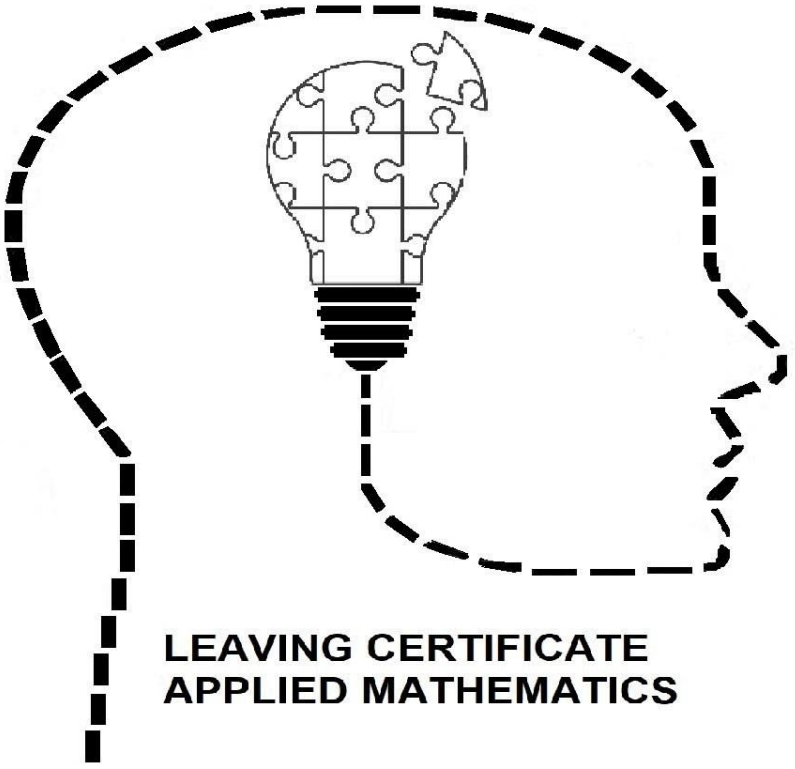
PDST
Professional Development Service for Teachers | An tSeirbhís um Fhorbairt Ghairmiúil do Mhúinteoirí

An Roinn Oideachais agus Scileanna
Department of Education and Skills



National Seminar 1

2021

Booklet

Introduction to Mathematical Modelling, Networks and Algorithms

Applied Mathematics National Seminar 1 Booklet

Session 1

Aims and Objectives

Aim

Leaving Certificate Applied Mathematics aims to develop the learner's capacity to use mathematics to model real-world problems. By focusing on all aspects of the problem-solving cycle it is envisaged that learners will move beyond calculating procedures and gain experience in asking appropriate questions, formulating mathematical representations of problems, and interpreting and verifying results. Through Applied Mathematics, students should learn to appreciate the extent to which mathematics is relevant in everyday life, generating engagement and interest in the process. It is anticipated that digital technology will be used as a learning tool in some aspects of this course.

Objectives

The objectives of Leaving Certificate Applied Mathematics are to develop applied mathematical problem-solving skills so that students will be able to:

- Formulate a problem: Consider the scope and detail of a real-world problem, and to define manageable questions to address
- Translate the problem into mathematics: Create or choose a suitable mathematical model, and then formulate the question as a mathematical problem within the model
- Compute a solution: Use mathematical techniques to solve the mathematical problem
- Evaluate the solution: Interpret the mathematical solution in the original context.

Key Messages

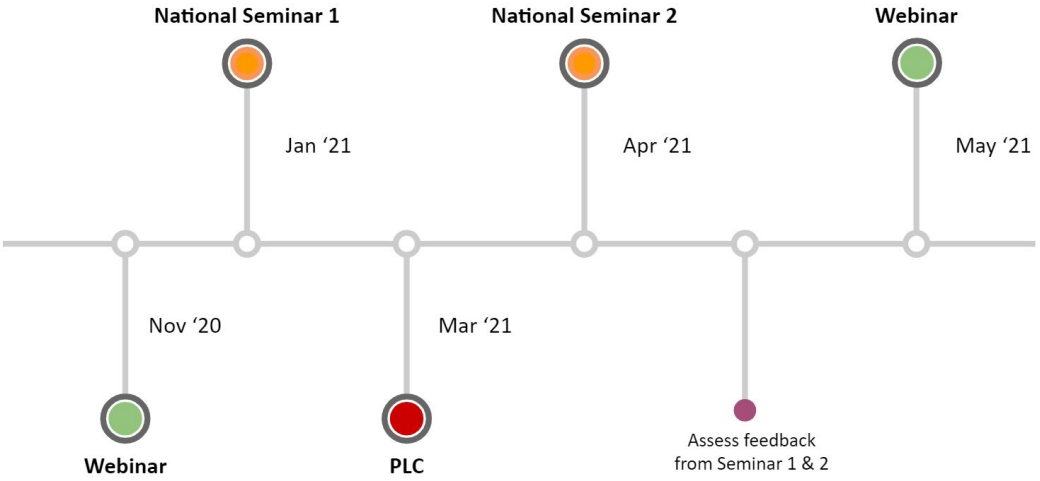
1. Core to the specification is a non-linear approach which promotes the making of connections between various concepts.
2. Strand 1 of the specification is a unifying strand and emphasises the importance of utilising modelling across all learning outcomes.
3. Applied Maths is rooted in authentic problems and plays a key role in the development of problem solving skills which are applicable to a variety of disciplines.

Overview of Professional Development (PD) and Supports Available

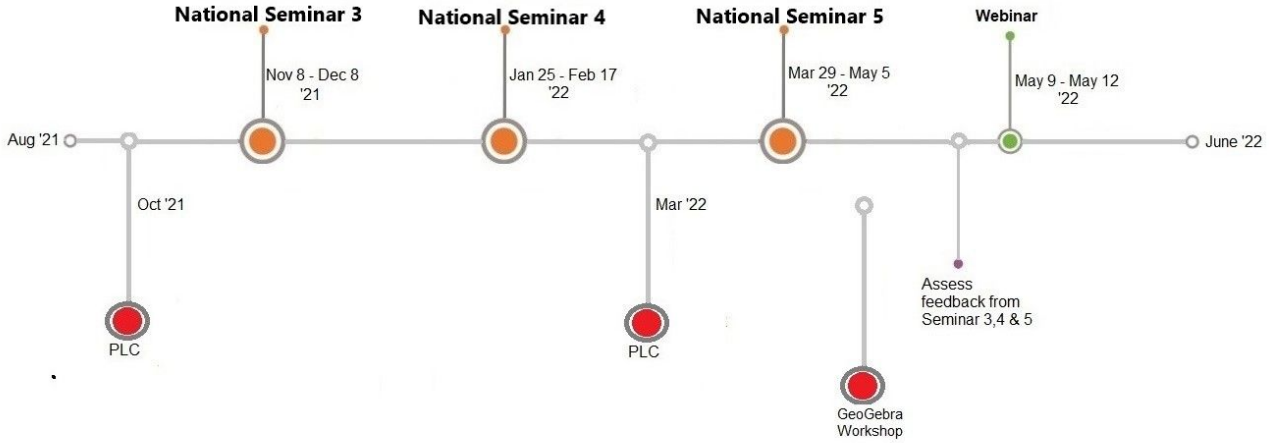
- **9 X 1 day seminars**
  - Full day PD workshops
- **5 X Professional Learning Communities (PLCs)**
  - Autumn and Spring each year in response to teacher PD needs and relevant to the particular community.
- **4 X Webinars**
  - Live events discussing new material with Q&A
- **2 X Geogebra Workshops**
  - Exploring the use of technology in teaching Applied Maths.

Overview of Professional Development

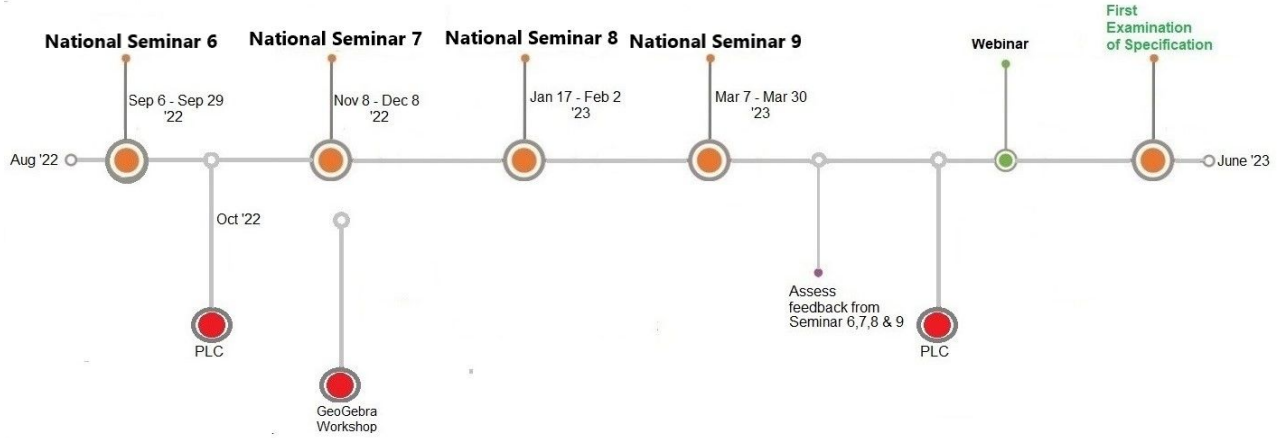
Year 1 Nov 2020 - June 2021



Year 2 Sep 2021 - June 2022



Year 3 Sep 2022 - June 2023



PDST Sustained Support

What is it?
The PDST Sustained support model invites schools to engage with bespoke support based on individual school context.

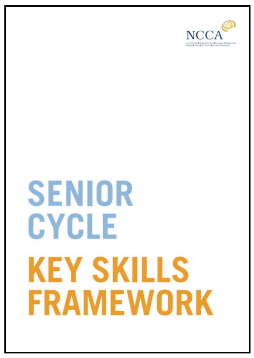
Schools will have access to a PDST advisor for a number of school visits during a school year.

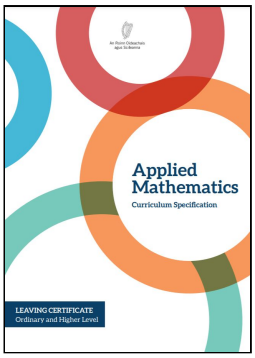
Sustained support is a collaborative process towards educational change and improvement in learner outcomes.

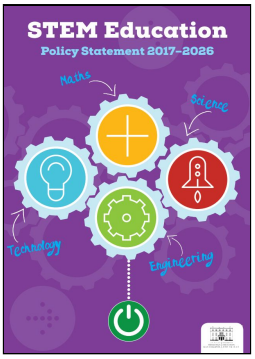
How do I apply?

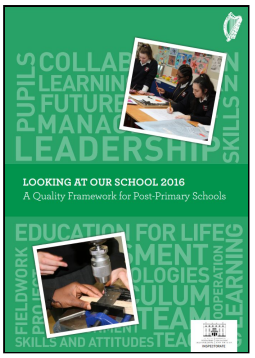
https://pdst.ie/schoolsupport

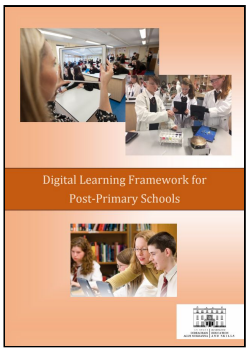
Overview of Key Policy Documents





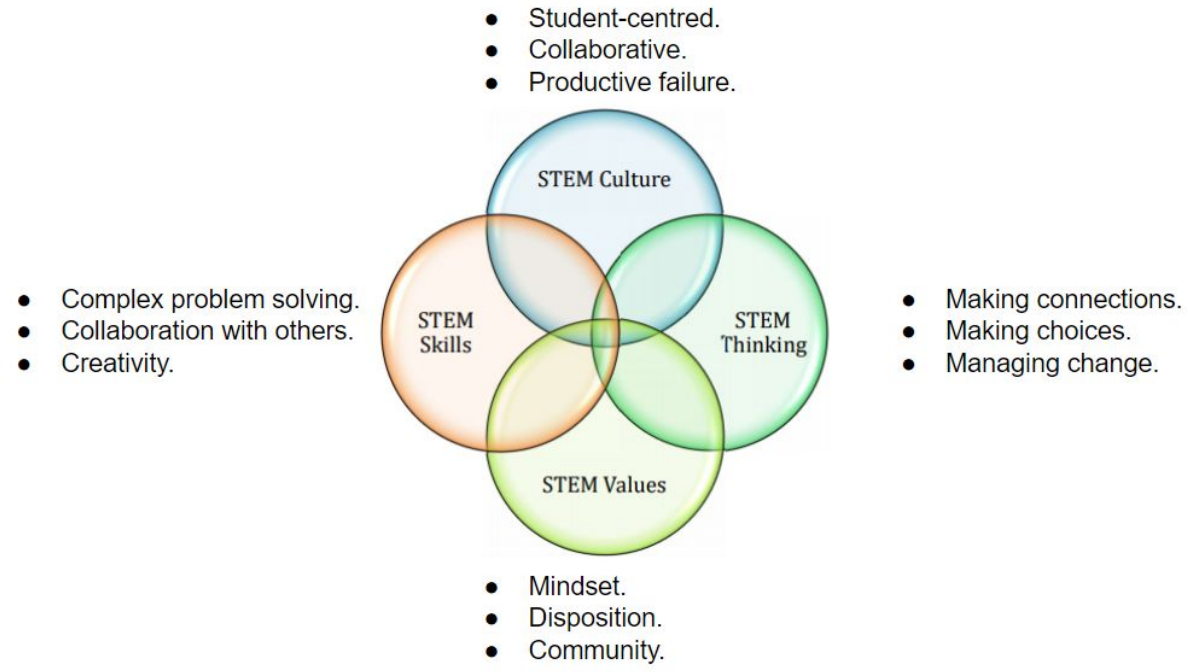






(Images are also hyperlinked for digital version)

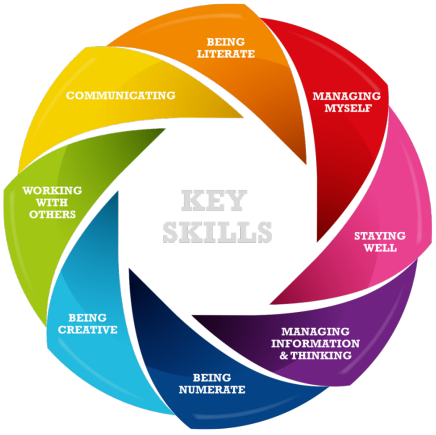
Pillars of STEM Education in Ireland



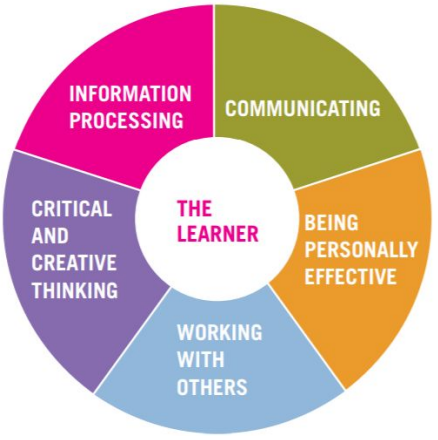
Task 1: Skills Analysis Padlet link: https://tinyurl.com/ycjjm49f

What skills does an Applied Maths student need for 21st century living?

Outline of Key Skills



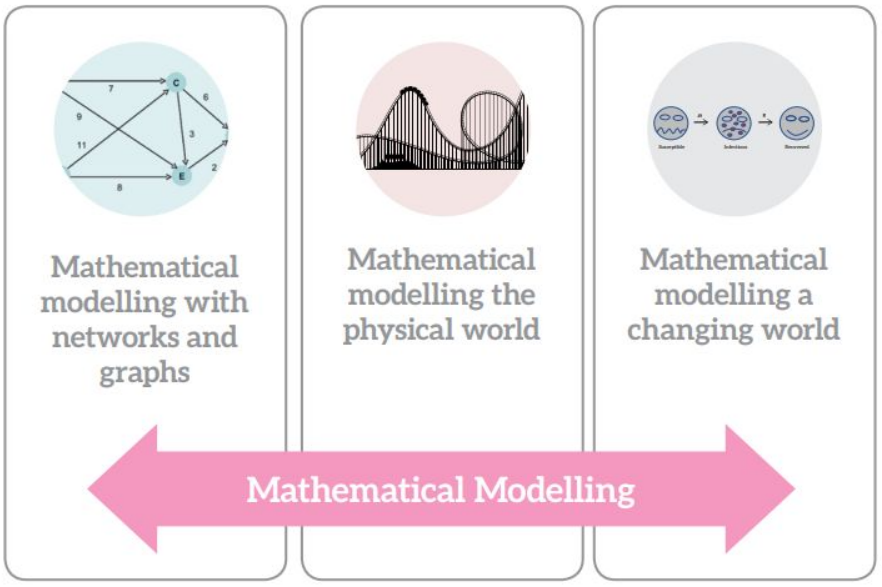
Junior Cycle



Senior Cycle

(Source: NCCA)

Structure of Curriculum

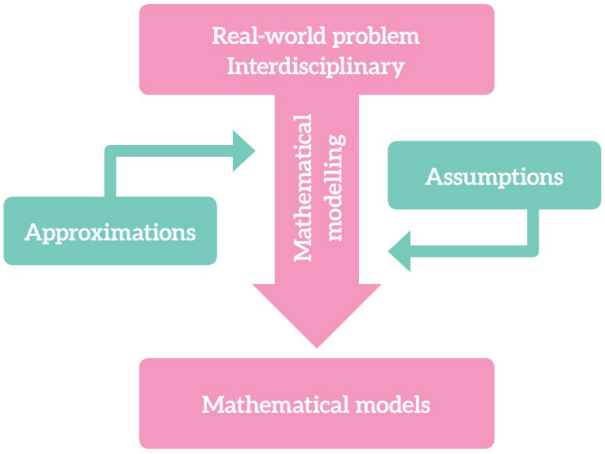


(Source: Applied Mathematics Specification p. 9)

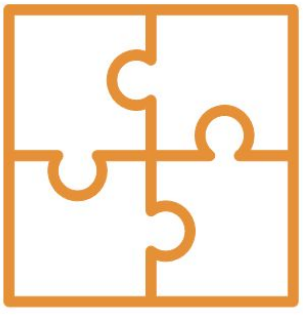
Strand 1: Mathematical modelling

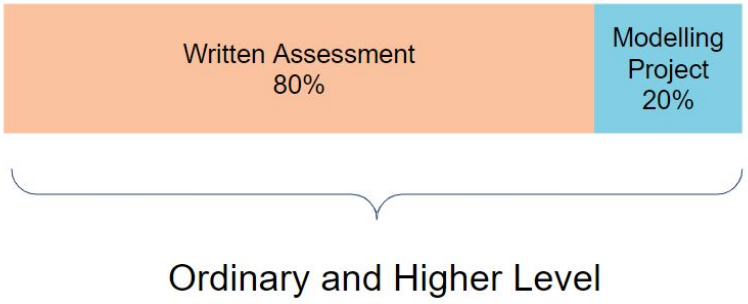
In this **unifying strand** students learn about mathematical modelling as a process that will develop skills such as:

- Formulating problems
- Translating problems into mathematics
- Computing solutions
- Evaluating solutions



Assessment and Coursework





Task 2: Specification Analysis

Padlet link: https://tinyurl.com/y7vjtmh7

Analyse your allocated topic based on the Group number you are assigned:

Group 1	Group 2	Group 3	Group 4
Aims & Objectives of Specification Pg 6-7	Overview & Structure Pg 9-10	Teaching & Learning Pg 13	Strand 1 Mathematical Modelling Pg 16

Share the key messages from your piece and one point about what this means for your teaching of Applied Mathematics with the group.

What student skills are being developed from your reading of the specification?

"Modelling problems require the solver to research the situation themselves, make reasonable assumptions, decide which variables will affect the solution and develop a model that provides a solution that best describes the situation
Specification p. 10

Modelling Project Overview

- Allows students to demonstrate proficiency in course content and skills.
- Students present a solution to an authentic modelling problem.
- Common brief issued annually by the State Examinations Commission (SEC) for both OL and HL.

*"**Word problems** explicitly include all the information needed to solve them. The solver needs only to determine the appropriate model to use to compute the one correct answer. **Modelling problems** require the solver to research the situation themselves, make reasonable assumptions, decide which variables will affect the solution and develop a model that provides a solution that best describes the situation"* Specification p. 10.

Task 3: Introduction to Mathematical Modelling

Padlet link: https://tinyurl.com/yaxc9llr

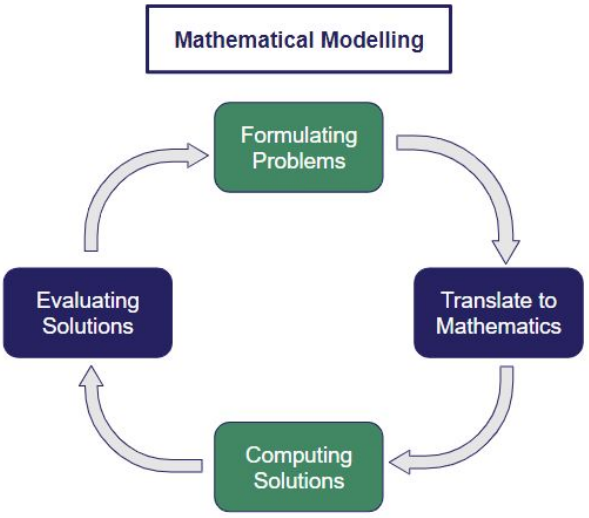
How far apart should speed bumps be placed so that traffic does not reach a speed greater than 50 km/h?

What factors will affect the formulating of this problem?

What assumptions will you make?

Extension Questions

1. Gather appropriate data on the lengths of cars and revise your solution to the speed bumps problem to take account of this factor.
2. Describe how changing the speed at which the vehicle crosses the bumps would change the solution to the problem.



Reflection: Session 1

What are your main takeaways from session 1?

What key messages have you taken from this session, regarding the teaching and learning of the specification?

Session 2: Networks and Graph Theory

Introduction

Mathematical modelling (Strand 1), should be central to all of our teaching and learning of the other three strands. It should be integrated into our day-to-day teaching to maximise student understanding and engagement.

In Strand 2,(Mathematical modelling with networks and graphs) students learn about networks or graphs as mathematical models which can be used to investigate a wide range of real-world problems. Graph Theory is a branch of Mathematics concerned with networks of points connected by lines.students are given further opportunity to consolidate their understanding that mathematical ideas can be represented in multiple ways. As students progress in their understanding they will explore and appreciate the use of algorithms in problem-solving as well as consider some of the wider issues involved with the use of such techniques.

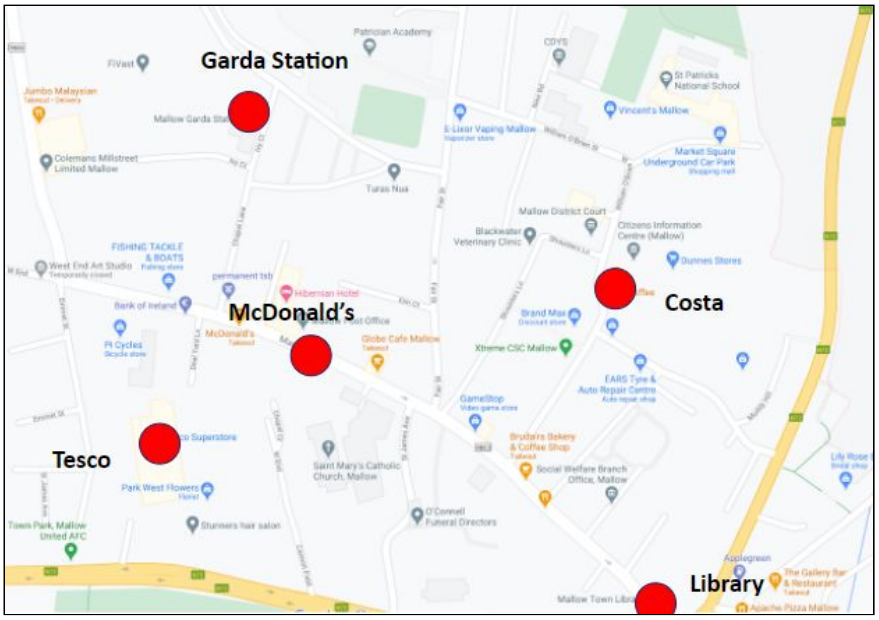
Task 4: New Broadband for Mallow!

Padlet Link: https://tinyurl.com/ydgnnmq9

Seán works for the County Council who are planning to connect a number of buildings in Mallow, Cork with an upgraded broadband network. They will connect the buildings by laying cables in the ground following the current road layout.

What information does Sean require in order to complete this plan?

Road distances between each building (to the nearest metre)					
	Garda St.	Costa	McDonald's	Tesco	Library
Garda St.	-	500	170	900	550
Costa	500	-	400	700	300
McDonald's	170	400	-	600	400
Tesco	900	700	600	-	700
Library	550	300	400	700	-



Additional Notes/Sketches:

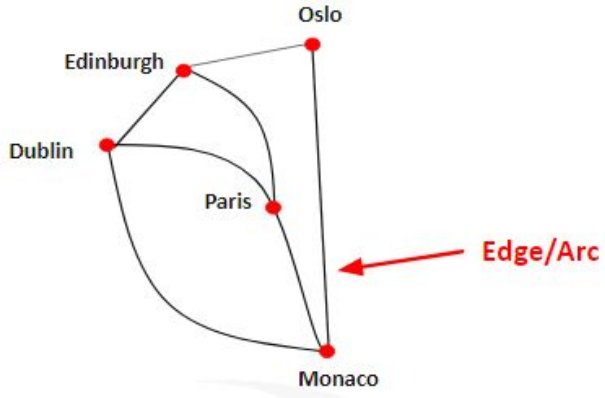
Task 4: Reflection on your Approach:

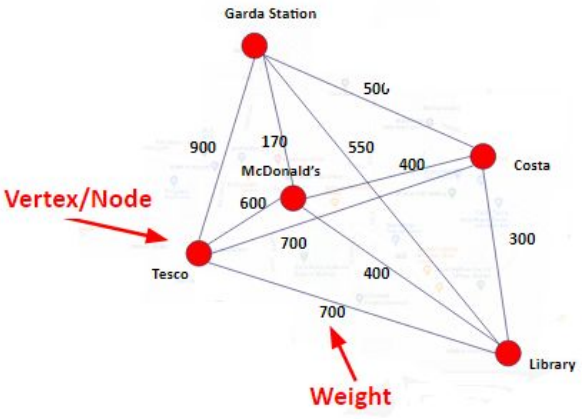
What factors influenced the decisions that you made?

Could your approach be improved?

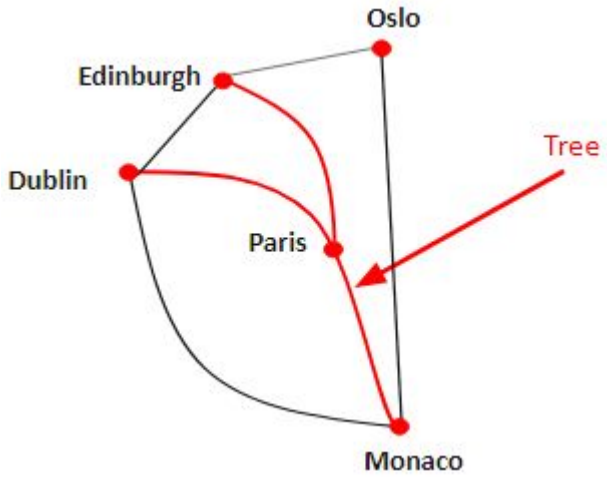
Give a reason why you didn't include both 400 metre options?

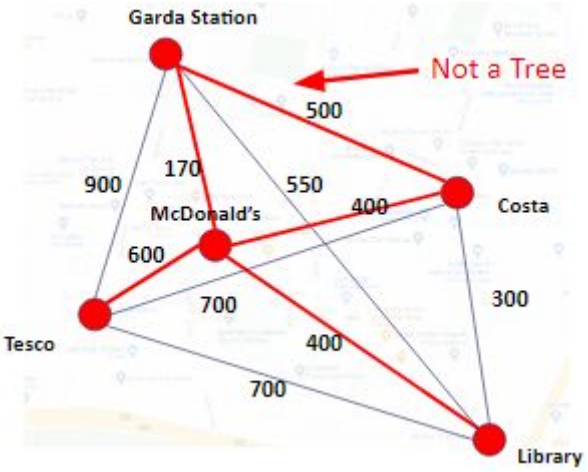
Understanding the Terminology





What defines a Tree?





Given your understanding to date can you explain the following:

What is a Spanning Tree ?

What is a Minimum Spanning Tree ?

Task 5: Describe the Best Approach

Padlet link: https://tinyurl.com/ydd2uyjm

Create a step-by-step approach to create a Minimum Spanning Tree (MST) for any network using the suitable terminology.

Can you suggest other real life applications that minimum spanning trees could be used for?

Algorithm

Explain in your own words what the term Algorithm means to you?

Kruskal's Algorithm

1. To begin, select the edge of least weight.
2. Find the next edge of least weight. If it would form a cycle with the edges already selected, don't choose it. If not then add it to the MST.
3. If there is a choice of equal edges, it has no effect which you choose first.
4. Repeat step 2 until all vertices are connected.

Note: Kruskal's algorithm is a greedy algorithm which is where it builds up a solution piece by piece, always choosing the next piece that offers the most obvious and immediate benefit.

The **purpose** of Kruskal's Algorithm is to find a subset of the edges that forms a tree and includes every vertex where the total weight of all of the edges is a minimum.

Reflection on Teaching and Learning: Session 2

Consider the approaches to teaching and learning used during this session.

Identify two ways in which these approaches support the aims of the specification.

How does this connect to key messages from your analysis of the specification in Session 1?

Padlet link: https://tinyurl.com/y4d9hxpx

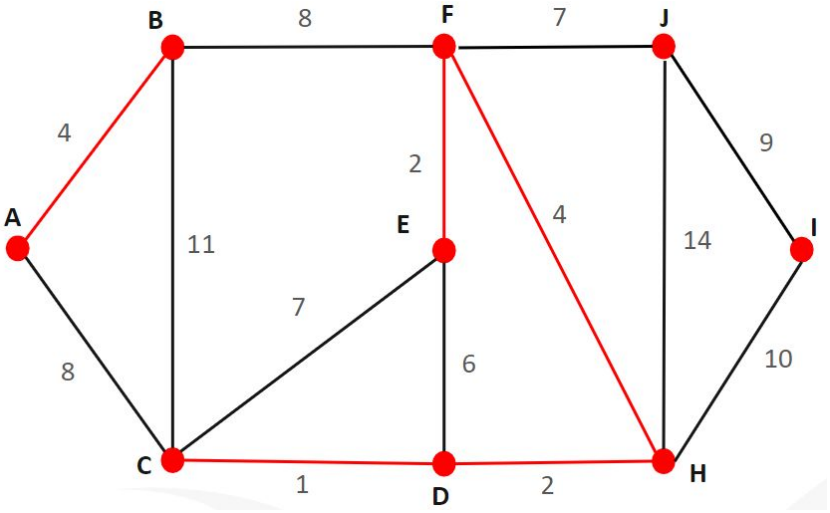
Session 3: Algorithms and PLCs

Task 6: Kruskal's Algorithm

Padlet link: https://tinyurl.com/yazqwtd4

For the network below, a student has commenced finding the minimum spanning tree (MST) using Kruskal's algorithm. The student's work is highlighted in red.

Is the student's work correct so far? If not, please correct it and then complete the MST to determine the weight of the tree.



Reflection on Teaching and Learning: Task 6

What did you notice about the teaching and learning approach used here?

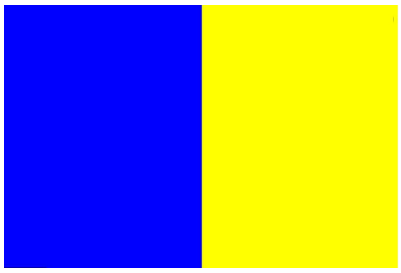
What are some of the benefits for students of using an approach like this?

Task 7: Plan Jennifer's Day

Padlet link: https://tinyurl.com/yccxgdon

Jennifer is a GAA coach and wishes to increase the uptake of camogie amongst primary school children in different counties. On Tuesday she will visit Tipperary to deliver equipment and she is hoping to get all of her visits done in one day. She must visit one school in each of : Nenagh, Thurles, Clonmel, Carrick-on-Suir, Cashel and Roscrea.

She will start in Carrick-on-Suir school and will not return to Carrick-on-Suir.



What information would she need to ensure that her day is completed as effectively as possible?

Task: Use digital technology to find any unknown times, all times can be rounded to the nearest minute.

(a) Create the overall network, using the direct connections as shown in the table above..
(b) Identify the most efficient way for her to visit all of the towns if she must start in Carrick-on-Suir.
(c) Determine her total time driving in visiting all of the towns.

Time to Travel between Towns (in minutes)						
	Nenagh	Roscrea	Thurles	Cashel	Clonmel	Carrick-On -Suir
Nenagh	-	25	39	-	-	-
Roscrea	25	-	39	-	-	-
Thurles	39	39	-	21	44	57
Cashel	-	-	21	-	30	-
Clonmel	-	-	44	30	-	?
Carrick-On -Suir	-	-	57	-	?	-

Prim's Algorithm

Prim's Algorithm was originally discovered in 1930 by Vojtech Jarnik and was then independently discovered by Robert Clay Prim in 1957.
Prim's starts by picking a vertex.

The **purpose** of Prim's Algorithm is to find a subset of the edges that forms a tree which includes every vertex where the total weight of all of the edges is a minimum.

1. To begin, pick any vertex/node (unless a predetermined one is indicated).
2. Find all the edges that connect the tree to new nodes, select the minimum and add it to the tree, ensuring to avoid cycles.
3. Keep repeating step 2 until we get a minimum spanning tree with all nodes connected and cycles avoided.

Note: It can be helpful to write a visited list to keep track of nodes that are already in the minimum spanning tree.

Task 8: Connect the Villages

Padlet link: https://tinyurl.com/ybtlsamb

An energy supply company wishes to connect six villages in Connemara.
The company will need to build a substation at one of the villages where this cost of building is the same at each village. The cost of connecting each village is outlined in the table below.

Cost of connection between each village						
	Cleggan	Moyard	Letterfrack	Tullycross	Garraun	Kylemore
Cleggan	-	€5,000	€8,000	€19,000	-	-
Moyard	€5,000	-	-	€8,000	€10,000	-
Letterfrack	€8,000	-	-	€11,000	-	€13,000
Tullycross	€19,000	€8,000	€11,000	-	€6,000	€25,000
Garraun	-	€10,000	-	€6,000	-	€21,000
Kylemore	-	-	€13,000	€25,000	€21,000	-

a) Use Prim's algorithm to calculate the minimum cost energy supply network that would connect all 6 villages separate to the substation cost and show the minimum spanning tree.

b) Use Kruskal's algorithm to calculate the minimum-cost energy supply network that would connect all 6 villages separate to the substation cost and show the minimum spanning tree.

c) A new minimum spanning tree is required which includes the links between Tullycross and Garraun as well as Garraun and Kylemore.
In groups,
1. Select and justify the most appropriate algorithm to solve this problem
2. Determine the new cost.
3. Consider possible extensions to this problem.

Reflection: Teaching and Learning in Algorithms

Consider the approaches to teaching and learning used during this session.
Identify two ways in which these approaches support the aims of the specification.

Professional Learning Communities (PLC)

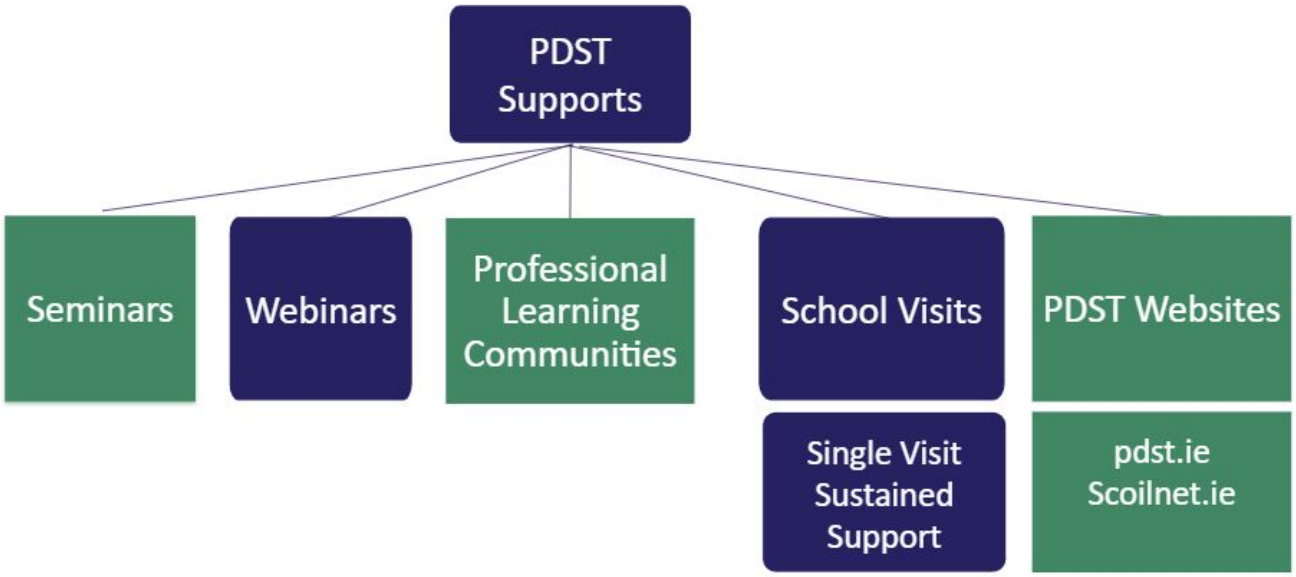
- Teacher lead with support from PDST.
- Teachers work/support each other in the development of knowledge, teaching and learning approaches etc.
- Long-term support network.
- Evening sessions (Face to face/online).

"None of us is as smart as all of us" - Japanese Proverb

Next Steps?

- Next meeting - PLC 1 (March 2021).
- Focus of PLC 1 may be Mathematical Modelling depending on feedback from teachers as noted in their evaluations.
- Teachers and PDST will work together to develop understanding of the modelling cycle and formulating problems.
- Why not try today's modelling task with one of your classes (e.g. Transition Year)
- Try the **takeaway problem** in the booklet yourself (page 21).

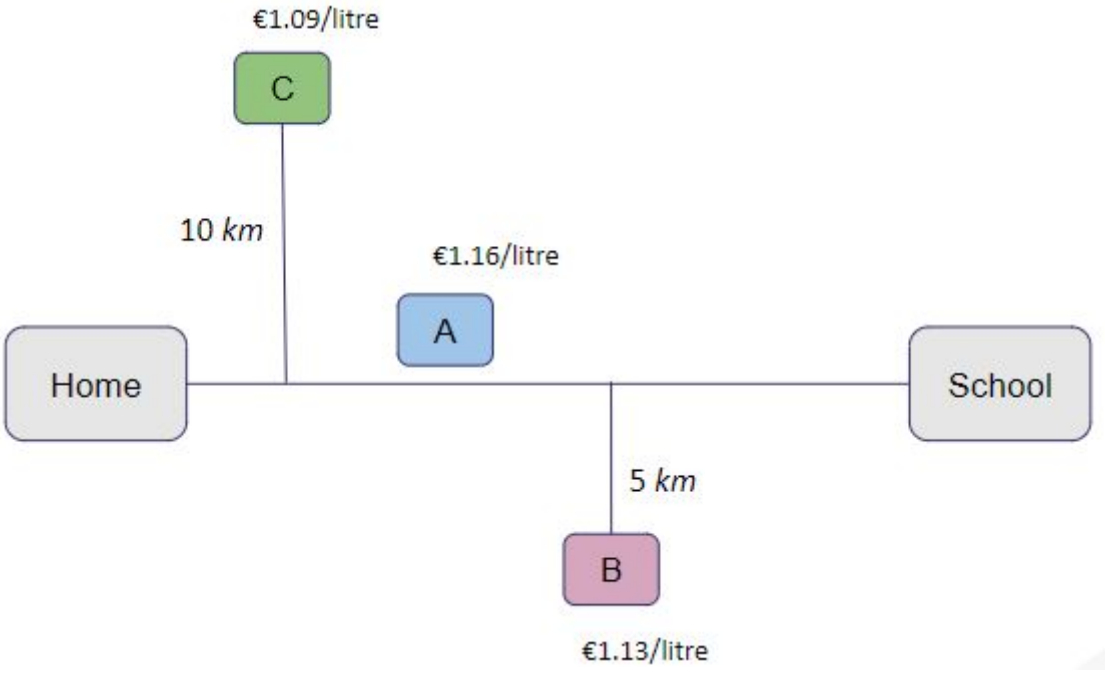
Supports Provided by PDST



Takeaway Problem: Driving for Fuel

Station A is on your normal route home from school and is selling petrol this week for €1.16 per litre while Station B, which is 5 km off your normal route is selling petrol for €1.13 per litre. Station C has the least expensive petrol but is 10 km off your route.

Your car uses 5 litres/100km and your neighbour's car uses 14 litres/100km. Should either of you drive to Station B or C for Petrol. Explain your decisions.



Source: GAIMME 2016, p. 181

Reflection Questions:
What might students find challenging about this question?
If you were doing this problem with your students, how might you guide them?
How could students improve on their initial solution/assumptions?

Reflection: National Seminar 1

What are the implications of the new specification on your teaching of Applied Maths?

What effect will the specification have on students' learning experience?

What are the subject specific skills that we looked at today?

What are the transferable skills?

Reading List:

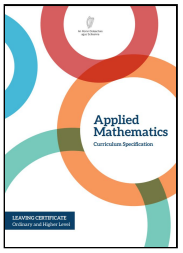
Mathematical Modelling:

GUIDELINES FOR ASSESSMENT & INSTRUCTION IN MATHEMATICAL MODELING EDUCATION	https://tinyurl.com/yd2zy8tc	
Teaching mathematical modelling: a framework to support teachers' choice of resources	https://tinyurl.com/ydf4lpxa	
Mathematical Modelling: Can It be Taught and Learnt?	https://tinyurl.com/ycyvp5rj	

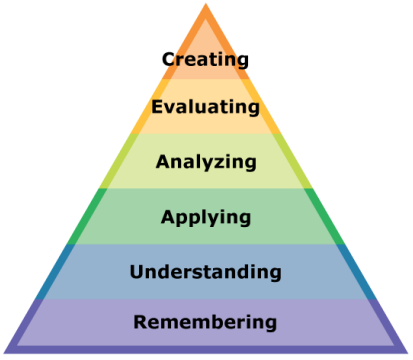
Graph Theory:

Graphs and Networks	https://tinyurl.com/yazvq4he	
An Introduction to networks	https://tinyurl.com/y8bvszan	
Introduction to Graph Theory	https://tinyurl.com/y5hl5uec	
Describing graphs - Khan Academy	https://tinyurl.com/y9kn94wn	

Differentiation in Teaching and Learning:



Students vary in the amount and type of support they need to be successful. Levels of demand in any learning activity will differ as students bring different ideas and levels of understanding to it. The use of strategies for differentiated learning such as adjusting the level of skills required, varying the amount and the nature of teacher intervention, and varying the pace and sequence of learning will allow students to interact at their own level. (Specification, p. 14).



Bloom's Taxonomy - Higher Order questioning

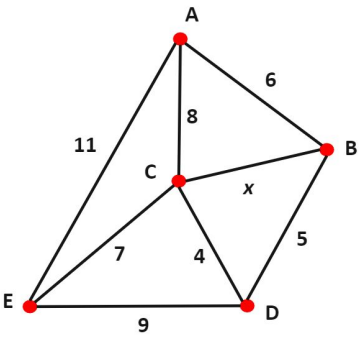
Additional questions:

Question 1

Seán's local football team are in the county final and his housing estate want to show their support by lining the roads with bunting. They want each house to have bunting but they want to use the least amount of it.

(a) The number on each edge below represents the distance, in metres, between each house. If $x = 4$, How much bunting do they need?

(b) What could the variable, x, represent in a different scenario? What might affect it?



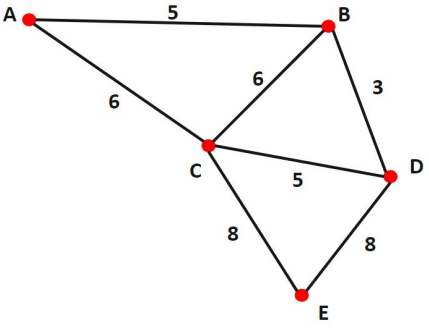
Question 2

In New York, there is a park with famous statues that has many visitors each day. Lighting is to be installed at 5 places in the park (as shown) with the places being connected either directly or indirectly by cabling following the given paths.

The values on each edge represent the distance (in m) between each light.

(a) Calculate the shortest length of cabling required and show the minimum spanning tree (MST).

(b) State two differences between Kruskal's algorithm and Prim's algorithm for finding a minimum spanning tree.



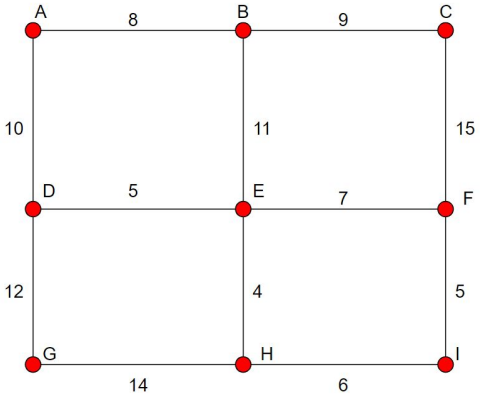
Question 3

(a) Explain the terms:

(i) Tree

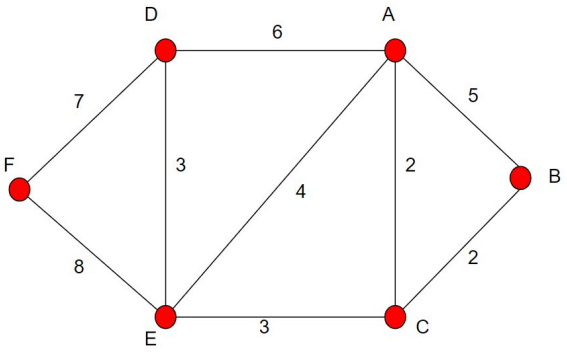
(ii) Spanning tree

(b) An electrician wants to wire sockets for a house using the least amount of wiring ensuring that they are all connected to each other. Draw the minimum spanning tree of the following graph and determine the total length of wiring needed. All distances are in metres.



Question 4

Pauline is a gardener and has created a sprinkler system in the given diagram. Using Prim's algorithm, determine the network that will connect all of the sprinklers with the least amount of piping and determine the total length of piping needed. Each vertex represents a sprinkler and the weight of each edge represents the distance in *m*.

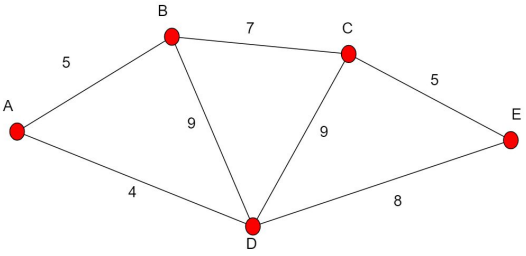


Question 5

The management of a Kerry campsite wants to connect each mobile home with running water in the easiest way possible. Each mobile home is represented by a letter and the weight on each edge represents the distance between the mobile homes in metres.

(i) Determine the Minimum Spanning Tree so that every mobile home is connected to running water using the least length of piping.

(ii) Calculate the total length of pipe needed.



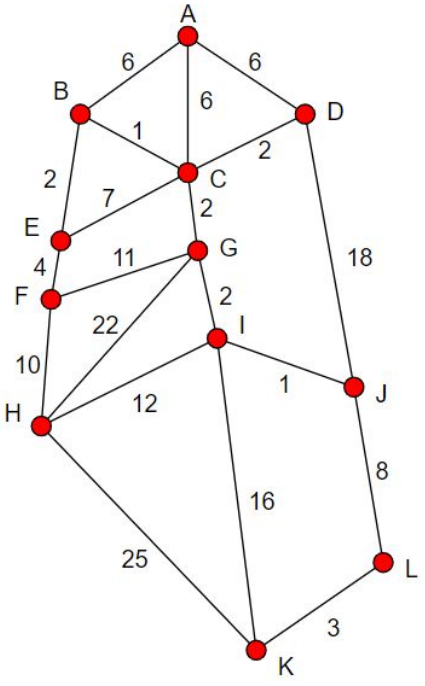
Question 6

Draw a network in which:

1) The three shortest edges form part of the minimum spanning tree.
2) Not all of the three shortest edges form part of the minimum spanning tree.

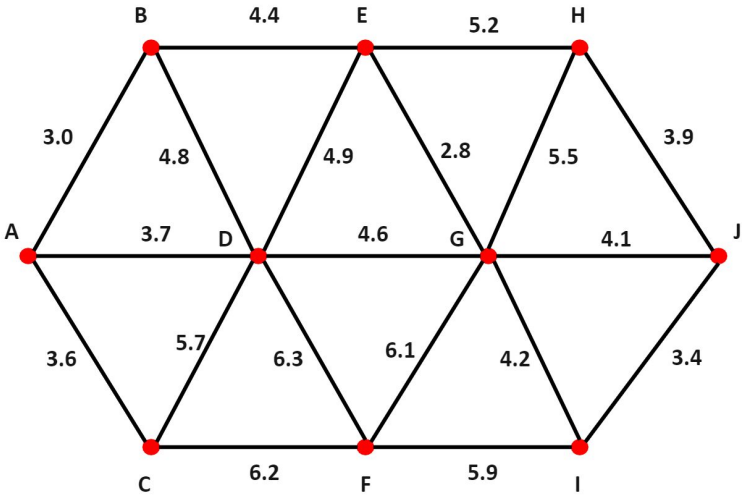
Question 7

A new broadband company wishes to lay new fibre optic cables in County Cork so that every house has access to broadband. It costs €350 per km to lay the cable. Calculate the total cost of the project. Each weight on the diagram represents distance in *km*.



Question 8

The network below shows the distances in metres between sprinklers on the ceiling of a room.



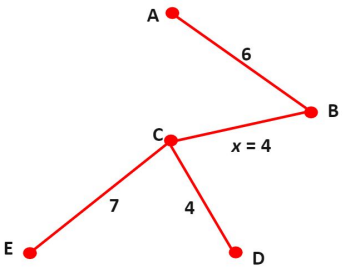
(a) (i) Use Kruskal's algorithm to find the minimum spanning tree for the network, showing the order in which you selected the edges.
(ii) Find the length of your minimum spanning tree.

(b) The same minimum spanning tree can be found using Prim's algorithm when starting at different points. State the final edge that would be added to complete the minimum spanning tree if the starting point were:
(i) A
(ii) B

Additional Questions - Solutions

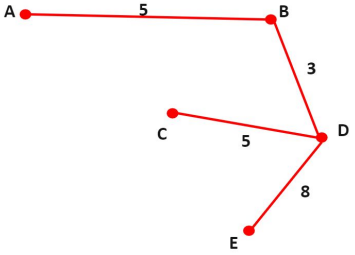
Question 1

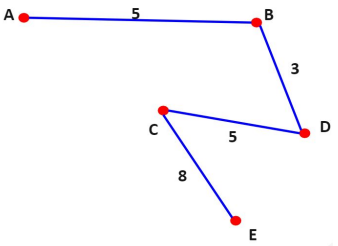
(a) Total length = 21 *m*
(b) In a different context the weight could represent cost, time, etc.



Question 2

(a) Weight = 21 *m*



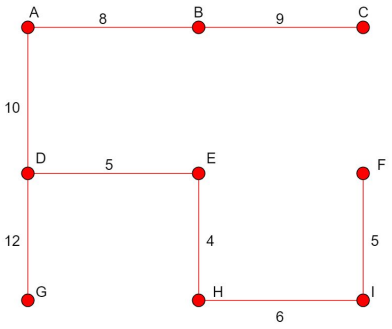


(b) Prim's starts with a node. Kruskal's starts with an edge.
Prim can start with any vertex while Kruskal must start with the edge of least weight.
The tree grows in a connected fashion when using Prim while Kruskal's can grow separately.

Question 3

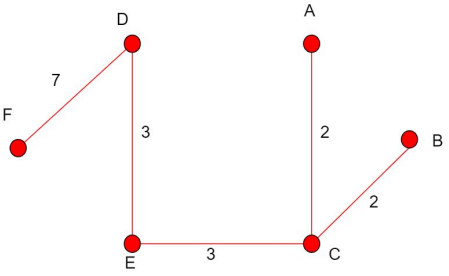
(a) (i) (ii) A connected graph with no cycles.
(iii) All nodes are connected and is also a tree.

(b) Weight = 59 *m*



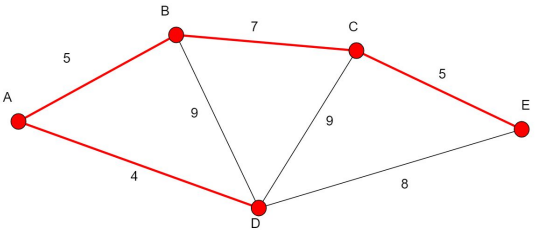
Question 4

Total distance = 17 *m*



Question 5

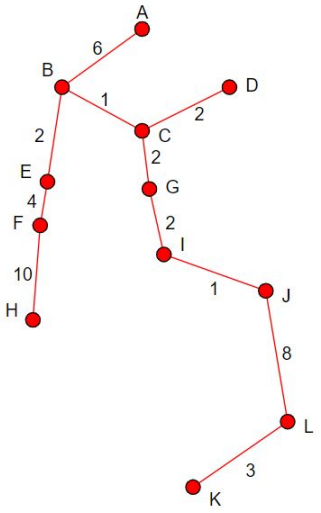
Length of pipe needed = 21 m



Question 7

Weight = 41 km, Cost = €14,350

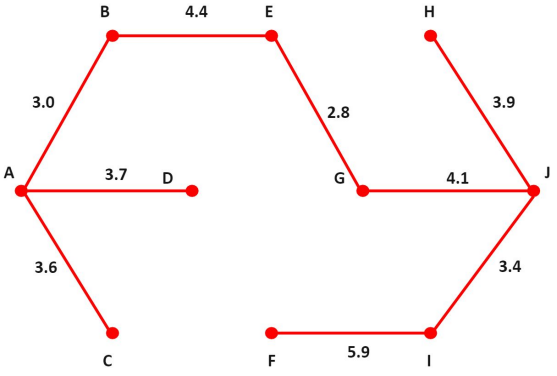
Note: This is one of many valid solutions.



Question 8

(a) (i) EG - AB - IJ - AC - AD - HJ - GJ - BE - FI
2.8 - 3 - 3.4 - 3.6 - 3.7 - 3.9 - 4.1 - 4.4 - 5.9
(ii) 34.8 m

(b) (i) FI
(ii) DA



Further Information:

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Additional Notes:

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