Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2017

Mathematics

Paper 1

Ordinary Level

Friday 9 June Afternoon 2:00 – 4:30

300 marks

Examination number

Centre stamp

Running total

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the Formulae and Tables booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if you do not show all necessary work.

You may lose marks if you do not include appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) A new machine is bought for €30 000. Its value **depreciates** by 15% each year for five years. Find the value of the machine at the end of the five years.

(b) A sum of money was invested for two years at 3% compound interest per year. At the end of the two years it amounted to €30 000. Find the sum invested.

(c) A company invested €25 000 for three years at a fixed rate of compound interest. At the end of the three years it amounted to €26 530·20. Find the rate of interest.

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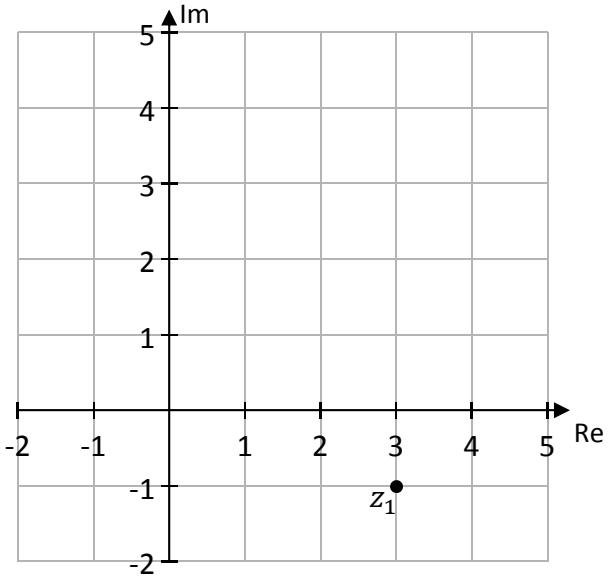
Question 2 **(25 marks)**

(a) The complex number $z_1 = a + bi$, where $i^2 = -1$, is shown on the Argand Diagram below.

(i) Write down the value of a and the value of b.

$a =$ _____ $b =$ _____

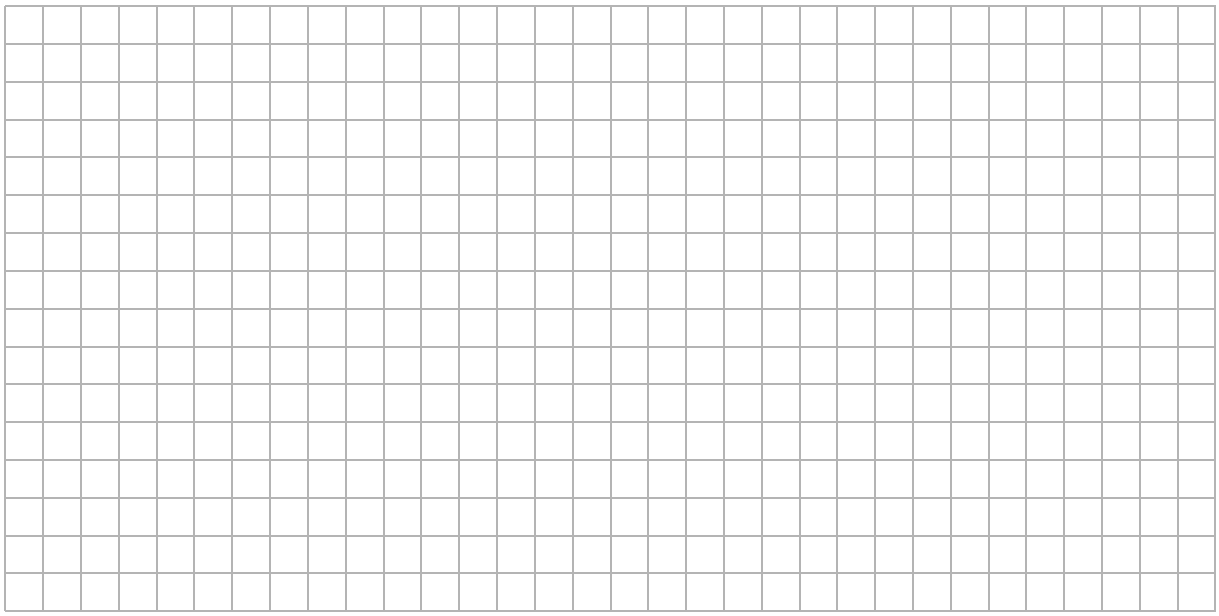


(ii) $z_2 = -1 + 2i$. Plot z_2 on the Argand Diagram.

(iii) $z_3 = \frac{z_1}{z_2}$. Write z_3 in the form $x + yi$, where $x, y \in \mathbb{R}$.

(b) Solve for z:

$$2z - 6(4 - 6i) = (-1 + i)(4 - 2i).$$



Question 3 **(25 marks)**

(a) Find the two values of x for which $3x^2 - 6x - 8 = 0$.
Give each answer correct to 1 decimal place.

(b) Find the co-ordinates of the minimum point of the function
$f(x) = 3x^2 - 6x - 8$, where $x \in \mathbb{R}$.

Question 4 **(25 marks)**

(a) Solve for x:

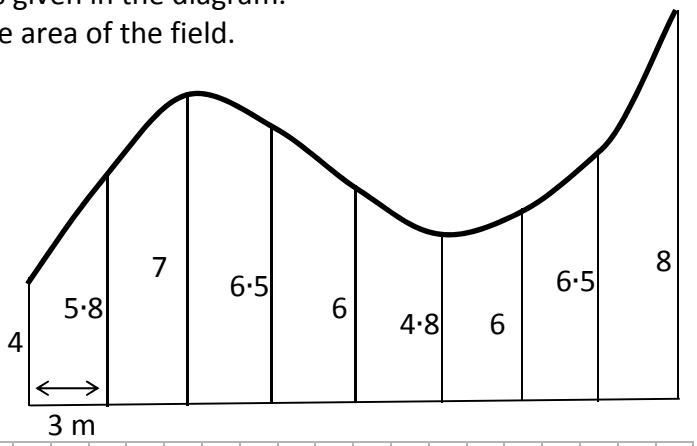
$$11x - 5(2x - 1) = 3(6 - x) + 3.$$

(b) Solve the simultaneous equations:

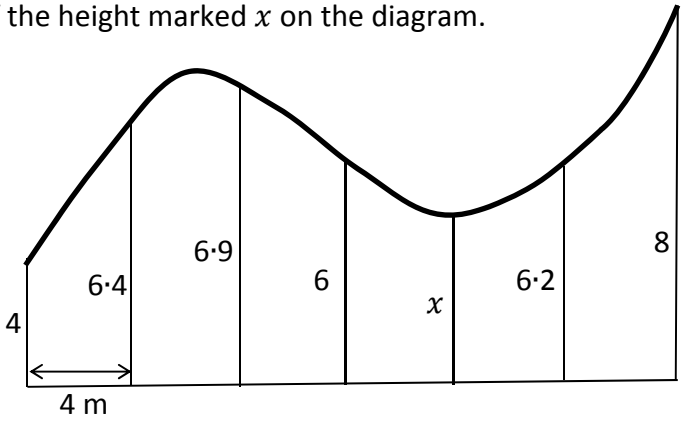
$$y + 5 = 2x$$
$$x^2 + y^2 = 25.$$

Question 5 **(25 marks)**

(a) A field is divided into eight sections as shown below. The width of each section is 3 metres. The height, in metres, of each section is given in the diagram.
Use the Trapezoidal rule to estimate the area of the field.



(b) The area of the same field was re-estimated by applying the Trapezoidal rule again.
This time, a different section width (4 m) and a different set of section heights were used, as shown below. The area was found to be 145·6 m^2.
Use this information to find the value of the height marked x on the diagram.



Question 6 **(25 marks)**

(a) A salesman earns a basic salary of €150 per week. In addition, he gets commission of 20% on sales up to the value of €1000 in the week and 30% commission on any sales above this. Find his total income for a week when his total sales amount to €3000.

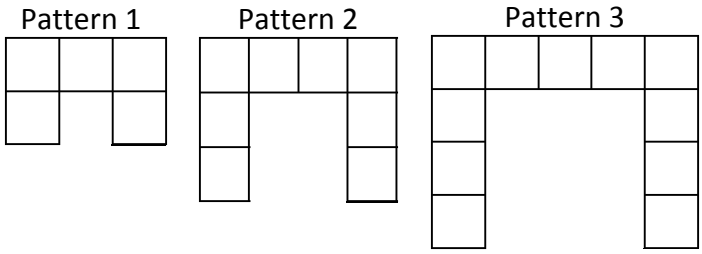
(b) On a different week his total income is €1160. Find his total sales for this week.

Section B	Contexts and Applications	150 marks

Answer **all three** questions from this section.

Question 7 **(50 marks)**

The first three patterns in a sequence of patterns of tiles are shown in the diagram below.



(a) Draw the next pattern of tiles onto the diagram above.

(b) Based on the patterns shown, complete the table below.

Pattern number (n)	Number of Tiles
1	5
2	
3	
4	
5	

(c) **(i)** Assuming the pattern continues, the number of tiles in the n^{th} pattern of the sequence is given by the formula $T_n = pn + q$, where p and $q \in \mathbb{N}$.
Find the value of p and the value of q.

(ii) How many tiles are in the 20th pattern?

(iii) Find which pattern has exactly 290 tiles.

(d) **(i)** Show that $S_n = \frac{3n^2+7n}{2}$ is a formula for the total number of tiles needed to build the first n patterns.

(ii) Find the total number of tiles needed to build the first 30 patterns.

Question 8 **(45 marks)**

A company makes and sells fibre optic cable. It can sell, at most, 200 kilometres of cable in a week. For a certain range of its production the company has found that profit can be modelled using the function:

$$P(x) = 275x - x^2 - 2000, \text{ where } x \leq 200.$$

In the function, x is the number of kilometres of fibre optic cable sold and $P(x)$ is the profit in euro.

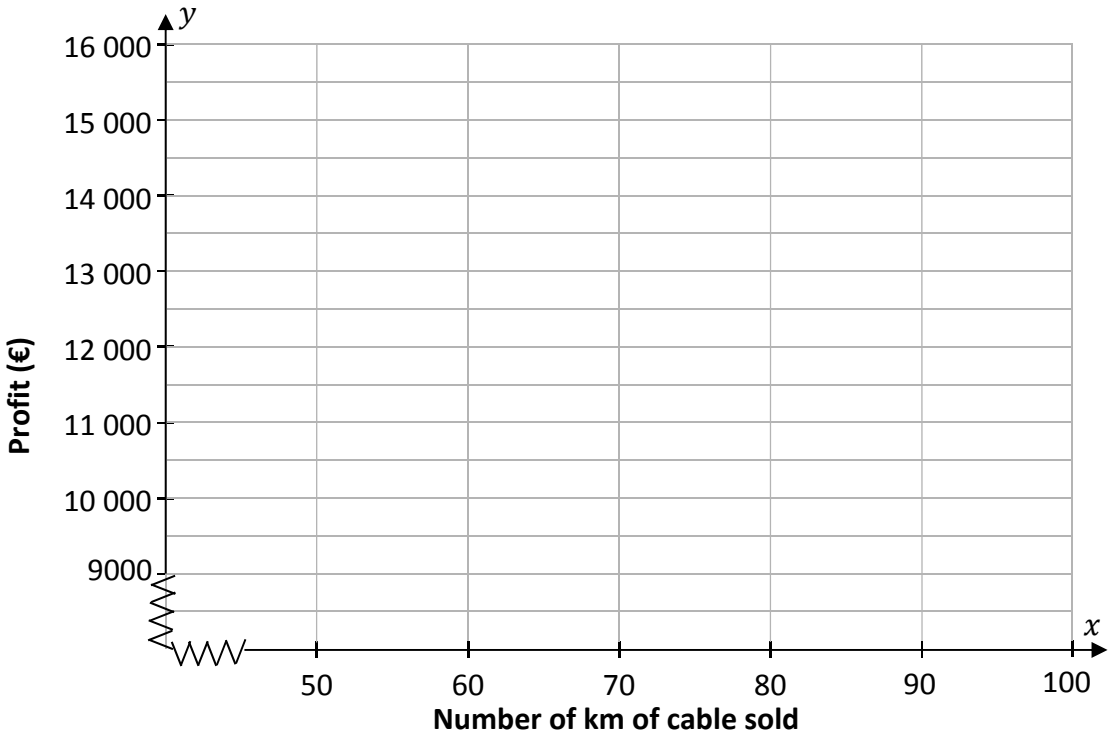
(a) Use the profit function, $P(x)$, to find how much money the company loses if it does not sell any cable.

(b) In a particular week the company made a profit of €8350.
Find the number of kilometres of cable it sold that week.

(c) **(i)** The table below shows some of the data representing the profit on sales.
Use the profit function, $P(x)$, to complete the table.

Number of km of cable sold (x)	50	60	70	80	90	100
Profit (€)			12 350			

(ii) Use the data in the table to draw the graph of the profit function on the axes below for $50 \leq x \leq 100,\ x \in \mathbb{R}$.



(iii) Use your graph to estimate the lower and upper range of sales (in km of cable) in order to make a profit of between €10 000 and €14 000 in a particular week.
Show your work on the graph above.

Lower = Upper =

(d) Use calculus to find the number of kilometres of cable sold when the profit is increasing at a rate of €105 per km.

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Question 9 **(55 marks)**

Forensic scientists can estimate the height of a person from the lengths of their bones.
One method uses a function which relates the length of the femur bone, x, to the height of the person. Using this method the heights of males and females are estimated using the following functions:

$$\textbf{Male: } m(x) = 2{\cdot}3x + 65{\cdot}53,$$

where $m(x)$ is the height and x is the length of the femur, in cm.

$$\textbf{Female: } f(x) = 2{\cdot}5x + 54{\cdot}13,$$

where $f(x)$ is the height and x is the length of the femur, in cm.

(a) Use the functions above to estimate the height of a male and the height of a female each of whose femur is 47·54 cm in length. Give both answers correct to 2 decimal places.

Male height = Female height =

(b) Use $m(x)$ to estimate the femur length of a male whose height is 184 cm.
Give your answer correct to 2 decimal places.

(c) Conor's femur length is 44·2 cm. His height is 171 cm.
Find the percentage error in using $m(x)$ to estimate his height.
Give your answer correct to 2 decimal places.

(d) Find the length of a femur for which the estimated height of a male and the estimated height of a female are the same **and** find this estimated height.

Length of femur =

Height =

This question is continued on the next page

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(e) The Ponderal Index is a number which relates a person's height to their weight.
The formula for the Ponderal Index, P, is

$$P = \frac{M}{h^3}$$

where M is weight in kg and h is height in metres.

(i) Find the Ponderal Index for a person who is 1·60 m tall and weighs 72·5 kg.
Give your answer correct to 1 decimal place.

(ii) Rearrange the formula $P = \frac{M}{h^3}$ to give a formula which will give the height of a person in terms of their weight and Ponderal index.

$h =$

(iii) Mary has a Ponderal Index of 13 and a weight of 67·5 kg. Find her height.
Give your answer in metres, correct to 2 decimal places.

You may use this page for extra work.

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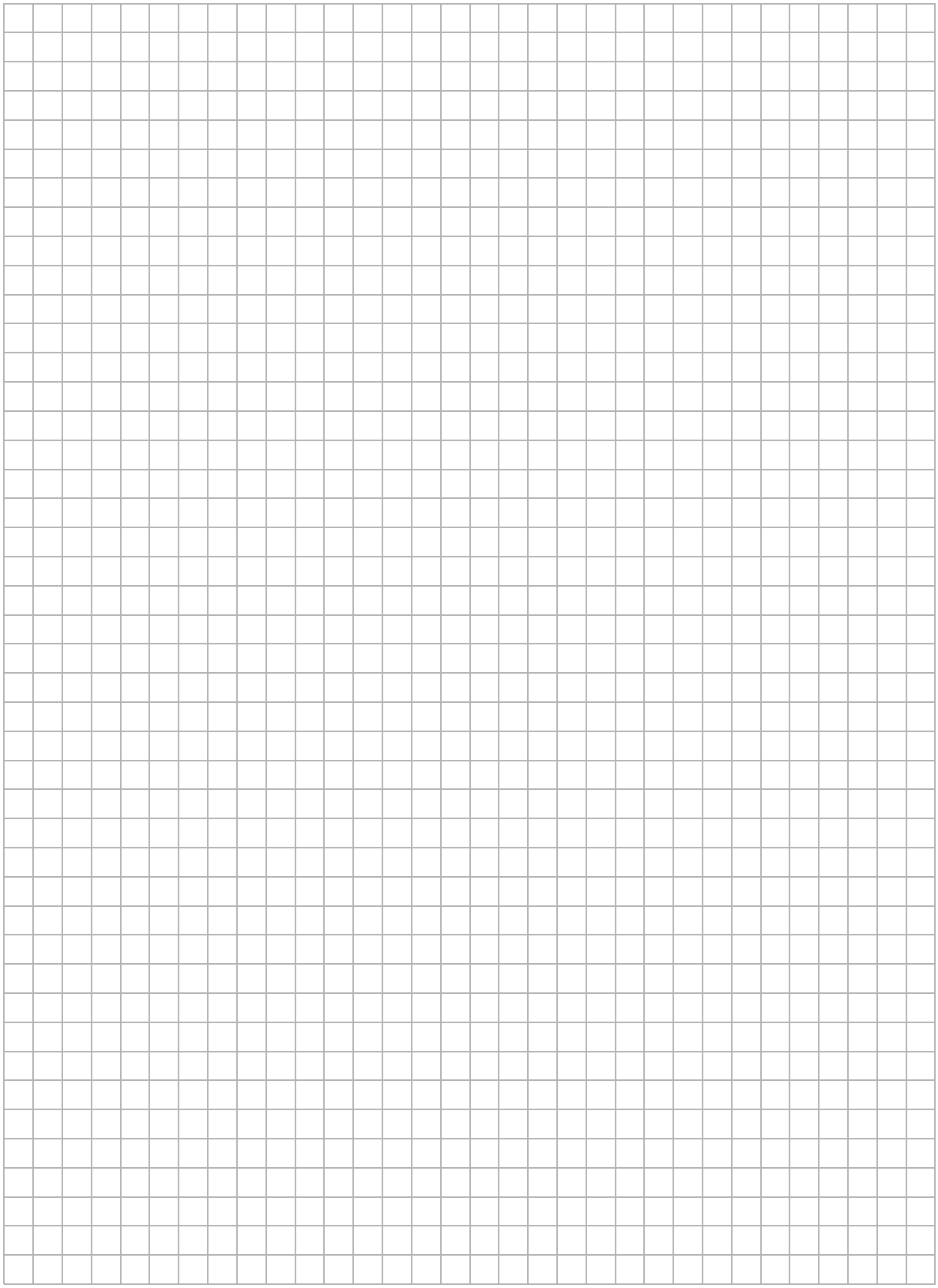

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