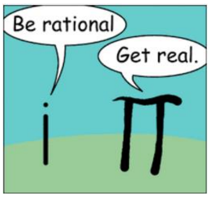
Chapter 7



Complex numbers

(Usually Q3 or Q4 on Paper 1)

This revision guide covers

- Real and imaginary part to complex numbers
- **Plotting complex numbers on a graph (Argand diagrams)**
- Adding/ Subtracting complex numbers **(Put in brackets)**
- Multiplying complex numbers
- The conjugate
- **Dividing complex numbers   (Can never have i in the denominator, so multiply by denominators conjugate)**
- The modulus **[a+bi] means get the** $\sqrt{a^2+b^2}$
- Simplify complex numbers
- **Quadratic equations with complex numbers**
- **Transforming complex numbers**

Date	**How many pages I got done**	

○ Identify the real and imaginary parts of the complex number:

Complex number	Real part	Imaginary part (i)
3 + 2i	3	+2
8-6i		
4+3i		
5-6i		
4i		
5 + 8i		
6		

○ Plotting these complex numbers on an Argand diagram:

The x-axis (real axis) with real numbers and the y-axis (imaginary axis) with imaginary numbers. (3, 8)

3 + 8i

Real + Imaginary

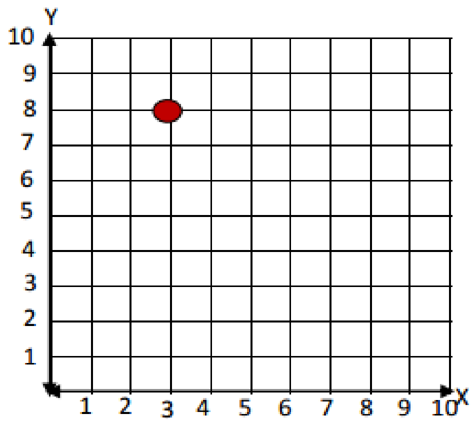
3 = Real

8 = Imaginary

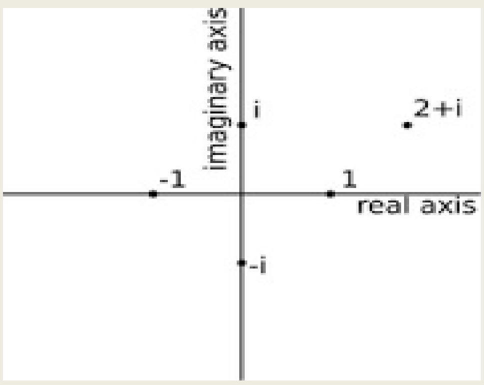
x = real

y = imaginary

(3, 8)

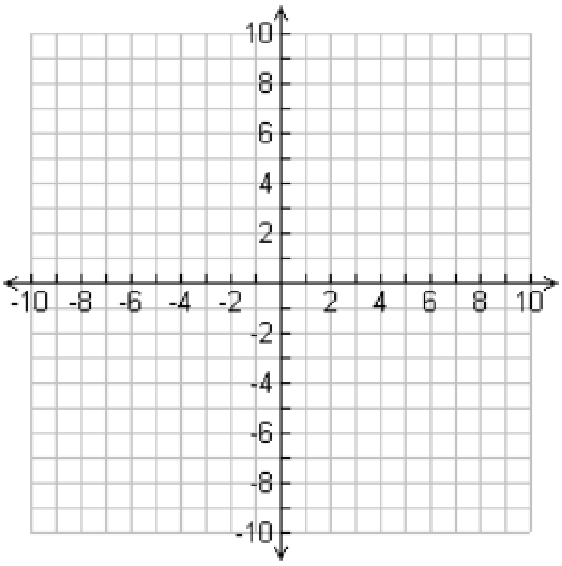


The complex number is represented by the point or by the vector from the origin to the point.

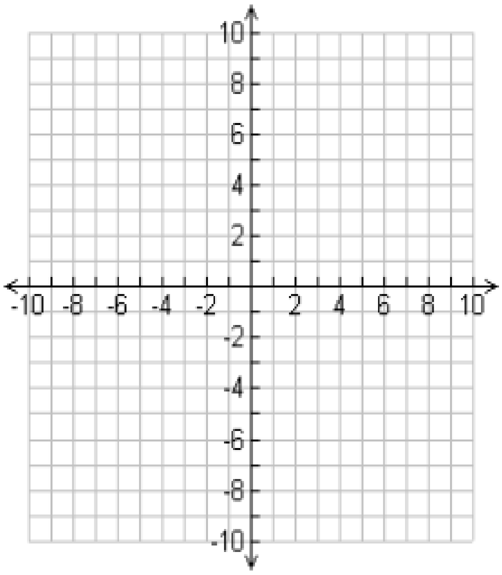


NOTE:

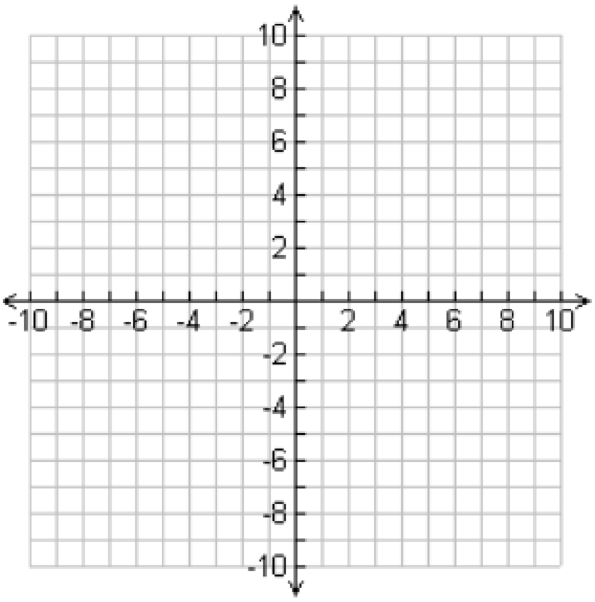
-4 + 4i



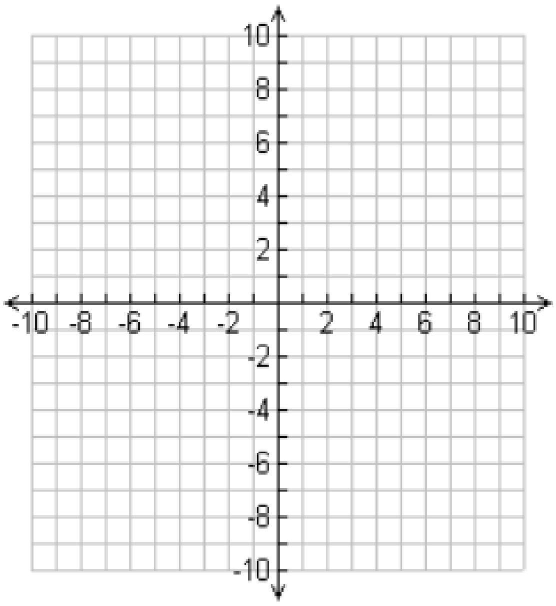
3 + 8i



-5 + 2i



7 + 4i



- Adding complex numbers **(Put in brackets)**

Solve $(5 + 20i) + (10 + 5i)$

Group the real part of the complex number and the imaginary part of the complex number.

$(5 + 20i) + (10 + 5i)$

$= 15 + 25i$

Combine the like terms and simplify.

Answer is: $15 + 25i$

Solve the following questions:

Q1. $(4 + 8i) + (9 + 10i)$

Step 1: Group the real parts together: _______

Step 2: Group the imaginary parts together: ________

Step 3: Put together (real part first, imaginary second): __________

Q2 $(7 + 22i) + (15 - 4i)$

Step 1: Group the real parts together: _______

Step 2: Group the imaginary parts together: ________

Step 3: Put together (real part first, imaginary second): __________

Q3. $(7 + 5i) + (6 + 4i)$

Step 1: Group the real parts together: _______

Step 2: Group the imaginary parts together: ________

Step 3: Put together (real part first, imaginary second): __________

Q4. $(2 + 15i) + (5 + 5i)$ **Answer: _______________**

- Subtracting complex numbers **(Put in brackets)**

$z_1 = 9 - 18i$ $z_2 = 12 - 6i$ What is $z_1 - z_2$?

Answer:

(9 – 18i) – (12 – 6i)

Group the real part of the complex number and the imaginary part of the complex number.

= 9 – 18i – 12 -6i

Combine the like terms and simplify.

= 9 - 12 - 18i + 6i

= -3 – 12i

Note: PUT THE COMPLEX NUMBERS IN BRACKETS BEFORE SUBTRACTING!! This will avoid errors.

Q1. Solve $z_1 - z_2$ when $z_1 = 3 - 13i$ $z_2 = 14 + 5i$

Step 1: Put complex number in brackets: ____

Step 2: Multiply out the second bracket by the minus sign:______________________________

Step 3: Put the real numbers together: _________

Step 4: Put the imagery numbers together: ____________

Step 5: Put together; real number first, imaginary number second: _____________________

Q2. Solve $z_1 - z_2$ when $z_1 = 9 - 17i$ $z_2 = 13 - 5i$

Step 1: Put complex number in brackets: ____

Step 2: Multiply out the second bracket by the minus sign:______________________________

Step 3: Put the real numbers together: _________

Step 4: Put the imagery numbers together: ____________

Step 5: Put together; real number first, imaginary number second: _____________________

Q3. Solve $z_1 - z_2$ when $z_1 = 15 - 3i \quad z_2 = 18 + 3i$

Step 1: Put complex number in brackets: ____

Step 2: Multiply out the second bracket by the minus sign:_________________________________

Step 3: Put the real numbers together: __________

Step 4: Put the imagery numbers together: _____________

Step 5: Put together; real number first, imaginary number second: _______________________

Q4. Solve $z_1 - z_2$ when $z_1 = -2 + 2i \quad z_2 = -1 - 6i$

Step 1: Put complex number in brackets: ____

Step 2: Multiply out the second bracket by the minus sign:_________________________________

Step 3: Put the real numbers together: __________

Step 4: Put the imagery numbers together: _____________

Step 5: Put together; real number first, imaginary number second: _______________________

Q5. Solve $z_1 - z_2$ when $z_1 = 0i \quad z_2 = 4 + 8i$

Step 1: Put complex number in brackets: ____

Step 2: Multiply out the second bracket by the minus sign:_________________________________

Step 3: Put the real numbers together: __________

Step 4: Put the imagery numbers together: _____________

Step 5: Put together; real number first, imaginary number second: _______________________

Q6. Solve $z_1 - z_2$ when $Z = 3 + 4i \quad W = 5 - 9i$

Answer: ______________

○ Multiplying complex numbers

Solve 4i(10+12i)

4i (10 + 12i)

4i(10) +4i (12i)

40i + 48i^2

40i + 48(-1)

40i -48

-48 +40i

Note to remember:

$$i^2 = -1$$

$$i^3 = -1\,i$$

$$i^4 = 1$$

i^2=-1
complex

Q1: Solve $(z_1)(\,z_2)$ **when** $z_1 = 4i \quad z_2 = 4 + 8i$

Step 1: Sub in the complex numbers: ____________________

Step 2: Multiply out: _________________________

Step 3: Note $i^2 = -1$, sub in for i^2 :______________________

Answer: ____________________

Q2: Solve $(z_1)(\,z_2)$ **when** $z_1 = -3i \quad z_2 = -1 - 2i$

Step 1: Sub in the complex numbers: ____________________

Step 2: Multiply out: _________________________

Step 3: Note $i^2 = -1$, sub in for i^2 :______________________

Answer: ____________________

Q3: Solve $(z_1)(\,z_2)$ **when** $z_1 = 5i \quad z_2 = 5 - 6i$

Step 1: Sub in the complex numbers: ____________________

Step 2: Multiply out: _________________________

Step 3: Note $i^2 = -1$, sub in for i^2 :______________________

Answer: ____________________

Q4 Solve: $(6 - 3i)(3 - i)$

Step 1: Re-write out the brackets so first part by last bracket and second part of first by last bracket:

____

Step 2: Multiply out: _________________________

Step 3: Note $i^2 = -1$, sub in for i^2 :______________________

Answer: ____________________

Q5 Solve: $(8 - 4i)\ (6 + 3i)$

Step 1: Re-write out the brackets so first part by last bracket and second part of first by last bracket:

____

Step 2: Multiply out: _________________________

Step 3: Note $i^2 = -1$, sub in for i^2 :______________________

Answer: ____________________

Q6 Solve: $(4 - 2i)^2$

Step 1: Remove square by rewriting in brackets: ____________________________________

Step 2: Re-write out the brackets so first part by last bracket and second part of first by last bracket:

____

Step 2: Multiply out: _________________________

Step 3: Note $i^2 = -1$, sub in for i^2 :______________________

Answer: ____________________

○ The conjugate

Explanation#1

To find the conjugates remember: The conjugate of a + bi = a - bi

$$-9i = 9i$$

Explanation#2

We will follow a very similar procedure to number 1.

Using: a + bi = a - bi

$$5 + 20i = 5 - 20i$$

Q1. Write the conjugates:

Complex number	The conjugate
3-4i	3+4i
6-2i	
5+6i	

Q2.

Find the complex conjugate of the following numbers and check your answers using the interactive file.

		Calculate $\bar{z}$.
a.	$z_1 = 3 + 2i$	
b.	$z_1 = 2 + 3i$	
c.	$z_1 = 1 - 3i$	

- VERY IMPORTANT QUESTION!

  Dividing complex numbers (Can never have i in the denominator, so multiply by denominators conjugate)

Solve $\frac{z_1}{z_2}$ **where** z_1 **= 7 and** $z_2 = 4 + 3i$

Explanation:

To finding conjugates remember: The conjugate of a + bi = a - bi

Original number: 4 + 3i

Step 1) Determine the conjugate of the denominator.

Conjugate: 4 – 3i

$$\frac{(7)}{(4+3i)} \times \frac{(4-3i)}{(4-3i)} = \frac{(7)(4-3i)}{(4+3i)(4-3i)}$$

Step 2) Multiply the top and bottom by the conjugate.

$$\frac{28-21i}{16-9i^2} = \frac{28-21i}{16-9(-1)}$$

Step 3) Simplify

$$\frac{28-21i}{16+9}$$

$$\frac{28-21i}{25} = \frac{7(4-3i)}{25}$$

So the answer is $\frac{7(4-3i)}{25}$

Solve $\frac{z_1}{z_2}$ **where** $z_1 = 2$ **and** $z_2 = 2 - 3i$

Step 1: Substitute in complex number using brackets:

Step 2: Note: An 'i' cannot be in the denominator so you will need to multiply the top and bottom by the conjugate: **the conjugate is ______________**

Step 3: Multiply top and bottom by conjugate

(ensure conjugate is in brackets)

Step 4: Multiply out the brackets on top and bottom:

Step 5: Add the real number together and imaginary numbers together on top and bottom line.

Step 6: Note $i^2 = -1$, sub in.

Step 7: Answer: ________________________ Make sure there is no 'i' in the denominator position.

Solve $\frac{z_1}{z_2}$ where z_1 = 2+4i and $z_2 = 1 - 2i$

Step 1: Substitute in complex number using brackets:

Step 2: Note: An 'i' cannot be in the denominator so you will need to multiply the top and bottom by the conjugate: **the conjugate is ______________**

Step 3: Multiply top and bottom by conjugate

(ensure conjugate is in brackets)

Step 4: Multiply out the brackets on top and bottom:

Step 5: Add the real number together and imaginary numbers together on top and bottom line.

Step 6: Note $i^2 = -1$, sub in.

Step 7: Answer: _________________________ Make sure there is no 'i' in the denominator position.

Solve $\frac{z_1}{z_2}$ where z_1 = 6+5i and $z_2 = 2 - 1i$

Step 1: Substitute in complex number using brackets:

Step 2: Note: An 'i' cannot be in the denominator so you will need to multiply the top and bottom by the conjugate: **the conjugate is _______________**

Step 3: Multiply top and bottom by conjugate

(ensure conjugate is in brackets)

Step 4: Multiply out the brackets on top and bottom:

Step 5: Add the real number together and imaginary numbers together on top and bottom line.

Step 6: Note $i^2 = -1$, sub in.

Step 7: Answer: __________________________ Make sure there is no 'i' in the denominator position.

Solve $\frac{z_1}{z_2}$ where z_1 = 1-2i and $z_2 = 4 - 1i$

Step 1: Substitute in complex number using brackets:

Step 2: Note: An 'i' cannot be in the denominator so you will need to multiply the top and bottom by the conjugate: **the conjugate is _______________**

Step 3: Multiply top and bottom by conjugate

(ensure conjugate is in brackets)

Step 4: Multiply out the brackets on top and bottom:

Step 5: Add the real number together and imaginary numbers together on top and bottom line.

Step 6: Note $i^2 = -1$, sub in.

Step 7: Answer: __________________________ Make sure there is no 'i' in the denominator position.

Solve $\frac{z_1}{z_2}$ where z_1 = 3 + 1i and z_2 = 3-3i

Step 1: Substitute in complex number using brackets:

Step 2: Note: An 'i' cannot be in the denominator so you will need to multiply the top and bottom by the conjugate: **the conjugate is _______________**

Step 3: Multiply top and bottom by conjugate

(ensure conjugate is in brackets)

Step 4: Multiply out the brackets on top and bottom:

Step 5: Add the real number together and imaginary numbers together on top and bottom line.

Step 6: Note $i^2 = -1$, sub in.

Step 7: Answer: __________________________ Make sure there is no 'i' in the denominator position.

- The modulus $|a + bi|$ **means get the** $\sqrt{a^2 + b^2}$

Find $|3 + 4i|$

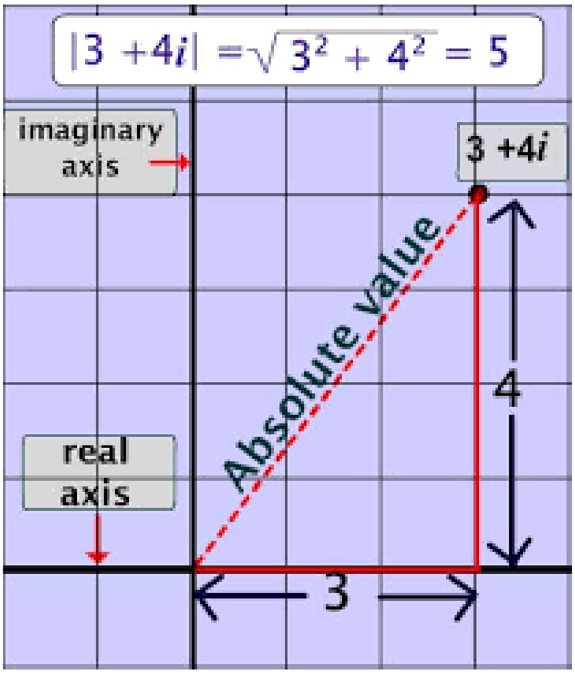
Answer:

$\sqrt{3^2 + 4^2}$

$\sqrt{9 + 16}$

$\sqrt{25}$

=5



Q1: Solve $|5 + 5i|$

Step 1: Find $\sqrt{a^2 + b^2}$ = ______________________________

Answer: ____________

Q2: Solve $|2 + 8i|$

Step 1: Find $\sqrt{a^2 + b^2}$ = ______________________________

Answer: ____________

Q3: Solve $|6 + 4i|$

Step 1: Find $\sqrt{a^2 + b^2}$ = ______________________________

Answer: ____________

Q6: Solve $|9 + 6i|$

Answer: ____________

- Simplify complex numbers

- VERY IMPORTANT QUESTION:

  Quadratic equations with complex numbers

Note: $\sqrt{-b} = \sqrt{b}\ i$

Verify that 4-3i is a root of $z^2 - 8z + 25 = 0$

$$z^2 - 8z + 25 = 0$$

Step 1: Use $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Step 2: Write out values for a, b , c:

a=1 b= -8 c=25

Step 3: sub values into formula to find roots. $\frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(25)}}{2(1)}$

Step 4: Simplify . $\frac{8 \pm \sqrt{64-100}}{2}$ = $\frac{8 \pm \sqrt{-36}}{2}$ $= \frac{8 \pm \sqrt{36}i}{2}$ $= \frac{8 \pm 6i}{2}$

Step 5: Write down the two possible roots:

4+3i or 4-3i

Verify (2+3i) is a root of the complex number z^2 – 4z +13 = 0

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Step1: Use formula:

Step 2: Write out values for a, b , c: a=_____ b =______ c= _____

Step 3: Sub the values into the formula:

Step 4: Simplify:

Step 5: Note $\sqrt{-b}$ = $\sqrt{b}$ i.

Write down the 2 possible values of the roots:

_________________ and _____________________

Step 6: Verified? Tick if yes:

Solve by Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1.) $z^2 + z = 12$	**2.)** $3z^2 = 7 - 2z$	**3.)** $z+1 = z^2$
Roots: ______________ **and** ______________	**Roots:** ______________ **and** ______________	**Roots:** ______________ **and** ______________

Discriminant = $b^2 - 4ac$

If $b^2 - 4ac < 0$, then the equation has 2 imaginary solutions

If $b^2 - 4ac = 0$, then the equation has 1 real solution

If $b^2 - 4ac > 0$, then the equation has 2 real solutions

Find the discriminant of the quadratic equation and give the number and type of solutions of the equation.

1) $3z^2 - 5z = 1$

2) $z^2 = -3z - 7$

- **Transforming complex numbers**

Rotating a complex number involves multiplying the number by i:

Rotate by 90 degrees: Multiply the complex number by i

Rotate by 180 degrees: Multiply the complex number by i^2

Rotate by 270 degrees: Multiply the complex number by i^3

Rotate the complex number 2+4i by 90 degrees:

Step 1: Multiply the complex number by i: ___________________

Step 2: Note $i^2 = -1$, sub in and solve: ____________________________

Rotate the complex number 3+4i by 180 degrees:

Step 1: Multiply the complex number by i^2: ___________________

Step 2: Note $i^3 = -1i$, sub in and solve: ____________________________

Rotate the complex number 5-6i by 270 degrees:

Step 1: Multiply the complex number by i^3: ___________________

Step 2: Note $i^4 = 1$, sub in and solve: ____________________________

<u>Notes to self on complex numbers :</u>