Algebra

Simple Equation

$3(2x-1)=4x$

$6x-3=4x$
$6x-4x=3$
$2x=3$
$x=\frac{3}{2}$

Multiply to get rid of brackets, x's to one side, numbers to the other….

Inequalities

$5x+1\leq 4x+3 \quad , x\in N$

$5x-4x\leq 3-1$

$x\leq 2$

Treat this like an equation with x's to one side and numbers to the other.
$x\in N$, $x\in Z$ means dots.
$x\in R$ means shading

Compound Inequalities

$-2\leq 5x+3<18 \quad , x\in R$

Split into two inequalities and solve as before.

$-2\leq 5x+3$	$5x+3<18$
$-2-3\leq 5x$	$5x<18-3$
$-5\leq 5x$	$5x<15$
$-1\leq x$	$x<3$

Substitution

Calculate the value of p^2-3rq
For $p=3, q=-4$ and $r=7$

Sub in values for p, q and r
$(3)^2-3(7)(-4)$
$=93$

MATHS POINTS

Multiplying Brackets (the opposite of Factorising)

$(3x+2)(4x-3)$
$=3x(4x-3)+2(4x-3)$
$=12x^2-9x+8x-6$
$=12x^2-x-6$

Manipulate Formulae (Rearranging)

This is taking a formula and changing its subject (the letter by itself) through rules of maths. (Multiplication, factorising etc)

$M=\frac{1}{S+P}$

Rearrange the formula to make P its subject.

$M(S+P)=1$

$MS+MP=1$

$MP=1-MS$

$P=\frac{1-MS}{M}$

Factorising – 4 types

Highest Common Factor

$6x^2-15xy$
$3x(2x-5y)$

Difference of Squares

$4x^2-81$
$(2x+9)(2x-9)$

Quadratics

$6x^2-5x-14$
$(3x-7)(2x+3)$

Grouping

$9a^2-6ab+12ac-8bc$
$3a(3a-2b)+4c(3a-2b)$
$(3a+4c)(3a-2b)$

Combinations

Forming a Quadratic Equation

$x^2-(sum\ of\ the\ roots)x+(product\ of\ the\ roots)=0$

Form the equation with roots $x=4$ and $x=7$

$x^2-(4+7)x+(4)(7)=0$
$x^2-11x+28=0$

Note – '**roots**' are values for x that satisfy the equation.

Solving Quadratics

$f(x)=2x^2-4x-6=0$

factorise $(2x-6)(x+1)=0$

or – b formula

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(2)(-6)}}{2(2)}$
$x=\frac{4\pm\sqrt{16+48}}{4}$
$x=\frac{4\pm\sqrt{64}}{4}$
$x=\frac{4\pm 8}{4}$
$x=\frac{4+8}{4}$ & $\frac{4-8}{4}$
$x=3$ & -1

Laws of Indices

Properties of Indices

1. $a^m . a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $(a^m)^n = a^{m+n}$
4. $(ab)^n = a^n b^n$
5. $a^{-n} = \frac{1}{a^n}$
6. $a^{\frac{1}{n}} = \sqrt[n]{a}$
7. $a^0 = 1$

Indices Example

$$\frac{8^x}{\sqrt{2}} = 4^{2x+1}$$

$$\frac{(2^3)^x}{2^{\frac{1}{2}}} = (2^2)^{2x+1}$$

$$\frac{2^{3x}}{2^{\frac{1}{2}}} = 2^{4x+2}$$

$$2^{3x-\frac{1}{2}} = 2^{4x+2}$$

$$3x - \frac{1}{2} = 4x + 2$$

$$2 - \frac{1}{2} = 4x - 3x$$

$$1\frac{1}{2} = x$$

Properties of Surds

Properties of Surds:

1. $\sqrt{ab} = \sqrt{a}\sqrt{b}$
2. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
3. $\sqrt{a}\sqrt{a} = a$

Surds Example

$$(x + \sqrt{x})(x - \sqrt{x}) = 6$$

$$x(x - \sqrt{x}) + \sqrt{x}(x - \sqrt{x}) = 6$$

$$x^2 - x\sqrt{x} + x\sqrt{x} - \sqrt{x}^2 = 6$$

$$x^2 - x = 6$$

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

$x = 3$ $\quad$ $x = -2$ not a solution

Verify

This means check your solutions.
From above:
$x = 3$

$$(3 + \sqrt{3})(3 - \sqrt{3}) = 6$$

$$9 + 3\sqrt{3} - 3\sqrt{3} - \sqrt{3}^2 = 6$$

$$9 - 3 = 6$$ Verified

Writing Expressions/ Solving Problems

The length of a rectangle is 5 times its width. The perimeter of the rectangle is 120m.

$width = x$ $\quad$ $length = 5x$

$$x + x + 5x + 5x = 120$$

$$x = 10$$

Express as a Single Fraction

Write as a single fraction

$$\frac{3x-2}{4} + \frac{2x}{3} - \frac{5x-1}{12}$$

Find a common denominator.

$$\frac{3(3x-2) + 4(2x) - 1(5x-1)}{12}$$

$$\frac{12x-5}{12}$$

Equations Using Fractions

$$\frac{2}{3x-4} - \frac{1}{2x+1} = \frac{1}{2}$$

Similar to above but we can drop the denominator. Remember to put any whole number over 1.

$$\frac{2(2)(2x+1) - 1(2)(3x-4) = 1(3x-4)(2x+1)}{2(3x-4)(2x+1)}$$

$$8x + 4 - 6x + 8 = 6x^2 - 8x + 3x - 4$$

$$0 = 6x^2 - 8x - 8x + 6x + 3x - 8 - 4 - 4$$

$$6x^2 - 7x - 16 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-16)}}{2(6)}$$

$$x = \frac{7 \pm \sqrt{433}}{12}$$

$$x = \frac{7 + \sqrt{433}}{12} \text{ and } x = \frac{7 - \sqrt{433}}{12}$$

$$x = 2.3 \text{ \& } -1.2$$

Simultaneous Equations – 2 unknowns (linear)

Solve $\quad 4x + 16y = 20$
$\quad\quad\quad 2x - 3y = -1$

Multiply one or both lines to make co-efficients of one of the variables the same. Cancel down and solve.

$4x + 16y = 20$
$2x - 3y = -1 \quad \times -2$

$4x + 16y = 20$
$-4x + 6y = 2$
$22y = 22$
$y = 1$

$4x + 16y = 20$
$4x + 16(1) = 20$
$4x + 16 = 20$
$4x = 4$
$x = 1$

Also occurs in co-ordinate geometry to find where lines intersect

Algebra Division

Divide $2x^3 + x^2 - 13x + 6$ by $x + 3$

$$\begin{array}{r|l} & 2x^2 - 5x + 2 \\ x + 3 & 2x^3 + x^2 - 13x + 6 \\ & 2x^3 + 6x^2 \\ \hline & -5x^2 - 13x + 6 \\ & -5x^2 - 15x \\ \hline & 2x + 6 \\ & 2x + 6 \\ \hline & 0 \end{array}$$

Writing Expressions/ Solving Problems II

x is a real number.
One new number is formed by increasing x by 1.
A second new number is formed by decreasing x by 2.
The product of the 2 new numbers is 1
Find x.

$$(x + 1)(x - 2) = 1$$

$$x(x - 2) + 1(x - 2) = 1$$

$$x^2 - 2x + x - 2 - 1 = 0$$

$$x^2 - x - 3 = 0$$

Use –b to finish to give $x = 2.303$ & $x = -1.303$