Modelling

How far apart should speed bumps be placed so that traffic does not reach a speed greater than 50 km/h?



What factors will affect the formulating of this problem?

What assumptions will you make?

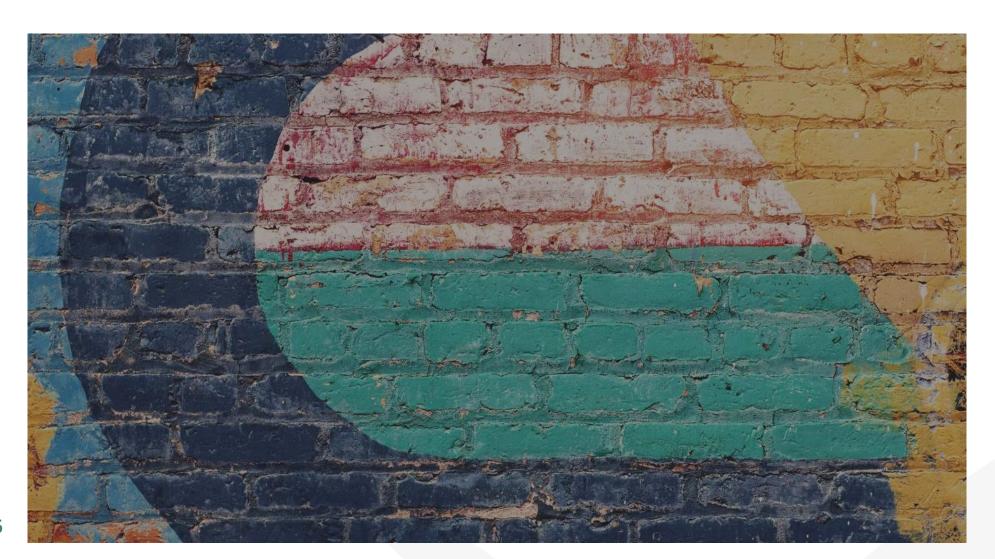
Padlet link: https://tinyurl.com/yaxc9llr







Feedback from Groups



"Students learn the importance of assumptions to the modelling process, and how they affect the validity of a model." Specification p. 16





Modelling Task: Speed bump design

Sample of assumptions made:

- A car is a particle.
- The road conditions are good.
- The road is horizontal.
- The speed of the car is approximately zero when it crosses the bumps.
- All cars slow down according to the table of stopping distances outlined by the RSA.
- All cars speed up at the same rate as they slow down.





RSA Total Minimum Stopping Distance (m)





Modelling Task: Speed bump design

First Considerations for a Solution:

Based on our assumptions we formulate the mathematical model:

Distance between speed bumps = 2 × stopping distance

Using this model we can calculate the required stopping distance as 50 m (the distance to speed up to 50 km/h plus to slow down from 50 km/h).

A person solving this problem in real life may be able to set up some experimental bumps and observe the cars between them.

There are many ways in which this model could be refined for further iterations, for example:

- Modelling the car as a rigid body that has length.
- Research alternative models to describe how a car gains speed and slows down.
- The car crosses the bumps at a low speed, for example 5 km/h.

"In strands 2, 3 and 4, students will encounter both modelling and word problems that cross many disciplines." Specification p. 10





Modelling Task: Speed bump design

Extension Questions?

Suggestions:

- Gather appropriate data on the lengths of cars and revise the solution to the speed bumps problem to take account of this factor.
- 2. Describe how changing the speed at which the vehicle crosses the bumps would change the solution to the problem.



"The student demonstrating high level of achievement: addresses the viability and reliability of the mathematical modelling solution"

Modelling Project Assessment Criteria, Specification p. 24





Supporting Students in Mathematical Modelling

How will we **adapt our teaching** to encourage students to create their own unique models?

What can we do to develop student agency and challenge them to reach a deeper level of understanding of mathematical modelling?

How can we **develop students' skills in the evaluation** and improvement of their model after iterations?





Reflection: Session 1

What are your main takeaways from Session 1?

What key messages have you taken from this session, regarding the teaching and learning of the specification?







By the end of this session you will have:

Experienced approaches to teaching and learning which support the aims of the specification

Discovered the uses of Graph Theory and Algorithms to solve real world problems.

Experienced Minimum Spanning Trees (MST) and their applications.

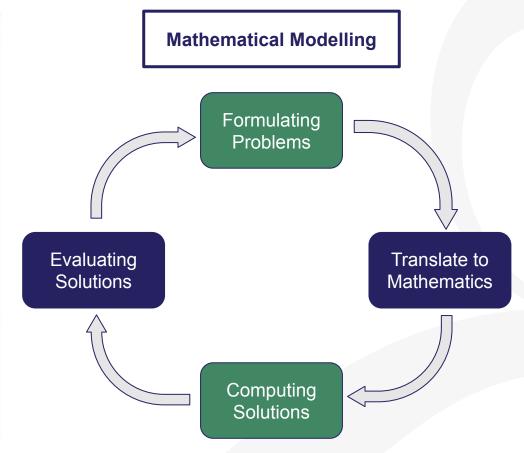




Networks and Graph Theory: Introduction

Graph Theory is a branch of Mathematics concerned with networks of points connected by lines.

STUDENTS LEARN ABOUT	STUDENTS SHOULD BE ABLE TO
Networks and their associated terminology Matrices, matrix algebra and adjacency	 represent real-world situations in the form of a network use and apply the following network terminology: vertex / node, edge/arc, weight, path, cycle distinguish between connected and disconnected graphs, and between directed and undirected graphs represent a graph using an adjacency matrix, and reconstruct a graph from its adjacency matrix perform multiplication of square matrices by hand, with the help of a computer for larger matrices interpret the product of adjacency matrices translate between multiple representations of mathematical ideas
Minimum spanning trees applied to problems involving optimising networks and algorithms associated with finding these (Kruskal, Prim)	 demonstrate an understanding of the concepts of tree, spanning tree, minimum spanning tree in appropriate contexts use appropriate algorithms to find minimum spanning trees



Task 4: New Broadband for Mallow!

£03503

"determine what assumptions are necessary to simplify the problem situation" Specification p. 16





Seán works for the County Council who are planning to connect a number of buildings in Mallow, Cork with an upgraded broadband network.

They will connect the buildings by laying cables in the ground following the current road layout.

What information does Sean require in order to complete this plan?

Padlet Link: https://tinyurl.com/ydgnnmq9









Response from Groups





New Broadband for Mallow!

Buildings to be connected:











McDonalds

Costa Coffee

Garda Station

Tesco

Library

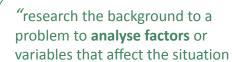
Map of Mallow



Task 4: New Broadband for Mallow!

Road distances between each building (to the nearest metre)

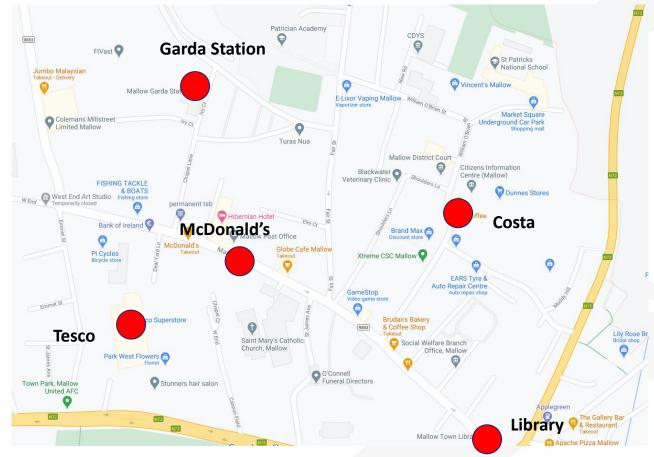
	Garda St.	Costa	McDonald's	Tesco	Library
Garda St.	-	500	170	900	550
Costa	500	-	400	700	300
McDonald's	170	400	-	600	400
Tesco	900	700	600	-	700
Library	rary 550 30		400	700	-



- **determine information relevant** to the problem
- -determine what **assumptions** are necessary to simplify the problem situation"

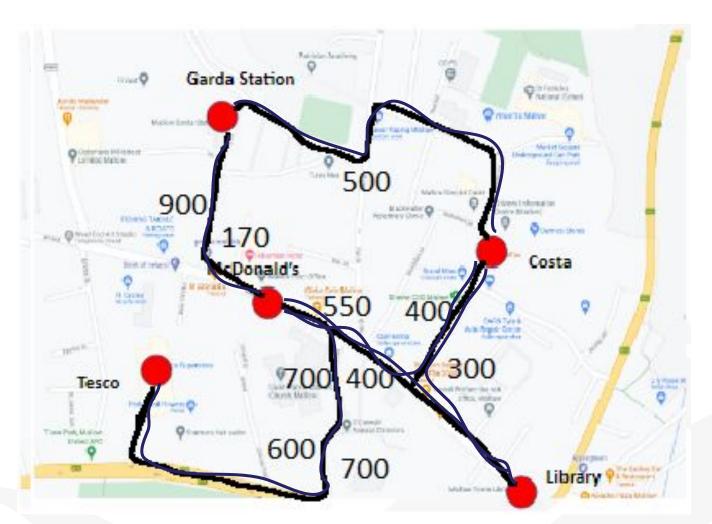
 Specification p. 16







Task 4: New Broadband for Mallow!



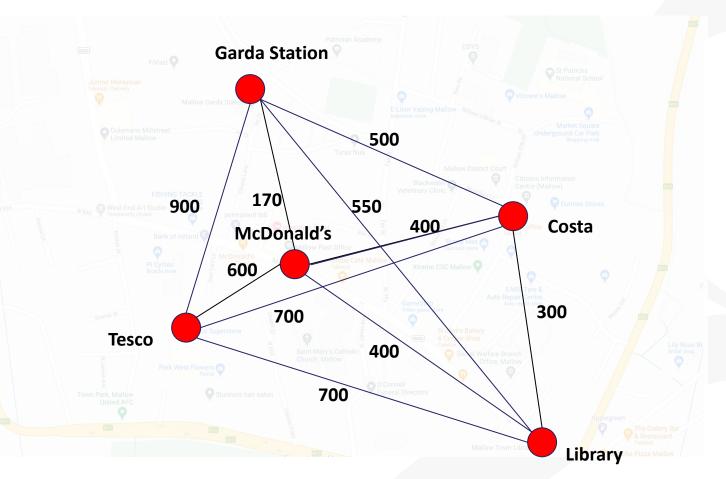


Task 4: New Broadband for Mallow!

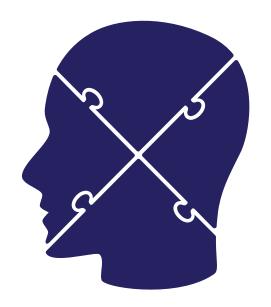
Total minimum length of Cable required: _____ m

Padlet Link: https://tinyurl.com/ydgnnmq9





Task 4: Review of Solutions



"One of the most beneficial outcomes of working with others is in identifying, evaluating and achieving collective goals. Students learn to negotiate and resolve differences of opinion as they discuss their different strategies and achieve compromise."

Specification p. 11







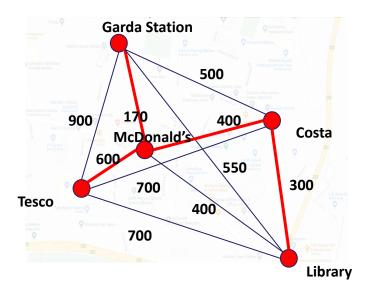
Task 4: Reflection on your Approach:

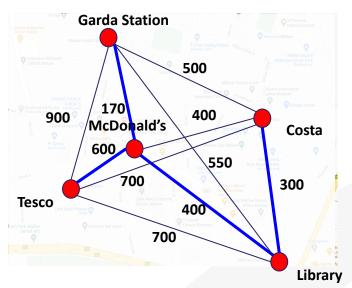


What factors influenced your decisions?

Could your approach be improved?

Give a reason as why you didn't include both 400 metre options?





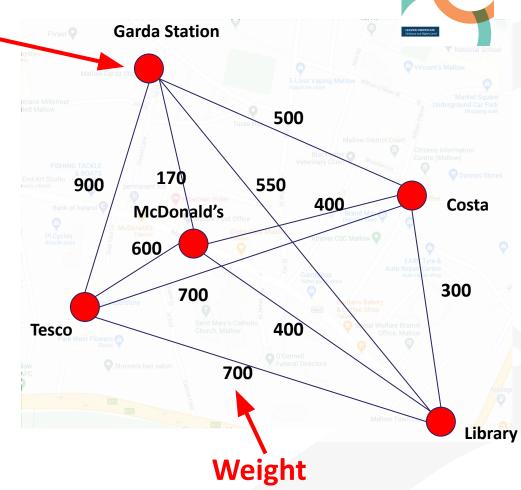
"Students should be able to: use and apply the following network terminology: vertex / node, edge/arc, weight, path, cycle."

Specification p. 17

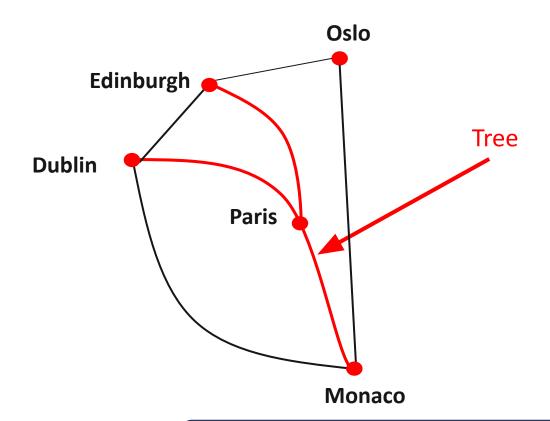
Understanding Terminology

Oslo **Edinburgh Dublin Paris** Edge/Arc **Monaco**

Vertex/Node



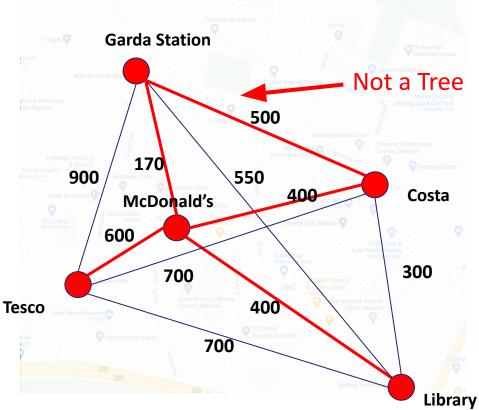
What defines a Tree?



"Students should be able to demonstrate an understanding of the concepts of tree, spanning tree, minimum spanning tree in appropriate contexts"

Specification p. 17



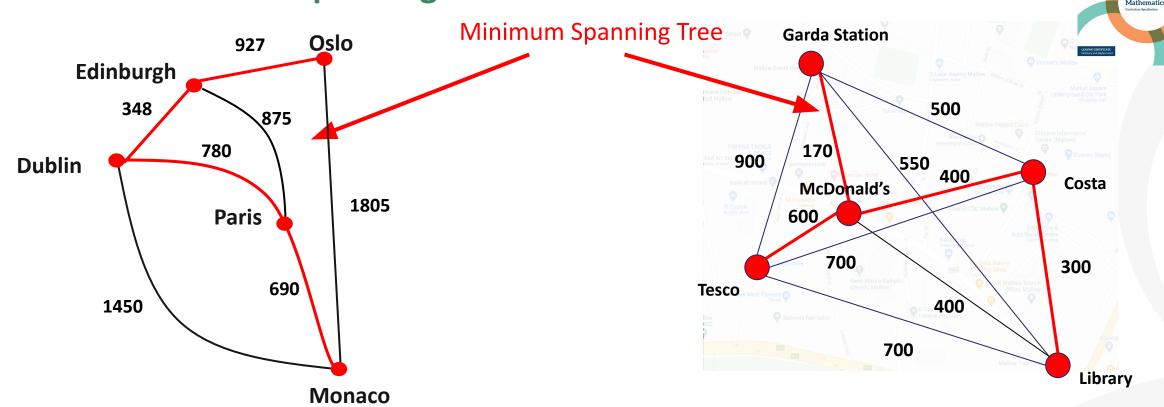


Given your understanding of what a tree is, can you explain what a spanning tree are?

"Students should be able to demonstrate an understanding of the concepts of tree, spanning tree, minimum spanning tree in appropriate contexts". Specification p. 17



What is a Minimum Spanning Tree?



A Minimum Spanning Tree (MST) is a _____ such that the total weight of its ____ is as ____ as possible.



Task 5: Describe the Best Approach

Create a step-by-step approach to create a Minimum Spanning Tree (MST) for any network using *suitable terminology*.

What other applications could this approach be used for?

Padlet link: https://tinyurl.com/ydd2uyjm





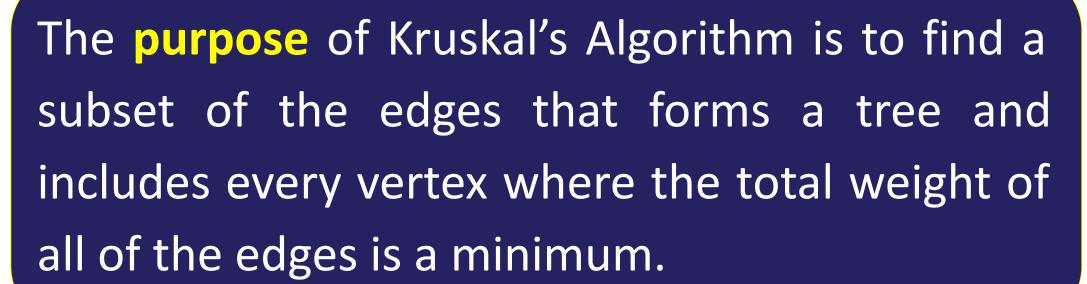
Response from Groups



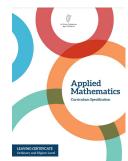


"use algorithms to solve problems" Specification p. 17

Kruskal's Algorithm



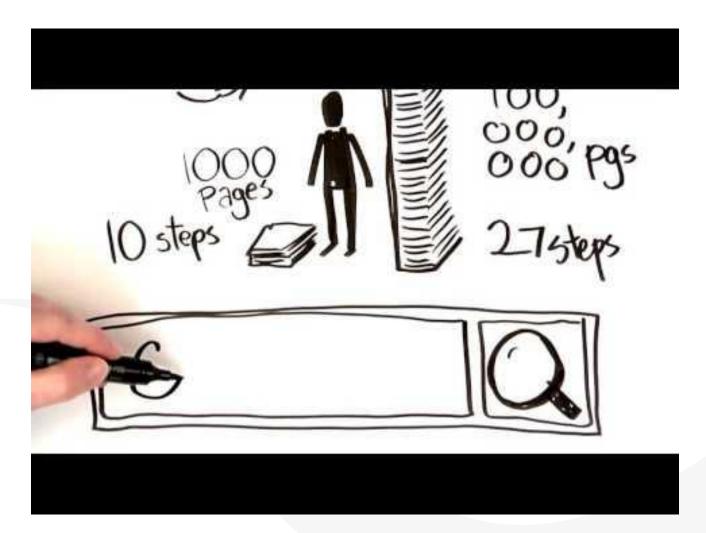
Note: Kruskal's algorithm is a *greedy* algorithm which is where it builds up a solution piece by piece, always choosing the next piece that offers the most obvious and immediate benefit.







What is an Algorithm?



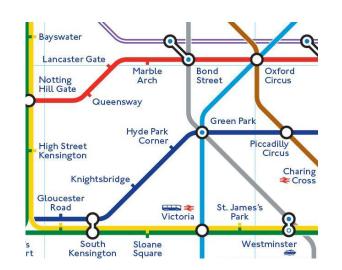
Applications of Graph Theory using Algorithms

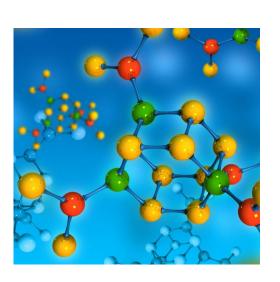
"Students should be able to: represent real-world situations in the form of a network."













Reflecting on each of the four images above, is it immediately clear how each one may have a link with graph theory?

Source: Mathigon



Reflection on Teaching and Learning: Session 2

Consider the approaches to teaching an arning tring this session.

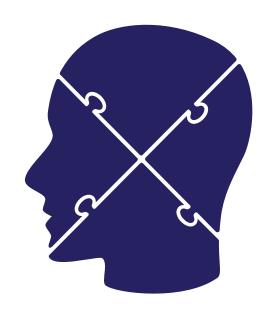
Consider how these approaches support the sime of the specification.

Padlet link: https://tinyurl.com/y4d9hxpx

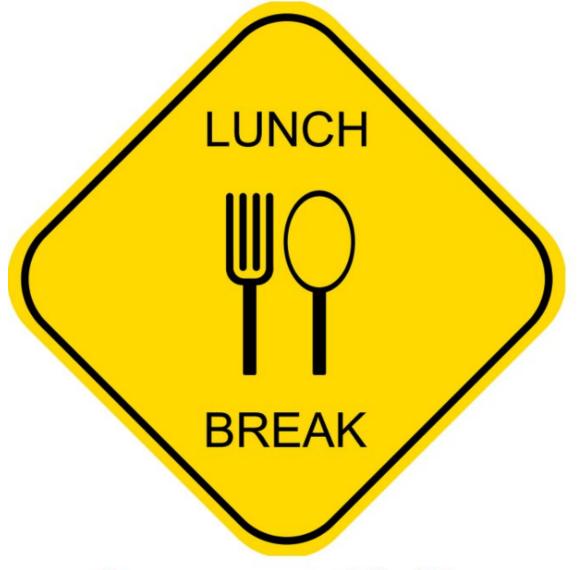




Response/feedback from group/







See you at 13:45





By the End of this Session you will have:

An understanding of Prim's algorithm and its distinctions with Kruskal's Algorithm.

Further developed your understanding of how to use algorithms and the role of mathematical modelling in doing so.

Experienced additional approaches to teaching and learning which support the aims of the specification.

Explored the benefits of Professional Learning Communities (PLCs) and the role they play in professional development.



Task 6: Kruskal's Algorithm



For the network below, a student has commenced finding the minimum spanning tree (MST) using Kruskal's algorithm. The student's work is highlighted in red.

Is the student's work correct so far? If not, please correct it and then complete the MST to determine the weight of the tree.

- To begin, select the edge of least weight.
- Find the next edge of least weight. If it would form a cycle with the edges already selected, don't choose it. If not then add it to the MST.

 3. If there is a choice of equal edges, it has no effect which you choose first.
- Repeat step & until all vertices are connected.

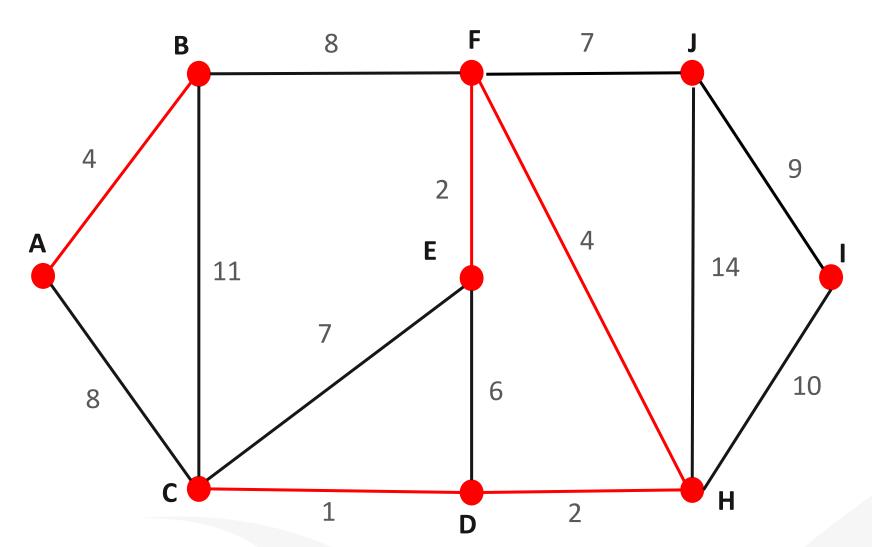
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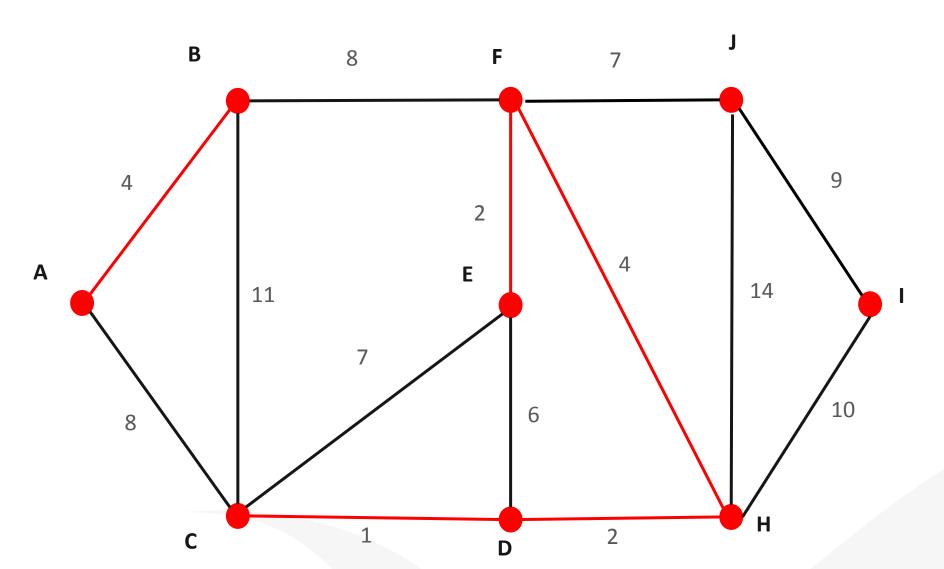


Response 1





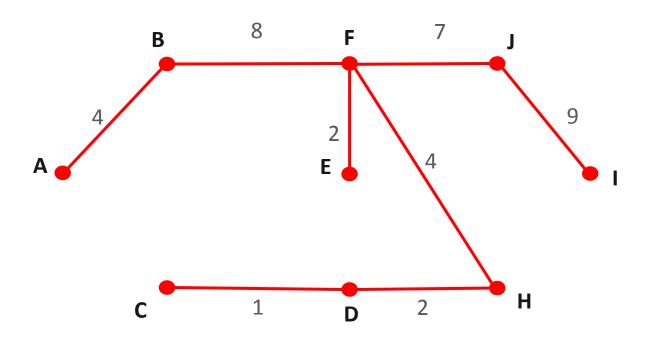
Response 2



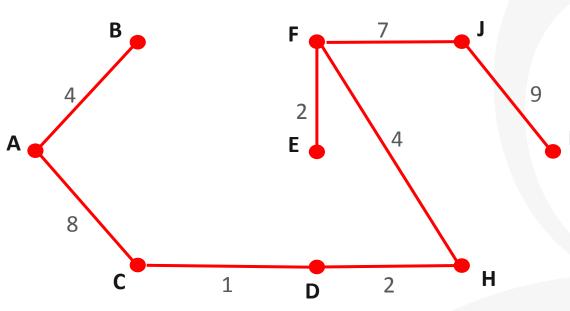


Task 6 - Solution





Solution 2



Weight = 37

Reflection on Teaching and Learning: Task 6

"The focus on the experiential approach to teaching and learning, which is central to applied mathematics, means that students can be engaged in learning activities that complement their own needs and ways of learning."

PDS Professional Development
Service for Teachers

An tSeirbhis um Fhorbairt
Ghairmiúil do Mhúinteoirí

Applied Mathematics
Constitution Residence

Applied Mathematics
Co

Specification p. 14

What did you notice about the teaching and learning approach used here?

What are some of the benefits for students of using an approach like this?



Further reading: Atkinson, R. K., Renkl, A. and Merrill, M. M. (2003) 'Transitioning from studying examples to solving problems: effects of self-explanation prompts and fading worked-out steps.

"determine what assumptions are necessary to simplify the problem situation"

Specification p. 16



Task 7: Plan Jennifer's Day

Jennifer is a GAA coach and is looking to increase the uptake of camogie amongst primary school children in different counties. On Tuesday she will visit Tipperary to deliver equipment and she is hoping to get all of her visits done in one day. She must visit one school in each of: Nenagh, Thurles, Clonmel, Carrick-on-Suir, Cashel and Roscrea.

She will start in Carrick-on-Suir school and will not return to Carrick-on-Suir.

What information is required to ensure that her journeys are completed as effectively as possible?

Padlet link: https://tinyurl.com/yccxgdon







Response from individual participants

VISITED LIST					
{ Letterfrack	Tullycross	Moyard	Cleggan	Garraun	Kylemore }



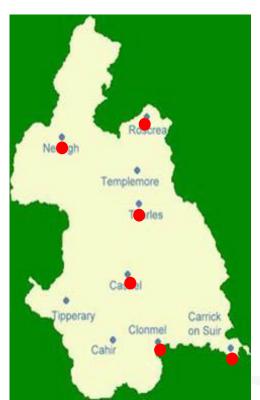
Task 7: Plan Jennifer's Day

"It is anticipated that digital technology will be used as a learning tool in some aspects of this course."

Specification p. 6







Time to Travel between Towns (in minutes)							
	Nenagh	Roscrea	Thurles	Cashel	Clonmel	Carrick-On- Suir	
Nenagh	-	25	39	-	-	-	
Roscrea	25	1	39	1	-	-	
Thurles	39	39	ı	21	44	57	
Cashel	-	1	21	-	30	-	
Clonmel	-	-	44	30	-	?	
Carrick-On- Suir	-	-	57	-	?	-	



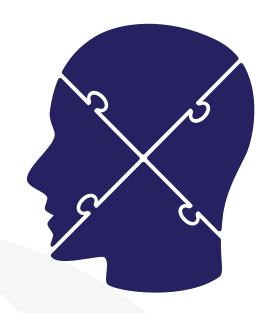
Task: Use digital technology to find any unknown times, all times can be rounded to the nearest *minute*.

- (a) Create the overall network, using the direct connections as shown in the table above.
- (b) Identify the most efficient way for her to visit all of the towns if she must start in Carrick-on-Suir.
- (c) Determine her total time driving in visiting all of the towns.



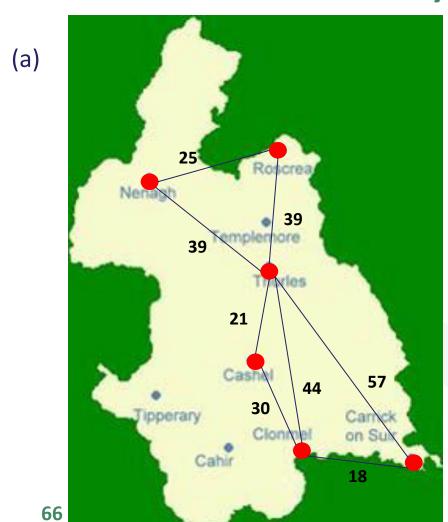


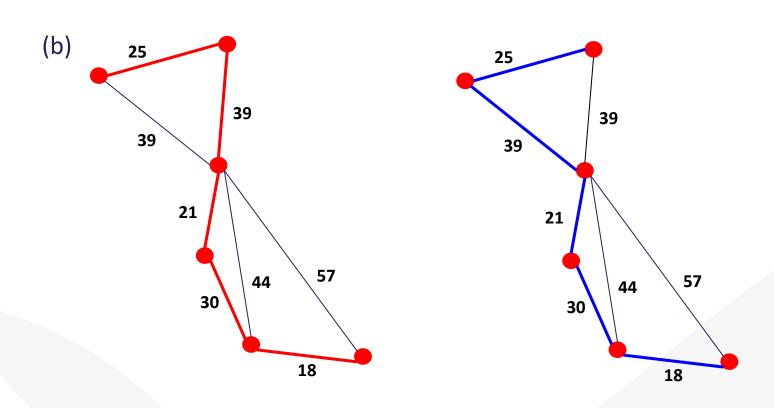
Response/feedback from group





Task 7: Plan Jennifer's Day

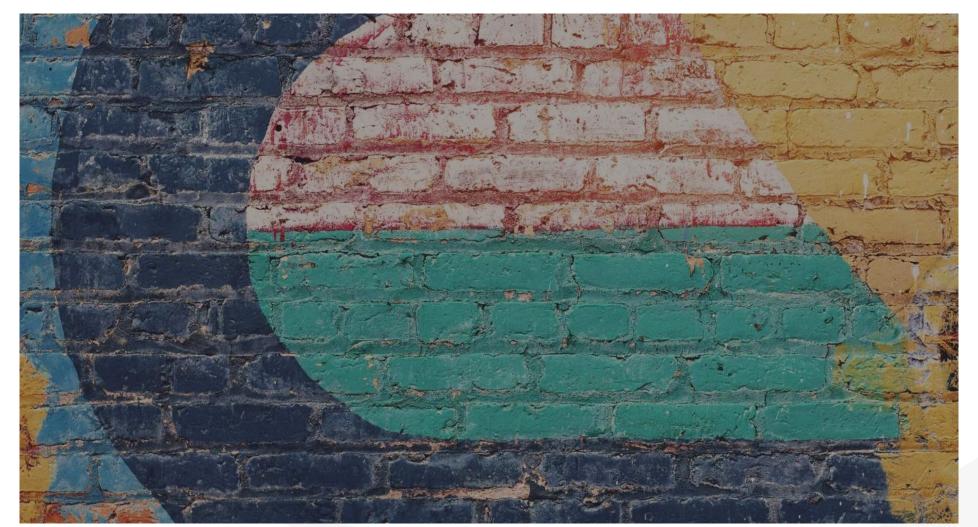




(c) Total time = 25 + 39 + 21 + 30 + 18 = 133 mins



Alternative Approaches to finding a Minimum Spanning Tree





"use appropriate algorithms to find minimum spanning trees"

Specification p. 17

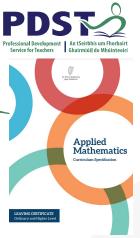
Prim's Algorithm

- 1. To begin, pick any vertex/node (unless a predetermined one is indicated).
- 2. Find all the edges that connect the tree to new nodes, select the minimum and add it to the tree, ensuring to avoid cycles.
- 3. Keep repeating step 2 until we get a minimum spanning tree with all nodes connected and cycles avoided.

It can be helpful to write a visited list to keep track of nodes that are already in the minimum spanning tree.



Note: Prim's algorithm is a *greedy* algorithm which is where it builds up a solution piece by piece, always choosing the next piece that offers the most obvious and immediate benefit.





Background to Prim's Algorithm

Prim's Algorithm:

Prim's Algorithm was originally discovered in 1930 by Vojtech Jarnik and was then independently discovered by Robert Clay Prim in 1957.

Prim's starts by picking a vertex.

The **purpose** of Prim's Algorithm is to find a subset of the edges that forms a tree which includes every vertex where the total weight of all of the edges is a minimum.

Task 8: Connect the Villages





An energy supply company wishes to connect six villages in Connemara. The company will need to build a substation at one of the villages where this cost of building is the same at each village. The cost of connecting each village is outlined in the table below.

a) Use Prim's algorithm to calculate the minimum cost energy supply network that would connect all 6 villages separate to the substation cost and show the minimum spanning tree.

Padlet link: https://tinyurl.com/ybtlsamb



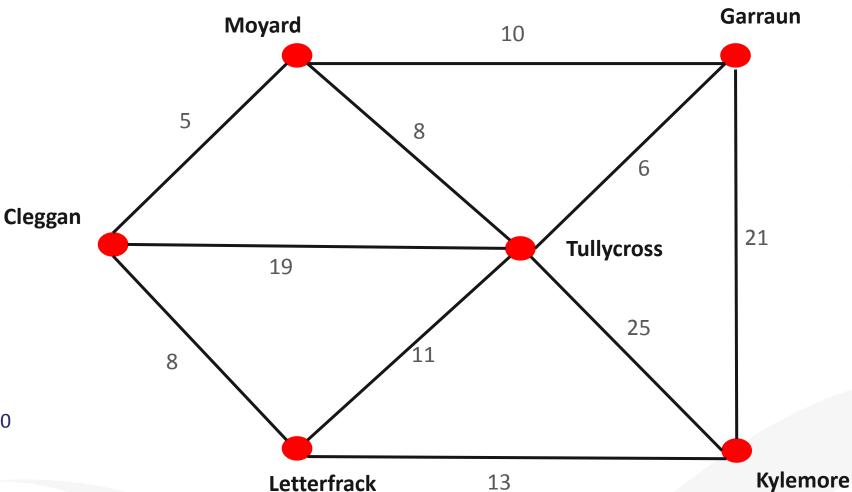
Cost of connection between each village							
	Cleggan	Moyard	Letterfrack	Tullycross	Garraun	Kylemore	
Cleggan	-	€5,000	€8,000	€19,000	-	-	
Moyard	€5,000	-	-	€8,000	€10,000	-	
Letterfrack	€8,000	-	-	€11,000	-	€13,000	
Tullycross	€19,000	€8,000	€11,000	-	€6,000	€25,000	
Garraun	-	€10,000	-	€6,000	-	€21,000	
Kylemore	-	_	€13,000	€25,000	€21,000	-	



Task 8: Connect the Villages. Worked solution (a)

The network shows 6 villages and the cost, in thousands of euros, of connecting them with a new energy supply

a) Calculate the minimum cost energy supply network that would connect all 6 villages and show the minimum spanning tree (MST).



Note: Weights are per €1,000



Task 8: Connect the Villages

The cost of connecting six villages in Connemara with a new energy supply is represented in the chart below.

b) Use Kruskal's algorithm to calculate the minimum-cost energy supply network that would connect all 6 villages separate to the substation cost and show the minimum spanning tree.

To an all and an all and a second a second and a second a	"translate the i the problem to assumptions in model that can
Applied Mathematics Curriculum Specification	
LEAVING CERTIFICATE	

"translate the information given in the problem together with the assumptions into a mathematical model that can be solved" Specification p. 16

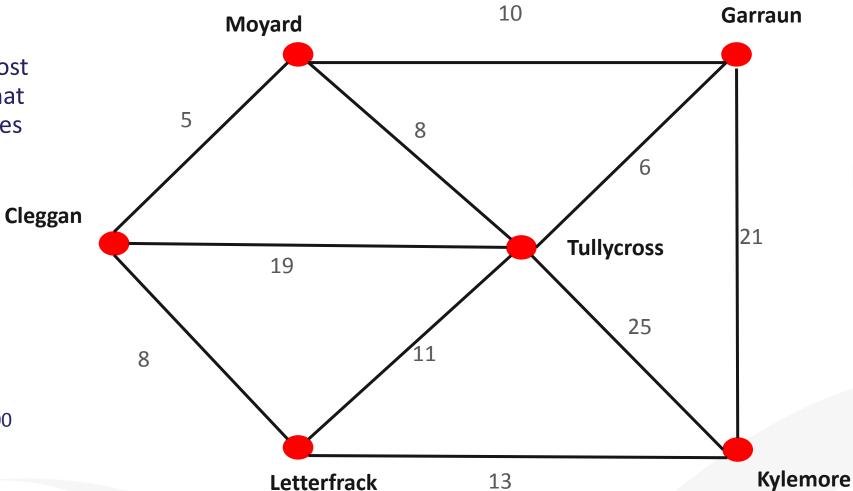
Cost of connection between each village								
	Cleggan	Moyard	Letterfrack	Tullycross	Garraun	Kylemore		
Cleggan	-	€5,000	€8,000	€19,000	-	-		
Moyard	€5,000	-	-	€8,000	€10,000	-		
Letterfrack	€8,000	-	-	€11,000	-	€13,000		
Tullycross	€19,000	€8,000	€11,000	-	€6,000	€25,000		
Garraun	-	€10,000	-	€6,000	-	€21,000		
Kylemore	-	-	€13,000	€25,000	€21,000	-		



Task 8: Connect the Villages. Worked solution (b)

The network shows 6 villages and the cost, in thousands of euros, of connecting them with a new energy supply

Calculate the minimum cost energy supply network that would connect all 6 villages and show the minimum spanning tree (MST).



Note: Weights are per €1,000

"interpret the mathematical solution in terms of the original situation"

Specification p. 16



(c) Selecting appropriate Algorithm - Group Task

Kylemore

A new minimum spanning tree is required which includes the links between Tullycross and Garraun as well as Garraun and Kylemore.

In groups,

- 1. Select and justify the most appropriate algorithm to solve this problem
- 2. Determine the new cost.
- 3. Consider possible extensions to this problem.

Cost of connection between each village						
	Cleggan	Moyard	Letterfrack	Tullycross	Garraun	Kylemore
Cleggan	-	€5,000	€8,000	€19,000	-	-
Moyard	€5,000	-	-	€8,000	€10,000	-
Letterfrack	€8,000	-	-	€11,000	-	€13,000
Tullycross	€19,000	€8,000	€11,000	-	€6,000	€25,000
Garraun	-	€10,000	-	€6,000	-	€21,000

€13,000

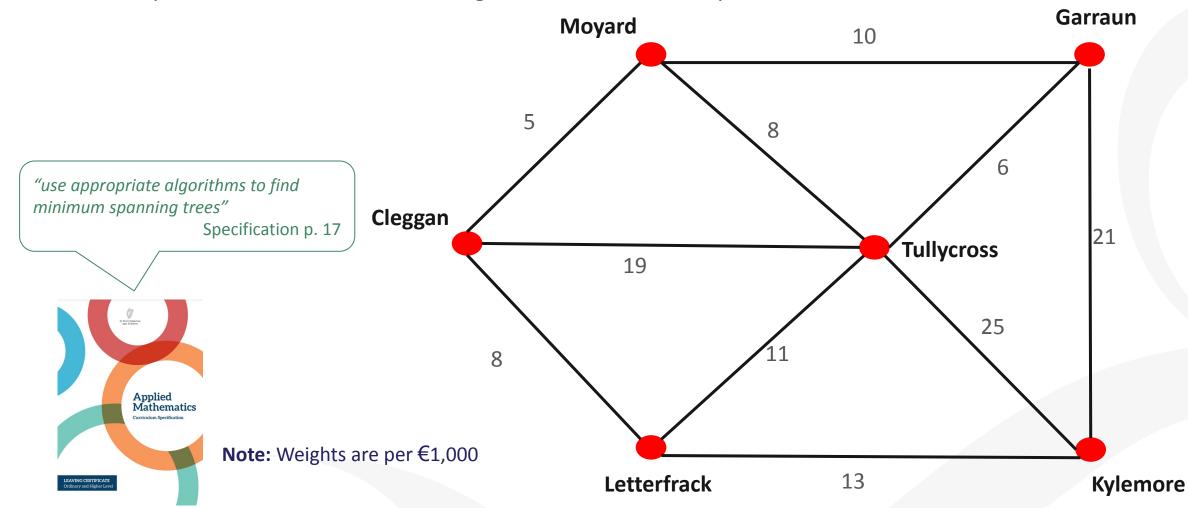
€25,000

€21,000



Task 8: Connect the Villages. Worked solution (c)

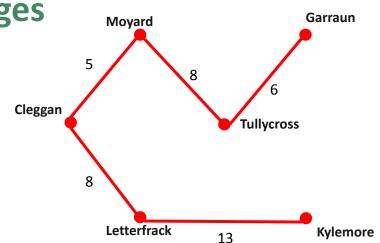
c) A new spanning tree is required which includes the links between Tullycross and Garraun as well as Garraun and Kylemore, which also has the lowest possible total weight. Explain which algorithm you would choose to complete the tree, and how the algorithm should be adapted.



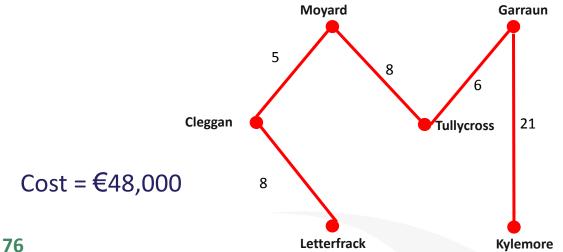


Task 8: Connect the Villages

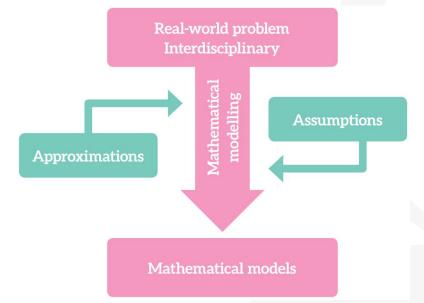
(a)/(b) Minimum Cost = €40,000



(c) Start off the tree with links between Tullycross and Garraun as well as Garraun and Kylemore and then apply Kruskal's algorithm.



Note: Weights are per €1,000





Reflection: Teaching and Learning in Algorithms

Consider the approaches to teaching and learning used during this session.

Identify how these approaches support the aims of the specification.





Reflection: Teaching and Learning in Algorithms

Did you enjoy the teaching and learning approaches in the previous tasks?

What role did the student play in the development of understanding?

How could you add to differentiated teaching & learning skills as you experienced them today?

What are the implications of the new specification on your teaching of Applied Maths?





Professional Learning Communities (PLC)

Teacher lead with support from PDST.

Teachers work/support each other in the development of knowledge, teaching and learning approaches etc.

Long-term support network.

Evening sessions (Face to face/online).



"None of us is as smart as all of us" - Japanese Proverb



Next Steps?

Next meeting - PLC 1 (March 2021).

Focus of PLC 1 may be Mathematical Modelling depending on feedback.

Teachers and PDST will work together to develop understanding of the modelling cycle and formulating problems.

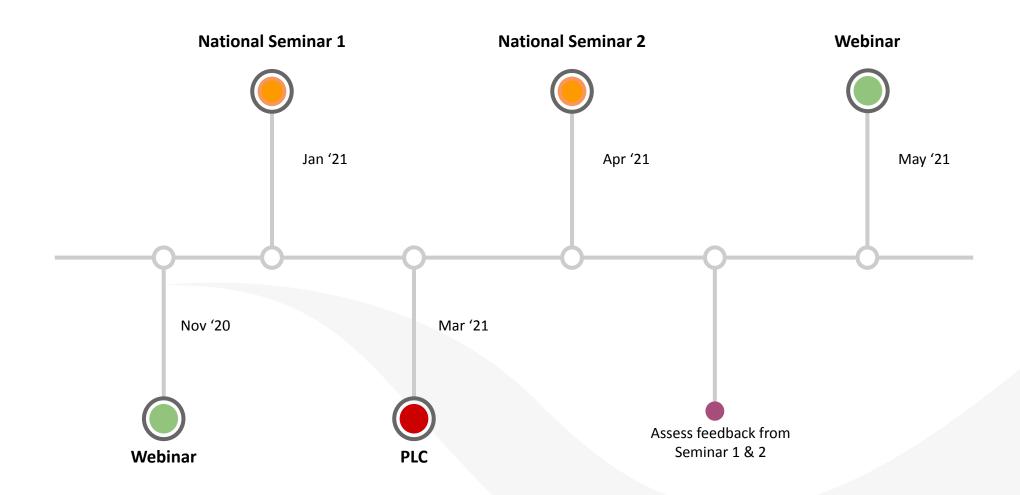
Why not try today's modelling task with one of your classes (e.g. Transition Year)

And/or try the takeaway problem in the booklet yourself.





What's next? Timeline 2020 - 2021





Supports Provided by PDST

PDST Supports

Seminars

Webinars

Professional Learning Communities

School Visits

PDST Websites

Single Visit Sustained Support

pdst.ie Scoilnet.ie



Evaluation





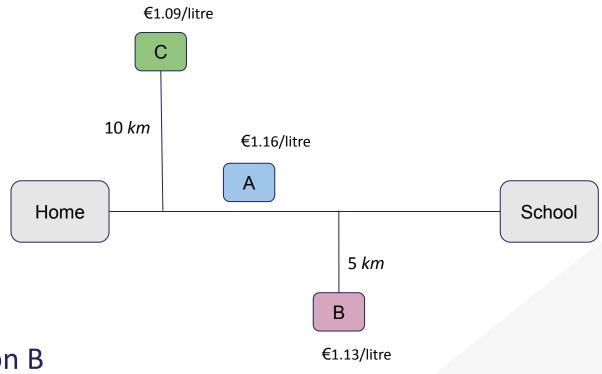


Takeaway Problem: Driving for Fuel

Station A is on your normal route home from school and is selling petrol this week for €1.16 per litre while Station B, which is 5 km off your normal route is selling petrol for €1.13 per litre. Station C has the least expensive petrol but is 10 km off your route.

Your car uses 5 litres/100km and your neighbour's car uses 14 litres/100km.

Should either or both of you, drive to Station B or Station C for Petrol. Explain your decisions.



Source: GAIMME 2016, p. 181



Questions?



Any further questions please contact: appliedmaths@pdst.ie



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