

Probability



This revision guide covers

- Probability Scale
- **Probability of an outcome with one event happening.**
- **Probability of an outcome with two events happening.**
- **Fundamental principle of counting.**
- **Arrangements – Permutations.**
- Expected frequency.
- Or Rule (add): Mutually exclusive events
- **And rule/ The multiplication rule – Bernoulli trial.**
- Tree diagram
- Expected values – Law of large numbers
- Venn diagrams
- **Margin error and hypothesis test**

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○ Probability Scale

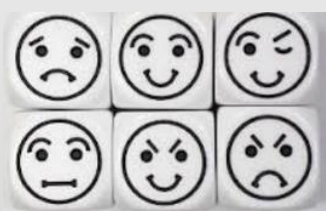
List the events using the probability scale;

certain (1), likely, even chance, unlikely, impossible (0)

Event	Probability
School will be closed tomorrow	
You will eat today.	
Your first child will be a girl.	
You will wake up a millionaire tomorrow	
You will sit your leaving cert in June	
Probability will be two questions on your maths paper.	
You will do an apprenticeship next year.	
You will go to college in Dublin	
It will be Friday in 4 days time	
If you drew a square, it would have four sides.	

The probability of any event; cannot be less than zero or more than 1.

The probability of throwing a facial expression dice and it landing on each of the following expressions is given in the table below:



Facial expression						
Probability	0.2	0.3	x	0.2	0.1	0.1

Answer: $0.2 + 0.3 + x + 0.2 + 0.1 + 0.1 = 1$

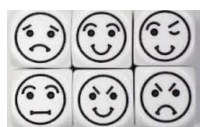
$$0.9 + x = 1$$

$$x = 1 - 0.9$$

$$x = 0.1$$



Q1. The probability of throwing a facial expression dice and it landing on each of the following expressions is given in the table below:



Facial expression						
Probability	0.03	a	0.16	0.14	0.23	0.02

Answer: _____

$$a = \underline{\hspace{2cm}}$$

Q2. The probability of throwing a facial expression dice and it landing on each of the following expressions is given in the table below:

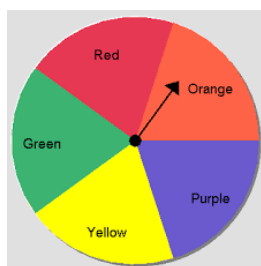


Dice number	1	2	3	4	5	6
Probability	0.1	0.15	0.2	0.05	b	0.3

Answer: _____

$$b = \underline{\hspace{2cm}}$$

Q3. The probability of the spinner landing on each colour is given in the table below:



Colour	Red	Orange	Purple	Yellow	Green
Probability	0.4	0.3	C	0.05	0.05

Answer: _____

$$C = \underline{\hspace{2cm}}$$



Question 4: If A is an event, it will either happen or not happen:

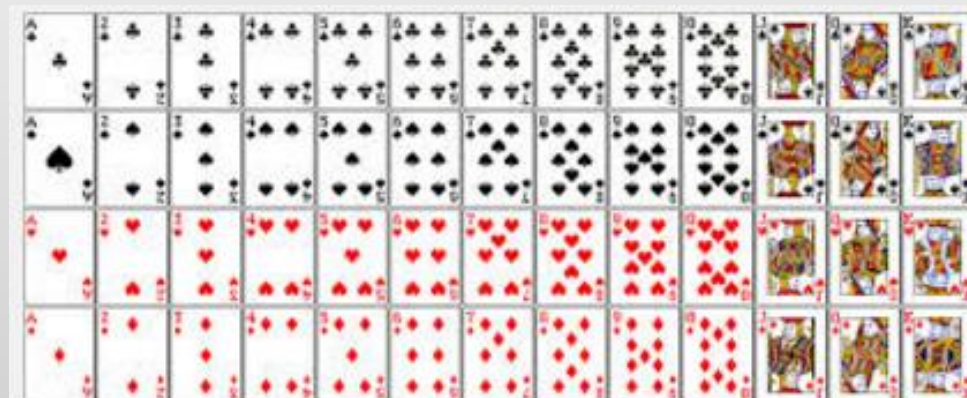
$$P(\text{A happening}) = 1 - P(\text{A not happening})$$

Event	Prob. of happening	Prob. of not happening
Getting 'tails' on coin	0.5	
Getting a Spade from pack of cards		0.75
Snow tomorrow	0.12	
Winning the lottery	0.001	
Passing maths exam		0.15
Spotting a red car	0.33	
Meeting Usain Bolt	$\frac{3}{4000}$	
Getting Homework today		$\frac{6}{7}$
Getting struck by lightning	$\frac{3}{72356}$	

○ **Probability of an outcome with one event happening.**

$$\text{Probability of event} = \frac{\text{Number of outcomes favourable to the event}}{\text{Total number of possible outcomes}}$$

Find the probability that the chosen card will be:



Note:

You are expected to know the cards in a deck of cards.

What is the probability of picking the following cards at random?

$$\text{A NUMBERED CARD ?} = \frac{36}{52}$$

$$\text{A RED jack?} = \frac{2}{52}$$

$$\text{A Queen?} = \frac{4}{52}$$



QUESTION 1: Find the probability of picking the following cards from deck of cards above:

Ace?	=	_____
A SEVEN?	=	_____
A PICTURE CARD?	=	_____
A one?	=	_____
A club?	=	_____
A heart?	=	_____
A diamond?	=	_____

Question 2: What is the probability of picking a **vowel** from the alphabet? _____

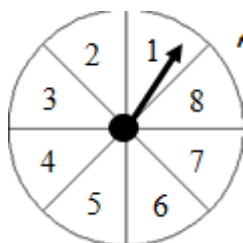
What is the probability of picking a **consonant** from the alphabet? _____

A B C D E F
G H I J K L
M N O P Q
R S T U V
W X Y Z

What is the probability of choosing the letter B at random from the word ALPHABET? _____

What is the probability of choosing the letter A at random from the word ALPHABET? _____

Question 3:



A spinner, numbered 1-8, is spun once. What is the probability of spinning...

A prime number: _____ A natural number: _____

An even number: _____ A rational number: _____

An odd number: _____ An irrational number: _____

A multiple of 3: _____ A multiple of 2: _____



○ Probability of an outcome with two events happening.

Q1. Write out the possible outcomes; if a coin and a dice is thrown:

	1	2	3	4	5	6
Head						
Tails						

- What is the number of possible outcomes? _____
- What is the probability of getting a head and an even number? _____
- What is the probability of getting a tail and an odd number? _____

Q2. Write out the possible outcomes; if two coins are thrown:

	Heads	Tails
Heads		
Tails		

- What is the number of possible outcomes? _____
- Work out the probability of getting two heads? _____
- Work out the probability of getting a head and a tail? _____

Q3. Write out the possible outcomes; if two DICES are thrown:

		2^{ND} DICE					
		1	2	3	4	5	6
<i>1st</i> <i>D</i> <i>I</i> <i>C</i> <i>E</i>	1						
	2						
	3						
	4						
	5						
	6						



Q4.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Determine the probability that the total score on the two dice is:

- (a) 5,
- (b) 7,
- (c) an even number,
- (d) a prime number,
- (e) a square number,
- (f) greater than 3 but less than 9.

Q5. Two spinners are spun together. Draw a table of the possible outcomes.

	1	2	3	4	5
1					
2					
3					
4					
5					

What is the probability of getting a 10? _____

What is the probability of getting a number less than 5? _____

What is the probability of getting a number greater than 6 ? _____

What is the probability of getting a prime number? _____



○ **Fundamental principle of counting.**

Need to learn this definition:

The Fundamental Counting Principle states that the total number of possible outcomes is the product of the number of items in each category.



Example: You have an option between 5 models of cars above. The colour option for each car is either Blue, Red or Black.

How many choices have you to pick between?

Answer: $5 \times 3 = 15$ (You have 15 choices.)

1. A restaurant offers four sizes of pizza, two types of crust, and eight toppings. How many possible combinations of pizza with one topping are there?

Answer: _____

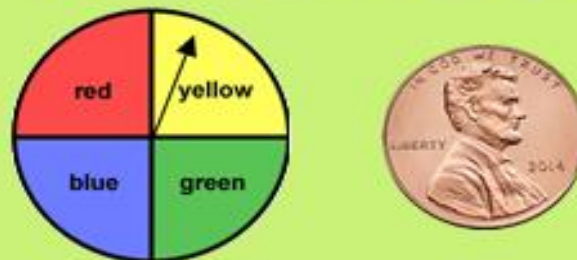
2. Rob has 4 shirts, 3 pairs of pants, and 2 pairs of shoes that all coordinate. How many outfits can you put together?

Answer: _____



Q3.

Consider: If you spin a spinner with 4 separate colors and flip a coin, how many different possible outcomes would there be?



Answer: _____

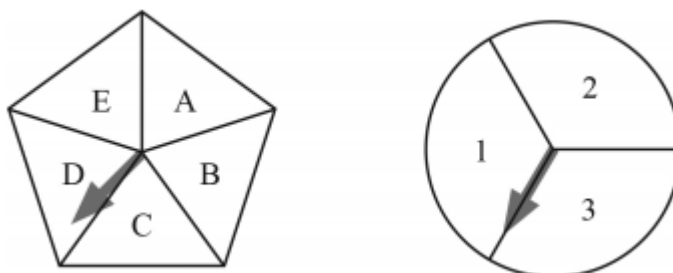
Q4. A coin is tossed and a digit from 1-9 is selected. How many different outcomes are possible?

Answer: _____

Q5. A lunch menu has 3 starters and 4 main courses. How many different two course meals are possible ?

Answer: _____

Q6. Two spinners are spun:



What is the possible number of outcomes? _____



Example: A team consists of 12 players. In how many ways can a captain and a vice captain be chosen?



Answer:

Captain	AND	Vice-captain	
11	x	10	= 110

There are 110 ways of selecting a captain and a vice-captain.

Q1. A committee consists of 10 people. In how many ways can a **chairperson** and a **secretary** be chosen?



Answer:_____



Q2. There are eight horses in a race. In how many ways can the first three places be filled?



Answer: _____

If the favorite wins the race. How many ways can second and third win the race? _____

Q3. There are six dogs in a race. In how many ways can the first three places be filled?



Answer: _____

If the favorite wins the race. How many ways can second and third win the race? _____

Q4. A code consists of a letter of the alphabet followed by two different digits from 1 to 9 inclusive. How many codes are possible?

The first box can be filled in _____ ways

The second box can be filled in _____ ways

The third box can be filled in _____ ways

Number of codes: _____ **Answer:** _____



○ Arrangements – Permutations.

Example: A mobile phone is locked. How many six digit numbers can be formed from the digits 1,2,3,4,5,6?

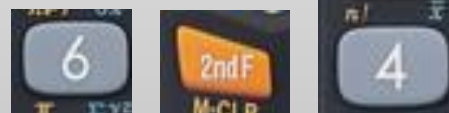


Answer:

The number of arrangement of numbers can be given by

$$6 !$$

(i.e: 6 factorial) **(Use calculator)**



$$= 720$$

Q1. Evaluate $5!$ _____

$$6! \text{ _____}$$

$$4! \times 3! \text{ _____}$$

$$\frac{6!}{3!} = \text{_____}$$

$$\text{Is } 8! = 5! + 3!$$



Q2. Seven horses run in a race. How many ways can they win?

_____ **Answer:** _____

Q3. In how many ways can the letter of the word EIGHTY be arranged? _____ **Answer:** _____

Q4. How many different four digit numbers can be formed using the digits 3,4,5,6 if no digit can be repeated in a number?

_____ **Answer:** _____

Q5. How many ways can five paintings be lined up on a wall? _____ **Answer:** _____

Q6. In how many ways can seven cars be parked in a line?

_____ **Answer:** _____

Q7. In how many ways can 10 people line up in a que for concert tickets? _____ **Answer:** _____

Example: These are the letters in the word TUESDAY.

How many of these arrangements begin with T and end in a VOWEL?



1 5 4 3 2 1 3

Answer: $1 \times 5 \times 4 \times 3 \times 2 \times 1 \times 3 = 360$

Q1. These are the letters in the word FRIDAY.

How many of these arrangements begin with F and end in a vowel?

Answer: _____

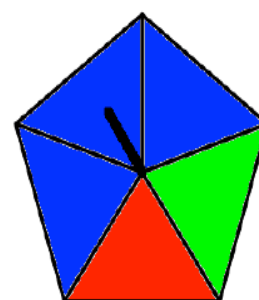


- Expected frequency.

$$\text{Expected Frequency} = \text{Probability} \times \text{number of trails.}$$

This pentagonal spinner is spun 100 times with the results recorded.
Here are the results.

Colour landed on	Frequency
Blue	55
Red	20
Green	25



- a) What is the probability of the spinner landing on the colour
- i) Red ii) Green iii) Blue iv) White ?
- b) How many times would you expect the spinner to land on each colour after 500 spins?
- c) How many times would you expect the spinner to land on each colour after 1000 spins?

Show workings:

A coin is biased so that the probability of tossing a head is 0.7.

How many heads would you expect when the coin is tossed 100 times?

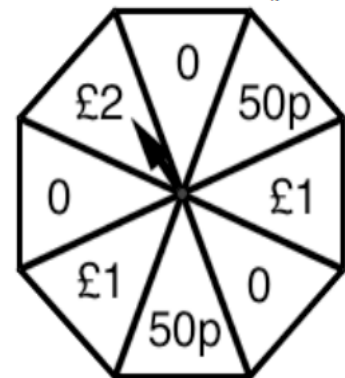
How many tails would you expect when the coin is tossed 300 times?



Players, at a village fayre, pay £1 to spin the pointer on the board shown. Players win the amount shown by the pointer.

The game is played 600 times.

Work out the expected profit or loss on the game.



A biased 6 sided dice has the following probabilities:

Number	1	2	3	4	5	6
Probability	0.15	0.4	0.2	0.1	0.05	0.1

If the dice is rolled 250 times what is the expected frequency of each number?



- **OR rule (add):** Mutually exclusive events.

KEYWORD TO LEARN: Mutually exclusive events are events that cannot happen at the same time.

EG: You can't turn left and turn right at the same time.

$$P(A \text{ or } B) = P(A) + P(B).$$

Q1. Which of these pairs of events are mutually exclusive?

- Winning a football match and drawing the same match
- Wearing one red sock and one blue sock
- Eating toast for breakfast and Chips for dinner
- Being on time and being late for a day at school

Example:

1. Rennin has a pair of dice and he throws them on the table then, what is the probability that the sum of the numbers appears is 6?

Answer: First dice probability = $\frac{1}{6}$

Second dice probability = $\frac{1}{6}$

Probability of mutually exclusive event = $\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$



Q1. A bag contains 4 red, 3 blue and 2 green marbles. If a marble is selected at random, what is the probability that it is

- a) A red **OR** a green marble? _____
- b) A red **OR** a blue marble? _____
- c) A blue **OR** a green marble? _____

Q2. A card is selected at random from a pack of 52 cards.

What is the probability that it is a spade **OR** a red picture card?

Step 1: Probability for spades: _____

Step 2: Probability for red picture cards: _____

Step 3: Add the probabilities: _____

Notes to self so far:



- **AND rule/ The multiplication rule – Bernoulli trial.**



Christy has a bag of candies. In the bag there are 5 red colors, 3 orange colors and 8 green colors. She takes one candy, record its color and put it back in the bag. She then draws another candy. What is the probability of taking out a green candy followed by the red candy?

Probability of GREEN candy: $\frac{8}{16}$

Probability of RED candy: $\frac{5}{16}$

Probability of GREEN **AND** RED: $\frac{8}{16} \times \frac{5}{16} = \frac{40}{256} = \frac{5}{32}$

2. Thomas has a box with 4 black color bottles and 8 gray color bottles. Two bottles are drawn without replacement from the box. What is the probability that both of the bottles are gray?



3. A jar contains colored stones that are 4 pink stones, 9 orange stones and 5 green stones. Ryan picks one stone, records its color and puts it back in the jar. Then he draws another stone. What is the probability of taking out an orange stone followed by the green stone?

4. Henry has 3 black shirts and 7 blue shirts in his wardrobe. Two shirts are drawn without replacement from the wardrobe. What is the probability that both of the shirts are black?

5. Andrew has a box which contains 4 pink blocks, 5 yellow blocks and 6 green blocks. He picks one block, records its color and puts it back in the box. He then draws another block. What is the probability of taking out a yellow block followed by the pink block?

6. Anna has 2 purple lipsticks, 3 red lipsticks and 3 pink lipsticks in her kit. She picks one lipstick, record its color, puts it back in the kit and draws another lipstick. What is the probability of taking out a purple lipstick followed by the red lipstick?

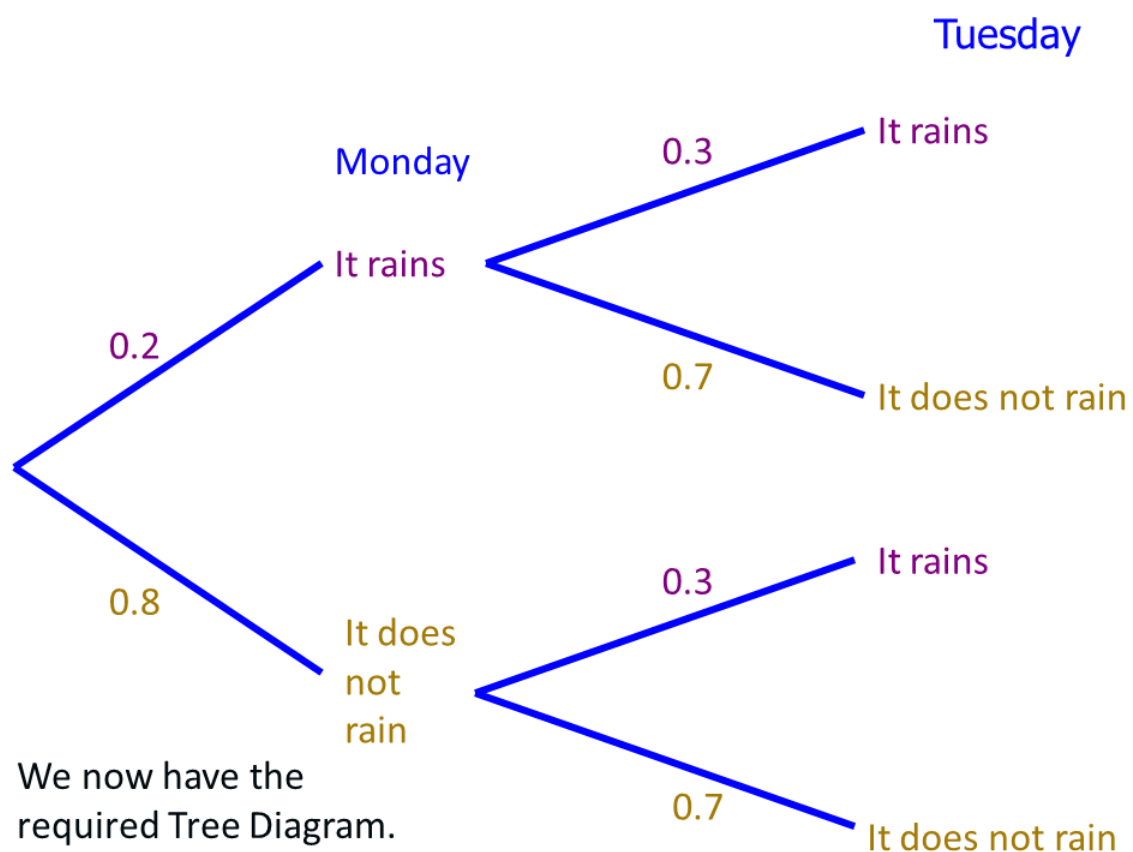


○ Tree diagram



- A tree diagram is a diagrammatical way of representing the probabilities of two or more events
- To calculate probabilities using a tree diagram, you **multiply along the branches**

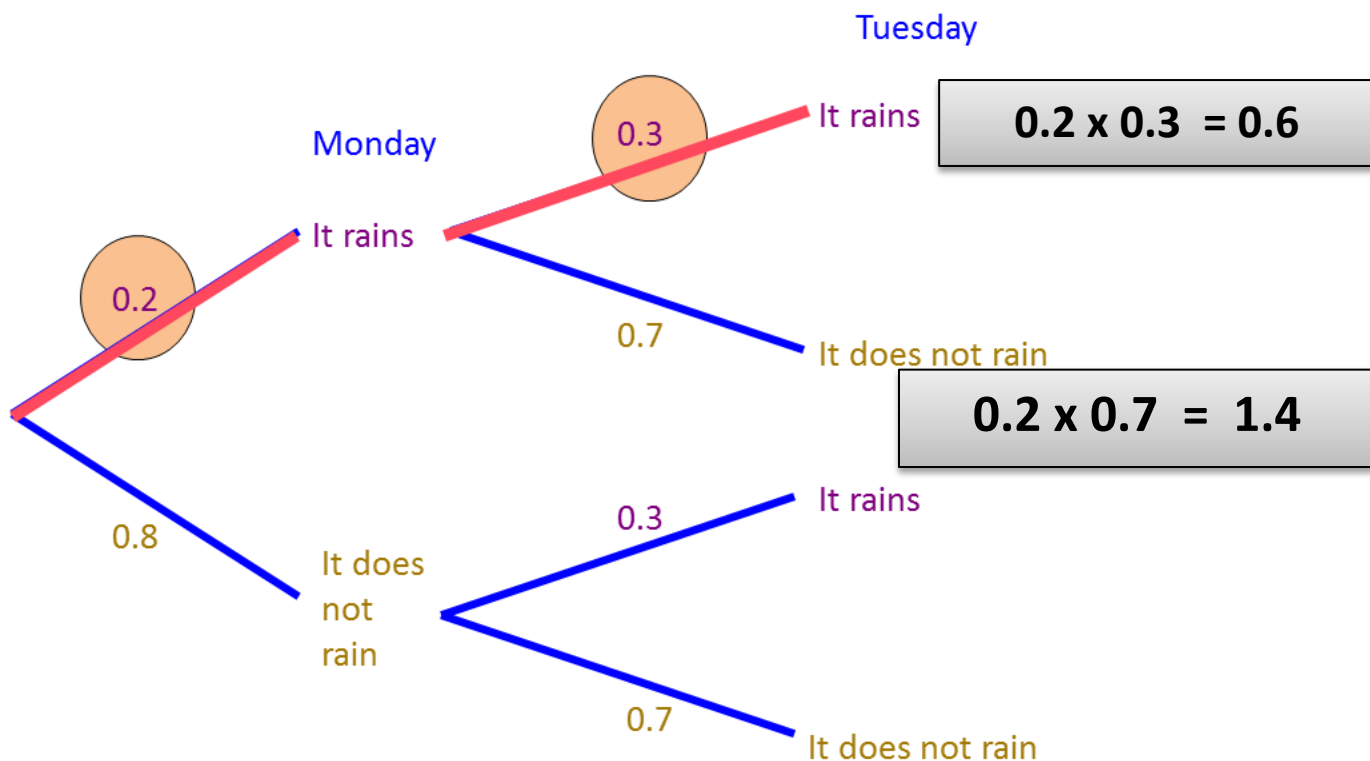
Example:



Note: Each vertical part adds up to 1



We wanted to know the probability that it rained on Monday and Tuesday.



Question 1: Calculate the probability that it is not going to rain on Monday AND will rain on Tuesday?

_____ **Answer:** _____

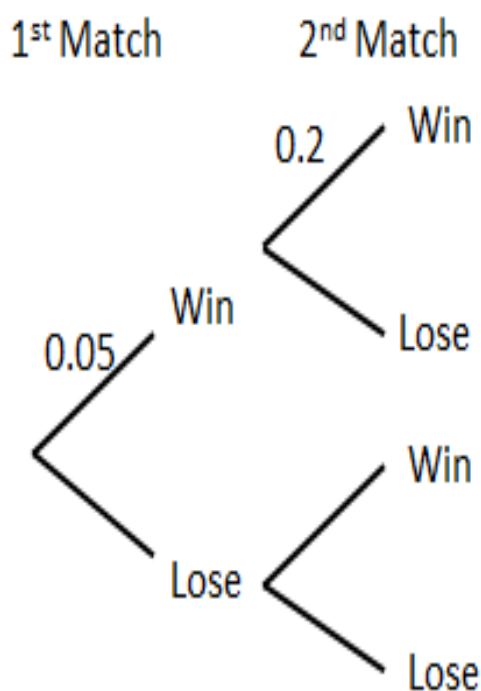
Question 2: Calculate the probability that it is not going to rain on Monday AND will not rain on Tuesday?


_____ **Answer:** _____




Question 3: I place a bet on the outcome of two football matches. The probability I win on the first match is 0.05 and the probability I win on the second match is 0.2.

A) What is the probability that I don't win on the first match?



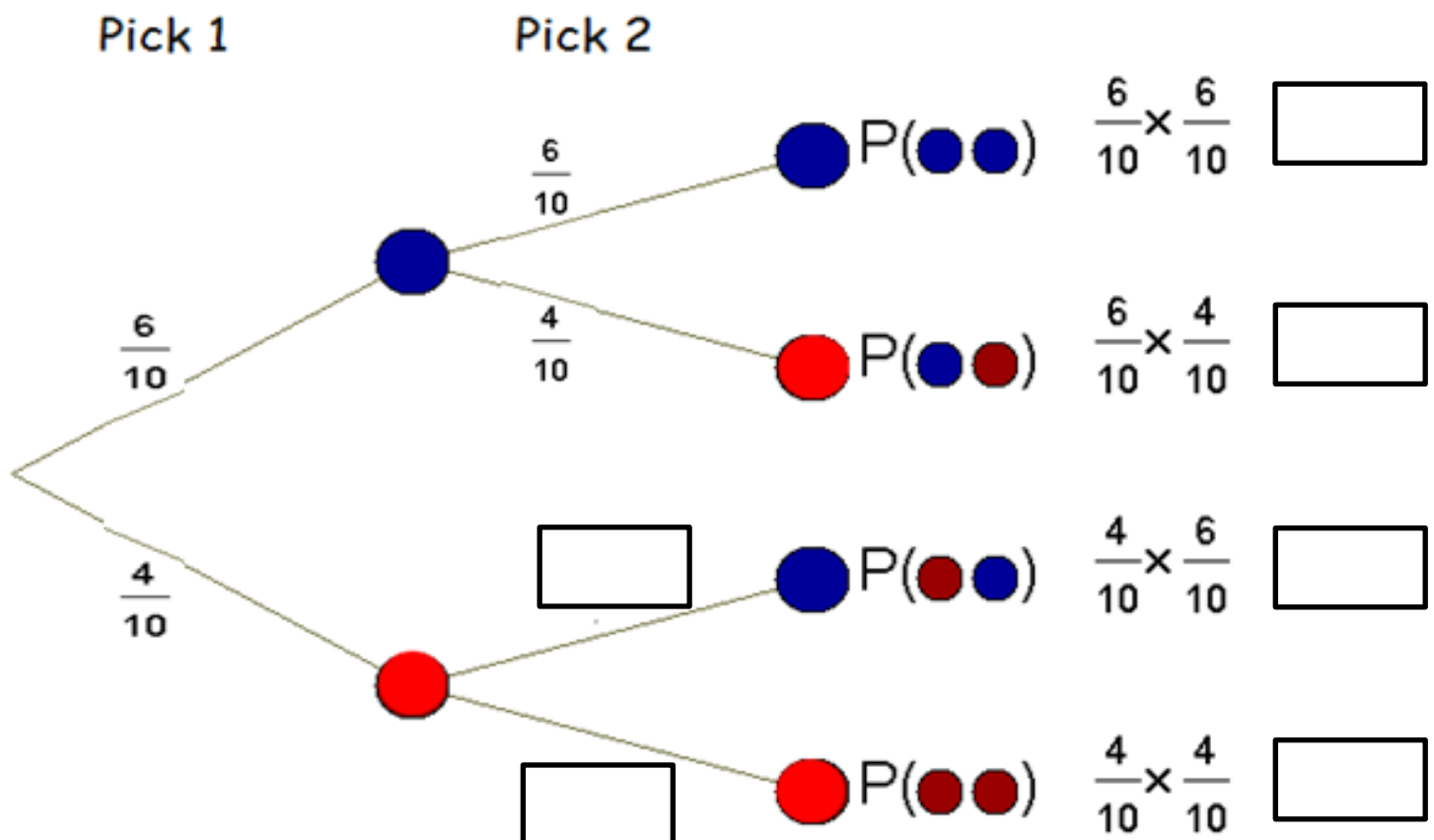
 Calculate the probability of me winning both my bets

 Calculate the probability that I win at least one of the bets



Question: Fill in the blanks:

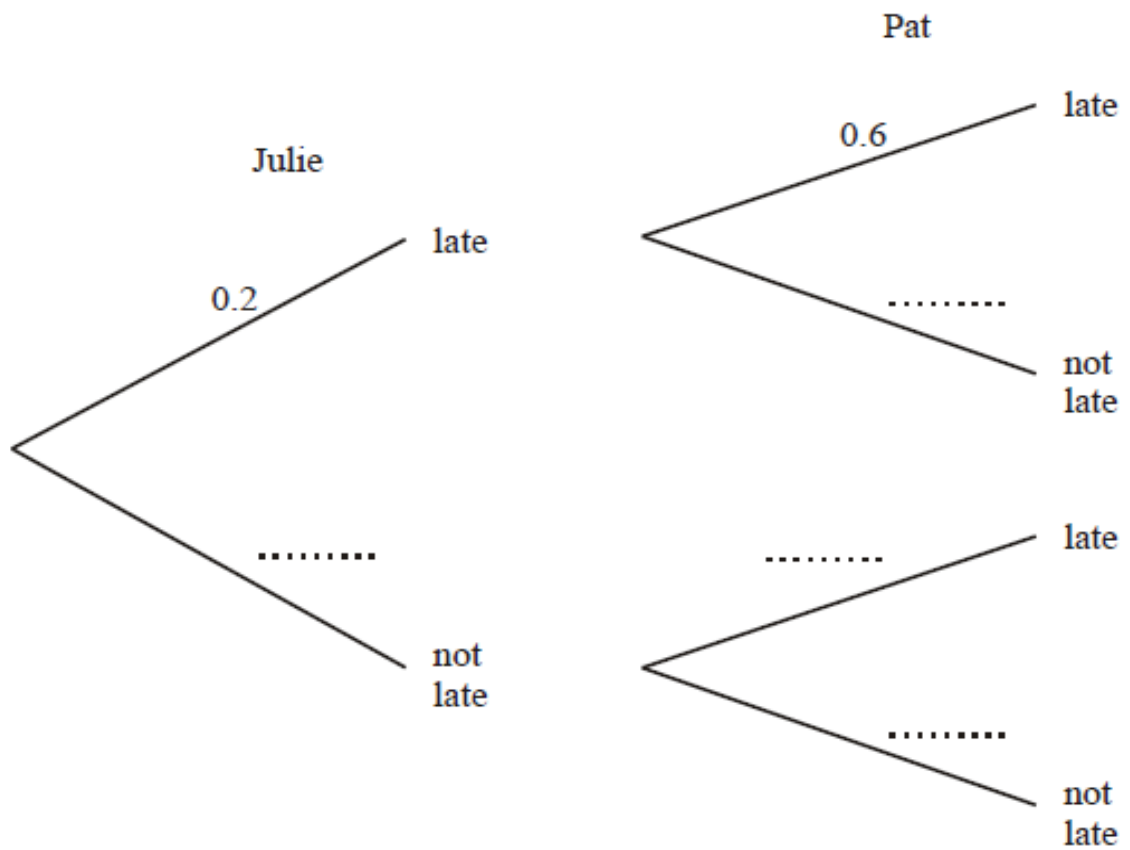
Tree diagram





Question:

- (a) Complete the diagram.



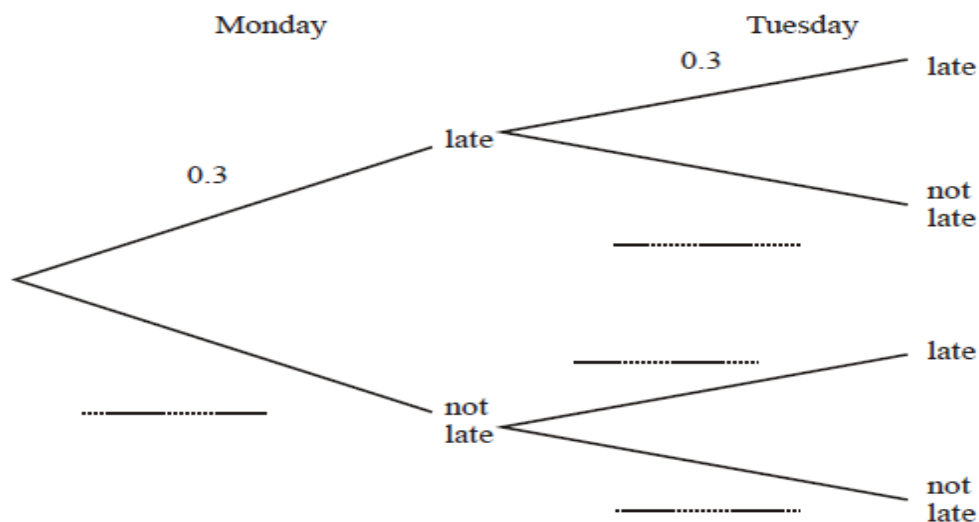
- (b) Work out the probability that Julie and Pat will both arrive late.



Salika travels to school by train every day.

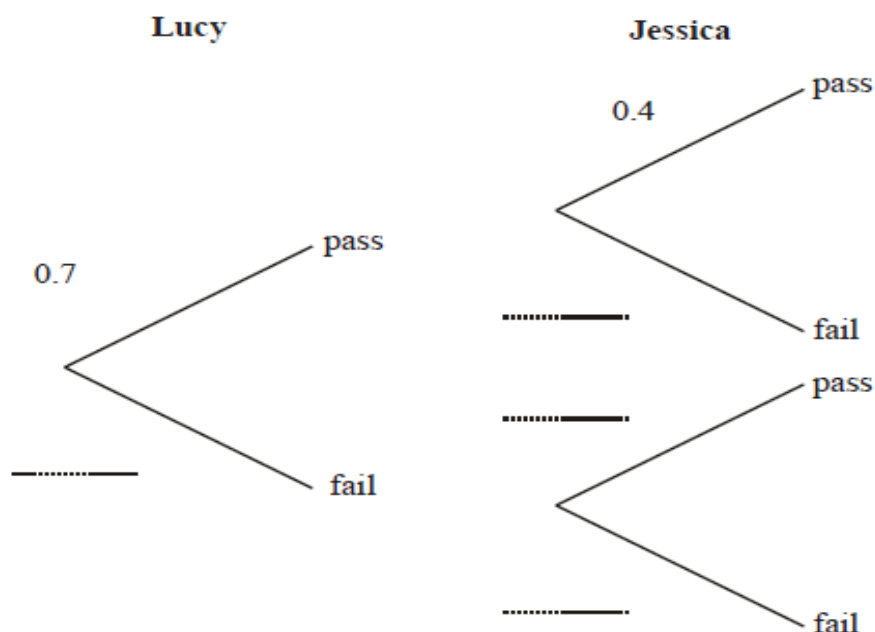
The probability that her train will be late on any day is 0.3

- (a) Complete the probability tree diagram for Monday and Tuesday.



- (b) Work out the probability that her train will be late on **at least one** of these two days.

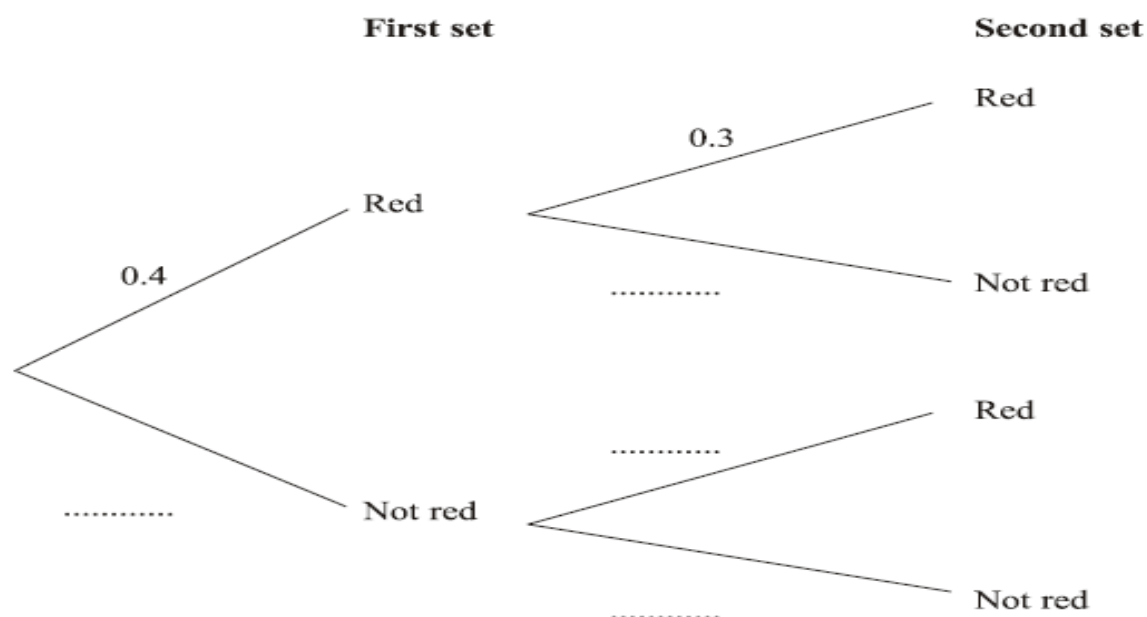
- (a) Complete the probability tree diagram.



- (b) Work out the probability that only one of the 2 girls will pass the test.

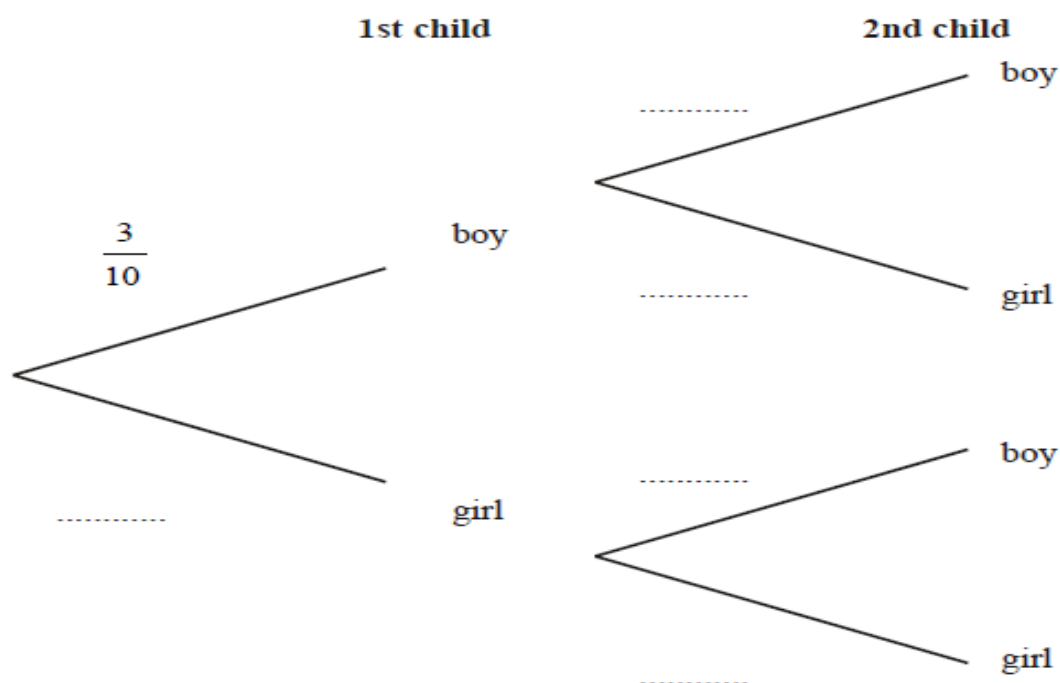


- (a) Complete the probability tree diagram.



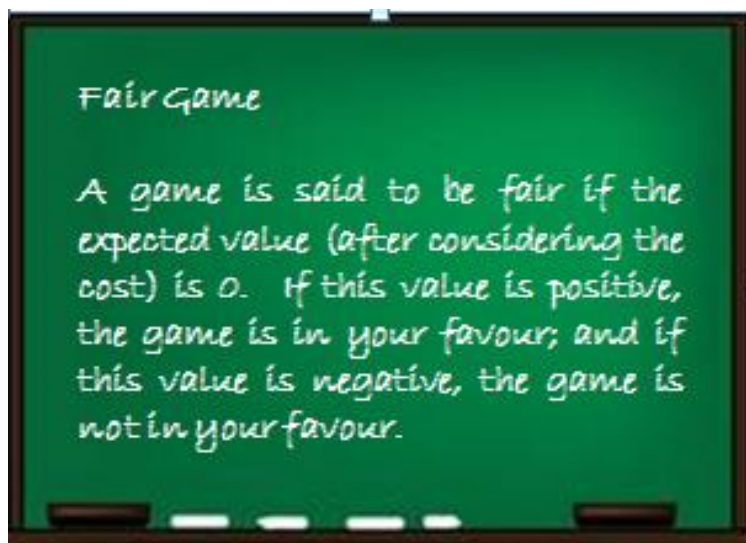
- (b) Work out the probability that both sets of traffic lights will be red.

- (a) Complete the probability tree diagram below.

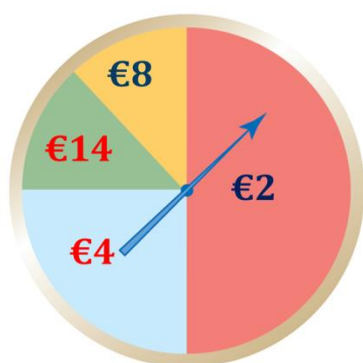


- (b) Work out the probability that Mrs Gold selects two girls.

- Expected values – Law of large numbers



Question 1:



Outcome	Probability	Outcome x Probability
2 euro	$\frac{1}{2}$	
4 euro	$\frac{1}{4}$	
14 euro	$\frac{1}{8}$	
8 euro	$\frac{1}{8}$	

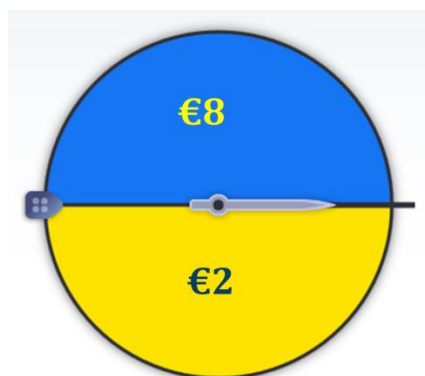
Expected value of payout = (Add you answers from the third column)

_____ Answer: _____



Is the expected value in your favour or not? _____

Question 2:



Outcome	Probability	Outcome x Probability

Expected value: _____

Answer: _____

Is the game in your favour or not? _____

Question 3:

X	500	1000	1500	2000	2500	3000	15,000
P(X)	6/36	7/36	8/36	3/36	2/36	1/36	9/36

Calculate the expected value:

_____ Answer: _____

Note:

$$E(X) = \sum x.P(x)$$



- **Margin error and hypothesis test**