Chapter 5

Arithmetic AND terminology used in paper

(Usually Q1 Paper 1)

This revision guide covers

- Rounding **(to one decimal place, to two decimal..etc)**
- **Numbers in the standard form**
  (write as $a\ x\ 10^n$ where $1 \leq a\ < 10$)
- BIMDAS
- Converting between units **(mm → cm →m→km)**
- Speed-Distance-Time
- **The different types of numbers (N,Z,R,Q,R/Q)**
- Surds **($\sqrt{ab} = \sqrt{a}\ \sqrt{b}$)**
- Equations with Surds (Sample Q c parts)
- Indices
- Equations with Indices **(Sample Q c parts)**

Date	**How many pages I got done**	

After completing booklet; practice answering exam paper questions – Questions 1

Highlight the topics you need to go over before the L.C exam.

BIMDAS:

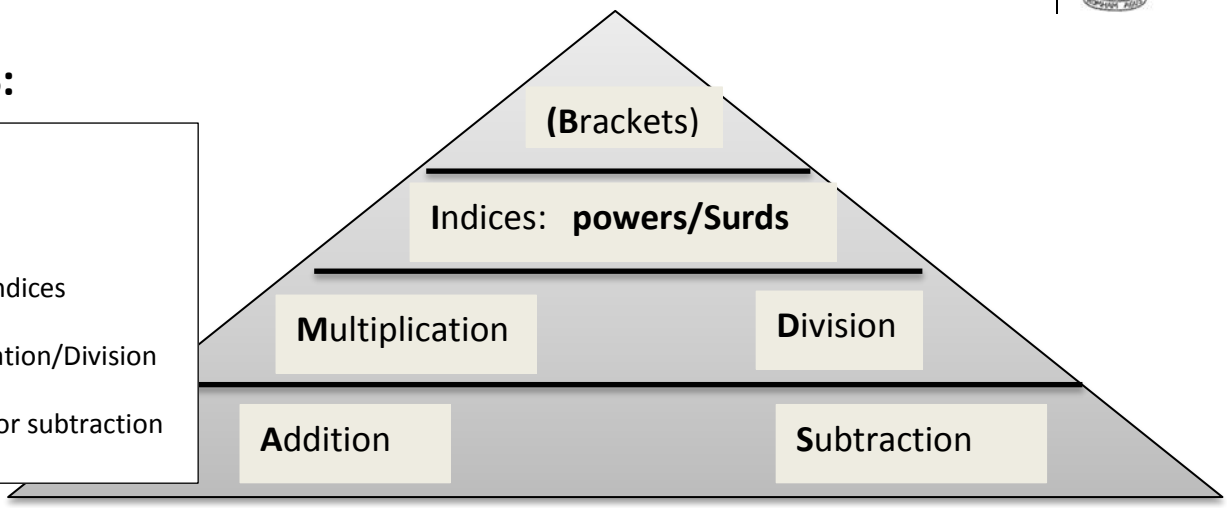
Order of calculations:

1st: Solve any brackets

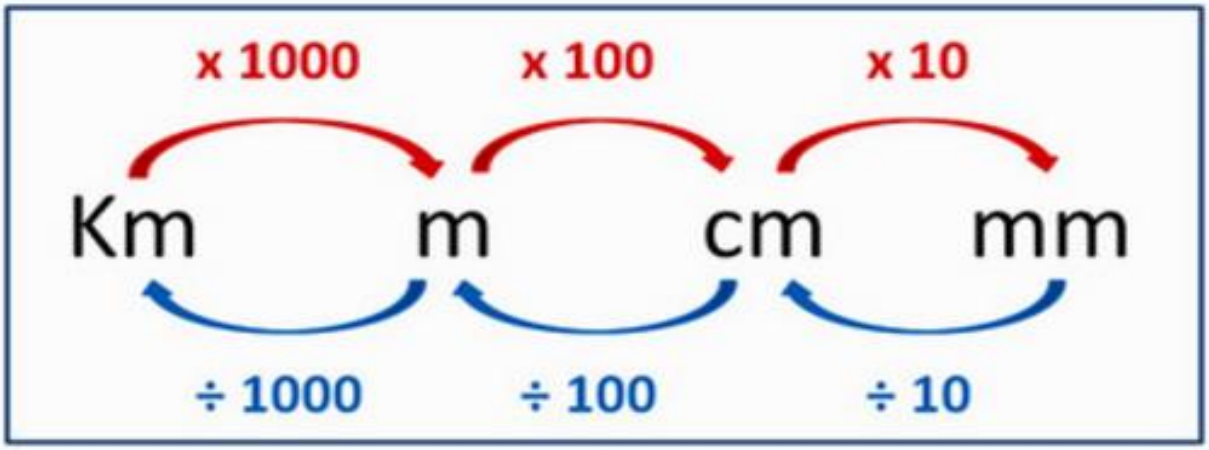
2nd: Solve any power/indices

3rd: Solve any Multipication/Division

4th: Solve the addition or subtraction



Converting between units.



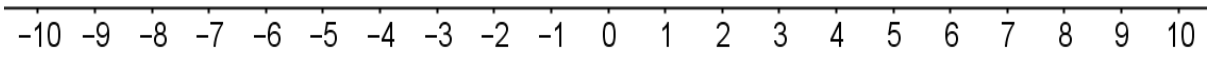
5km = ? m Need to x 1000 5 x 1000 = **5000m** ✓

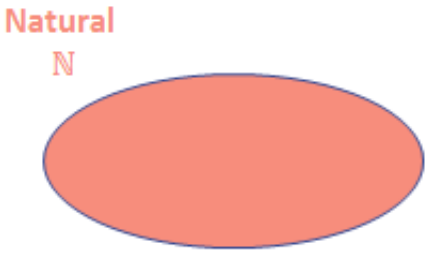
120cm = ? m Need to ÷ 100 120 ÷ 100 = **1.2m** ✓

The different types of numbers:

- **N=Natural Numbers:** Is the positive whole numbers.
  $\mathbb{N}$ = {1, 2, 3, 4, 5, ……} Note: Zero is NOT a natural number.

*Highlight the **Natural numbers** on the number line:*

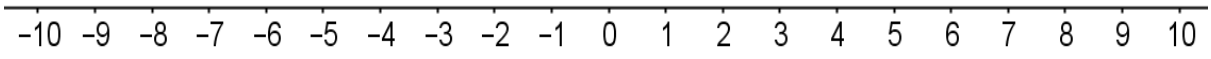


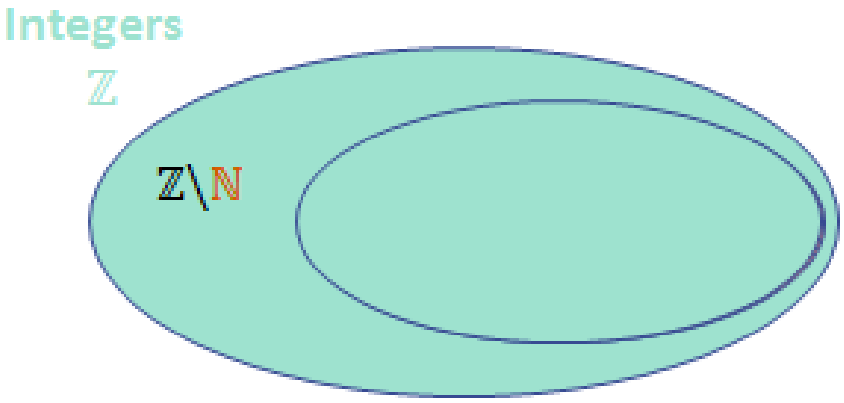


- **A Prime number:** Is a natural number that has only two factors.
- **A composite number:** Is a whole number that is not a prime number.

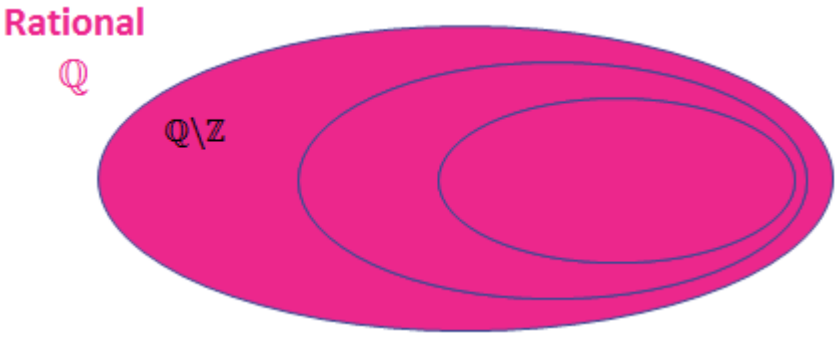
- **Z = Integers:** Is the negative whole numbers or positive whole numbers $\mathbb{Z}$ = {…..−3,−2,−1, 0, 1, 2, 3, …..} Note: Zero is an integer.

*Highlight the **Natural numbers** on the number line:*





- **Q= Rational number:** **is a number that can be written as a fraction. I.e in the form $\frac{p}{q}$, where p, q ∈ ℤ & q≠ 0.**



- **R/Q: Irrational number:** Is a number that cannot be written as a fraction/ ratio.

	Number	Decimals	Rational/ Irrational
(1)	$\sqrt{4}$		
(2)	$\sqrt{9/100}$		
(3)	$\sqrt{4/9}$		
(4)	$\sqrt{25/36}$		
(5)	$\sqrt{2}$		
(6)	$\sqrt{8}$		
(7)	$\sqrt[3]{5}$		
(8)	π		
(9)	$1-\sqrt{2}$		

(a) In the spaces provided, write down:

(i) 2 natural numbers ☐ and ☐

(ii) 2 negative integers ☐ and ☐

(iii) 2 prime numbers ☐ and ☐

Student Activity

Classify all the following numbers as **natural**, **integer**, **rational**, **irrational or real** using the table below. List all that apply.

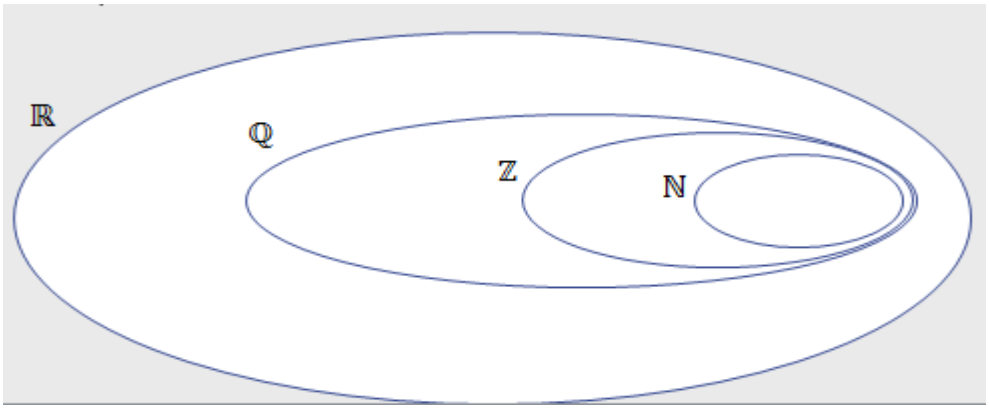
	Natural $\mathbb{N}$	Integer $\mathbb{Z}$	Rational $\mathbb{Q}$	Irrational $\mathbb{R}\setminus\mathbb{Q}$	Real $\mathbb{R}$
5					
$1+\sqrt{2}$					
$-9.6403915\ldots$					
$-\frac{1}{2}$					
$6.\dot{3}\dot{6}$					
2π					
-3					
$\sqrt[3]{8}$					
0					
$-\sqrt{3}$					

Question:

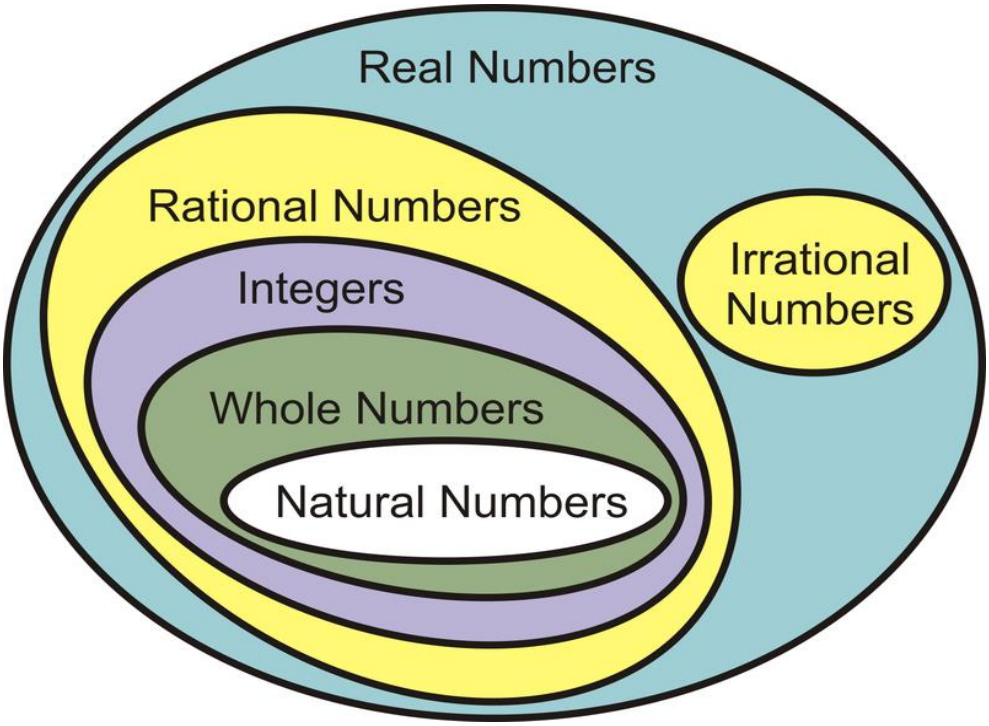
The diagram represents the sets: Natural Numbers $\mathbb{N}$, Integers $\mathbb{Z}$, Rational Numbers $\mathbb{Q}$ *and* Real Numbers $\mathbb{R}$.

Insert each of the following numbers in the correct place on the diagram:

5, 1+√2, $-9.6403915\ldots\ldots$, $-\frac{1}{2}$, $6.3\dot{6}$, 2π, -3, $\sqrt[3]{8}$, 0 and -√3.



Note: Be able to write numbers into the following categories;



Surds:

Question 1 HELP:

SIMPLIFY A SURD:

a) $\sqrt{12}$

A surd can be written as the factor of the inside number.

Factors of 12 = 6, 2 or 4,3

So $\sqrt{12} = \sqrt{6}\sqrt{2}$ **or** $\sqrt{12} = \sqrt{4}\sqrt{3}$

$\sqrt{12} = \sqrt{3}\sqrt{2}\sqrt{2}$ $\sqrt{12} = \sqrt{2}\sqrt{2}\sqrt{3}$

$\sqrt{12} = \sqrt{3}(2)$ $\sqrt{12} = (2)\sqrt{3}$

Simplify the following:

a. $\sqrt{12}$ b. $\sqrt{20}$ c. $\sqrt{18}$ d. $\sqrt{27}$

e. $\sqrt{8}$ f. $\sqrt{24}$ g. $\sqrt{28}$ h. $\sqrt{32}$

Question 2 HELP:

ADD OR SUBTRACT A SURD

A) $4\sqrt{2} + 3\sqrt{2}$

Surds can be added or subtracted once they have the SAME SURD attached to them both.

Since $\sqrt{2}$ **is common here…we can add.**

Answer = $7\sqrt{2}$

Question 2: Add the Surd

a) $8\sqrt{2}+2\sqrt{2}$ = _____$\sqrt{2}$

b) $\sqrt{5}+\sqrt{5}$ = ___$\sqrt{5}$

c) $6\sqrt{3}+2\sqrt{3}$ = _____

Subtract the Surd:

a) $5\sqrt{2}-3\sqrt{2}$ = ____$\sqrt{2}$

b) $\sqrt{5}-\sqrt{5}$ = _____

Question 3 HELP:

a) SIMPLIFY $\sqrt{8}+\sqrt{2}$

Step 1: We need to change $\sqrt{8}$ into its multiples.

Factors of 8 are 4, 2

Step 2: Sub in the re-arranged surd

$\sqrt{4}\sqrt{2}+\sqrt{2}$

Step 3: Note that $\sqrt{4}$ on the calculator = 2

$2\sqrt{2}+\sqrt{2}$

Step 4: Since they have the same number inside the surd, you can now add them.

Answer: 3 $\sqrt{2}$

Question 3: Simplify the following

a) $\sqrt{18}-\sqrt{2}$

Step 1: Rearrange $\sqrt{18}$ into its factors_____

Step 2: Sub into rearranged surd into equation _____

Step 3: Use calculator to get what $\sqrt{9}$ is and input: _____

Step 4: Subtract as same surds: _____

b) $\sqrt{125} - 5\sqrt{5}$

Step 1: Rearrange $\sqrt{125}$ into its factors _____

Step 2: Sub into rearranged surd into equation _____

Step 3: Use calculator to get what $\sqrt{25}$ is and input: _____

Step 4: Subtract as same surds: _____

c) $\sqrt{48} - \sqrt{12}$

Step 1: Rearrange $\sqrt{48}$ into its factors_____

Step 2: Rearrange $\sqrt{12}$ into its factors_____

Step 3: Sub into rearranged surd into equation _____

Step 4: Subtract as same surds: _____

Try this one on your own:

d) $\sqrt{45} - \sqrt{20}$

Question 4 HELP: MULTIPLICATION WITH SURDS

A) $2\sqrt{3}$ X $3\sqrt{5}$

Step 1: Multiply the numbers 2x3 = 6

Step 2: Multiply the numbers in the surds and keep the surd 3x5 = $\sqrt{15}$

Step 3: Put the values together $6\sqrt{15}$

Question 4: Simplify the following

a) $4\sqrt{6} \ x \ 2\sqrt{5}$

Step 1: Multiply the numbers _____

Step 2: Multiply the numbers in the surds and keep the surd _____

Step 3: Put the values together _____

b) $\sqrt{10}\times\sqrt{2}$

Step 1: Multiply the numbers in the surd and keep the surd: _____

Step 2: Simplify further by getting the factors of 20: 4,5 _____

Step 3: Use calculator to get $\sqrt{4}$ *and sub in*: _____

c) $\sqrt{5}\times\sqrt{10}$

Step 1: Multiply the numbers in the surd and keep the surd: _____

Step 2: Simplify further by getting the factors of 50: 25,5 _____

Step 3: Use calculator to get $\sqrt{25}$ *and sub in*: _____

d) $5\sqrt{5} \ x \ 7\sqrt{3}$

Step 1: Multiply the numbers _____

Step 2: Multiply the numbers in the surds and keep the surd _____

Step 3: Put the values together _____

Question to challenge you:

Show that $\sqrt{8} + \sqrt{18} = \sqrt{50}$

Hint: Get these surds have a common factor

Question 5 HELP: MULTIPLICATION WITH SURDS

Multiply out the brackets:

$$(2+\sqrt{2})(3+\sqrt{2})$$

Step 1: Rewrite to set yourself up for multiplying out the brackets:

$$\mathbf{2\,(3+\sqrt{2})\ +\ \sqrt{2}\,(3+\sqrt{2})}$$

Step 2: Multiply out the brackets.

$$\mathbf{6+2\sqrt{2}\ +3\sqrt{2}+\sqrt{2}\sqrt{2}}$$

Step 3: Note that $\sqrt{2}\,\sqrt{2}$ gives you a 2. Sub this in

$$\mathbf{6+2\sqrt{2}\ +3\sqrt{2}+2}$$

Step 4: Rearrange so that alike terms together.

$$\mathbf{6+2+2\sqrt{2}+3\sqrt{2}}$$

Step 5: Add the like terms

$$\mathbf{8+5\sqrt{2}}$$

Question 5: Multiply out the brackets.

a) $(4+\sqrt{3})(5+\sqrt{3})$

Step 1: Rewrite to set yourself up for multiplying out the brackets:

_____

Step 2: Multiply out the brackets. _____

Step 3: Note that $\sqrt{3}\,\sqrt{3}$ gives you a 3. Sub this in _____

Step 4: Rearrange so that alike terms together. _____

Step 5: Add the like terms _____

b) $(5 - \sqrt{3})(5 + \sqrt{3})$

Step 1: Rewrite to set yourself up for multiplying out the brackets:

_____

Step 2: Multiply out the brackets. Careful with the minus!

_____

Step 3: Note that $\sqrt{3}\sqrt{3}$ gives you a 3. Sub this in _____

Step 4: Rearrange so that alike terms together. _____

Step 5: Add the like terms _____

c) $(2 - 2\sqrt{5})^2$

Step 1: Note anything to the power of 2; means multiplied by itself. Rewrite without the power:

_____

Step 2: Rewrite to set yourself up for multiplying out the brackets:

_____

Step 3: Multiply out the brackets. Careful with the minus!

_____

Step 4: Note that $\sqrt{5}\sqrt{5}$ gives you a 5. Sub this in _____

Step 5: Rearrange so that alike terms together. _____

Step 6: Add the like terms _____

Challenge Question:

$$\left(\frac{1}{\sqrt{2}} + \sqrt{2}\right)\left(\frac{1}{\sqrt{2}} - \sqrt{2}\right)$$

Equations with Surds:

Solve the equation: $\sqrt{4x-3} = 3$

Step 1: To get rid of the surd; square both sides.

$$(\sqrt{4x-3})^2 = 3^2$$

Step 2: When you square a surd, you get what is inside the square: 4x – 3 = 9

Step 3: Rearrange equation, so that you have x on one side. 4x = 9 + 3

4x = 12 so x=3

a) Solve the equation: $\sqrt{4x+5} = 5$

Step 1: To get rid of the surd; square both sides. _____

Step 2: When you square a surd, you get what is inside the square: _____

Step 3: Rearrange equation, so that you have x on one side. _____

X=_____

b) Solve the equation: $-3 + \sqrt{2x-5} = 0$

Step 1: Rearrange do that the surd is on its own on one side. _____

Step 2: To get rid of the surd; square both sides. _____

Step 3: When you square a surd, you get what is inside the square: _____

Step 4: Rearrange equation, so that you have x on one side. _____

X=_____

c) Solve the equation: $x = \sqrt{4x-3}$

Step 1: To get rid of the surd; square both sides. _____

Step 2: When you square a surd, you get what is inside the square: _____

Step 3: Rearrange equation so that you have it in the form $ax^2 + bx + c$.

_____

X=_____ x= _____

CHALLENGE QUESTION:

$$2\sqrt{x-6} = \sqrt{8+x}$$

Indices: Multipication

Examples:

1. ***Solve*** $2^9 x\, 2^8$ Answer: **When multiplying** numbers with the same power, **ADD** the indices:
   Answer: 2^{9+8} = 2^{17}

a) $a^5 \times a^2$ **Step 1:** Add the indices _____

b) $2x^2 \times 3x^4$

Step 1: multiply the numbers _____

Step 2: Add the indices _____

Answer:_____

c) $\frac{1}{2}x^2 \times 8x^2$

Step 1: multiply the numbers _____

Step 2: Add the indices _____

Answer:_____

d) $x^{-2} \times 3x \times 2x$

Step 1: multiply the numbers _____

Step 2: Add the indices _____

Answer:_____

e) $(4p^3r^6) \times (3pr^2)$

Step 1: multiply the numbers _____

Step 2: Add the indices _____

Answer:_____

f) $(5c^6d^4) \times (4c^9d)$ Answer: _____

g) $(3x^2y^4) \times (2x^7y^5)$ Answer: _____

Indices: Division

Examples:

Solve the following $\frac{a^6 x\, a^4}{a^2}$

Step 1: Add the indices in numerator position

$$\frac{a^{10}}{a^2}$$

Step 2: Subtract the indices when dividing:

Answer= a^{10-2} = a^8

a) $\frac{a^6 \times a^8}{a^7}$

Step 1: Add the indices in numerator position _____

Step 2: Subtract the indices when dividing: _____Answer: _____

b) $\frac{(y+5)^6}{(y+5)^2}$

Step 1: Add the indices in numerator position _____

Step 2: Subtract the indices when dividing: _____ Answer:_____

c) $\frac{4X^2 \times 2}{2X^3}$

Step 1: Add the indices in numerator position _____

Step 2: Subtract the indices when dividing:

_____Answer:_____

Challenge Question:

Show that $\frac{a^4(a^2+a^3)}{a^6}$ can be simplified to (1+a)

Negative Indices:

Example: Write these as whole number **a)** 16^{-4}

Answer: Question wants you to get rid of the negative power.

Step 1: Get rid of negative power 16^{-4} can be written as $\frac{1}{16^4}$

Step 2: Use calculator to simplify $\frac{1}{16^4}$ = $= \frac{1}{65536}$ = 0.00001525878

Step 3: Can rewrite in form $a\ x\ 10^n$ **1.52 x 10^{-5}**

a) 5^{-2} Step 1: Get rid of negative power _____

Step 2: Use calculator to simplify _____

a) $8^{-\frac{1}{3}}$ Step 1: Get rid of negative power _____

Step 2: Use calculator to simplify _____

$c)\ 15^{-5}$ Step 1: Get rid of negative power _____

Step 2: Use calculator to simplify _____

Step 3: Can rewrite in form $a\ x\ 10^n$

Example: Write these as whole number **a)** $\frac{1}{4^{-4}}$

Step 1: Get rid of negative power $\frac{1}{4^{-4}}$ **can be written as 4^4**

Step 2: Use calculator to simplify **= 256**

d) $\frac{1}{6^{-3}}$ **Step 1:** Get rid of negative power

Step 2: Use calculator to simplify

e) $\frac{2}{3^{-2}}$ **Step 1:** Get rid of negative power

Step 2: Use calculator to simplify

Fractional Indices:

Write the following indices as surds: a) $3^{\frac{1}{2}}$ **b)** $3^{\frac{1}{3}}$ **c)** $6^{\frac{1}{4}}$ **d)** $3^{\frac{2}{3}}$

a) $3^{\frac{1}{2}} = \sqrt{3}$

b) $3^{\frac{1}{3}} = \sqrt[3]{3}$

c) $6^{\frac{1}{4}} = \sqrt[4]{6}$

d) $3^{\frac{2}{3}} = \sqrt[3]{3^2} = \sqrt[3]{9}$

Write these indices as surds:

a) $64^{\frac{1}{3}}$ b) $32^{\frac{3}{5}}$ c) $6^{\frac{1}{2}}$

Challenge Question:

1. Show that $\frac{\left(a\sqrt{a}\right)^3}{a^4}$ simplifies to $\sqrt{a}$.

2. Write 6^{-2} and $81^{\frac{1}{2}}$ without using indices.

$6^{-2} =$ $81^{\frac{1}{2}} =$

Equations with Indices:

Find the value of x: $4^x = 16$

Step 1: Write all numbers as indices with same base. $4^x=4^2$

Step 2: Since the base numbers are the same, they can be ignored. **X=2**

a) Find the value of x: $16^x = 64$

Step 1: Write all numbers as indices with same base. _____

Step 2: Since the base numbers are the same, they can be ignored. _____

b) Find the value of x: $4^{x+1} = 32$

1: Write all numbers as indices with same base. _____

Step 2: Since the base numbers are the same, they can be ignored. _____

c) Find the value of x: $4^{x-1} = 2^{x+1}$

Step 1: Write all numbers as indices with same base. _____

Step 2: Since the base numbers are the same, they can be ignored. _____

d) Solve the equation $49^x = 7^{2+x}$ and verify your answer.

Step 1: Write all numbers as indices with same base. _____

Step 2: Since the base numbers are the same, they can be ignored. _____

Step 3: Sub value of x back into question given to verify: _____

Challenge Question:

Find the value of x: $3^x = \frac{1}{27}$