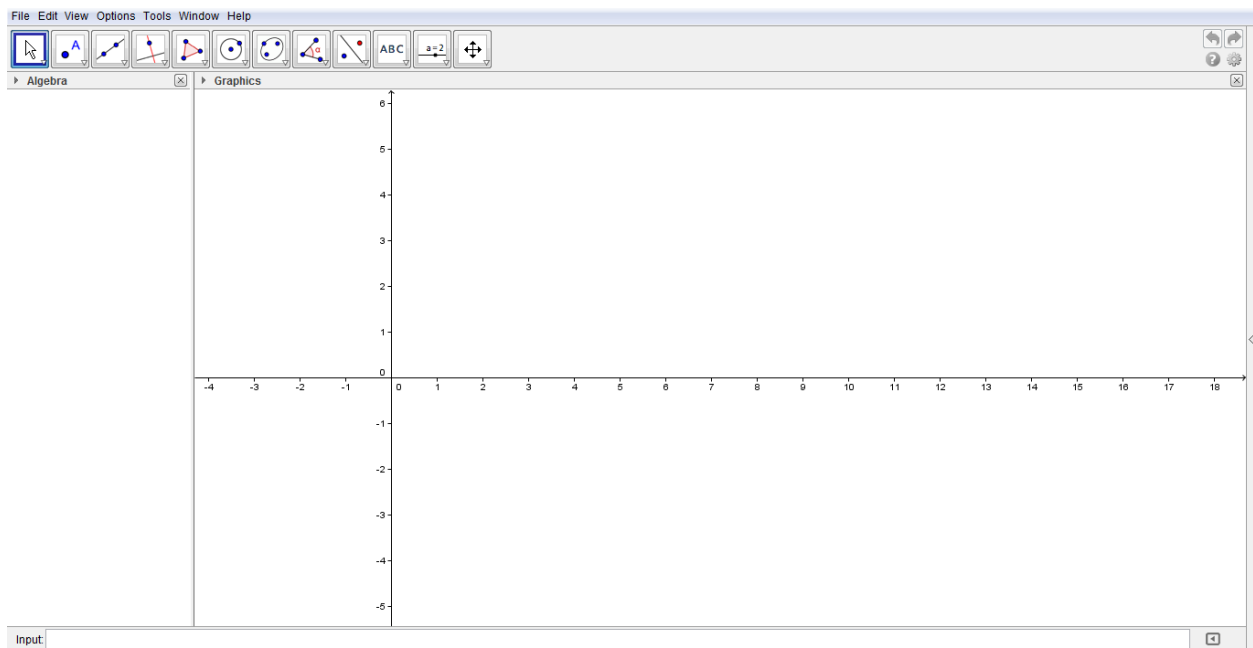


# GeoGebra Geometry



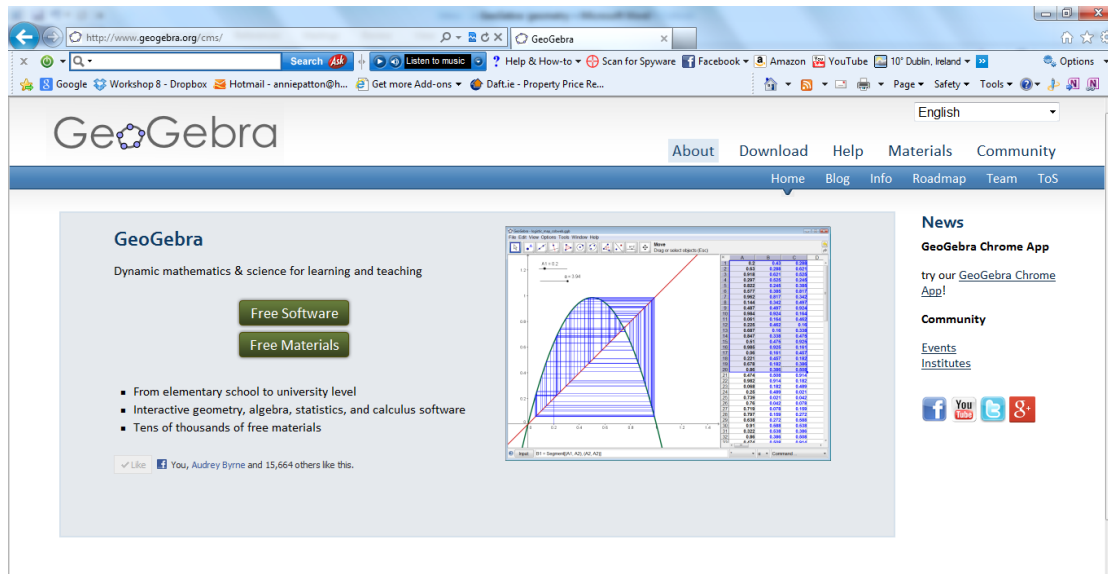
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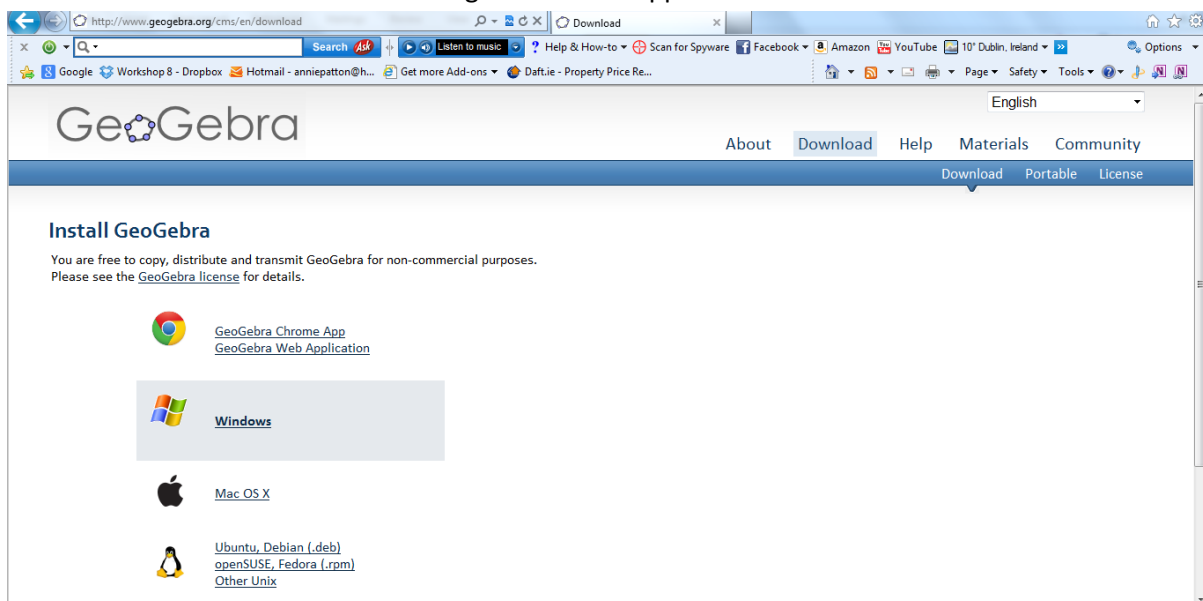
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## Activity 1: Introduction

To download GeoGebra go to [www.geogebra.org](http://www.geogebra.org) and the following window will appear.



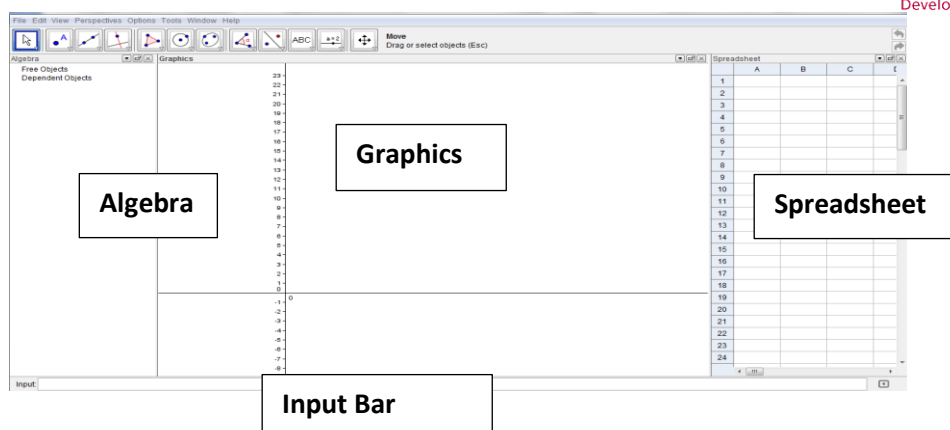
Click Free Software and the following window will appear.



Click the platform suitable for your computer and follow the indicated steps to download GeoGebra.

Note: In order to run GeoGebra it is essential that an up to date version of Java is available on your machine. This package is available at free at <http://www.java.com/en/>.

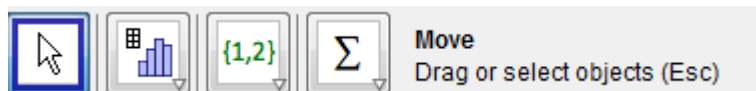
In GeoGebra there are three views, Algebra, Graphics and Spreadsheet. The bar containing File, Edit etc. is known as the Menu bar and depending if you have the cursor clicked in the Graphics or the Spreadsheet you get a different toolbar.



**Note:** When you click on the Graphics the following toolbar appears:



and when you click on the Spreadsheet the following toolbar appears:



**Note:** If the Spreadsheet is not visible go to View and click Spreadsheet and if the Graphics is not visible go to View and choose Graphics.

## Activity 2: To construct an equilateral triangle

1. Draw two points A and B using the New Point tool



2. Draw the line segment AB using the Segment between



Two Points tool

3. From A draw a circle through B using the Circle with Centre through a Point tool



4. From B draw a circle through A using the Circle with Centre through a Point tool



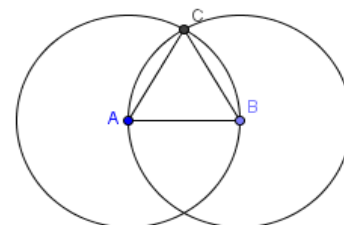
5. Find one intersection C of the two circles using the Intersect Two Objects tool



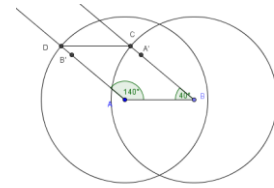
6. Draw the line segments AC and BC using the Segment between Two Points tool





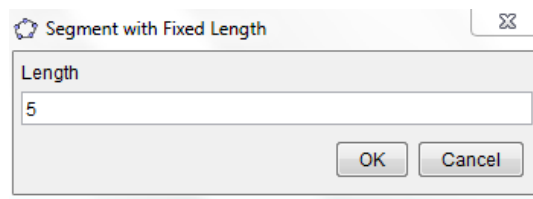
7. Hide the circles by right clicking them and deselecting Show Object.




**Activity 3: To construct a parallelogram ABCD, where the length of AB = 5 cm, the length of BC=6 cm and the angle ABC is 40°**





1. Create the point A using the New Point tool .
2. Select the Segment with Fixed Length from Point tool . Click on A. In the new dialogue box that appears type in the value 5 and click OK.





3. If the labels have not appeared on the points A and B, right click on these points and select Show label.


4. Select the Angle with Given Size tool . Click on the point A followed by the point B. In the new dialogue box that appears, replace the 45° with 40° and check that clockwise is selected. Click OK.


5. Select the Circle with Centre and Radius tool , click on the point B and in the new dialogue box insert 6 for the radius. Click OK.


6. Select the Ray through Two Points tool  and click on B followed by A'.
7. Find the point of intersection C of the ray and the circle using the Intersect Two

Objects tool .

8. Select the Angle with a Given Size tool , click on the point B followed by the point A. In the new dialogue box that appears, insert 140° and check that anticlockwise is selected. Click OK.


9. Select the Circle with Centre and Radius tool , click the point A and in the new dialogue box insert 6 for the radius. Click OK.

10. Select the Ray through Two Points tool  and click on A followed by B'.
11. Find the point of intersection D of the ray and the circle formed in No 9 and No 10.


12. Create the line segment DC using the Segment between Two Points tool .

13. Hide unnecessary objects by right clicking them and deselecting Show Object. Draw line segments as required to give the parallelogram ABCD.

## Activity 4: To construct an isosceles triangle

1. Select the  tool (Circle with Centre through a Point) and construct a circle centre A through point B. If the labels are not showing, right click, select Properties and with the Basic tab open, click on Show Label.


2. Select the Point on Object tool  and construct any point C on the circumference of circle c.

3. Select the Segment between Two Points  tool and construct [AC]
4. Construct [BC] and [AB].

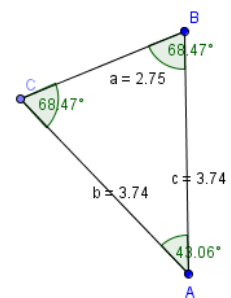
5. Right click on one side of the triangle, select *Properties*, and with the Basic tab open, click on the drop down arrow beside the Show Label box. Select Name and Value to show the name and length of this side of the triangle. Repeat for the other triangle sides.



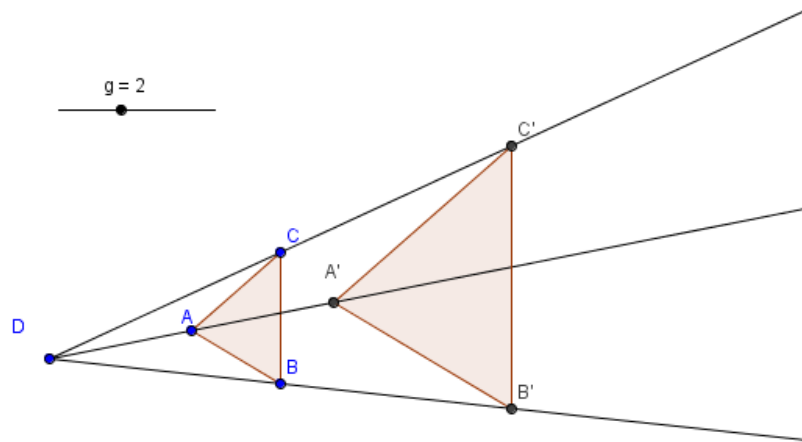
6. Drag each vertex of triangle ABC and note the length of its sides.
7. Hide the circle, by right clicking on it and clicking on *Show Object*.
8. Measure the 3 angles in the triangle using the Angle


tool  .


9. Drag any of the vertices of the triangle ABC and observe how the angle measures change.





## Activity 5: To demonstrate the enlargement of a triangle by the ray method, where the scale factor $k$ is greater than 0




1. Create the polygon ABC in the shape of a small triangle using the Polygon tool .  
**Note:** If A is for example the first point of the polygon, then A must be the last point clicked again at the end.

2. Create the point D using the New Point tool , to the left of A.

3. Using the Ray through Two Points tool , draw rays through DA, DB and DC.
4. Create a slider called  $g$  with min value 0, max 5 and increment 0.1.

**Note:** To create a slider: Go to the Slider tool  and click on the screen, where you want your slider located. Choose if you want it to be a Number, Angle or Integer. Give it a name. Choose the Min, Max values and the Increment you want it to go up in. Click Apply.





5. Set the slider to 2.

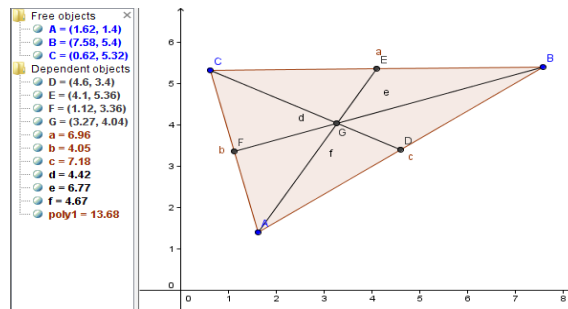
6. Select the Enlarge Object from Point by Factor tool . Click on the polygon that forms the triangle. Click on point D and in the new dialogue box that appears type in the name of the slider ( $g$  in this case) and click OK.
7. Move the original polygon to enable the enlargement to be demonstrated clearly.







## Activity 6: Constructing medians and constructing the centroid of a triangle


(A median is a line segment connecting any vertex of a triangle to the midpoint of the opposite side)

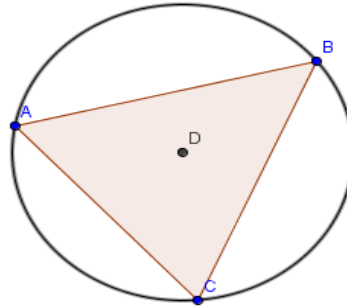
1. Click on File and select New Window.
2. Draw a triangle using Polygon tool  as above.
3. Using the Midpoint or Centre tool  and the Line Segment tool , construct the three medians of the triangle.
4. Construct the intersection of the medians by selecting the Intersect Two Objects tool .
5. Drag any of the vertices of the triangle and note that the 3 medians remain concurrent, at the centroid.



## Activity 7: Constructing perpendicular bisectors (mediators) and constructing the circumcentre and circumcircle of a triangle

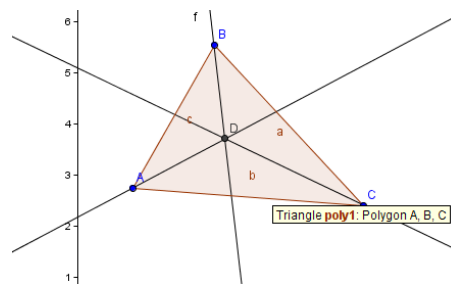
1. Click on *File, New Window*, and draw a triangle using the Polygon tool  as above.
2. Select Midpoint or Centre tool  and selecting each side of the triangle in turn, construct the midpoints of each side.
3. Using the Perpendicular Bisector tool, , select each side to construct perpendicular bisectors (mediators) of each side.
4. Select the Intersect Two Objects tool  and then 2 of the perpendicular bisectors to construct the circumcentre.
5. The equations of the 3 perpendicular bisectors are shown in the Algebra window.
6. Hide the perpendicular bisectors by right clicking on each one and clicking on Show Object. Drag the vertices to see the circumcentre change position.






7. Using the Circle through a Point tool , then the circumcentre D and one of the vertices of the triangle and construct the circumcircle. This passes through the 3 vertices.

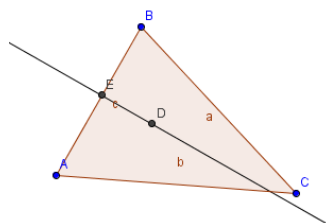



8. Drag the vertices of the triangle to confirm the construction.

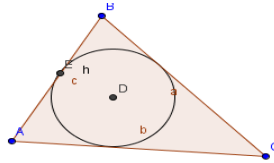
### Activity 8: Constructing the bisectors of the angles and constructing the incentre and incircle of a triangle.



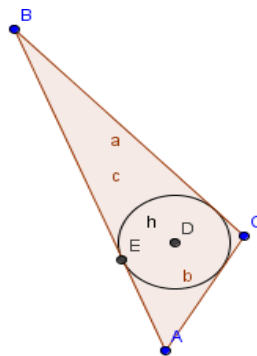
- Using the Polygon tool  construct a triangle ABC in a new window. Select , the Angle Bisector tool. Select the points B, A and C, in that order, to construct the angular bisector of  $\angle BAC$ . Repeat for the other two angles in the triangle.
- Select the Intersect Two Objects tool  and 2 of the angle bisectors to construct the incentre.
- Hide the angle bisector lines.
- Selecting the Perpendicular Line  tool, draw a perpendicular line from the incentre D, to line AB or any of the 3 sides of the triangle. With the Intersect Two Objects tool  selected construct the intersection E of side AB and this perpendicular line.






5. Hide the perpendicular line. Select , and with D as centre and E as the point on the circle, construct the incircle using the Circle with Centre through a Point tool





6. Drag the vertices to confirm the construction.






### Activity 9: To demonstrate reflection in a line

1. Create a polygon using the Polygon tool  and a line using the Line Through Two Points tool , if using a line other than the X or Y axis.
2. Go to the Reflect Object in Line tool  and click on the polygon plus the line or the X or Y axis whichever is being used.
3. Now drag the line and or the polygon to different locations and see the image change accordingly.

### Activity 10: To demonstrate reflection in a point

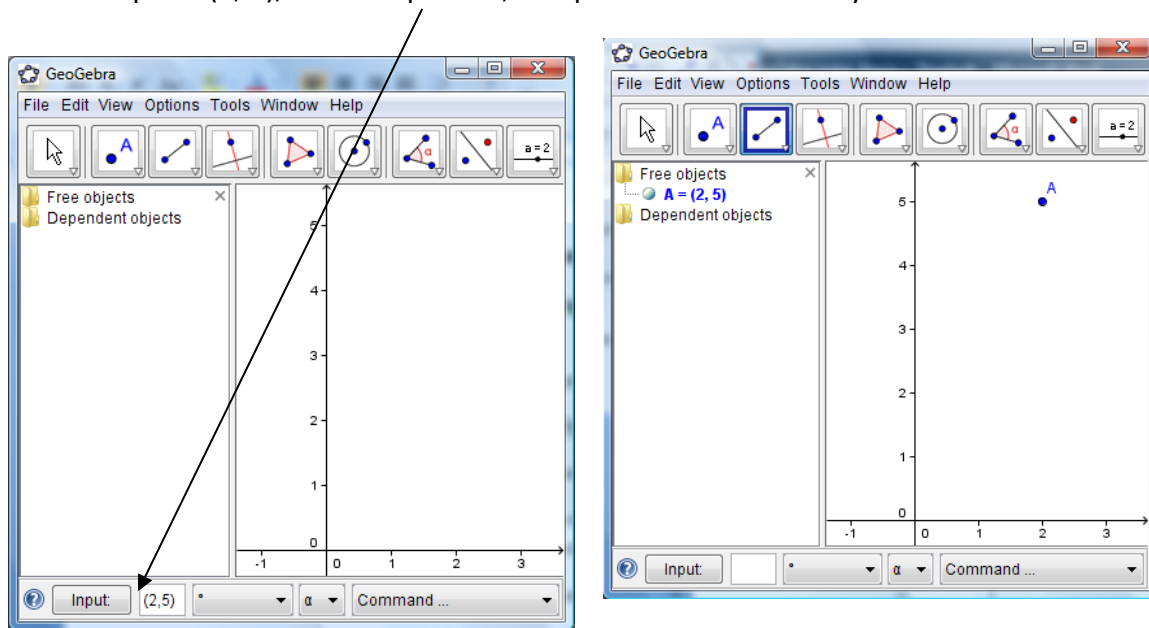
1. Create a polygon using the Polygon tool  and a point using the New Point tool. Click on the Reflect Object in point tool  and click on the polygon and the point.
2. Now drag the point and the polygon to different locations etc. and see the image change accordingly.

## Activity 11: To demonstrate how to use a vector to translate a polygon

1. Create a polygon using the Polygon tool  and create the vector that you wish to use for the translation using the Vector from Point tool .
2. Go to the Translate Object by Vector tool  and click on the polygon, followed by the vector. Both the vector and the polygon can be changed and see how the image changes accordingly.

## Activity 12: Plotting points with fixed coordinates, reading coordinates, drawing line segments and finding their lengths and midpoints

Enter the point (2, 5), on the Input Bar, and press Enter on the keyboard.




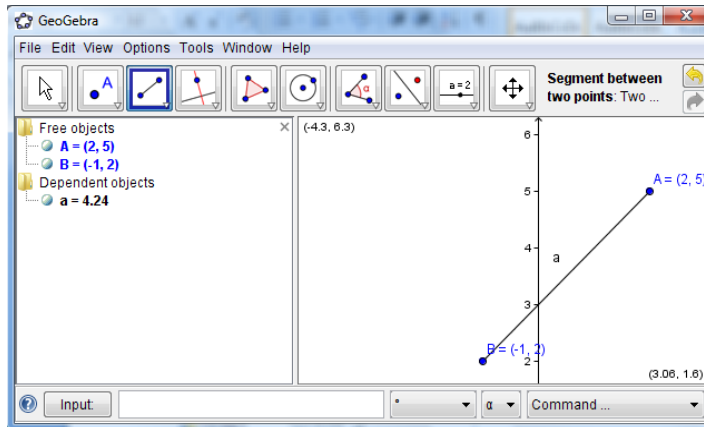
The coordinates of points are displayed in the Algebra window, once Enter is pressed.

Plot the point (-1, 2), displaying its coordinates on the Graphics.

To display the coordinates on the Graphics: Right click on the point, select Object Properties, and with the Basic tab open, and the box "Show Label" ticked, choose "Name and Value" from the drop down arrow beside it.




Select the Segment between Two Points tool  and then select points A and B to plot segment [AB].




The length of line segment [AB] i.e.  $|AB|$ , is displayed in the Algebra window as **a=**

(To display the length of a line segment, right click on it, choose Object Properties and with the Basic tab open, and the box “Show Label” ticked, choose “Name and Value” from the drop down arrow beside it.)



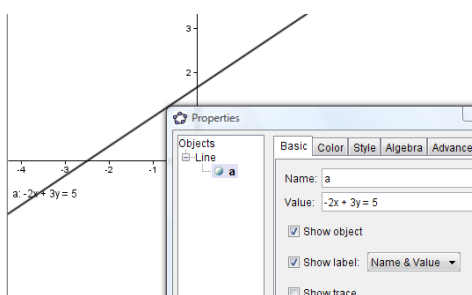
Select the Midpoint tool, . Then select the line segment [AB] or points A and B to get the midpoint, which is plotted and the coordinates are displayed in the Algebra window.

### Activity 13: Using the New Point tool to plot points with variable co-ordinates

With the New Point tool  selected and clicking anywhere on the Graphics View, points may be plotted whose coordinates are displayed automatically in the Algebra window. Dragging these points changes the co-ordinates.



### Activity 14: Plotting a line

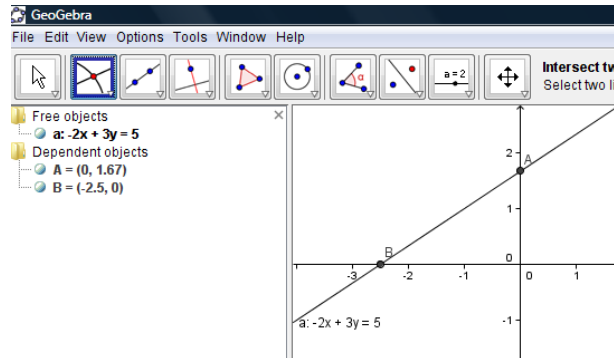
In the Input Bar type in  $2x-3y+5=0$ . In the Algebra window it will be written as  $-2x+3y=5$ .



Right click on the line and select Object Properties, and choose Name and Value, to have the equation appear in the Graphics (View).



## Activity 15: To find the points of intersection of a line with the axes

1. Draw a line  $a$  using the Line through Points tool .
2. Using the Intersect Two Objects tool , select the line  $a$  and the  $y$  axis. The point of intersection  $A$  is marked on the line and its co-ordinates are displayed in the Algebra window.

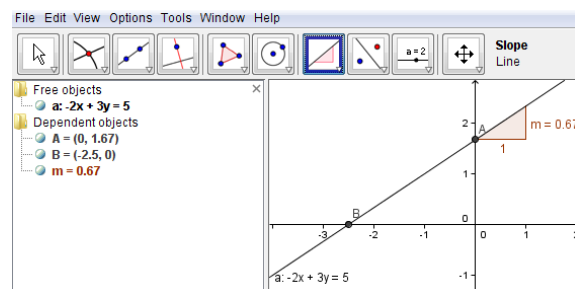


3. Repeat this for the point of intersection with the  $x$  axis.

## Activity 16: To find the slope of a line


1. Draw a line using the Line between Two Points tool .
2. Select the Slope tool  and click on the line, to find its slope.

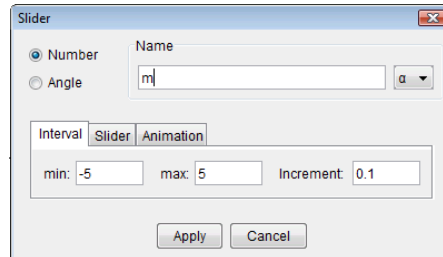
The value of the slope is displayed in the Algebra window.




## Activity 17: Using sliders to see the effect of varying $m$ and $c$ in the equation of the line $y=mx+c$


1. Go to File and select New Window.

2. Go to the Slider tool  and click on the screen, where you want your slider located.



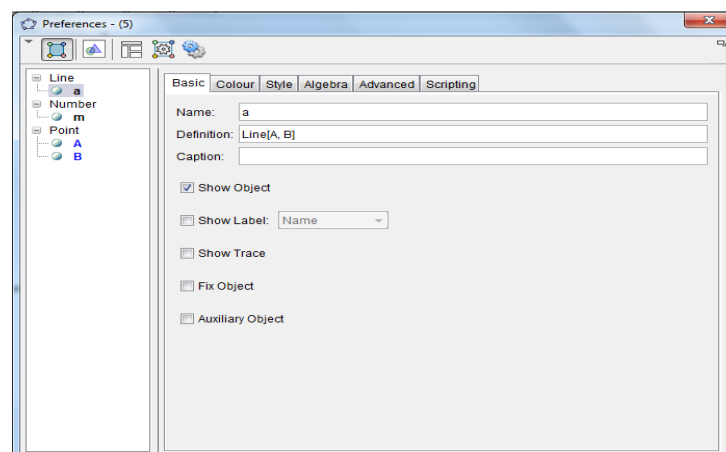
3. Choose Number rather than Angle. Give the slider a name  $m$ . Choose the min, max values and the increment you want it to go up in. Click Apply.

4. Go to the Slider tool  and click on the screen, where you want your slider located.
5. Choose Number rather than Angle. Give the slider a name  $c$ . Choose the min, max values and the increment you want it to go up in. Click Apply.


6. Type in  $y=mx+c$  into the Input Bar. Vary the value of  $m$  by selecting , and dragging the slider  $m$ . Observe the effect.
7. Investigate the effects of varying  $c$  by dragging  $c$  along the slider.
8. What do you conclude about  $m$  and  $c$ ?


### To change the sliders

Right click on slider  $m$ . Select Object Properties and the Slider tab to vary max and min values, the increment, and the width of the line representing the slider. Selecting the Colour tab allows you to vary the colour of the slider. The Style tab allows you to vary the thickness of the slider. Select the X at the top of the dialogue box when finished.

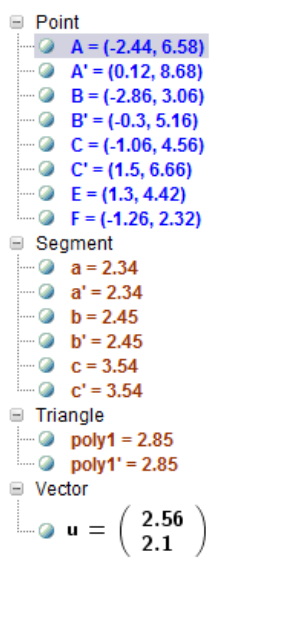


## Activity 18: Drawing triangles, finding area, finding area of the image of a triangle under a translation

Triangles may be plotted using the New point tool  or by typing in specific co-ordinates in the Input line and joining the points with line segments or else by using the Polygon tool.

Use the Polygon tool  to make a triangle ABC. The area is given in the Algebra window as poly = or alternatively you may click on Command on the Input Bar, select the word Area which will be followed by closed square brackets i.e. Area[]. Inside the brackets type A,B,C and the area is printed in the Algebra window as d= .

Using the instructions (How to use a vector to translate a polygon), translate the triangle under the defined translation. Drag the points of the original triangle and note the area of the original triangle and its image. The area of both triangles will be displayed in the Algebra window as “Poly 1 = ” and “Poly 1’ =”.



Point

- A = (-2.44, 6.58)
- A' = (0.12, 8.68)
- B = (-2.86, 3.06)
- B' = (-0.3, 5.16)
- C = (-1.06, 4.56)
- C' = (1.5, 6.66)
- E = (1.3, 4.42)
- F = (-1.26, 2.32)

Segment

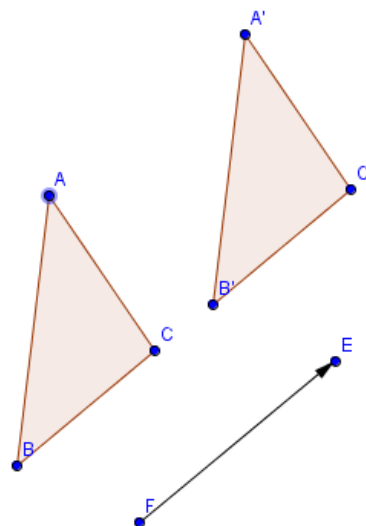
- a = 2.34
- a' = 2.34
- b = 2.45
- b' = 2.45
- c = 3.54
- c' = 3.54

Triangle

- poly1 = 2.85
- poly1' = 2.85

Vector


- $\mathbf{u} = \begin{pmatrix} 2.56 \\ 2.1 \end{pmatrix}$

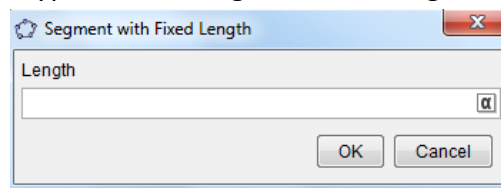




## Activity 19: To demonstrate a circle as a set of points equidistant from a given point

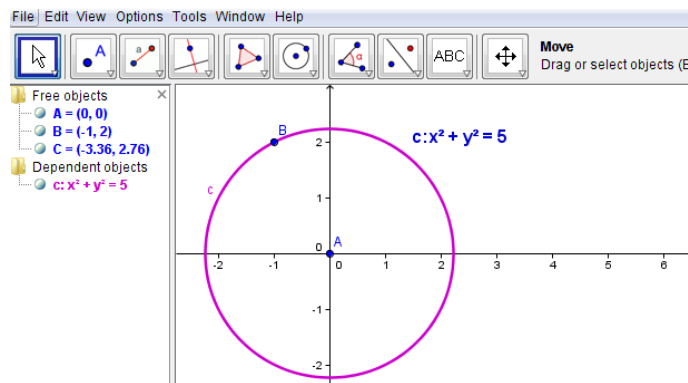
1. Plot the point (1, 2) by typing (1, 2) in the Input Bar and pressing Enter on the keyboard.
2. Display its co-ordinates on the Graphics – right click the point, select Object Properties and with the Basic tab open click the box beside Show Label and choose Name and Value.


3. Select the Segment with Fixed Length tool .
4. Click on the point A and type 4 in for length in the dialogue box which appears.



5. Click OK.
6. Point B is drawn 4 units away from A. Right click on point B and select *Properties, Show Trace*.
7. Drag B and observe.
8. *Select View* on the menu bar, then Refresh views to get rid of the traces.

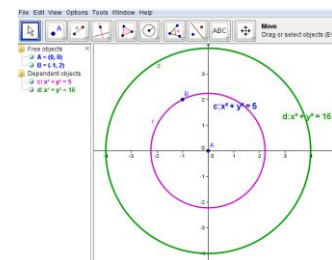
## Activity 20: Plotting a circle given the centre and one point



1. Plot points A(0,0) and B(-1,2)
2. Select  and the 2 points A and B and draw a circle

The equation of the circle is displayed in the Algebra window.

The circle is also displayed on the Graphics.




## Activity 21: Plotting a circle given a centre (0,0) and radius 4

1. In the Input Bar type (0,0) and press Enter on the keyboard.

2. Select Circle with Centre and Radius tool .

3. Type in 4 into the dialogue box which appears and click on Apply.

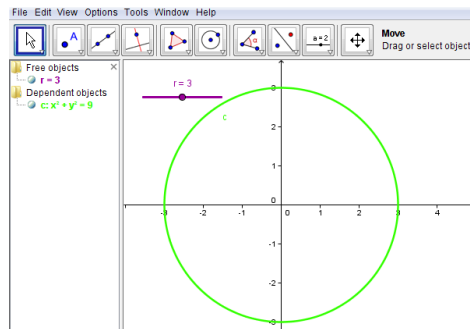
## Activity 22: Use a slider to vary r and to investigate equations of the type $x^2 + y^2 = r^2$

1. Select the slider tool  and click on the Graphics.

2. Type in r for the slider name, 0 for Min: and 6 for the Max: and 1 for the Increment:.

3. Type in  $x^2 + y^2 = r^2$  into the Input Bar and press Enter on the keyboard.

4. Drag the slider to vary the value of r.

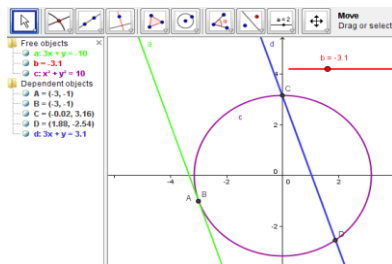


## Activity 23: Intersection of lines and circles

1. To plot the circle type  $x^2 + y^2 = 10$  in the Input Bar and press Enter on the keyboard.

2. To plot the line type  $3x + y + 10 = 0$  in the Input Bar and press Enter on the keyboard.

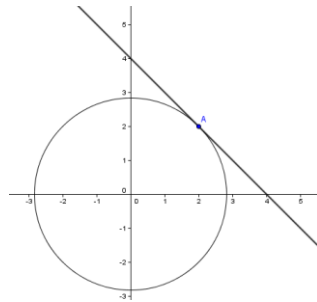
3. Using the Intersect two Objects tool  find where the circle and line meet and make a conclusion about the relationship between the line and the circle.



## Activity 24: To find the equation of the tangent to circle


1. Type  $x^2 + y^2 = 8$  into the Input Bar and press Enter on the keyboard to plot the circle, called c.
2. In the Input Bar type (2,2) and press Enter on the keyboard to create a point called A.
3. In the Input Bar type Tangent[A,c]

The tangent to the circle is then drawn and its equation is displayed in the Algebra window.



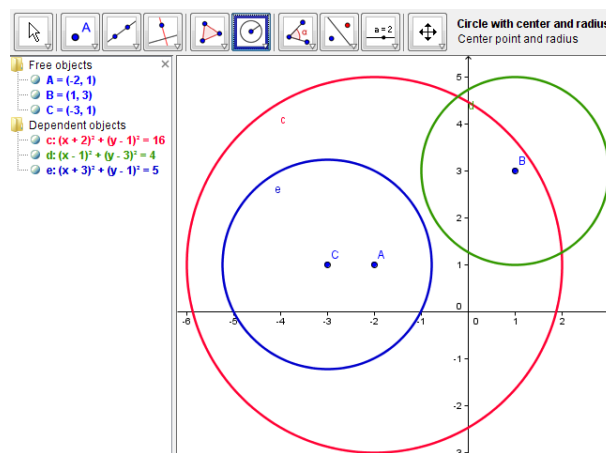
## Activity 25: Circles whose centres are not at (0,0)

To create the circle with centre at (2,1) and radius equal to 4.

1. Enter (2,1) in the Input Bar and press Enter on the keyboard.
2. Select the Circle with Centre and Radius tool .
3. Type 4 in the new dialogue box that appears and press OK.
4. Right click on the circle and choose Object Properties and with the Basic tab open click Show Label and from the drop down menu choose Name and Value.

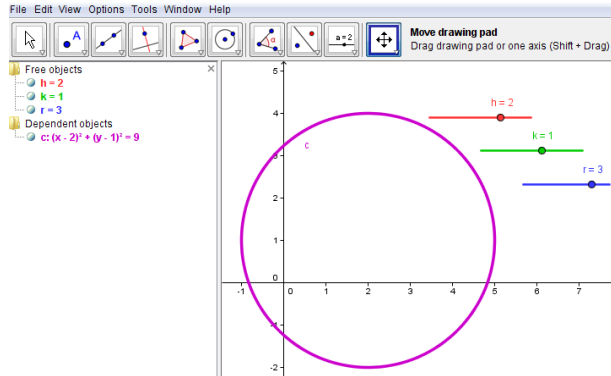
Plot the following circles and note their equations:

- (i) Centre (1,3) Radius 2
- (ii) Centre (5,-1) Radius 3
- (iii) Centre (-3,1) Radius  $\sqrt{5}$  (typed in as sqrt(5))



**To draw a circle with centre (h,k) and radius r, where h, k and r can vary?**

Set up sliders for h, k, and r and investigate the equations of circles with different values of h, k and r, typing in  $(x-h)^2+(y-k)^2 = r^2$  in the input Bar and pressing Enter on the keyboard.



What is the effect of varying h?

What is the effect of varying k?


To display the circle equation in the form  $x^2 + y^2 + 2gx + 2fy = -c$ , right click on the circle equation in the Algebra window and click on the option

$$ax^2 + bxy + cy^2 + dx + ey = f$$

**Activity 26: Transformations of the circle**


1. Plot the circle  $x^2 + y^2 + 4x - 6y + 4 = 0$ .
2. The equation will automatically be displayed in the Algebra window in the form  $(x-h)^2 + (y-k)^2 = r^2$ . By right clicking on the equation it is possible to toggle back and forth between the two forms of the equation.

**Transformations of the circle using Axial Symmetry**

Select the Reflect Object in a Line tool  and find the image of the circle under axial symmetry in the x – axis. Compare the equations of the circle and its image.


**Transformations of the circle using Central Symmetry**


Plot the point (0,0).

Select the Reflect Object in a Line  and find the image of the circle by central symmetry in the origin. Compare the equations of the circle and its image.

## Transformations of the circle using a Translation

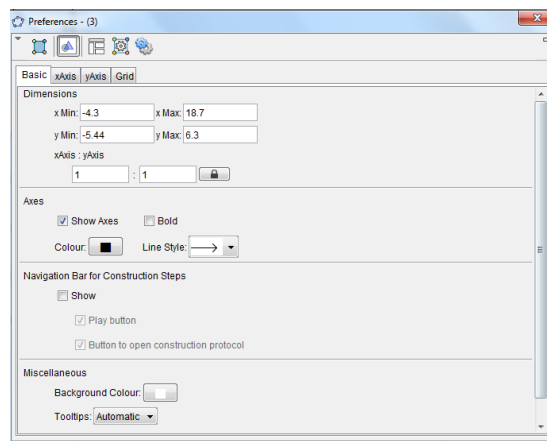
Plot the points  $(-2,1)$  and  $(5,3)$ .

Select the Vector between Two Points tool , and select each of the points  $(-2,1)$  and  $(5,3)$  to define the translation  $\overrightarrow{(-2,1) \text{ to } (5,3)}$ .

Select Translate Object by Vector , followed by the circle and the vector to translate the circle. Compare the equations of the circle and its image.

**Note: When displaying circles it is important that the ratio of the xAxis: yAxis is 1:1.**

This is achieved by right clicking on the Graphics and selecting Graphics. A new dialogue box appears.

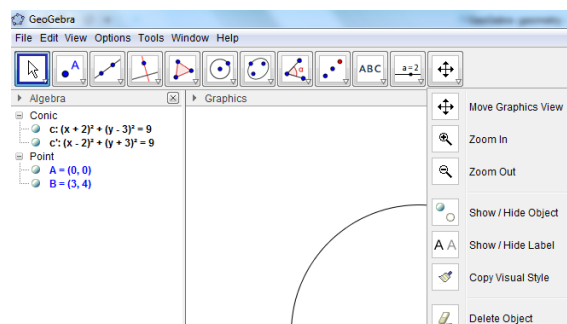


With the Basic tab open change the ratio of the xAxis to yAxis to 1:1.

The xMin, xMax, yMin and yMax values can now be changed to show the circle and its image if the Graphics window is not displaying both.

Click the X at the top of the dialogue box.

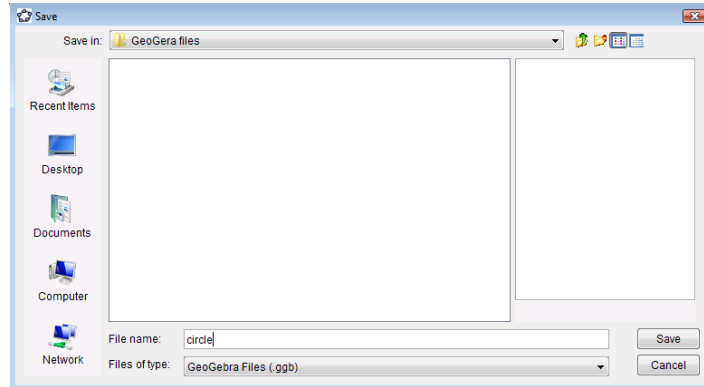
The Move Graphics View, Zoom In and Zoom Out tools can also be used.



## Appendix A

### A1: How to save a file as a GeoGebra file

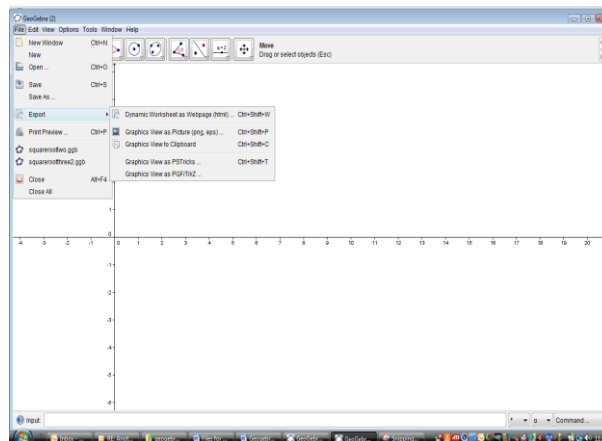
1. Go to the Menu and choose Save as from the drop down menu. A new dialogue box appears.

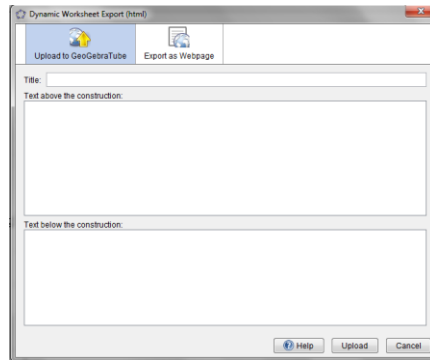


2. For Save in, navigate to the folder that you wish the file to be saved in.
3. For File name select the name you wish to give your file.
4. Click Save. A file with the .ggb extension is created. This extension identifies the file as a GeoGebra file and it can be opened again and adjustments made using the GeoGebra application.

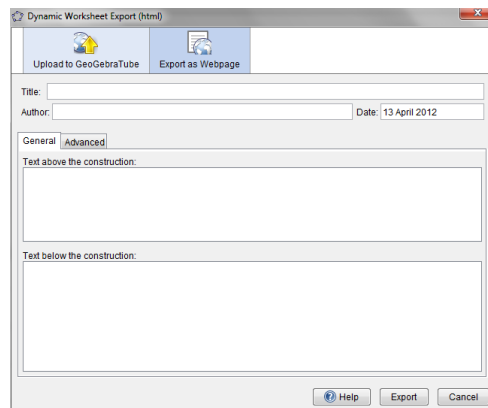
### A2: How to save a file as an interactive web page

1. First save your file as a GeoGebra file.
2. Go to File, from the drop down menu select Export and from the further menu select the option Export Dynamic Worksheet as Webpage (html). A new dialogue box appears.

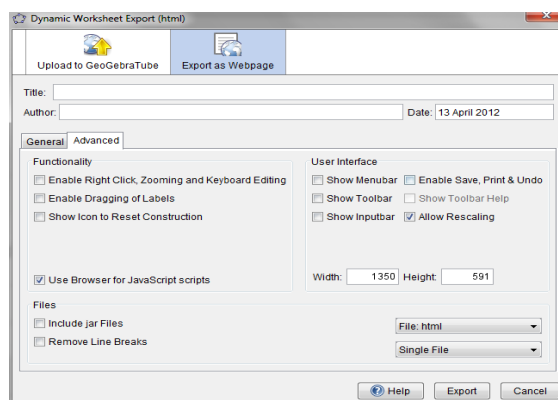




3. Click on the Export as Webpage tab.

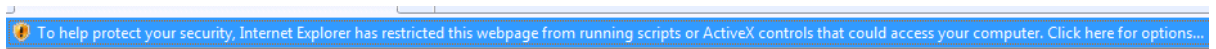


4. Add Title, Author, etc. Then click on the Advanced tab and a new dialogue box appears.



It is recommended that one clicks the Show Icon to reset construction button.


5. Click Export.
6. Navigate to the folder you want to save your file in. Give it the name of your choice.  
 Note: This file will have a .html extension and will be viewed using an Internet browser such as Internet Explorer 8.
7. Click Save and your default Internet browser will open.
8. When the webpage comes up you the following message will normally appear at the top of the screen.



9. Click on Click here for options and choose Allow Blocked Content... and click Yes.

## Appendix B

**B1: To insert an image:** Go to Insert Image tool  and click at the location on the Graphics where you want to place the image. A new box appears. Select your picture and click Open. Be careful as a lot of pictures are too big and will need to be edited to the size you require using alternative software.

**B2: Check Box to Show / Hide objects:** Click on the Check Box to Show / Hide Objects  and click on the Graphics at the location you require the check box to appear. In the new dialogue box that appears, type in your Caption for example “Click to hide or show the circle”. Follow the arrow under ‘Select objects in construction or choose from list’ in the dialogue box and in this case choose the circle. You can select more than one object here. Click Apply. With the Selection arrow clicked, click the check box and the circle disappears.

