MEASURES - INTRODUCTION

This manual has been designed by members of the Professional Development Service for Teachers. Its sole purpose is to enhance teaching and learning of the strand of Measures in Irish primary schools and will be mediated to practising teachers in the professional development setting. Thereafter it will be available as a free downloadable resource on www.pdst.ie for use in the classroom. This resource is strictly the intellectual property of PDST and it is not intended that it be made commercially available through publishers. All ideas, suggestions and activities remain the intellectual property of the authors (all ideas and activities that were sourced elsewhere and are not those of the authors are acknowledged throughout the manual).

It is not permitted to use this manual for any purpose other than as a resource to enhance teaching and learning. Any queries related to its usage should be sent in writing to:

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Contents

MEASURES - INTRODUCTION

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AIM

The aim of this Teacher's Handbook is to assist teachers in teaching the strand of Measures (infants to 6th class). This comprises the Strand Units of Length, Area, Weight, Capacity, Time and Money. This resource is intended to complement and support the implementation of the Primary School Mathematics Curriculum (PSMC) rather than replace it. By providing additional guidance in the teaching and learning of each strand unit of Measures, this resource attempts to illuminate an instructional framework for enhancing mathematical thinking. This instructional framework advocates methods of eliciting, supporting and extending higher-order mathematics skills such as reasoning; communicating and expressing; integrating and connecting; and applying and problem solving. Although this resource highlights the Measures Strand, this instructional framework can be used for all strands and strand units of the PSMC.

Repeated assessments of mathematics at primary level have revealed weak performance in important areas of the mathematics curriculum such as problem solving and measures (DES, 2011, p.13).

To that end a problem solving approach focusing on pupils' conceptual development of each strand unit of Measures, through engagement in practical hands-activities, is espoused throughout this resource. Enabling pupils to reason and communicate a variety of different approaches and strategies to solve problems, develops true mathematical understanding, rather than merely applying facts which have been learned 'by rote'. Engaging pupils in this way, enhances pupils' learning experiences, enables multiple access points to the same tasks and encourages reflection and opportunities for self-assessment and self-directed learning.

INSTRUCTIONAL FRAMEWORK

INSTRUCTIONAL STRATEGIES

Table 1.1 illustrates a framework for advancing mathematical thinking. Although it does not explicitly refer to concrete materials or manipulatives, the use of these are often a prerequisite for developing mathematical thinking and can be used as a stimulus for this type of classroom discourse.

DIFFERENTIATION

This manual is designed to support individual pupil difference in each classroom. The trajectory described for each strand unit, describes the development stages pupils progress though to become fluent in their mathematical understanding of concepts in Measures. Supporting pupils through the concrete, pictorial, abstract stages, ensures each pupil has the opportunity to understand the concept being taught at their own pace. Some pupils will need to spend more time engaging in a variety of hands-on approaches in order to consolidate conceptual understanding. This approach is also advocated by Reys et al.[1] in describing methods to support children with SEN in mathematics.

"Research indicates that lessons using manipulative materials have a higher probability of producing greater mathematical achievement than do lessons without such materials. Handling the materials appears to help children construct mathematical ideas and retain them" (p.192).

For most children, the content of each lesson on measures, will remain similar, however the support they receive, can be adjusted. This can include more individualised support from the teacher, the use of a variety of materials to scaffold understanding, more opportunities to practise key vocabulary in context, use of appropriate ICT resources, greater links between special and mainstream teachers and home school links which espouse opportunities for practising real-life skills in measures. The way in which individual pupils demonstrate their learning can also vary, including, for example, the use of oral descriptions, pictures and concept maps, video, photographs and other forms of ICT to support assessment and learning. Many extension activities can arise from pupils' active engagement and self-reflection on learning, thereby enabling many pupils to ask further questions, create new problems, and develop more in-depth linkage with other aspects of mathematics and the wider curriculum. It is important to allow opportunities for all pupils to share their learning and strategies with peers and with the whole class, as appropriate. The way in which class groupings are organised, including individual, pair-work and group-work, will also enable differentiation of measures concepts, particularly in terms of opportunities for scaffolding pupils' learning.

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[1] Westwood, P. (2003. Commonsense Methods for children with Special Educational Needs. 4th edn.

TEAM TEACHING FOR NUMERACY

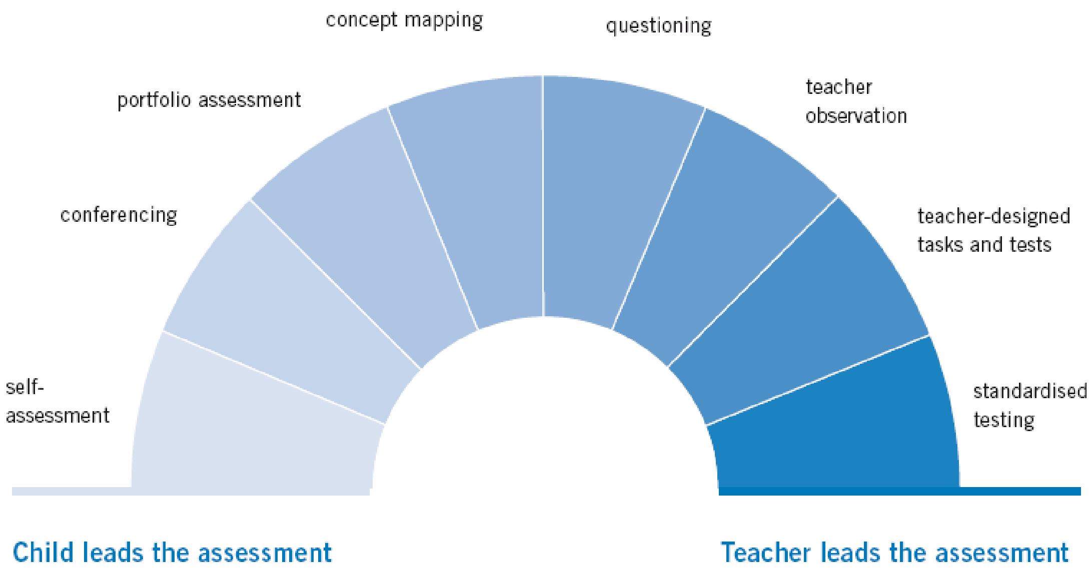
Teachers working collaboratively can help to support differentiated instruction as described above. Team teaching or co-teaching can enable two or more teachers providing in-class support in numeracy to facilitate active and engaging learning experiences for every child, including children with Special Education Needs (SEN). Team teaching can promote inclusive practice, by supporting pupil diversity, a variety of pupil learning styles and abilities, allowing for greater support for pupils in the whole class, while monitoring and supporting individual pupils. The learning experiences described in each strand unit of measures in this manual, and the instructional framework in Table 1.1., can effectively be used to facilitate teacher questioning and scaffold pupil learning, in whole class and team teaching contexts. Similarly, a variety of assessment opportunities can be used to inform teaching and learning for individuals and the whole class, and enable greater pupil participation in self-assessment strategies. Teachers may find it informative to engage in pre and post-team-teaching pupil assessment, to determine progress during a period of intervention, and the use of the PDST trajectories in each strand unit of measures can help in this regard.

Further information on Team Teaching in Numeracy, including Seminar Slides and Resources can be found at www.pdst.ie/numeracy

ASSESSMENT OPPORTUNITIES

A variety of assessment approaches is advocated in the Primary School Mathematics Curriculum (PSMC)[2] . "Teaching and the gathering and analysis of assessment information should run concurrently, with the results of assessment feeding back into the teaching and learning process. Assessment should be a positive experience for the child, as this makes his/her learning more effective" [3]

Multiple modes of assessment information garnered as pupils engage in the sample learning experiences described within this manual, will help to inform teaching and learning in each strand unit of Measures. This assessment icon is used throughout the manual to indicate opportunities for teacher and pupil led assessment. These could include pupils taking photographs to document their work, the use of ICT to record and reflect on learning, a variety of samples of pupils' work, including strategies used to solve problems or demonstrate their learning. Individual and group outcomes could include projects combining other aspects of the curriculum, for example 'The Perfect Suitcase Project' in the strand unit Weight.



A range of assessment tools, including pupil-led assessments, are recommended, in order to monitor pupil engagement and progress in learning. The use of journals in mathematics can be an effective pupil self-assessment tool, which can increase a pupil's awareness of how they learn and remember, as the art of writing is in itself a reflective process, so pupils have to think about the strategies and ideas used. Journals also serve to provide a record of thinking and learning and as a tool for supporting future conferencing with their teacher or indeed discussion with parents. Journals can also help pupils to identify challenges and set goals for future learning.

[2] NCCA (1999). Primary School Mathematics Curriculum.,(p.114)
[3] NCCA (1999). Primary School Mathematics Curriculum., (p.114)

Table 1.1 Strategies for Supporting and Developing Mathematical Thinking

Eliciting	Supporting	Extending
Facilitates pupils' responding Elicits many solution methods for one problem from the entire class *for example "Who did it another way?; did anyone do it differently?; did someone do it in a different way to X?; is there another way of doing it?"* Waits for pupils' descriptions of solution methods and encourages elaboration Creates a safe environment for mathematical thinking *for example all efforts are valued and errors are used as learning points* Promotes collaborative problem solving *Orchestrates classroom discussions* Uses pupils explanations for lesson's content Identifies ideas and methods that need to be shared publicly *for example "John could you share your method with all of us; Mary has an interesting idea which I think would be useful for us to hear."*	*Supports describer's thinking* Reminds pupils of conceptually similar problem situations Directs group help for an individual student through collective group responsibility Assists individual pupils in clarifying their own solution methods *Supports listeners' thinking* Provides teacher-led instant replays *for example "Harry suggests that ...; So what you did was ...; So you think that ...".* Demonstrates teacher-selected solution methods without endorsing the adoption of a particular method *for example "I have an idea ...; How about ...?; Would it work if we ...?; Could we ...?".* *Supports describer's and listeners' thinking* Records representation of each solution method on the board Asks a different student to explain a peer's method *for example revoicing (see footnote on page 8)*	*Maintains high standards and expectations for all pupils* Asks all pupils to attempt to solve difficult problems and to try various solution methods *Encourages mathematical reflection* Facilitates development of mathematical skills as outlined in the PSMC for each class level *for example reasoning, hypothesising, justifying, etc.* Promotes use of learning logs by all pupils *for example see Appendix for a sample learning log* *Goes beyond initial solution methods* Pushes individual pupils to try alternative solution methods for one problem situation Encourages pupils to critically analyse and evaluate solution methods *for example by asking themselves "are there other ways of solving this?; which is the most efficient way?; which way is easiest to understand and why?".* Encourages pupils to articulate, justify and refine mathematical thinking *Revoicing can also be used here* Uses pupils' responses, questions, and problems as core lesson including student-generated problems *Cultivates love of challenge*

This is adapted from Fraivillig, Murphy and Fuson's (1999) Advancing Pupils' Mathematical Thinking (ACT) framework.[4]

[4] Fraivillig, J.L., Murphy, L.A. & Fuson, K.C. (1999). Advancing pupils' mathematical thinking in everyday mathematics classrooms. Journal for Research in Mathematics Education, 30(2) 148-170.

CLASSROOM CULTURE

Creating and maintaining the correct classroom culture is a pre-requisite for developing and enhancing mathematical thinking. This requires the teacher to:

- cultivate a ‘have a go’ attitude where all contributions are valued;
- emphasise the importance of the process and experimenting with various methods;
- facilitate collaborative learning through whole-class, pair and group work;
- praise effort;
- encourage pupils to share their ideas and solutions with others;
- recognise that he/she is not the sole validator of knowledge in the mathematics lesson;
- ask probing questions ;
- expect pupils to grapple with deep mathematical content;
- value understanding over ‘quick-fix’ answers; and
- use revoicing[5] (reformulation of ideas) as a tool for clarifying and extending thinking.

In this type of classroom pupils are expected to:

- share ideas and solutions but also be willing to listen to those of others; and
- take responsibility for their own understanding but also that of others.

[5] Revoicing is ‘the reporting, repeating, expanding or reformulating a student's contribution so as to articulate presupposed information, emphasise particular aspects of the explanation, disambiguate terminology, align students with positions in an argument or attribute motivational states to students' (Forman & Larreamendy-Joerns, 1998, p. 106).

MEASURES BACKGROUND KNOWLEDGE FOR TEACHERS

FUNDAMENTAL FACTS REGARDING MEASURES

Non-Standard Units

Measurement involving the use of non-standard units is an important part of the development of measurement skills. Non-standard units are useful for a number of reasons:

- Non-standard units make it easier to focus directly on the attribute being measured.
- Non-standard units provide a good rationale for using standard units. A discussion of the need for a standard unit can have more meaning after groups have measured the same objects with their own units and arrived at different and sometimes confusing answers.
- Using non-standard units can be motivating.[6]

Historically, all units developed in this way. For example, originally body lengths, such as the cubit, were used to measure cloth (the cubit is the length from the elbow to the end of the middle finger). However, different people have different lengths for the parts of the body. So it was not uncommon for someone to use different people to measure the cloth depending on whether they were buying or selling. As a result, in order to stop cheating, it was eventually decided that uniform standard units should be used[7].

Metric and Imperial Systems of Measurement

There are two systems of standard measurement units that are used in Ireland. This can prove confusing for both pupils and teachers. The metric system or Systeme Internationale d'Unités (SI) is the official system and so is the one with which pupils engage with in the curriculum; however, the traditional imperial system is still in general use outside of school, for example:

- in baking (ounces, pounds);
- in body weight (pounds, stones)
- in distance (inches, feet, miles);
- in area (square inches, acres);
- in capacity (pints, gallons);
- etc.

The advantage of the official metric system is that it uses a base 10 system with standard prefixes to indicate the size of the units, for example, kilo (thousands), milli (thousandths) and centi (hundredths). However, the ongoing widespread use of both measurement systems can cause confusion for pupils and may impede their conceptual development of measures. The resources available on the bbc skillswise site[8], including a short

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[6] John A. Van De Walle et al, Elementary and Middle School Mathematics Teaching Developmentally, 7th Edition, 2010 p. 372
[7] http://nzmaths.co.nz/resource/dino-cylinders
[8] http://www.bbc.co.uk/skillswise/topic/imperial-and-metric

video, may prove beneficial for both classroom teachers and parents in helping pupils to understand the day to day use of these two systems. Senior pupils will find this story of how a metric mishap caused the loss of a NASA orbiter interesting and can explore further instances where metric and imperial measurement can cause confusion or worse.[9] There are opportunities for linkage and integration with science and other areas of the curriculum through these investigations.

Basic Ideas and Progression[10]

The three basic ideas in measurement are

- (i) The choice of unit: a unit is chosen
- (ii) Comparison: the unit chosen is compared with the object to be measured
- (iii) Counting: the number of times the unit is required is called the measurement of the object.

In developing the various aspects of measurement three main stages emerge[11]

Stage 1: Premeasuring Stage

- Comparisson of objects of unequal capacity
- Ordering of objects
- Equivalence of objects
- Conservation experiences

Stage 2: Arbitrary Units

- Use of arbitrary units

Stage 3: Standard Units

- Use of standard Units

Area, Perimeter & Volume

Area, perimeter and volume are related. For example, as the shapes of regions or three-dimensional objects change while maintaining the same areas or volumes, there is an associated effect on the perimeters and surface areas[12].

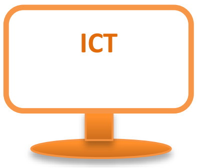
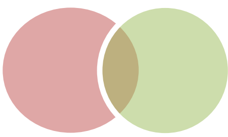
[9] http://edition.cnn.com/TECH/space/9909/30/mars.metric.02/

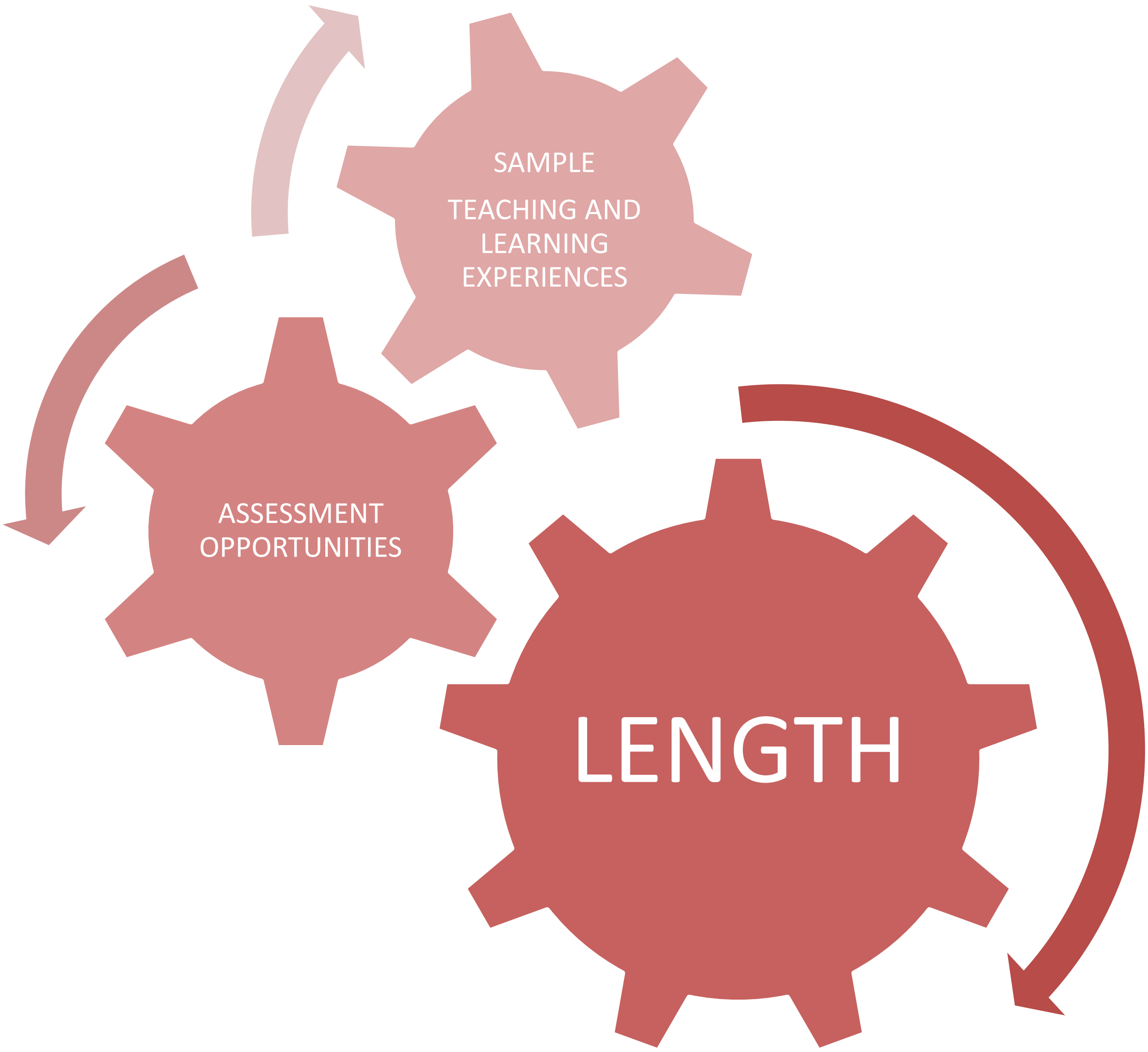
[10] Deboys and Pitt, (1980: 68). *Lines of Development in Primary Mathematics* (3rd ed.). Belfast: Blackstaff Press [for] the Queen's University of Belfast, Teachers' Centre

[11] Deboys, and Pitt,. (1980: 68). *Lines of Development in Primary Mathematics* (3rd ed.). Belfast: Blackstaff Press [for] the Queen's University of Belfast, Teachers' Centre

[12] John A. Van De Walle et al, Elementary and Middle School Mathematics Teaching Developmentally, 7th Edition, 2010 p. 369

GUIDE TO MEASURES MANUAL HEADINGS AND SYMBOLS

Symbol	Explanation
TEACHING NOTES	These notes provide essential pedagogical knowledge on conceptual development of the specific objective. Information on the mathematical content of the objective may also be contained in these notes. These notes precede the *Sample Learning Experiences.*
SAMPLE LEARNING EXPERIENCES	These are child-centred activities designed to develop conceptual understanding. A selection of sample learning experiences are outlined for each objective in the learning trajectory. This is not intended to be an exhaustive list of learning experiences. These learning experiences are colour coded according to the levels: **Level A** **Level B** **Level C** **Level D**
CONSOLIDATION ACTIVITIES	These consolidation activities follow on from the sample learning experiences. There are no new concepts introduced during these activities, rather, their purpose is to support and consolidate pupil understanding.
	This symbol signals important points to be aware of when teaching a particular concept.
	The speech bubble contains possible language that can be used by the teacher when teaching a particular concept. This language is intended to activate and extend pupils' mathematical thinking. For more information on Teacher Questioning please see the 'Instructional Framework' in this manual. The *NCCA Primary Assessment Guidelines* also have a section on Teacher Questioning.
	This is an interactive activity which is hyperlinked and so can be accessed online by clicking on the icon. The website address for each interactive activity is provided as a footnote, so that teachers using the hard copies of this Measures manual, can also access these activities.
	This icon highlights the opportunity for linkage or integration with another strand/strand unit or subject.
	This icon highlights opportunities for assessing pupil understanding. For more information on assessment, please see the *NCCA Primary Assessment Guidelines*.


SAMPLE
TEACHING AND LEARNING EXPERIENCES
ASSESSMENT OPPORTUNITIES
LENGTH

MEASURES - LENGTH

LENGTH

LENGTH - BACKGROUND KNOWLEDGE FOR TEACHERS

What is Length?

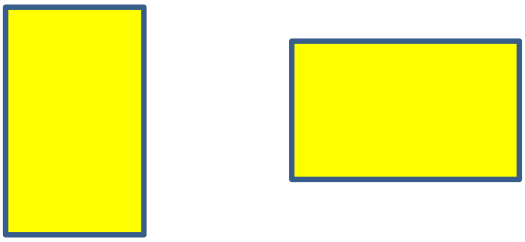
There are several conceptual definitions for Length including those for length, height and distance:

- The **length** of the object is the number of standard units (such as centimetres) which can be laid in a straight line along or beside the object. It is the minimum distance between the two ends of an object – so there can be no gaps or overlaps and they must be in a straight line.
- The **height** of a person is the length of an imaginary line from the top of their head straight through them to the floor between their feet. This cannot be measured directly so another line of the same length is taken, perhaps down a wall.
- The **distance** between two points on a piece of paper is the shortest possible distance between them or the length of a straight line between them.[13]

LENGTH - POSSIBLE PUPIL MISCONCEPTIONS

Conservation of Length

The attribute does not change if the orientation changes. Piaget found that this principle develops slowly, for example, he studied children's knowledge of conservation regarding a length of a piece of wood when placed horizontally and vertically – young children did not realise that the length would be the same. Children may be surprised to discover that their height is the same whether they are standing up or lying down.



Lining up ends when measuring length

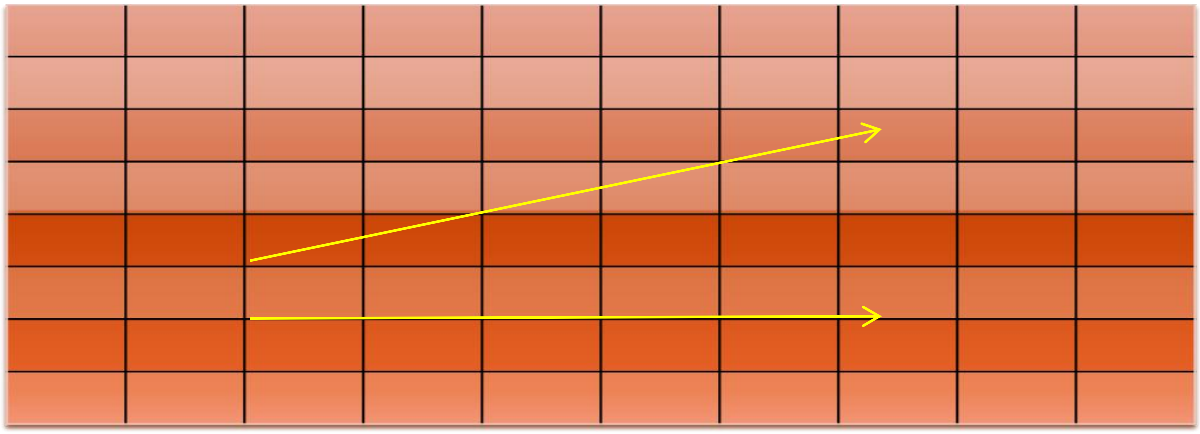
Many pupils fail to line up ends correctly when they are measuring length and so may believe that the one which 'sticks out' the farthest is the longest, regardless of the starting position.

The distance between two points

Children may assume that the lengths of both lines are the same because they start and end at the same point. This is because they may be concentrating on the position of the ends; instead of concentrating on the distance

[13] Suggate, Davis and Goulding (2010, p.180)

between the ends. The understanding that this is not the case will need a lot of exploration, practical experience and discussion.



Using a ruler effectively

Some pupils are unsure of where to begin measuring, for example, they often don't begin measuring at 0, and instead they will begin measuring at the end of the ruler. Similarly, pupils sometimes put the 1 mark beside the end of the object. Furthermore, some pupils find it difficult to begin measuring at a point other than 0 in order to work out the length of an object; for example, beginning to measure the object from 5 cm and finishing at 9cm tells us that the object is 4cm long. Finally, some pupils find it difficult when the object is longer than the ruler.

The 'bit' left over

The 'bit' left at the end when measuring using non-standard units can cause difficulty for pupils. If children say it is '3 and a bit matches' then it might be useful to say something like 'yes - it's more than 3 matches but less than 4 matches'. This will later lead on to degrees of accuracy, for example, the measurement might be to the nearest centimetre.

Real life maths

Centimetres and metres are used in school; however, the building trade uses metres and millimetres. Furthermore, many people still use miles and yards outside of school instead of using kilometres and metres.

LENGTH - PARENTAL INVOVLEMENT

The following are a list of ideas that may be shared with parents to help develop pupils' conceptual understanding of length. Some of the following ideas are taken from the NCCA's tip sheets for parents available through the NCCA website.[14]

[14] http://www.ncca.ie/en/Curriculum_and_Assessment/Parents/

MEASURES - LENGTH

Here you will also find some short videos of parents working with their children to develop their mathematical understanding.

Level A:

- When young children are measuring encourage them to use the language of length, for example, 'this pencil is long', 'this crayon is longer', 'you have the shortest pencil'.
- Ask your child to help you sort the washing: Find the long trousers. Can you find some that are shorter? Which trousers are the shortest?
- Build a tower with your child using blocks or boxes: Can you make a taller tower? A shorter tower?

Level B:

- Ask who is the shortest/tallest in our house?
- About how long is our garden?
- Make a paper aeroplane and test it to see how far it flies. Measure the distance in metres and centimetres using a measuring tape.

Level C:

- Look for examples of length in the environment at home, for example, written on items. Convert the measurements into centimetres/millimetres.
- Track growth of plant, for example, cress seeds
- Measure short distances in your home/garden using a measuring tape.

Level D:

- Involve your child in planning short and long journeys. Use Google Maps to find out distances.
- Look for examples of distances with your child, for example, 10km walk, a marathon, 100km cycle.
- Figure out distances with your child and create simple benchmarks. I wonder how far it is from here to Grannys' house?
-

LENGTH - TEACHING NOTES

Development stages in length

Deboys and Pitt [15] outline three stages in the development of *Measures* concepts. This development can be applied to *Length*.

- Stage 1: Pre-measuring. Pupils need to develop and extend vocabulary, order three or more objects, explore equivalence of two objects and have conservation experiences.
- Stage 2: Use of arbitrary or non-standard units
- Stage 3: Use of standard units

[15] Deboys and Pitt, 1979, p.68

MEASURES - LENGTH

What tools are useful when measuring length?

There are many tools which are useful for pupils to experience when measuring length. These tools can be generally categorised into tools for measuring length using non-standard units and those for measuring length using standard units. The tools include but are not limited to:

Non-standard units	Standard units
• straws • lollipop sticks • pipe cleaners • cocktail sticks • matchsticks • cubes • paperclips • pegs • paper strips • books • copies • pencils • hands • legs • arms • bodies • skipping ropes • etc.	• rulers • metre sticks • tape measures • trundle wheel • cuisenaire rods

Teaching Standard Units of Measure

According to Van De Walle et al[16], teaching standard units of measure can be organised around three goals:

- Familiarity with the unit
- Ability to select an appropriate unit
- Knowledge of relationships between units

Standard units of measurement

For length the International System of Units (SI) is the metre**.**

Length prefix naming system	
Millimetre	Thousandth of a metre
Centimetre	Hundredth of a metre
Decimetre	Tenth of a metre
Metre	
Decametre	10 metres
Hectometre	100 metres
Kilometre	1000 metres

[16] Van de Walle, J.(2007). Elementary and Middle School Mathematics Teaching Developmentally (p.378). 6th edn. Pearson: Allyn and Bacon

MEASURES - LENGTH

LENGTH - LEARNING TRAJECTORY

The learning trajectory is based on the objectives for Measures in the Primary School Mathematics Curriculum. In some instances, similar objectives at the same class level have been collapsed into one objective. Objectives that only refer to problem solving have not been included as discrete objectives because a problem solving approach is advocated throughout all of the teaching and learning experiences. Problem solving is viewed in this manual as a fundamental, integral part of mathematics teaching and learning that pupils should experience every day. The same colour coding from the curriculum is used – infants (green); first and second (red); third and fourth (blue); fifth and sixth (orange).

LENGTH LEARNING TRAJECTORY LEVEL A[17]

Trajectory Levels	Concept	Developmental Experiences		
		Concrete	Pictorial	Abstract
	Level A.1 Develop an understanding of the concept of length through exploration, discussion, and use of appropriate vocabulary			
	Level A.2 Compare and order objects of different length, width and height			
	Level A.3 Estimate and measure length in non-standard units			
	Level A.4 Select and use appropriate non-standard units to measure length, width or height. Discuss reasons for choice.			

[17] This level is generally aligned with the objectives for Junior and Senior infants.

LENGTH LEARNING TRAJECTORY LEVEL B [18]

Trajectory Levels	Concept	Developmental Experiences: Concrete	Pictorial	Abstract
	Level B.1 Estimate, compare, measure and record length using non-standard units and standard units (metre and centimetre) and deepen understanding of the conservation of length.			
	Level B.2 Select and use appropriate non-standard units to measure length, width or height and discuss reasons for choice.			

[18]This level is generally aligned with the objectives for First and Second class.

MEASURES - LENGTH

LENGTH LEARNING TRAJECTORY LEVEL C [19]

Trajectory Levels	Concept	Developmental Experiences: Concrete	Pictorial	Abstract
	Level C.1 Estimate, compare, measure and record lengths of a wide variety of objects, using appropriate metric units, and selecting suitable instruments of measurement			
	Level C.2 Rename units of length in m and cm and using decimal or fraction form			
	Level C.3 Understand, estimate and measure the perimeter of regular 2-D shapes			

[19] This level is generally aligned with the objectives for Third and Fourth class.

MEASURES - LENGTH

LENGTH LEARNING TRAJECTORY LEVEL D [20]

Trajectory Levels	Concept	Developmental Experiences		
		Concrete	Pictorial	Abstract
	Level D.1 Estimate and measure length using appropriate metric units (mm, cm, m, km) and appropriate instruments of measurement			
	Level D.2 Rename measures of length as fractions, decimal fractions and appropriate metric units			
	Level D.3 Estimate and measure the perimeter of regular and irregular shapes			
	Level D.4 Use and interpret scales on maps and plans			

[20] This level is generally aligned with the objectives for Fifth and Sixth class

MEASURES - LENGTH

LEVEL A.1

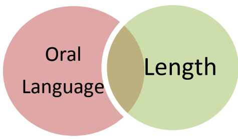
DEVELOP AN UNDERSTANDING OF THE CONCEPT OF LENGTH THROUGH EXPLORATION, DISCUSSION AND USE OF APPROPRIATE VOCABULARY

TEACHING NOTES

The following teaching and learning experiences are designed to support development of the vocabulary of length. Guided discussion and a hands-on approach are essential to enhance pupils' learning. It is easier at first for pupils to use the words long, short etc. They should then move quickly to an ability to use the comparative forms – longer, shorter etc.[21]

Remember to use precise language when helping students make comparisons. Avoid using 'bigger than', and instead use language such as 'longer than' or 'holds more than' (Van de Walle et al.,2007, p.376).

SAMPLE LEARNING EXPERIENCES



Worms[22]

Give each pupil a ball of play dough and ask them to make a worm. Get the pupils to bring their worms to the mat. Ask the pupils to describe their worm. Discuss what is different about the worms and elicit vocabulary associated with length. Now ask the pupils to make a worm that is short. Observe and discuss the short worms.

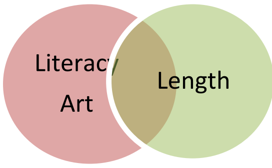
How would you describe your worm? How do you know that your worm is short? Tell me about your worm. How could we make different worms? What other kinds of worms could we make today?

Ask the pupils to think of other worms that could be made, for example. long, wriggly and thin. Choose a word and ask pupils to make a worm that fits the description. Repeat with other descriptive words and create a word bank that is displayed with illustrations. Conclude by asking the pupils to draw their favourite worm.

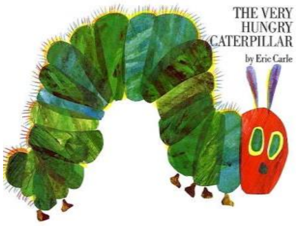
[21] Deboys, M. and Pitt, E. (1979)

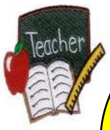
[22] From: http://nzmaths.co.nz/resource/worms-and-more

Caterpillars [23]



After reading the 'The Very Hungry Caterpillar'[24][25], provide children with modelling clay or playdough and pictures of caterpillars, and invite them to create their own caterpillars.





What will your caterpillar be like? How can you make one? Tell me about your caterpillar. How is your caterpillar similar to/different from Aoife's? Who has made the longest caterpillar? How do you know? Who has made the shortest caterpillar? How do you know? How could you make a longer one? How could you make a shorter one?

Pupils order their caterpillars by length. Discuss the importance of using a baseline, for example, the edge of the desk to line up the caterpillars.

The following activities can be used to extend and consolidate the vocabulary for length:

- Making patterns using long and short lines of objects (cubes, lollipop sticks, links etc.)
- Using thick and thin attribute blocks to make a pattern
- I Spy with my little eye something that is longer than/shorter than/taller than etc.

What am I?

Pupils take turns to select an item and use the language of length to describe the object. For example 'I am longer than a pencil but shorter than a hurley. What am I?' or 'I am wider than the door but narrower than the blackboard. What am I?' Encourage the other pupils to ask questions. 'Are you taller than the door?' Whoever guesses correctly takes the next turn.

Children's Literature for Developing the Vocabulary of Length

A picture book can be used as an introduction to a concept, as a launch pad to further explore an idea, or as a prompt for a discussion and debate. Using picture books with mathematical content can be an effective way

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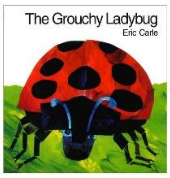
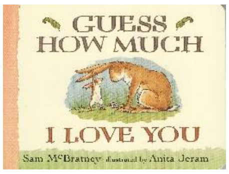
[23] From: http://nrich.maths.org/content/id/8861/Making%20caterpillars.pdf

[24] Carle, E. (1979) The very hungry caterpillar. New York: Collins Publishers.

to engage pupils and explore problems within entertaining contexts.[26] The following children's books may be used to prompt discussion and exploration of vocabulary relating to length.

- *'The Grouchy Ladybug* [27] The bad tempered insect is looking for a fight and encounters various creatures but none are big enough to fight! Pupils can discuss, compare and order the animals according to their size.





ICT Link
[Guess How Much I Love you Lesson]

- *'Guess How Much I Love You'* [28][29] The above ICT link, features a lesson based on this picture book. The book provides the opportunity to explore comparative measures and statements and link the language of measurement to an engaging story.

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[26] http://nzmaths.co.nz/picture-books-mathematical-content
[27] Carle, E. (1979) The grouchy ladybug. New York: Harper Collins
[28] http://nzmaths.co.nz/resource/guess-how-much-i-love-you
[29] McBratney, S. (1994) Guess how much I love you. Cambridge: Candlewick Press.

LEVEL A.2:

COMPARE AND ORDER OBJECTS OF DIFFERENT LENGTH, WIDTH AND HEIGHT

TEACHING NOTES

When comparing objects in terms of length, width or height, the comparison should initially involve objects which are clearly **only** different in one attribute, any of length, width or height. Later, other differences should be included. For example, as pupils gain confidence, comparisons should be extended to involve more than one attribute, such as length and width. In asking questions it is important to elicit from the pupils all attributes related to the comparison. For example, if the blue pencil is longer than the red pencil, then it follows that the red pencil is shorter than the blue pencil [30]

The use of appropriate language to support these experiences is crucial. Good use of language by the teacher will support the children's emerging ideas of length and will provide a model for the development of their language. As children use the language of length correctly in practical situations they demonstrate their developing grasp of this important concept (Eunice Pitt, Ready, Set, Go – Maths; Measures, Shape and Space and Visual Representation, 2005).

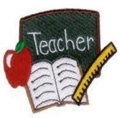
SAMPLE LEARNING EXPERIENCES

Compare Two Objects of Different Length, Width or Height

1. Pupils initially compare objects which are different only in length, width or height.

What do you notice about these pencils? How could we describe these pencils? How are these pencils the same? How are these different?

2. Pupils compare objects which are different only in length, width or height. Questioning focuses on the use of comparative language.



*Find some things **longer** than this straw.*
*Find some things **shorter** than this straw.*
How will we check if we are correct?
*Whose ribbon is **wider**? Darragh's or Sophie's?*
*Whose ribbon is **narrower?***
How can you be sure?

[30] Deboys and Pitt, 1979.

Pupils should handle a wide variety of objects of varying length, width and height. They should make comparisons by eye and then check practically.

3. Pupils compare objects with more than one difference, for example length **and** width.

*Look at the ribbon and the string. Which is **wider?** How can you tell? Are the ribbon and the string the same length? Let's check. Which is **shorter?** So which is **longer?***
So what can we say about the ribbon and the string?

Order Three Objects of Different Length, Width or Height.

1. Pupils will order objects where the difference is only length, width or height.

Which of the three skipping ropes is the longest? How can we compare the ropes? How can we be sure of a fair test?

2. Pupils will order objects with more than one difference for example width and height.

Let's compare this ribbon and this piece of wool. Which is the longest object? How do you know? How can we check?

Find an Object of Similar Length, Width or Height

Length Station[31]

Create a length station in your classroom. Sort objects as longer than, shorter than or roughly equal to a reference item. The pupils can contribute items to the length station. The pupils can sort objects as longer than, shorter than or roughly equal to a reference item. This station can also be used for width, height etc.

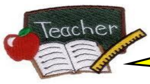
[31] Van de Walle et al., 2007, p.378

Teddy Bears & Friends[32]

Pupils bring a soft toy or teddy to class. Pupils introduce their teddy to the class. One pupil puts her teddy in the middle of the circle.

Who has a teddy that is different to Sarah's teddy? How is it different? How can we check?

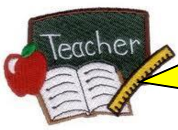
Let the pupils take turns bringing their teddies into the centre to compare. Put taller teddies in one group, shorter teddies in another and teddies of the same height in the third group. After heights have been compared invite pupils to suggest other ways that the teddies could be compared. For example: bigger or smaller feet, longer or shorter legs. In groups of three, pupils put their teddies in order. Elicit from the pupils their reasoning for ordering their teddies in such a way.



Why did you line up your teddies in that order? Why does Joe's teddy come first? How is it different to Michael's teddy?

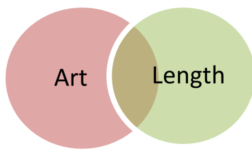
Trains

Pupils create trains using blocks or cubes. Pupils compare these trains by the number of blocks using the appropriate language for example longer, shorter, the same. Encourage pupils to explain their comparisons.



How do you know that your train is shorter or longer? How could you make your train longer than Sarah's? How could we record our trains after we put away our blocks?

Rainbow Cloud



Supply the pupils with crepe paper in rainbow colours. Pupils create cloud rainbows by sticking strips of coloured paper onto cloud-shaped white card. Pupils order their rainbow strips from the longest to the shortest and stick them onto the cloud.

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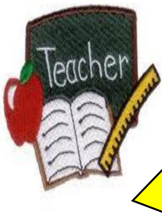
32 http://nzmaths.co.nz/resource/teddy-bears-and-friends



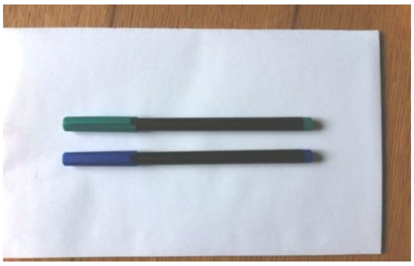
What colour is the longest strips? Why is the blue strip at the end? How do you know it's the shortest? Which strip is longer than the green one but shorter than the yellow one? How do you know? What can you ask your partner about their rainbow cloud?

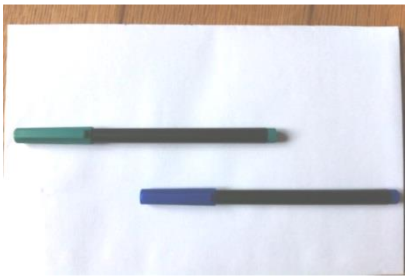
Conservation of Length

Conservation is a difficult, but very important concept for young children. Once children have grasped the idea of equivalence of length they should begin to explore the notion of **conservation**. Essentially they need to appreciate that length **remains constant** even though a change in form or arrangement occurs. Pupils need to display an understanding of conservation of length before progressing any further. If not, pupils need further opportunities for comparing, ordering and equivalence (Deboys & Pitt, 1979).

Conservation of Length Assessment

Take two pens of different colour, otherwise identical.

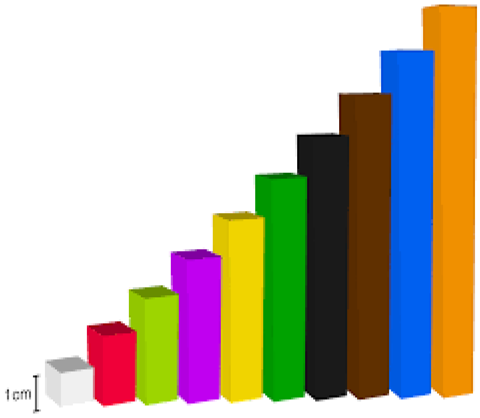




Is the green pen the same length as the blue pen, or are they different lengths?
Watch carefully. The green pen stays where it is and I move the blue pen. Is the green pen the same length as the blue pen or are they a different length? Why do you say that?

Once pupils have developed an understanding of conservation of length, they may benefit from experiences using Cuisenaire rods.

CONSOLIDATION ACTIVITIES



The following Cuisenaire activities are from the Ready, Set, Go Maths Manual. [33] For each of the games, you will need a collection of rods spread randomly across the table.

Match the Rod

The first pupil closes the eyes while the second pupil places one of the rods in his hand behind his back. The pupil then opens his eyes, carefully feels the rod behind his back (no peeping!) and with the other hand explores the rods on the table to choose a rod that is just the same. The pupils check the rods together. The game continues with the roles reversed.

Fill the Gap

This time two rods are placed behind the back of the first pupil. The pupil compares the rods by feeling them, focusing on the 'gap' or difference between the rods. He then chooses a rod from the table to 'fill the gap'. The pupils check the rods together and the game continues.

Two for One

One rod is placed behind the back of the first pupil. This time the game is to choose two rods that together would match the one that is hidden. Once again the pupils check the rods together and the game continues.

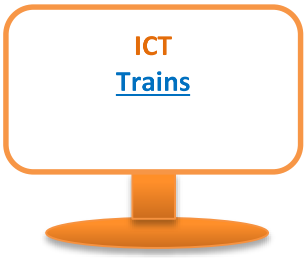
Trains [34]

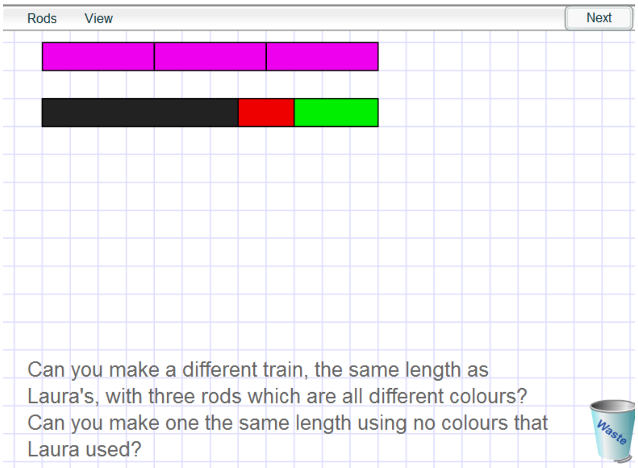
This activity is from the Nrich website and an interactive Cuisenaire version is available through the ICT link. Show the pupils a 'train' made from three pink Cuisenaire rods.

With your partner, make a train the same length as mine, using three different Cuisenaire rods. How did you do it? How can we be sure they are the same length? Now make a train the same length as mine using four Cuisenaire rods.

[33] Pitt, E. (2001) Ready Set Go Maths, p.88
[34] http://nrich.maths.org/4331

This activity can be repeated with various combinations of rods. Pupils can work in pairs to test out their solutions and check if they are correct.





Length Scavenger Hunt

In pairs or small groups, pupils find an object that is the same length as a given object or set of objects. This task can be completed in a variety of settings for example at desks, in the classroom, in the school hall, at home. Pupils must justify and explain how they know it is the same length as the given object. This task can be replicated for other attributes such as width and height.

LEVEL A.3

ESTIMATE AND MEASURE LENGTH IN NON-STANDARD UNITS

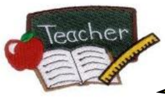
TEACHING NOTES

Situations should be presented to help pupils see the need for some form of measuring. One example could be comparing the heights of pupils. Who is taller? Right from the start, the pupils should be encouraged to be as accurate as possible. At this stage, parts of a unit may be spoken of as 'nearly', 'just over', 'and a bit'.[35]

SAMPLE LEARNING EXPERIENCES

Progression in Measuring

Step 1. Use different units to measure different objects



Let's measure the length of the table. What objects can we use to measure the length of the table? With your partner, estimate and record your estimation. Measure and record your answers.

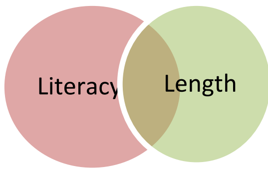
Step 2. Use different units to measure the same objects.

Step 3. Use the same unit with different objects. Pupils should compare their measurements for the different objects.

Note: The same type of measuring activities can be carried out using body type measurements: width of finger, hand span, cubit and foot.

A cubit is an ancient unit for measuring length based on the forearm length from the elbow to the tip of the middle finger.

The Giant's Foot



After reading 'Jack and The Beanstalk' or a story about a giant, pupils use a 'giant' footprint to estimate and measure various items in the classroom.

What do you notice about the giant's footprint? With your partner, estimate how many giant's footprints would fit across your table? How can we check? Why do you think so? If we measured using your footprints, would we get the same answer? Why?

[35] Deboys & Pitt 1979, p.78

CONSOLIDATION ACTIVITIES

Estimate and Measure with Chef Pierre[36]

This interactive activity requires pupils to estimate and measure in non-standard measuring units.

ICT
Estimate & Measure with Chef Pierre

Who has the Tallest Dog?[37]

This activity could be used on an interactive whiteboard. Pupils must estimate, measure, compare and order the puppies. Blocks are provided as a non-standard unit of measurement.

ICT
Puppy problem solving

[36] http://www.compasslearningodyssey.com/sample_act/math_k/grade/subject/mak_04_03_03.html
[37] http://resources.hwb.wales.gov.uk/VTC/2008-09/maths/puppies/FullRelease-v104/tallestPuppy-en.html

MEASURES - LENGTH

LEVEL A.4

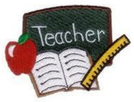
SELECT AND USE APPROPRIATE NON-STANDARD UNITS TO MEASURE LENGTH, WIDTH OR HEIGHT. DISCUSS REASONS FOR CHOICE

TEACHING NOTES

At this level, pupils measure length, width or height of objects and choose which non-standard unit of measurement to use.

SAMPLE LEARNING EXPERIENCES

Provide a variety of objects to choose from, for example, lollipop sticks, matchsticks, toy cars, cubes, links. Pupils measure the length of an object and choose which unit to use.



Aoife, estimate how many links you will need to measure the length of your copy. How many links did you need? Why did you use the links? Darragh you chose the cubes to measure the same copy. Is your answer different or the same? Why?

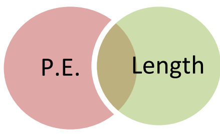
Making Lengths[38]

Draw a line on the ground using chalk or else mark it out with masking tape. Pupils work in pairs or small groups to measure the line using a non-standard unit of measurement. Pupils choose from a collection of different non-standard units to measure a line, for example Cuisenaire rods, cubes, blocks, links, straws, pencils, crayons etc. Pupils use the units to make a line the same length as the one they are measuring on the floor. It is essential that pupils choose and use a set of identical units. Record which units were used and how the line was measured.

CONSOLIDATION ACTIVITIES

Beanbag Throw

Four or five children will take turns to throw a different coloured beanbag. The other children must decide which beanbag was thrown the furthest.



Which beanbag was thrown the furthest? How do you know? Why is it important to throw from a starting line? How can we show that Molly's beanbag is the furthest away? What could we use to measure it? Which is the best unit to measure the distance? Why do you think this? How can we record the distance thrown?

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[38] http://www.schools.nsw.edu.au/learning/7-12assessments/naplan/teachstrategies/yr2014/img/teaching_measurement_es1_s1.pdf p.33

MEASURES - LENGTH

LEVEL B.1

ESTIMATE, COMPARE, MEASURE AND RECORD LENGTH USING NON-STANDARD UNITS, AND DEEPEN UNDERSTANDING OF THE CONSERVATION OF LENGTH

TEACHING NOTES

The learning experiences using non-standard units of measurement for Level B.1 are the same as for A.3. Pupils should be provided with opportunities to estimate, compare, measure and record length using non-standard units. Through measuring activities with non-standard units of length, ample discussion and questioning, pupils will come to realise the need for a standard unit of measurement.

The amount of time that should be spent using non-standard unit models varies with the age of the children and the attributes being measured...When non-standard units have served their purpose, move on. Non-standard units provide a good rationale for using standard units. A discussion of the need for a standard unit can have more meaning after groups have measured the same objects with their own units and arrived at different and sometimes confusing answers (Van de Walle et al., 2007, p.377).

MEASURES - LENGTH

LEVEL B.2

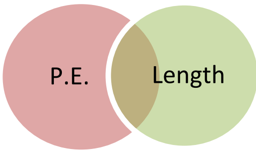
SELECT AND USE APPROPRIATE NON-STANDARD MEASURING UNITS AND INSTRUMENTS AND DISCUSS REASONS FOR CHOICE

TEACHING NOTES

The learning experiences at this level are similar to those contained within Level A.4. Some further learning experiences are outlined below. Some useful non-standard units of length are: clothes pegs, playing cards, links, cubes, shoe-laces, pipe cleaners, straws, pencils, lollipop sticks, craft match sticks.

SAMPLE LEARNING EXPERIENCES

Mini Olympics

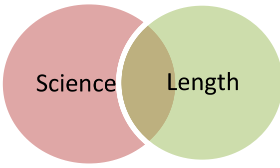


Pupils decide how best to measure the length of various activities which they engage in, for example, long jump, triple jump, ball throwing, bean bag throwing. Each pupil keeps a record of these measurements and can include them in their personal benchmarks notebook. The various lengths can be ordered from shortest to longest and the distance between lengths can be calculated. Pupils should discuss how best to measure the distances to ensure accuracy and fairness.

Tracking Growth[39]

Pupils track the length of various items as they grow, for example cress seeds. Using yoghurt pots, wet cotton wool and cress seeds, pupils can grow their own cress seeds in class. Pupils must decide how they will measure the growth of their seeds. Pupils could also estimate how much growth they expect in a week.


[40]



Making Rulers

Pupils make their own measurement device for length that use 10 of a certain item, for example 10 lollipop sticks. This measuring device can be the called a 'decalollipop' and used to measure classroom objects, for example, the desk is 3 and a bit decalollipops. Other suggestions can be a decacube, decastraw. This activity could lead to informal conversation about the accuracy of these measurements and the need for a standard unit of measurement.

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[39] Department of Education and Training in Western Australia (2005,p.40)

[40] http://ichef.bbci.co.uk/childrens-responsive-ichef/r/720/1x/cbeebies/cbeebies-mrbloomsnursery-img-makes-cressheadsmain_432_243.jpg

MEASURES - LENGTH

LEVEL B.3

ESTIMATE, MEASURE AND RECORD LENGTH USING STANDARD UNIT (THE METRE, HALF METRE AND CENTIMETRE)

Before introducing the standard unit of length, pupils should be very familiar with the vocabulary associated with measurement of length.

Teaching standard units of measurement can be organised into three learning objectives:

1. Familiarity with the unit of measurement.
2. Ability to select an appropriate unit. Pupils need practise in selecting appropriate units of measurement and monitoring the level of precision.
3. Knowledge of relationships between units ([41]).

Always begin a measuring activity with pupils making an estimate. This is true with standard and non-standard units (Van de Walle et al., 2007, p.379).

SAMPLE LEARNING EXPERIENCES

Introducing the Metre[42]

Show pupils a metre long strip of card or paper and ask the pupils to consider how long the piece of paper is. Challenge the pupils to think about **how** they came up with their estimate.

How could we check our answers? Using the equipment we have in the room, what would give us the most accurate answer?

Measure the strip of paper with the metre stick or rulers if suggested by pupils. Pupils should be encouraged to see that a metre stick or ruler will provide them with a quick, accurate and consistent measurement.

What could we do to measure the strip of paper if we didn't have standard measuring tools like a ruler or metre stick?

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[41] Van de Walle et al., 2007, p.378

[42] http://nzmaths.co.nz/resource/making-benchmarks-length

The discussion should focus on developing personal benchmarks for a metre. Pupils may need specific prompts such as ‘How could we use parts of our body to show 1 metre in length?’ or ‘Where in the classroom can we find a benchmark for 1 metre?’

Pupils work in pairs or groups. Provide each group with a metre length piece of string/paper. They will use this to experiment, finding 1 metre lengths on their bodies to be used as personal benchmarks. Pupils record their findings in their personal benchmarks notebook or learning journal.

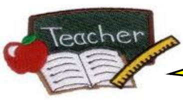
Is your foot/arm/hand span more than or less than 1 metre? Measure your leg? Is it longer than or shorter than 1 metre? Measure the length of your friend’s arm span? Is it longer than or shorter than a metre? Is there any measurement on your body about 1 metre long? In pairs, measure each other’s stride. Is it more than, less than or about 1 metre long?

Further Development:

Pupils use their personal benchmarks to find objects in the environment that are longer than 1 metre, shorter than 1 metre and about the same length as 1 metre. This can be followed by pupils using standard measuring devices to check their estimated lengths of given objects. The pupils can record their findings in their personal benchmarks notebooks or learning journal. For example, a door way is more than two metres and a door handle is about 1 metre from the ground.

Benchmark-Half Metre

The aim of the following activity is to equip pupils with other key benchmarks like ½ a metre so they can be more accurate in estimations and measurements of length.



What other benchmarks for length do you think it would be useful to have? Why do you think it would be useful to have that benchmark?

Brainstorm possibilities ensuring that benchmarks are just that – a few key measurements to aid estimating of various lengths. Discuss half a metre. Using their personal benchmarks for a metre, ask the pupils to locate some objects that might measure half a metre. How can we check for accuracy? Elicit suggestions. Using pieces of string or paper measuring 50cm, ask the pupils in pairs/groups to find and record personal benchmarks for ½ metre.

Further Development:

Pupils will use their personal benchmarks to estimate and measure the length of various items.

Exploring Estimation Strategies

Begin with a problem like "How far is it from one end of the mat to the other end of the mat?". (It could be a distance between 2 objects anywhere between 4-8 metres). Encourage pupils to suggest how they would estimate the length.

Pupils share their estimate and estimation strategies.

Anna thinks the distance is 2 ½ metres. How did you get that estimate? Liam, can you now explain how Anna came up with her estimate? Did anybody use a different strategy to Anna?

- Some pupils may act it out and use their metre benchmark as a non-standard unit of measure.
- Others may look at the distance and visualise a metre.
- If there were carpet tiles on the ground, another pupil may estimate the length of one carpet tile and multiply it by the number of tiles from one side of the mat to the other.
- Another pupil may know that the length of 2 desks is a metre and visualise the number of desks it would take to get from one end of the mat to the other.

Provide opportunities for many pupils to share their strategies. Summarise the discussion by saying that we have on-board benchmarks that include body parts but there will be other benchmarks that we can use, for example visualising the length of a 30 cm ruler.

Organise pupils into groups of 3 or 4 to discuss other possible benchmarks that may be useful to add to your benchmark 'belt'. Allow pupils 2-3 minutes to discuss and record other possibilities. Record the responses under the heading 'other possible benchmarks'. For example, pupils may come up with ideas like visualising a 30 centimetre ruler, knowing the door is 2 metres etc. Test out pupils' benchmarks by asking the various strategies and benchmarks they could use to work out the height of the chair and the table.

When standard units are used, estimation helps develop familiarity with the unit. If you estimate the height of a door in metres before measuring, you must think about the size of a metre (Van de Walle et al., 2007, p.379).

Using Metres for Measuring

Pupils should have plenty of opportunities to estimate, measure, discuss and check the length of distances and objects in metres and half metres.

Let's measure the width of the classroom using our metre strips. How many metres wide do you think it is? Estimate first. Billy, how wide do you think the classroom is? How did you get that estimate? Mary, your estimate is different. Why do you think it is about ____ metres wide?

Accept a wide range of estimates. Discuss how different students made their estimations. This will confirm that there is no single right way to estimate while reminding pupils of other approaches

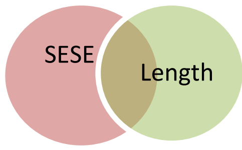
(Van de Walle et al., 2007 p.380).

	Estimate	**How I Estimated? (Strategy or Benchmark)**	**Measurement**
My height			
The distance from the door to the window			
The height of the door			
The length of the classroom			

The Decametre

While the decametre is a rarely used measurement of 10 metres, at this level it can be useful to teach pupils about decametres as it provides a sense of distance. In Level D, pupils often grapple with the size of a kilometre so it may be useful to introduce the decametre when measuring larger distances at this level. For example, pupils can measure out pieces of string up to 10 metres. Pupils should be encouraged to find distances around the school that equate to a decametre and record them in their personal benchmarks notebook. For example, the distance from our classroom to the front door. Pupils can use the trundle wheel to mark out a decametre.

Extension Activities:



Pupils could also find animals which grow to the length of a decametre or discover how many metres tall/ decametres tall are the Pyramids/The Eiffel Tower/The Spire etc. Pupil- friendly search engines such www.scoilnet.ie www.duckduckgo.com and www.kidrex.com may be useful for these tasks.

Introducing the Centimetre (2nd Class)

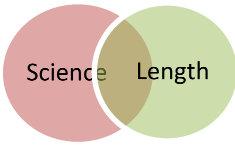
While estimating and measuring, situations will arise when it is necessary to measure with a unit smaller than a metre. The size of the centimetre unit can be established by constructing it, for example by cutting 1-centimetre pieces of paper or straws. An appreciation of the size of the unit can be built up through lots of experience in measuring everyday objects. The pupils should be encouraged to develop their own reference for a centimetre, for example, a fingernail.[43] These can be recorded in pupils' personal benchmark notebooks.

Using Centimetres for Measuring

Pupils should estimate, measure, check and discuss the length of various objects in centimetres. A ruler is a useful way of measuring the length of smaller objects. Pupils should also use their rulers to measure lengths and discuss how that can be done. Provide pupils with rulers. Discuss where you begin measuring from.

Many students have misconceptions where you begin measuring from. Reinforce that they start measuring from the 0 and not the edge of the ruler. (http://nzmaths.co.nz/resource/making-benchmarks-length)

Pupils can then measure accurately a list of given items. Compare the measurements pupils come up with to ensure all pupils are accurately using the measuring tape or ruler.



Make a Decimetre[44]

Pupils work in pairs and make a ruler using 1 cm grid on light card. Cut out a 10 cm strip and colour one long edge as the measuring edge. Label the centimetre end points from 1 to 10. Find and record objects which measure longer than 10 cm, less than 10 cm or longer than 100 cm.

Crooked Paths [45]

Make some crooked or curvy lines on the ground outside using chalk or inside using tape. Ask for suggestions for measuring the crooked lines.

Can anybody suggest how we could measure the crooked lines? Why is it not as easy as measuring a straight line? Does anybody have a different idea about measuring the crooked line? How long do you think it might be? Record your estimate before we measure. Anna, how did you get your estimate? Billy, how did you come up with your estimate? What strategy did you use?

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[43] http://nzmaths.co.nz/length-units-work
[44] http://www.schools.nsw.edu.au/learning/7-12assessments/naplan/teachstrategies/yr2014/img/teaching_measurement_es1_s1.pdf p.42
[45] Van de Walle et al., 2007,p.381

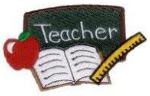
Encourage the pupils to share their solution strategies for solving the problem. Repeat this activity by comparing straight lines and crooked lines and asking pupils to estimate which is longer.

We have a straight line and a crooked line. I wonder which one is longer. Which do you think is longer Liam? Why? Anna thinks this one is longer. Why? Use a strategy to estimate the length of the lines. How can we see which is longer? Measure both lines and compare your estimates.

Research indicates that when students see standard rulers with the numbers on the hash marks, they often believe that the numbers are counting the marks rather than indicating the units or spaces between the marks. This is an incorrect understanding of rulers that leads to wrong answers (Van de Walle et al., 2007, p.384).

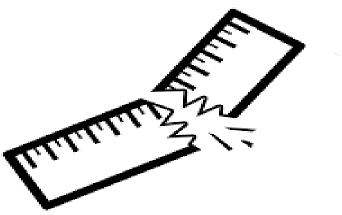
The Broken Ruler[46]

The purpose of this activity is to measure various small objects with a 'broken' ruler. Templates for centimetre rulers can be found through the ICT link[47]. These can be printed out, variously altered to be 'broken rulers' and laminated for use by the pupils. Pupils suggest how to measure the same items (for example a textbook or a paper clip) using a broken ruler.



We are going to measure the length of our paper clips/notebooks. Estimate first. How long do you think it is Mary? How did you get that estimate? The rulers we are using are broken. There is no 0 mark on any of them. Can we still measure the length? How?

Elicit various suggestions from the pupils. The pupils should compare their answers from the broken ruler with an unbroken ruler. Were the answers the same? If not, why not? If pupils are struggling, encourage them to measure using the unbroken ruler and count the centimetre segments along the object being measured. Eventually pupils describe length as the total number of unit segments needed to match the length of an object.



[46] Van de Walle et al., 2007, p.384
[47] https://www.teachervision.com/tv/printables/scottforesman/Math_2_TTM_29.pdf

CONSOLIDATION ACTIVITIES

Gummy Worms

Pupils work in pairs to measure the length of gummy worms in centimetres. Pupils record the length of the worms. Pupils should now stretch their gummy worms without breaking them. How far can they stretch them? How much longer are the worms after being stretched?

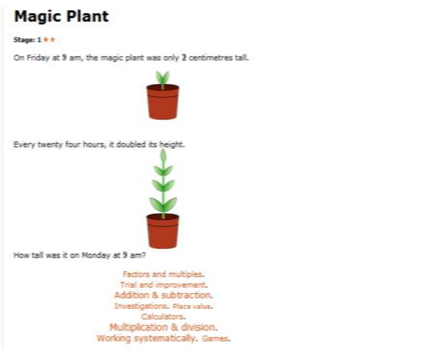
Length Scavenger Hunt[48]

Provide teams or pairs of pupils with a list of measurements and ask them to find objects or distances that match the measurements given. The list could also include some non-standard units of measurement and personal benchmarks as well as standard units of measurement.

- A length of 10cm
- Something that is the same length as your Maths book
- A length of 1 metre
- Something that is more than 20cm but less than 1 metre
- Something that is around the same length as your finger

The Magic Plant[49]

This problem from the Nrich website requires pupils to calculate in centimetres the growth of a magic plant.



ICT
[The Magic Plant](http://nrich.maths.org/145)

Robot Monsters[50]

Pupils are required to create a robot monster using parts of robots which are of different length. Pupils can find the tallest/smallest robot possible to make an experiment with various heights of monsters.

ICT
[Robot Monsters](http://nrich.maths.org/2404)

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[48] Van de Walle et al.,2007, p. 381.
[49] http://nrich.maths.org/145
[50] http://nrich.maths.org/2404

MEASURES - LENGTH

LEVEL C.1

ESTIMATE, COMPARE, MEASURE AND RECORD LENGTHS OF A WIDE VARIETY OF OBJECTS, USING APPROPRIATE METRIC UNITS, AND SELECTING SUITABLE INSTRUMENTS OF MEASUREMENT

TEACHING NOTES

At this level, pupils should be familiar with the standard units of measurement for length (metres & centimetres). However some experience of measurement using non-standard units is necessary to help pupils appreciate the need for standard units 2[51]).

SAMPLE LEARNING EXPERIENCES

Measurement Using Spans

Pupils make a cardboard cut-out of their hand span. Pupils then use the cut-out to estimate and measure the length of objects in the room. Pupils could also use cardboard cut-outs of their feet or else a piece of string that represents the length of their stride. After measuring in non-standard units, it is important to afford time to compare pupils' answers through discussion to raise awareness of the necessity for standard units of measurement.

	Estimate in spans	Measurement in spans
Width of desk		
Length of whiteboard		
Length of school bag		

Estimation & Measuring to the Nearest Metre

Pupils should estimate, measure, compare and discuss objects and distances in the school to the nearest metre. These measurements could be displayed throughout the school. Sample objects could be the height of doors, length of corridors and width of classroom. Pupils should be encouraged to share their estimations and also how they reached that estimate. Did they use a benchmark? (See Level B.3)

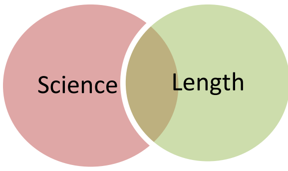
	Estimate in metres	Measurement in metres
Width of classroom		
Length of corridor		
Height of door		

[51] Deboys and Pitt, 1979

A trundle wheel can be introduced to mark out specific distances. Children should begin from a definite mark and walk in to a straight line with a partner marking the finishing point correctly.

Trundle Wheel Activities

- Pupils stand in a line side by side. The pupils will estimate how long the line will be before the line is measured to the nearest metre using a trundle wheel. Then pupils line up with arms outstretched palm to palm. Discuss how this will alter the measurement of the line.
- Pupils estimate, measure, compare and discuss the length of the playground/flower beds/garden etc. using a trundle wheel.
- Elicit from pupils in what professions might someone use a trundle wheel.



Paper Skeleton[52]

Pupils work in groups of 4 to make a skeleton outline of a member of their group. Pupils measure out various parts of a group member's body with paper. Remind the pupils to label the strips so they don't mix them up. Pupils can measure and cut the following:

- the length of your back from the top of your neck to the bottom of your back
- length of skull from the top of the head to the bottom of your neck
- distance between shoulder blades
- length of arm from shoulder to elbow
- length of arm from elbow to hand
- length of hand (length of fingers!)
- length of leg from hip to knee
- length of leg from knee to foot
- length of foot

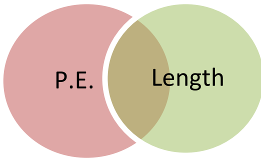
Pupils can then assemble the life size 'skeleton' on black sugar paper.

The Hectometre

From Level B, pupils may be familiar with the decametre. While not commonly used, the decametre is equivalent to 10 metres. It may be useful to revisit the decametre (Level B.3). Pupils can use the trundle wheel to mark out a hectometre (100 metres). This can be marked in the yard or recorded in the pupils' personal benchmarks notebooks. Pupils should discuss whether various distances are greater than or less a hectometre, for example the door of the school to the school gate, the length of the hurling field.

[52] http://www.primaryresources.co.uk/maths/pdfs/15skeleton.pdf

100m Sprints



- Mark out a 100m section suitable for running. (Alternatively a distance of 50m could be ran twice) The pupils take turns running a hectometre and record their time. Compare their time to the world record holders.
- Investigate what the Olympic Record for the hectometre is. Why does it differ from the World Record?
- The pupils can investigate the Irish and World Records for 200m & 400m sprints. Further investigation could involve 4 x 100m relay races.
- Using a stopwatch, have pupils see how they can run in 9 or 10 seconds and measure using a trundle wheel to the nearest metre.

Estimating and Measuring to the Nearest Centimetre

Pupils should estimate, measure, compare and discuss objects and distances in the school to nearest centimetre. Pupils should be encouraged to share their estimations and also how they reached that estimate. Did they use a benchmark? (See Level B3)

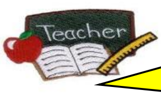
	Estimate in cm		
Copy book			
Pencil case			
Paint brush			

Paper Planes[53]

Pupils construct simple paper planes. Discuss flying the planes with pupils.

How could we measure how far our planes will fly? What will we need to be careful of when measuring the distance? How can we be sure we are measuring the distance accurately?

Discuss the merits of using non-standard units of measurement and standard units. Which would be better to use? Why would it not be fair to use spans or strides to measure? Show the pupils a variety of measuring tools for example ruler, tape measure, trundle wheel.

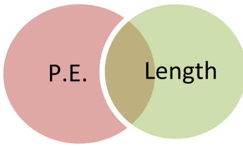


Which of these measuring tools do you think would be best to measure the distance of our plane's flight? Why? What other tools could we use?

[53] http://nzmaths.co.nz/resource/paper-planes-level-2

The pupils should estimate how far in metres and centimetres their plane is going to fly. Have pupils experiment with a variety of measurement tools to measure the flights of their paper planes. As they work, encourage estimation and reinforce the correct use of measurement tools to ensure measurements are accurate to the nearest metre and cm.

CONSOLIDATION ACTIVITIES

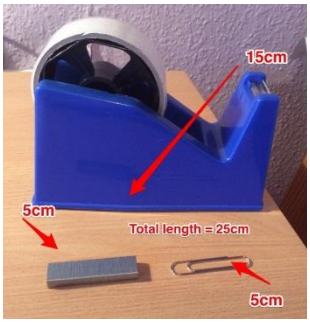


Mini Olympics (2)

Pupils decide how best to measure the length of their various activities which they engage in, for example, long jump, triple jump, ball throwing, bean bag throwing. The various lengths can be ordered from shortest to longest and the distance between lengths can be calculated. Pupils should discuss how best to measure the distances in metres & centimetres to ensure accuracy and fairness. Alternatively pupils could design a simple mini-Olympics for younger classes and take charge of recording their results of various activities such as bean bag throwing, long jump, welly-throwing. Pupils should pay close attention to accuracy of lengths.

Any Three Items[54]

Pupils work in pairs to find three items in the classroom which have a total length of 25 centimetres. Pupils record their findings by drawing the items, labelling with the measurements in centimetres, and showing how the three lengths were added to make a total of 25 centimetres.



How do I use a ruler to measure a length? What mistakes could I make? What does 25 cm mean? What objects would be shorter than 25 centimetres? How will you make a total length of 25 centimetres? How will you draw and record your measurements? What units of measure will you use?

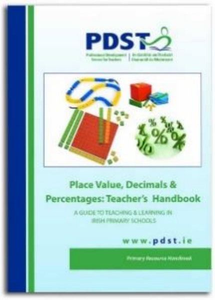
Extension: Replace one of the three items for two items so that these four items also measure 25 cm.

[54] http://www.schools.nsw.edu.au/learning/7-12assessments/naplan/teachstrategies/yr2014/img/TM2-3.pdf p.

LEVEL C.2

RENAME UNITS OF LENGTH IN M AND CM AND USING DECIMAL OR FRACTION FORM

TEACHING NOTES



PDST have produced a resource to assist teachers in teaching place value, decimals & percentages. Teacher may find the activities contained within this manual useful at this level. This manual is available free to download from www.pdst.ie

The relationship between centimetres and metres is clearly marked on most metre sticks. There are usually 10 divisions with 10cm in each. [55] Pitt suggests pupils engage in practical activities to develop understanding of the relationship between centimetres & metres. Understanding of the role of the decimal point as indicating the units' position is a powerful concept for making metric conversions.

> Avoid mechanical rules such as 'To change centimetres to metres, move the decimal point two places to the left.' Instead create, conceptual, meaningful methods for conversions rather than rules that are often missed and forgotten (Van de Walle et al., 2007, p.379).

In Level B, pupils were introduced to decametres, the equivalent of 10 metres. Earlier in Level C, pupils became familiar with hectometres or distances of 100 metres. The mathematical term for 10cm or one tenth of a metre is a decimetre.

> Derek Haylock suggests that since there are 100cm in a metre, the measurement of length in centimetres and metres offers a close parallel to the recording of money. In this context, it is helpful to exploit children's familiarity with money notation and press the parallel strongly, following the same convention of writing two digits after the point when expressing length in metres, for example writing 3.20m instead of 3.2m (Haylock, 2014, p.226).

SAMPLE LEARNING EXPERIENCES

Metres & Fractions

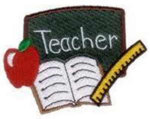
Give pairs of pupils, three 1 metre long strips of card. Pupils represent halves on one strip, quarters on another strip and tenths on the final strip.

[55] Deboys and Pitt, 1979. P175

*The first metre strip is divided into two parts. How many cm in each half? How did you divide your paper strip into quarters? How many cm in a quarter of a metre? How **many** cm in three quarters of a metre? How did you get your answer? Did someone do it a in a different way to Anna?*

How many equal parts have you altogether on your third metre? So there are 10 equal parts. On that paper strip, show me 3/10 of a metre. How many centimetres is that? How do you know? Show me 5/10 of a metre. Does anybody know a different name for 5/10 of a metre? Which is greater: ¼ of a metre or ½ a metre? Explain. What is the difference in centimetres? How did you get your answer?

The above activities can also be completed using lengths of string and paper clips/pegs. Here the focus could be on estimation (pupils estimate where to mark the fraction rather than folding). Pupils could construct their own metre sticks using decimetres or lengths of 10cm to be used as a basis for discussion.



Show me 0.60 of a metre using your metre stick. How many centimetres is there in 0.60 metre? How many centimetres in 0.90 of a metre? Where on your number line would you find 0.55m? Billy, how did you get that answer? Did anybody do it a different way? Look at where I am pointing on my metre strip. What decimal fraction of a metre could be here? Why do you think so?

CONSOLIDATION ACTIVITIES

Going the Distance

Working in pairs or groups, pupils will measure how far they run in 10 seconds. Pupils can record their distance in metres and record the distance in decimal notation.

Oral Maths Activities

Number Fans: What is 0.40 of a metre in centimetres? What is 0.32 of a metre in centimetres? How many centimetres in 1.56m? How many centimetres in 1.06m? What must I add to 0.65m to make 1 metre?

Counting Stick: Pupils can count in tenths of a metre, forwards & backwards. Show me 0.75 of a metre. Where would 0.35 of a metre be on the counting stick?

Loop cards- (See the resources in Appendix A for an example).

MEASURES - LENGTH

Target Board

1m	½ m	90cm	25cm	0.10cm
20cm	2m	10cm	3.20m	50cm
¼ m	1 ¼ m	2 ½ m	1.30m	99cm
1.10m	0.1m	0.60m	85cm	0.70m

1. What is the longest measurement on target board? How many centimetres altogether?
2. What is the shortest measurement? How do you know?
3. What is the difference between the longest and shortest measurement?
4. What must I add to each measurement to make a metre?
5. Can you find matching pairs of measurements?
6. Which measurements are less than 1 metre/ ½ metre/ ¼ metre?
7. Which measurements are more than 1 metre? By how much?
8. Change all the measurement to centimetres?
9. Add all of the measurements in the first/second/third row. How did you do it? Does anybody have a different way of adding these lengths mentally?
10. Rename the measurements as a decimal fractions.
11. Find a measurement that is greater than ¾ metre.
12. Find a measurement that is less than 0.1metre.
13. Find two lengths with a difference of ¼ of a metre.

Human Number Line – A group of pupils are given a card each with a decimal fraction or a fraction of a metre written on it and stand in a line. Activities can include:

- Swap with the person who is holding 110cm, swap with the person holding 1 ½ metres etc.
- Round each number to the nearest metre.
- Order the measurements in centimetres and metres and in decimal form from the least to greatest, allowing pupils time to explain their reasoning and how they got their answers.

LEVEL C.3

UNDERSTAND, ESTIMATE AND MEASURE THE PERIMETER OF REGULAR 2-D SHAPES

TEACHING NOTES

Perimeter is the distance around a two-dimensional shape. At this point, it is assumed that pupils understand the need for standard units and can use tools that measure length. [56] As perimeter is a length, it is measured in units of length.

> It is widely noted that area and perimeter are a continual source of confusion for students. This may be because students are taught the formula for both concepts at the same time and tend to get each formula confused. Teaching these two concepts during a close time frame is particularly challenging for students with disabilities (Van de Walle et al., 2007, p.388).

> To avoid confusion between area and perimeter, use the illustration of fields and fences to differentiate between these concepts. Area is the size of the field. Perimeter is the length of the fencing (Haylock, p.348).

Confusion can arise between perimeter and area. However, if the children have had enough practical experience to find that length is a one-dimensional attribute and area a two-dimensional one, differences should be obvious (See pupil misconceptions).

The PDST have produced a comprehensive resource on Shape & Space that is available for free at http://www.pdst.ie It may be appropriate here to revise the characteristics of regular 2-D shapes. *Level C.2: Explore, describe, compare and classify the properties of 2-D Shapes.*

Within this level, pupils should be given the opportunity to measure the perimeter of 2-D shapes. Rather than be given a formula, pupils should discover that with regular shapes, it is only necessary to measure one side and multiply it by the number of sides ([57].

SAMPLE LEARNING EXPERIENCES

Physical Activities

Physical activities to develop the concept of perimeter may be carried out such as walking the perimeter of the classroom, perimeter of the school, running the perimeter of the school hall or sports field. Distribute squares of equal size to the pupils. Pupils come up with ways to measure the length of the sides or the perimeter of the square. Take various suggestions from the pupils.

[56] Van de Walle et al., 2007, p.387).
[57] Deboys and Pitt, 1979, p.306

Let's look at the square. How could we find the perimeter of the square? What do we need to find? What is the length of one of the sides? So how can we find the perimeter of the square now? Mary, how did you find the perimeter of the square? Did anybody do it a different way? Anna, did you get the same answer a different way?

While eliciting various solution methods from the pupils, accept all strategies. Allow pupils sufficient time measuring various squares to see that you multiply one side by 4 to get the perimeter. This activity can be extended to rectangles.

Let's look at a rectangle now. How could we find the perimeter of a rectangle? Could we try some of the same ways we used with a square? Why not? John, you added up the all 4 sides and found the perimeter. Did anybody else use a different way and get the same perimeter?

Fixed Perimeters[58]

Give pupils a piece of string that is 24cm long and 1 cm grid paper[59] (alternatively you could just use grid paper). The task is to see what different sized rectangles can be made with a perimeter of 24cm. Each different rectangle can be recorded on grid paper. Pupils can label the length of the sides of the rectangle and check their rectangles have a perimeter of 24cm.

ICT
Printable Grid Paper

Rectangles & Squares in the Environment

Pupils find, estimate, measure, compare and discuss the perimeter of rectangles and squares in the environment

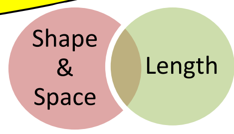
Object	**Estimate**	**Strategy**	**Perimeter**
Desk top			
Maths book cover			
School hall			

Pupils should also find and discuss the perimeter of regular triangles, pentagon, hexagons and octagons. Through discussion and practical experience, pupils should realise it is only necessary to measure one side of the shape and multiply it by the number of sides.

[58] Adapted from Van de Walle et al.,2007, p.388
[59] http://www.activityvillage.co.uk/grid-paper-1cm-square

What are we measuring the perimeter of now? That's right, it's a pentagon. How do you know it's a pentagon? Is it regular or irregular? How do we know? How could we find the perimeter of the pentagon? Now John suggested we could add up all the sides of the pentagon. Is there another way of finding the perimeter of a regular pentagon? Would that way work with an irregular pentagon? Explain.

Shape Constructions with Anglegs or Geostrips



Pupils will create 2-D shapes with Anglegs and find the perimeter of the shapes by measuring the sides. This activity can also be completed with strips of paper.

Aoife has constructed a shape with a perimeter of 24cm. Has anybody else constructed a shape of similar perimeter? Is it the same shape? How can we record our findings?

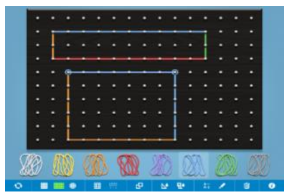
This activity can be extended to discuss types of angles in the constructed shapes.

Interactive Geoboards[60]

The following activity can be carried out on laptops, tablets or the IWB. Alternatively pupils can use geoboards & elastic bands. Ensure that pupils understand that the perimeter on the geoboards in measured in units. Ask pupils to work with a partner to make various shapes. Tasks can be given to pupils such as:

- Construct a square with a perimeter of 16 units
- Construct a rectangle of sides 4 units and 5 units. Find the perimeter.

ICT
[Geoboard App]



Open-ended tasks with more than one solution will allow for discussion and extension.

- Construct a 4 sided shape with a perimeter of 24 units
- Construct four different shapes on the geoboard and record the perimeters in your copy.

CONSOLIDATION ACTIVITIES

Room for Elbows[61]

Pupils design a dinner table which will seat four pupils along each side, with enough space to eat comfortably. Pupils draw a diagram of the table with listed reasons for the dimensions.

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[60] http://catalog.mathlearningcenter.org/apps
[61] http://www.schools.nsw.edu.au/learning/7-12assessments/naplan/teachstrategies/yr2014/img/TM2-3.pdf p.34

LEVEL D.1

ESTIMATE AND MEASURE LENGTH USING APPROPRIATE METRIC UNITS (MM, CM, M, KM) AND APPROPRIATE INSTRUMENTS OF MEASUREMENT

TEACHING NOTES

At this level, pupils should be afforded more opportunities to estimate, measure, compare and discuss lengths involving metres & centimetres. Refer to Level C.1 for sample learning experiences involving hectometres, metres and centimetres. Knowing, understanding and having some 'feel' for the basic units of measurement are important skills. Similarly, knowing how to measure these units is important too.[62]

SAMPLE LEARNING EXPERIENCES

How Far is a Kilometre?[63]

From level C, pupils may be familiar with the concept of a hectometer (100m). Begin by questioning the pupils to establish the need for a unit larger than a metre:

How far do you think it is from the school gate to the hurling field/community centre/next village? Is it more or less than a hectometre? How could we measure?

Discuss the impracticality of using a metre and the large number of units that would be required and introduce the concept of a kilometre as 1000 metres.

Measure out a route in your car from the school gates which is just over a kilometre. Make a note of a landmark at the 100m and 1 km mark. Give the pupils a simple map with the route marked on it. Ask the pupils to mark where they think the 1 kilometre mark will be. Walk with the pupils around the route. Pupils should attempt to point out where the 100m mark is. Pupils can make a second mark on their map if they change where they think the 1 kilometre mark is while they are walking. Make a note while walking of how long it takes the students to reach the 1 kilometre mark. When you return to the class ask the pupils to compare their estimates with other class members. Tell the pupils where the 1 km mark was and see who was close with their estimate.

Over the next few days, pupils work in pairs to develop a sense of the length of a kilometre. Using their maps with the kilometre walk marked on it, pupils can estimate whether local landmarks (shops, bus stops, church, own house) are more or less than a kilometre away from school. Pupils can record their findings in their personal benchmarks notebook.

[62] http://nzmaths.co.nz/resource/how-far-km
[63] http://nzmaths.co.nz/resource/how-far-km

Tell the pupils how long they took to walk the kilometre in the first session. Using this information pupils may be able to estimate how far distances they walk from home are (to school, to a friend's house, to the shops). If you have pedometers the pupils could measure distances using these. Pupils could then design a 1 kilometre circuit around the school and design a fitness activity. During the sessions reinforce the pupils' developing sense of the size of a kilometre.

Further Development

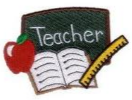
Brainstorm everything you know about 1km. How many times around the field/block is 1km? How long it takes to walk/run 1 km? How long does it take a car to go 1km? How does a car measure 1km? How many kilometres is it between two local destinations? How long would it take us to walk there? Or drive there? What is the longest trip you've gone on? What is the farthest you have biked, or walked or run?

Desks over the Horizon[64]

Pupils estimate, then calculate how many desks aligned end to end would fit into a line 1 kilometre long. Pupils record measurements and calculations. *Variation:* Pupils calculate how many times their body length would need to be repeated to measure 1 kilometre or how many times the length of a pair of pupils would need to be repeated.

Measurement using millimetres

The millimetre is the smallest recognised submultiple of the metre. Pupils should engage in discussion to come to an understanding for the need for a unit of measurement smaller than a centimetre.



I want to measure the length of a paper clip. How could I do it? Will I get an exact measurement if I measure in cm? Why not? How can we measure very small objects? What other objects would we need to measure in units smaller than centimetres? Can anyone think of someone who would measure in mm in their job?

Pupils should be afforded opportunities to estimate, measure, compare and discuss lengths of small objects in millimetres.

Object	Estimate in mm	Strategy	Measurement
Eraser			
Calculator			
Fingernail			

Pupils should also draw lines of given lengths in millimetres.

[64] http://www.schools.nsw.edu.au/learning/7-12assessments/naplan/teachstrategies/yr2014/img/TM2-3.pdf p.41

Ensure pupils use sharp pencils to draw lines to ensure accuracy while measuring in millimetres.

CONSOLIDATION ACTIVITIES

Length Scavenger Hunt

This activity builds on the scavenger hunt found in Level B.3. Provide teams or pairs of pupils with a list of measurements and ask them to find objects or distances that match the measurements given. The list could also include some non-standard units of measurement and personal benchmarks as well as standard units of measurement. Sample items could include:

- A length of 15mm
- A distance of 5km
- A length of 2 metres
- A distance that is the same as Cork to Dublin to the nearest kilometre.

Measurement Trail[65]

The mathematics trail can be made to accommodate any level of measurement expertise and any range of measurement topics. To make a mathematics trail (which can be inside the classroom or school buildings or outside, or a combination of both), the steps are:

1. Select 4 to 10 'stations' to form a route round the 'trail'. At each station students will have one or more questions to answer on a worksheet.
2. Make up questions for each station or site, which can ONLY be answered at that site.
3. Either have the questions fixed at each station OR have all questions included on the worksheet with space for answers.
4. Print copies for each group.

- **Points to remember:**
- Use the questions to practice or consolidate aspects of Measurement appropriate to the class
- Make sure the questions could only be answered while at the Station.
- Use a variety of types of question: closed and open; exact measurements and estimations; one-word answers and questions needing explanations; cover a range of types of measurement
- Make sure questions do not need elaborate equipment; for example each group could take only pencil, worksheet, a tape measure or ruler, and a calculator.
- Make sure you have all the answers!
- The Maths Trail should be easy to complete within one lesson

[65] http://nzmaths.co.nz/resource/make-measurement-maths-trail

MEASURES - LENGTH

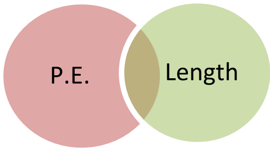
Suggestions for Measurement Questions (Of course, questions do not need to be all measurement)

- Stand at the station. Write your estimate of the distance in metres to the edge of the grass. Now measure and record your measurement.
- Estimate the height of the tree to your left. Explain your estimation strategy. Think about the strategy you used and how useful it was/was not.
- Draw and name an object about ten metres from the station.
- How high is each brick in the wall? How could this information help you to calculate the height of the wall?
- Calculate the perimeter of the door/window.

This trail could also encompass other aspects of Measurement:

- Estimate the area of the (irregular) flowerbed. Explain your estimation strategy.
- Estimate the weight of the flower tub.

Marathon



Show pupils a copy of the route of the Dublin Marathon or another marathon. Discuss the length of the marathon. Pupils could research the origin of the marathon and marathon world records. Using Google Maps, groups of pupils can plan a marathon route which should be as close to as a possible to 42.195 kilometres. The marathon route could begin from a local landmark or else begin/finish at the school. Extension: Pupils can determine the average speed of marathon record holders.

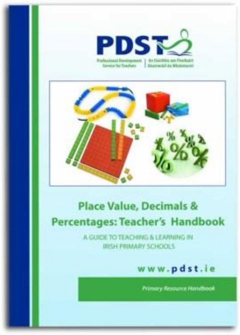
MEASURES - LENGTH

LEVEL D.2

RENAME MEASURES OF LENGTH AS FRACTIONS, DECIMAL FRACTIONS AND APPROPRIATE METRIC UNITS

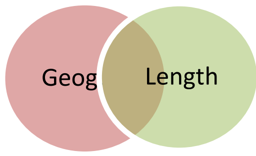
TEACHING NOTES

At this level, pupils should have a solid conceptual understanding of millimetres, centimetres, metres and kilometres. PDST have produced a resource to assist teachers in teaching place value, decimals & percentages. Teachers may find the activities contained within this manual useful at this level. This manual is available free to download from www.pdst.ie



While the metre is the standard unit for length, the metre is rather long for many situations in the classroom so the centimetre is the most frequently used unit in primary schools. However this can lead to difficulties as in building trades measurement is always given in millimetres. Thus 60cm is written as 0.6m or 600mm. This means that children need to aware of the relationship of metres and centimetres and between centimetres and millimetres.

SAMPLE LEARNING EXPERIENCES



Converting Kilometres to Metres

Pupils work in pairs to discover distance in kilometres using Google Maps.

Journey	**Distance in kilometres**	**Distance in metres**
Limerick – Nenagh	41.4km	41,400 metres
Tralee – Ballybunion		
Belfast – Dublin		
Galway – Dublin		
Thurles – Croke Park		
Tayto Park - Athlone		

[66]Converting Millimetres to Decimals[67]

ICT
[Converting metres to kilometres]

The metre stick is a useful model for thousandths when marked in millimetre increments. The length of the whole doesn't change and students can see that each interval can be subdivided (decimetres into centimetres, centimetres into millimetres) while the whole stays the same.

Figure these out with measurement ***less than 1 metre***

1. Given the fraction, write the decimal equivalent
2. Given the decimal, write the fraction equivalent
3. Given a fraction and decimal, determine if they are equivalent representations
4. Include fractional amounts greater than one for example write the decimal equivalent for $2\frac{56}{100}$ metre.

Various Values[68]

Kilometre	Hectometre	Decametre	Metre	Decimetre	Centimetre	Millimetres
			3	8	5	

How many ways can you represent the above information in different unit lengths using the decimal place? Explain and justify your answer. What is the function of the decimal place? What, if anything, does the decimal place tell us about the numbers sitting to the right/left of it?

Decimal Pairs Task[69]

Provide pupils with eight pairs of decimals and ask them to decide, for each pair, which decimal measurement is larger. It is important that pupils give you a rationale for this. The comparison should be made in their heads. It is not a pen and paper exercise. This activity should encourage the use of estimation and rounding.

Decimal Pairs	**Which is larger? Record your strategy**
1.1m or 101cm	
66mm or 0.06m	
12m or 0.12km	
0.5m or 60cm	
2.5km or 25mm	

[66] http://resources.woodlands-junior.kent.sch.uk/maths/practice/metres.html
[67] Adapted from PDST Place Value, Percentages & Decimals Manual page 84
[68] Van de Walle et al.,2007
[69] Adapted from PDST Place Value, Decimals & Percentages Handbook

CONSOLIDATION ACTIVITIES

Oral Maths Activities

Number Fans: How many millimetres in 3.5cm? Convert 4cm to millimetres. What is 26mm in centimetres?

Loop cards- (See the resources in Appendix A for an example).

Human Number Line – A group of pupils are given a card each with a decimal fraction or a fraction of a kilometre written on it and stand in a line. Activities can include:

- Swap with the person who is holding 1200m, swap with the person holding 1 ½ kilometres.
- Round each number to the nearest kilometre.
- Order the measurements from the least to greatest, allowing pupils time to explain their reasoning and how they got their answers.

Decimals Trail

Pupils take a photograph of a measurement of length found in their home/environment. Use the photographs as a basis for discussion.

LEVEL D.3

ESTIMATE AND MEASURE THE PERIMETER OF REGULAR AND IRREGULAR SHAPES

TEACHING NOTES

From activities contained within Level C.3, pupils will come to understand how to calculate the perimeter of regular shapes. It is important that pupils engage in practical activities and discussion to understand the concept of perimeter.

A common error is to confuse the formulas for area and perimeter. Such results are largely due to an overemphasis on formulas with little or no conceptual background (Van de Walle et al., 2007, p.388).

The Sample Learning Experiences used in Level C.3 can be used as a starting point within this level. The PDST have produced a comprehensive resource on Shape & Space that is available for free at http://www.pdst.ie/Shape-and-Space It may be appropriate here to revise the characteristics of regular 2-D shapes. Within this level, pupils should be given the opportunity to estimate and measure the perimeter of regular and irregular 2-D shapes.

SAMPLE LEARNING EXPERIENCES

Find the Perimeter of Shapes

Distribute a number of irregular shapes with straight sides only. Elicit from the pupils how they could find the perimeter of the shapes.

I want to find the perimeter of these 2-D Shapes. How could we do? What measuring instruments could we use? What unit of measurement will we use to record the perimeter?

This activity could be extended to find the perimeter of shapes with curved lines and straight lines. Elicit ways to find the perimeter of the shapes from the pupils. Pupils could use string and rulers/measuring tape to find the perimeter. Pupils should compare answers to check for accuracy.

Irregular Shapes in the Environment

Pupils could use trundle wheels or tape measure to find the perimeter of large irregular spaces such as the classroom, school building, playground, flower beds etc. Pupils should estimate, measure, compare and discuss their findings.

	Estimate in metres	**Measurement in metres**	**Difference**
Playground			
School garden			

Fixed Perimeters

This activity is an extension of the activity found in Level C.3. Give pupils a piece of string that is 24cm long and 1 cm grid paper (alternatively you could just use grid paper). The task is to see what different regular and irregular 2-D shapes can be made with a perimeter of 24cm. Each different shape can be recorded on grid paper. Pupils can label the length of the sides of the shapes and check their shapes have a perimeter of 24cm. This activity can be repeated with various lengths of perimeters.

CONSOLIDATION ACTIVITIES

Perimeter Hunt

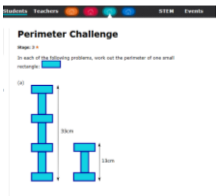
The pupils are involved in a Perimeter Hunt. This involves them finding objects whose perimeter is of a certain length (in either metres or centimetres). Give each pair of pupils a simple folded booklet into which to record the results of their perimeter hunt. Ask them to select 10 perimeters to find and head up each page with a different perimeter range.

- 0 - 10 cm
- 10 - 20 cm
- 20 - 30 cm
- 30 - 40 cm
- 40 - 50 cm
- 50 – 60 cm
- 60 – 70 cm
- 70 – 80 cm
- 90 – 100 cm
- 1 – 2 metres
- 2 – 3 metres
- 5 – 10 metres
- 10 – 20 metres
- Over 20 metres

Pose the perimeter hunt challenge: Find objects that have perimeters which match the measure on each page. The pairs should use measuring tools and record their findings on each page. Draw the object, labelling the measurement for each side, and calculate its perimeter.

Perimeter Challenge[70]

Pupils must use reasoning and logic to deduce the perimeter of a small rectangle.



[70] https://nrich.maths.org/11119

MEASURES - LENGTH

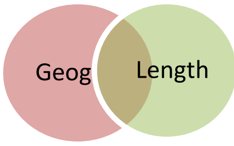
LEVEL D.4

USE AND INTERPRET SCALES ON MAPS AND PLANS

With primary school children, use only simple scales in scale drawing, such as 1:2, 1:5 or 1:10. You can also use simple scales with different measurement units such as 1 centimetre represents 1 metre, or 1 centimetre represents 10 metres (Haylock 2014, p.377).

TEACHING NOTES

Maps provide a two dimensional representation of the real world. By looking at a map pupils should be able to anticipate the landmarks they will see from a given location and in which direction (N, S, E, W) those landmarks will be seen. By using maps of their school or local area pupils will be able to check their thinking by matching the map with the real world. Pupils begin to use the map to help them follow and give directions. They start to use directions involving left and right turns and use landmarks to clarify pathways. Pupils also begin to use distances in whole numbers of metres. [71]



School Map[72]

Pupils are introduced to using a map to locate landmarks and identify views from different locations. Give the pupils copies of a school map with the outline of main buildings and features marked on it. Only label some of the buildings and features. Work with pupils to label their classroom and to orientate the map. Pupils are to label the buildings and features on the map. Pupils then take their map and walk around the school to check their labels and to add 2 or 3 new landmarks to the map. Back in the classroom ask the students to use the map to answer questions that require them to describe different views from locations on the map.

Which classroom has the best view of the playground? What building can you see from the garden? What building can you see from the office window?

[71] http://nzmaths.co.nz/resource/maps

Tayto Park Map

Distribute copies of the map of Tayto Park[73] to the pupils and use the map to stimulate discussion on maps. Alternatively the map can be viewed on the IWB or on tablets.

Can you find two paths from the entrance to the Outback Restaurant? Look at the little symbols around the Park? What are these for? What symbols do you see? What do they stand for? How tall must you be to go on the Cú Chulainn rollercoaster? How many centimetres is this? What other rides have height restrictions? What are they?

Questions about the map could include various routes, distance, key, cost of rides etc. Ask the pupils about the size of Tayto Park.

How big is Tayto Park? Does this map give us an idea about the size of Tayto Park? Where could we find this information?

Highlight to the pupils that this map is not drawn to scale and is a representative map of Tayto Park.

ICT
Map of Tayto Park

Map of the Classroom

What are maps? Who uses maps? When have you ever used a map? What is the name for a person who makes maps professionally? How are maps created? Why is it important that maps are accurate?

Discuss the importance of maps and in particular the accuracy in maps and introduce the concept of scale.

First we need to take some measurements in the classroom. How can we find out the width/ length of the classroom? What features will it be important to show on our map? Door/Windows/Desks/Teacher's Desk/Whiteboard

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[73] http://www.taytopark.ie/uploads/TaytoParkFinalMap.pdf

MEASURES - LENGTH

Introduce the concept of 1cm: 1m. Explain to the pupils they are going to draw a simple map of the classroom to scale, using 1cm: 1m. As the map is a bird's eye view, there is no need to measure height of objects, just the width. The pupils can work in pairs/groups to measure various distances and objects in the room. Pupils should record their answers.

Part of room	**Measurement in metres**	**Scale Measurement**
Width of classroom	**8.5m**	**8.5cm**
Length of classroom	12.25m	12 ¼ cm
Length of teacher's desk		

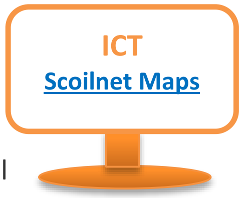
Pupils should compare and discuss actual measurement and scale measurements. Pupils can discuss the distance of objects from each other for example the distance from the teacher's table to the whiteboard. Encourage pupils to draw a rough outline map of the classroom to give a general sense of how big the objects in the classroom are and their general position. The pupils should now be able to draw an outline map of the classroom on 1cm square paper to scale. Pupils can also come up with a simple key to the features of the classroom.

CONSOLIDATION ACTIVITIES

Scale Map Investigations

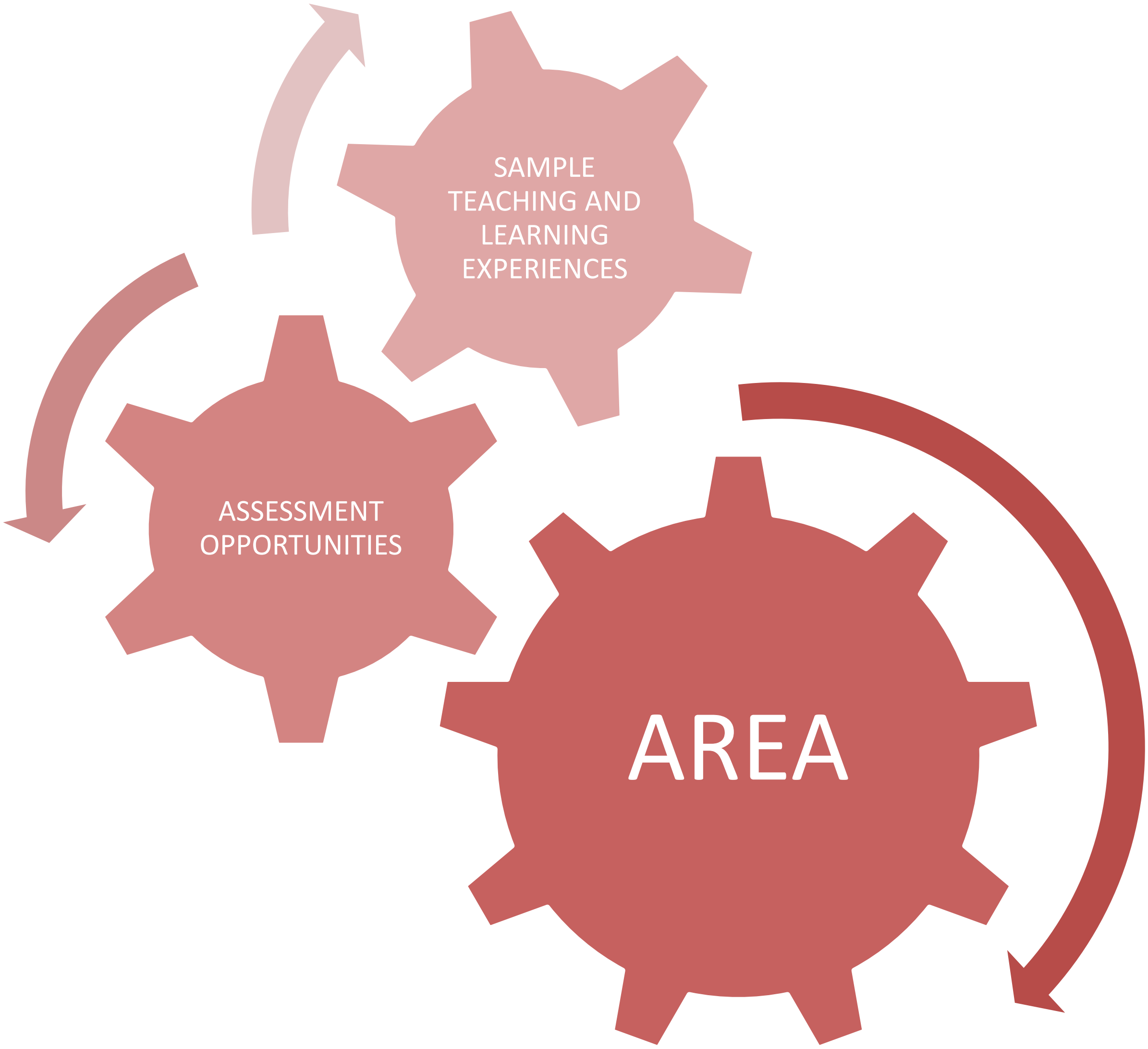
Using a scale map of 1cm: 1m, pupils could complete a map at home based on their bedroom/ideal bedroom/ideal classroom etc.

Further Activities



Further mapping activities can be carried with Scoilnet Maps[74] — the web mapping website for Irish schools. Here you will find detailed Ordnance Survey Ireland maps for Ireland as well as a range of world maps. Scales within atlases can be explored and used to calculate the distance between locations.

[74] http://maps.scoilnet.ie/


SAMPLE TEACHING AND LEARNING EXPERIENCES
ASSESSMENT OPPORTUNITIES
AREA

AREA

AREA -- BACKGROUND KNOWLEDGE FOR TEACHERS

Definitions

Area is the measurement of the surface covered by a certain region. Van de Walle et al.[75] state that '*area is the two dimensional place inside a region.'* The NCCA glossary defines area as *'the amount of a plane enclosed by a 2-D shape measured in square units.'*

Surface area: The surface area of a solid object is the sum of the areas of all its surfaces measured in square units

AREA - POSSIBLE PUPIL MISCONCEPTIONS

1. Conservation of area. The conservation of area means that if a 2-D shape is cut up and rearranged, its area is unchanged. However, it has been shown that not all children grasp this. Pupils sometimes assume that area can change if the shape is laid out differently. In an investigation reported in Dickinson, Brown, Gibson (1984), it was found that one third of 11 year olds did not conserve area consistently.

2. Relationship of area and perimeter. Pupils often mix up area with perimeter. Perimeter is the total length all the way around a boundary enclosing an area, like the length of fencing enclosing a field. Van de Walle et al., suggest that students often experience this confusion due to being taught formulas for both concepts at the same time. Teaching these two concepts during a close time frame is particularly difficult for pupils with learning difficulties. Haylock[76] notes that there is no direct relationship between perimeter and area. Haylock suggests using the illustration of fields and fences to differentiate between these concepts. The area is the size of the field and perimeter is the length of the fencing.

3. Developing formulas for area. It is a common mistake for pupils to mix up the formula for area and perimeter. Van de Walle et al., state such mistakes are largely due to an overemphasis on formulas with little or no conceptual understanding. Haylock says the formula 'area = length x width' only applies to rectangles and is considered incorrect or *'sloppy and wrong'*! [77]. Area of a triangle is ½ x base x perpendicular height. Area of a circle is πr^2. Ideally children should discover a formula for themselves. It should be regarded as a short method of finding an answer. In area work the children may realise as they count squares to find the area of a rectangle that it would be quicker to find the number of squares in one row and multiply this by the number of rows. In the same way, the children

[75] Van de Walle, et al. (2007) Elementary and Middle School Mathematics Teaching Developmentally. 8th edn.
[76] Haylock, D. (2010). Mathematics explained for primary teachers. 4th edn.
[77] Haylock, D. (2010). Mathematics explained for primary teachers. 4th edn.

might find a formula for calculating the area of a right-angled triangle as this shape can be seen by them as half a rectangle.

AREA - PARENTAL INVOVLEMENT

The following are a list of ideas that may be shared with parents to help develop pupils' conceptual understanding of area. Some of the following ideas are taken from the NCCA's tip sheets for parents available on the NCCa website[78]

Here you will also find some short videos of parents working with their children to develop their mathematical understanding.

Level B:

- Find out with your child whose foot/hand from the family covers the most space.
- During art activities, ask your child about covering surfaces, for example, how many sheets of newspaper will we need to cover the table?

Level C & D:

- Ask your child to help you with gardening. For this potato patch we need 12 square metres. What size of rectangles could we make?
- The tin says this paint will cover 10 square metres. Is that enough for the whole fence?
- Involve your child in DIY projects. How much carpet will we need to order for the sitting room? How much is it per square metre? What will the total cost be?

AREA - TEACHING NOTES

Pupils are usually introduced to the concept of area by superimposing areas. Rectangular areas are used so that pupils develop an understanding of the structure of the unit covering (array) in area. Suggested resources include:

• Playing cards	• Unifix cubes
• Post-its	• Centimetres Cubes
• Napkins	• Cuisenaire Rods
• Cardboard squares	• Dienes Blocks
• Books	• Attribute Blocks
• Copies	• Envelopes
• Pages	• Newspaper Sheets

The 1999 Primary School Mathematics Curriculum (PSMC) recommends the introduction of square metres and square centimetres in 4th class or Level C.2. When pupils can measure areas effectively using non-standard

[78] http://www.ncca.ie/en/Curriculum_and_Assessment/Parents/

units, they are ready to move on to the use of standard units. The motivation for moving to this stage, often follows from experiences where the pupils have used different non-standard units for the same area and have realised that consistency in the units used would allow for the easier and more accurate communication of area measures.

Standard units of measurement

For measuring area, the standard unit is always square units.

Square centimetre: A unit of area equivalent to that contained in a square region with sides of length 1 centimetre.

Square metre: A unit of area equivalent to that contained in square region with sides of length 1 metre.

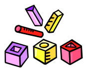
Hectare: a metric unit of square measure, equal to 2.471 acres or 10,000 square metres.

A rectangle with sides measuring 5cm and 6cm has an area of 30 square centimetres. The area can be abbreviated to $30cm^2$ but should be read as 30 square centimetres. Haylock (2015) suggests if it is read as '30 centimetres squared', it could be confused with the area of a 30 –cm square which actually would have an area of $900cm^2$.

AREA - LEARNING TRAJECTORY

The learning trajectory is based on the objectives for Measures in the Primary School Mathematics Curriculum. In some instances, similar objectives at the same class level have been collapsed into one objective. Objectives that only refer to problem solving have not been included as discrete objectives because a problem solving approach is advocated throughout all of the teaching and learning experiences. Problem solving is viewed in this manual as a fundamental, integral part of mathematics teaching and learning that pupils should experience every day. The same colour coding from the curriculum is used – infants (green); first and second (red); third and fourth (blue); fifth and sixth (orange).

AREA LEARNING TRAJECTORY LEVEL B[79]

Trajectory Levels	**Concept**	**Developmental Experiences** Concrete	Pictorial	Abstract
	Level B.1 Develop an understanding of the concept of area through exploration, discussion, and use of appropriate vocabulary			
	Level B.2 Estimate and measure area using non-standard units			

[79] This level is generally aligned with the objectives for Second class.

MEASURES - AREA

AREA LEARNING TRAJECTORY LEVEL C[80]

Trajectory Levels	Concept	Developmental Experiences: Concrete	Pictorial	Abstract
	Level C.1 Estimate, compare and measure the area of regular and irregular shapes using non-standard units of measurement			
	Level C.2 Estimate, compare and measure the area of regular and irregular shapes using standard units of measurement: square centimetres & square metres			

[80] This level is generally aligned with the objectives for Third and Fourth class.

AREA LEARNING TRAJECTORY LEVEL D[81]

Trajectory Levels	Concept	Developmental Experiences: Concrete	Pictorial	Abstract
	Level D.1 Estimate, calculate and measure the area of regular and irregular 2-D shapes			
	Level D.2 Compare visually square metres and square centimetres, identify the relationship between square metres and square centimetres and calculate area using square centimetres and square metres			
	Level D.3 Discover that the area of a rectangle is length by breadth			
	Level D.4 Recognise that the length of the perimeter of a rectangular shape does not determine the area of the shape			
	Level D.5 Measure the surface area of specified 3-D shapes			
	Level D.6 Find the area of a room from a scale plan			
	Level D.7 Calculate area using acres and hectares			

[81] This level is generally aligned with the objectives for Fifth and Sixth class

MEASURES - AREA

LEVEL B.1

DEVELOP AN UNDERSTANDING OF THE CONCEPT OF AREA THROUGH EXPLORATION, DISCUSSION, AND USE OF APPROPRIATE VOCABULARY

TEACHING NOTES

Area is the measurement of the 'surface covered' by a certain region. [82]Pupils are usually introduced to the concept of area by superimposing areas. In covering activities at this level, rectangular areas are used so that pupils develop an understanding of the structure of the unit covering (array) in area.

SAMPLE LEARNING EXPERIENCES

Pupils' first experience of area arises quite naturally from activities in painting, picture and pattern-making. Discussion will be largely based on shape and size.

Starting points for discussion may include:

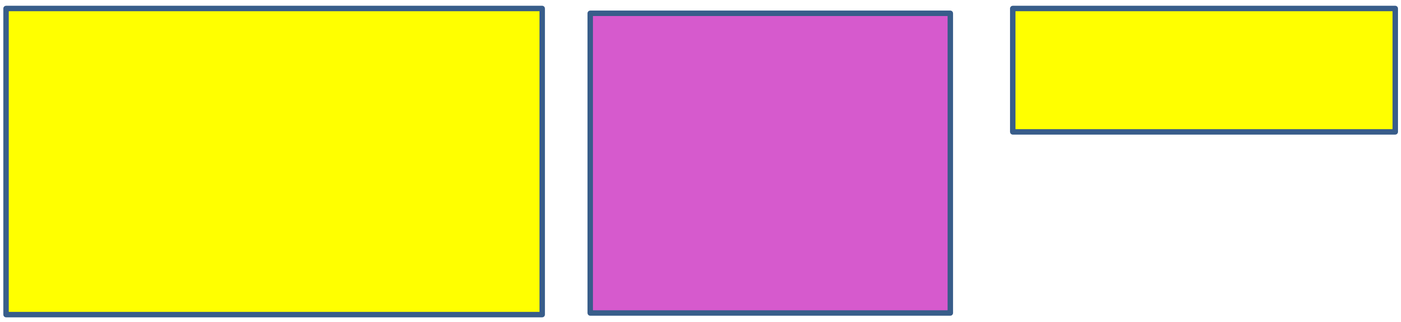
- Will this table cloth cover the teacher's desk? Will it cover your desk?
- Will this chart fit onto this space on the wall?

Before we paint, we need to cover our desks? How many sheets of newspaper will we need for each desk/group of desks? How do you know? Will there be gaps? Will I need a full sheet of newspaper to cover the leftover part? How could I cover the table without overlapping the newspaper?

Writing Notes

The purpose of this activity is to introduce the concept of area through exploration and discussion.

Show three completely different sized post-it notes to the pupils. Pupils discuss which sized post-it would be best for writing the longest note on. Elicit various responses from the children comparing the sizes of the notes for example taller, longer and wider.



[82] Pitt and Deboys, 1979 p.73

What is similar about these notes? What is different about these notes? How can we prove which note would be useful for a writing the longest note?

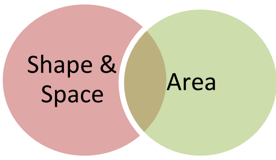
Pupils may demonstrate this by superimposing the notes or by placing one on top of another. Then introduce the word **area** and explain that it is the space inside a region. Give each pair of pupils a post-it note and challenge them to find a piece of paper with a larger surface area or smaller surface area than their post-it note.

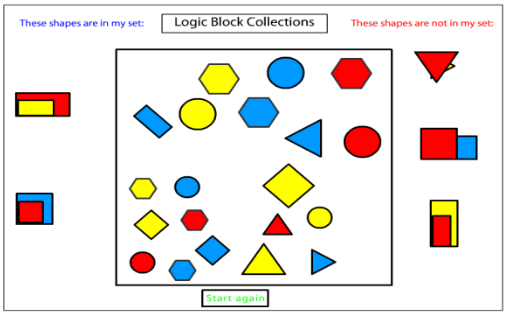
Let's Compare

The purpose of this activity is to provide opportunities for pupils to use the appropriate vocabulary when comparing the areas of 2-D shapes. Pupils take various 2-D shapes or attribute blocks from a bag and superimpose these shapes, by laying them on top of each other, to find out which has the largest or smallest areas. Pupils then order these shapes based on area.

Whose shape has the largest area? Whose shape has the smallest area? Whose shape has a larger area than Liam's shape? Whose shape has a smaller area than Rachel's shape? How do you know? Can you prove it.

Following on from this activity, pupils could compare and discuss the area of 2-D shapes. Alternatively, pupils could use the interactive logic blocks available on the Nrich website.





[83]

ICT
[Logic Block Collection](http://nrich.maths.org/content/id/6032/JulySh.swf)

[83] http://nrich.maths.org/content/id/6032/JulySh.swf

Whose shape has a great surface area than the red rectangle? How do you know? Find a shape with a smaller surface area than the blue square. I am a mystery shape. I am a red shape with a greater surface area than the blue rectangle.

Extension: Encourage pupils to give each other clues regarding a mystery shape.

Area is a two dimensional quantity and has to be identified as a property of a three dimensional object.

Wrapping Paper

Show the pupils three pieces of wrapping paper that differ in size. Using a variety of objects, pupils choose which piece of wrapping paper they would use to wrap the item.

Which piece of wrapping paper would you use to wrap the box of cereal? Why would you pick that one? Why wouldn't you use the smaller piece? Will this one cover the box entirely? Do you think there would be much of it left uncovered? How would you check?

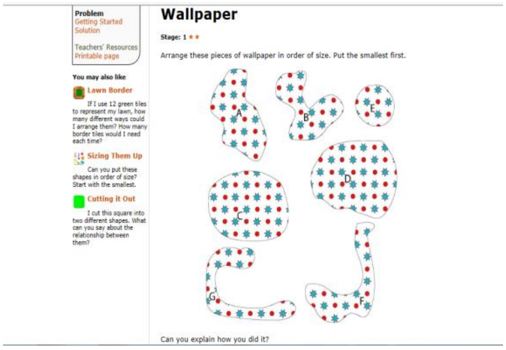
Next, using various sizes of wrapping paper, pupils work in groups to select a few items from the classroom they could wrap. Allow time for discussion focusing on covering the items, leaving gaps and overlapping of paper.

Wallpaper[84]

This problem from the Nrich website challenges pupils to compare and order pieces of wallpaper. Having completed the Nrich activity, the following questions could be used to extend pupils' reasoning.

Which piece of wallpaper would cover the most area on the wall? How do you know? Which piece would cover the least? Convince me.

[84] http://nrich.maths.org/4964



ICT
[Wallpaper]

Discussion is very important at this stage to lead children to a clear understanding of the meaning of area (Deboys & Pitt, 1979, p.73).

CONSOLIDATION ACTIVITIES

Order the Faces[85]

The purpose of the activity is to provide an opportunity for pupils to compare and order the surface area of 3-D shapes. Pupils bring in a box from home, (for example cereal, tissue, toothpaste, stock cubes). Pupils select and paint one of the faces of their box. In small groups, pupils compare and order the areas of the selected faces. Pupils can record their findings by writing or drawing how they ordered the faces.

Assessment

Whose surface has the greatest area? How do you know? How many of the faces of your box are the same area? Which surface has the smallest area? Prove it.

Area Hunt

The purpose of this activity is to provide pupils with the opportunity to compare the area of small objects. Give pairs of pupils a small square for example, a post-it. Pupils must find three objects in the classroom with a smaller area than their shape/with a larger area than their shape. Pupils record the items they find and order them according to their area. Pupils can share their strategies for example, directly comparing, comparing visually or superimposing. Pupils may record their strategies and findings in a learning log.

Assessment

[85] http://www.schools.nsw.edu.au/learning/7-12assessments/naplan/teachstrategies/yr2014/img/teaching_measurement_es1_s1.pdf

Conservation of Area

The conservation of area means that if a 2-D shape is cut up and rearranged, its area is unchanged. However it has been shown that not all children grasp this (Suggate et al 2014, p.190). Pupils need to be tested for conservation of area. If children are unsure, they will need further experience and development of the concept of area.

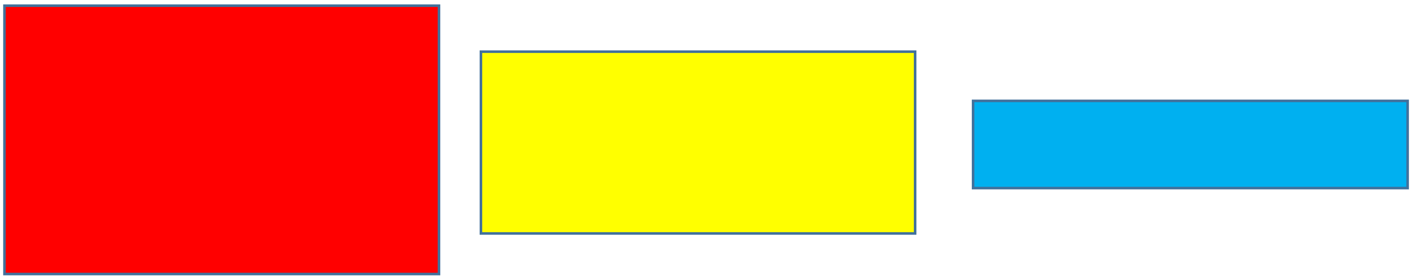
Conservation Assessment[86]

Take three rectangles.

Question: Does the red rectangle have the same amount of surface area as the yellow and blue rectangles altogether? You can move the shapes if you like.

OR

Question: There are three fields. Does this field have more grass than these two fields? You can move the shapes if you like.



[86] Deboys & Pitt 1979, p. 74

MEASURES - AREA

LEVEL B.2

ESTIMATE AND MEASURE AREA USING NON-STANDARD UNITS

TEACHING NOTES

Pupils should have opportunities to estimate and measure area using non-standard units. Deboys and Pitt(1979), suggest that pupils seem happiest handling 3-D units to cover surface area and move onto 2-D units.

The use of informal or non-standard measuring units leads to the need for standard units for better precision and unambiguous communication.

SAMPLE LEARNING EXPERIENCES

In this activity, pupils are using different non-standard units to measure the same surface.

How many shoe boxes will cover the table? How many Irish books will cover the table? What else could you use to measure the area of the table?

In this activity pupils guess first and then measure using different non-standard units (for example, playing cards, napkins, A4 sheets etc.) to measure the same surface (for example, table mat, press).

How many cards do you think will cover the table? Sarah why do you think it will take that many? Explain your thinking. Does anybody have a different guess? Measure and compare your guess to the actual measurement.

Using the same unit with different surfaces

CONSOLIDATION ACTIVITY

Hands & Feet[87]

Pupils estimate whether their hand or their foot covers the greatest area. Pupils then draw around one hand and cut out the outline, and draw around one foot and cut out the outline. Pupils share their ideas about how they could compare the area of their hands and feet, for example, using counters, tiles, blocks, directly comparing the outlines. Pupils decide how to record whether their hand or foot is larger and explain how they judged their hand or foot to have the larger area.

[87] http://nzmaths.co.nz/resource/great-cover

LEVEL C.1

ESTIMATE, COMPARE AND MEASURE THE AREA OF REGULAR AND IRREGULAR SHAPES USING NON-STANDARD UNITS OF MEASUREMENT

TEACHING NOTES

According to Deboys and Pitt [88]the concept of area as 'the amount of space covered' develops slowly through a variety of formal and informal experiences. Informally, teachers and pupils can discuss the area of objects through art and crafts, for example the amount of paint required to cover an area, the amount of paper needed to cover a display board. Pupils must also have practical experience of investigating area.

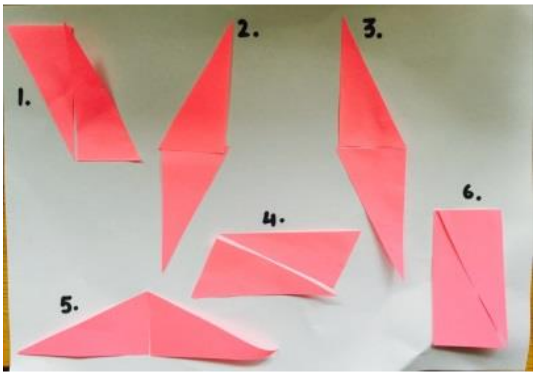
At this level, pupils will deepen their understanding of area through estimating, comparing and measuring the area of regular and irregular shapes using non-standard units of measurement. Learning experiences from Level B.2 could be used to further develop pupils' concept of area by estimating, measuring and comparing area using non-standard units of measurement.

SAMPLE LEARNING EXPERIENCES

Many 8 or 9 year olds do not understand that rearranging areas into different shapes does not affect the area. Cutting a shape into two parts and reassembling into a different shape can show that the before and after shapes have the same area even though they are different shapes (Van De Walle 2007, p.384).

Two Piece Shapes[89]

The purpose of this activity is to further develop pupils' conservation of area. Cut a number of rectangles (5cm x 12cm). Give each pair of pupils 6 rectangles. Pupils fold and cut the rectangles on the diagonal making two equal triangles. Pupils rearrange the triangles into different shapes, including back into the original rectangle, the rule being that only sides of the same length can be matched up. Pupils glue the shapes together to record their findings. Discuss findings.



Does one shape have a greater area than the rest? How do you know? What can we say about all the shapes?

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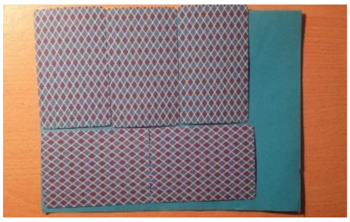
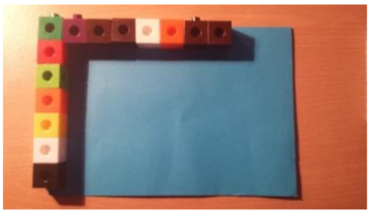
[88] Deboys M. & Pitt, E. (1979). Lines of Development in Primary Mathematics.
[89] Van De Walle 2007, p.385

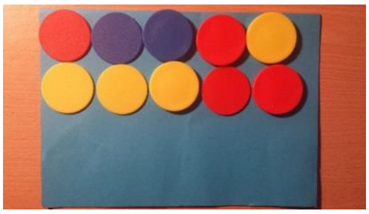
How will I Cover[90]

The purpose of this activity to discuss the suitability of units for measurement of area. Pairs/small groups of pupils choose units (tessellating and non-tesselating) for example, cubes, circles, coins, dienes blocks, cuisenaire rods, napkins, playing cards, from a collection to cover a shape such as a rectangular piece of paper. Discuss with the pupils the suitability of the units in terms of no gaps or overlaps.

Which units were the most useful to use when covering the rectangle? Why do you think is so? Explain your strategy for dealing with any leftover parts that were not covered by your unit. How can we record our findings?

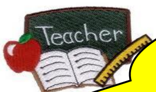






Introducing Square Units

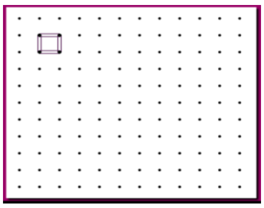
The purpose of this activity is to introduce square units for measurement of area. Provide pupils with a variety of square units to work with (post-its, square cubes, cardboard squares etc.). Pairs of pupils choose 12 of the same unit and make a shape with an area of 12 square units. Pupils record the shapes they made and their pattern.



What is similar about all the shapes you made? Tell me about the rows and columns in the shapes you made. These two groups used the same unit but made different shapes. Which has a larger area?

How would your shape be different if you used 12 larger or smaller tiles?

An extension of this activity or a follow up activity would be to ask pupils to choose a second lot of 12 (smaller or larger) tiles to make more shapes with an area of 12 tiles. Discuss whether different shapes with the same number of tiles had a different area.



90 http://www.schools.nsw.edu.au/learning/7-12assessments/naplan/teachstrategies/yr2014/img/teaching_measurement_es1_s1.pdf

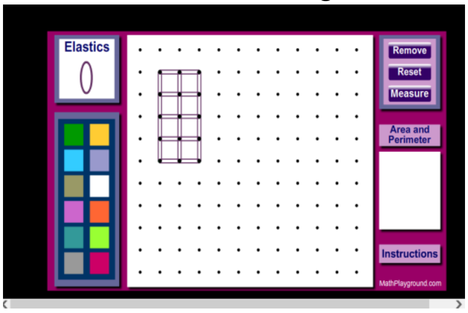
Area Using Geoboards[91]

Pupils investigate the concept of area using geoboards and elastic bands. The unit of area on the geoboard is the smallest square that can be made by connecting four nails.

How many different sized squares can you make on the geoboard? Record the area of each square on dotted paper.

How many different sized rectangles can you make? Record the size of each shape on dotted paper and write the area of each in square units? How many shapes can you make with an area of 24 squares?

The hyperlink below leads to an interactive geoboard[92] which can be used to further develop the activity outline above.



ICT
[Interactive Geoboards]

When children discover that squares are the most suitable units for measuring area, a variety of irregular shapes may be drawn on square paper. In considering shapes that are not rectangular it is hoped that children's attention is being focused on the size of the surface and that this is the concept which is being developed **not** the formula length x breadth = area (Deboys & Pitt 1979, p.168).

Area of Irregular Shapes[93]

The purpose of this activity is for pupils to share their strategy of measuring area using partial units. Distribute copies of a leaf outline to pupils on square paper. In pairs, pupils find the area of the leaf in square units. Discuss with pupils how to cope with partial squares.

What is the area of the leaf? Did anybody get a different answer? Why do you think we have so many different answers? What did you do with the partial squares?

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[91] Deboys & Pitt 1979, p.169
[92] http://www.mathplayground.com/geoboard.html
[93] Deboys & Pitt, p. 168

Give ample time to discuss pupils’ suggestions for exploring area with partial squares. The area of an irregular shape may be found by counting whole squares and the squares whose area is greater than half of a square. Provide further opportunities for pupils to find the area of irregular shapes using square paper. Examples include:

- Size of hands (drawn on square paper)
- Size of feet (drawn on square paper)
- Pupils’ socks/gloves/hats (drawn on square paper)
-

Rectangle Comparison with Square Units

It is not the goal of this activity to develop an area formula. While some pupils will use a multiplicative approach, others will count the squares. Other pupils may draw in the squares. By having pupils share their strategies, pupils can be exposed to the use of multiplication in this context (Van de Walle 1979, p.386).

The purpose of this activity is to explore multiple ways of practically finding the area of rectangles and to allow the pupils opportunities to justify their solutions. Using squared or dotted paper, give pupils pairs of rectangles that are very close in area. The pupils must decide, in any way, that they can, which rectangle is larger in area or whether they are the same. They should use words, pictures or numbers to explain and justify their answer. Some suggested pairs are:

- 4 squares x 10 squares / 5 squares by 8 squares
- 5 squares by 10 squares/ 7 squares by 7 squares
- 4 squares by 6 squares/ 5 squares by 5 squares

Assessment

CONSOLIDATION ACTIVITY

Roll the Dice Twice[94]

Pupils will need square paper and a dice for this game. The first pupil throws the dice and this is the number of squares to be drawn along the top row. The second pupil throws the dice to decide how many rows to make. Pupils will then predict how many squares they will need to draw after the second throw. Pupils make the array and count the tiles. Pupils record the area of each shape in square units. What is the area of the largest shape made by the pupils?

[94] http://www.schools.nsw.edu.au/learning/7-12assessments/naplan/teachstrategies/yr2014/img/TM2-3.pdf

MEASURES - AREA

LEVEL C.2

ESTIMATE, COMPARE AND MEASURE THE AREA OF REGULAR AND IRREGULAR SHAPES USING STANDARD UNITS OF MEASUREMENT

TEACHING NOTES

When pupils can measure areas effectively using non-standard units, they are ready to move on to the use of standard units. The motivation for moving to this stage, often follows from experiences where the pupils have used different non-standard units for the same area and have realised that consistency in the units used would allow for the easier and more accurate communication of area measures.[95] The Primary School Mathematics Curriculum[96] recommends the introduction of square metres and square centimetres in 4th class.

It is important to highlight the importance of using the correct language when referring to square metres and square centimetres. Avoid referring to the standard units of measurement as metres squared or centimetres squared.

SAMPLE LEARNING EXPERIENCES

Standard Units

The purpose of this activity is for pupils to come to the realisation that a standard unit is needed to measure area accurately. Pupils work in groups to measure the area of a common object for example, book/desk using square units. Give each group a set of different squares, for example, 10cm side length, 5cm side length etc. Give the pupils time to measure the area of the object and elicit answers from the groups.

Aoife's group measured the area as 5 ½ squares but John's group's answer is 12 square units. Why did the groups get different answers? Who is right? How could we come up with an answer that we all agree with?

Highlight to the pupils the need for a standard unit of measurement when measuring area.

Pentominoes[97]

The purpose of this activity is two-fold. Firstly, pupils will explore the conservation of area.

Show the class a 3-D shape created using 5 centimetre cubes. Pupils record the shape on centimetre squared paper. Ask the pupils what is the area of the shape they have drawn. If pupils say 5 squares or 5 square units, tell them each unit square is a square centimetre.

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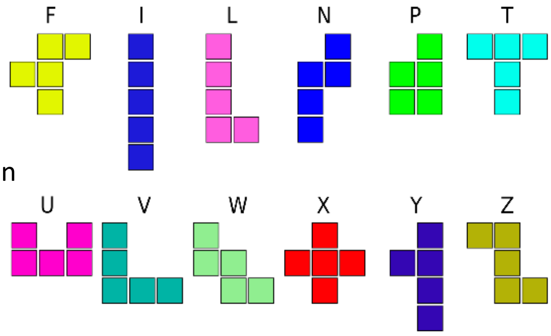
95 http://nzmaths.co.nz/resource/outlining-area
96 NCCA (1999). Primary School Mathematics Curriculum
97 http://nzmaths.co.nz/resource/outlining-area

Why do you think it is called a square centimetre? How could we prove it is a square centimetre?

Give the 5 centimetre cubes to one pupil and ask them to make a different shape with the 5 cubes. The shape must be flat and the whole sides of the square must touch. Ask pupils to record the shape they made on their square cm paper.

What is the area of this shape?

In pairs/groups, give the pupils 5 centimetre cubes and challenge them to find shapes that can be made with the 5 cubes. The pupils will record the shapes of the cm grid paper. These shapes are called pentominoes and there are 12 different shapes that can be made. When the pupils share the shapes they made, check that the pupils recognise the area of each shape is 5 square centimetres.



Measuring using Square Centimetres

This activity replicates previous activities where pupils measured area using non-standard units. In this activity, pupils measure area using standard units of measurement. Using a common object like a small book, challenge the pupils to find its area. Provide pairs/groups of pupils with centimetre cubes, square paper and rulers. Ask the pupils to work out the area but let them make the decision about how they will measure the area. Share answers and approaches used.

Mary, how did you and partner find the area of the book? What materials did you use? Lauren, revoice Mary's strategy? What was difficult about your method? Is there a more efficient method? Why is it more efficient?

Some pupils may have used the centimetre cubes, others may have traced the shape onto grid paper, and other pupils may have taken the dimensions with a ruler and drawn the shape. Some pupils may use the array method to find the area of the object. (See Level C.1) Repeat this activity with other small objects and encourage pupils to use a different strategy to find the area. Discuss which strategies are most effective and why.

Area of my Hand[98]

The purpose of this activity is to allow pupils to calculate the area of an irregular shape.

How could we find out the area of your hand? About how many square cm do you think it might be? Why do you think that?

Using square cm paper, pupils can draw the outline of their hands.

How many full squares have you coloured in? Is this the full area of your hand? Why not? What could we do with the partial squares? Should we count them? Why or why not?

Pupils should be encouraged to count and colour the squares that are more than half way covered included in the outline.

Will this give us an exact measurement of the area? Why?

Pupils can repeat this exercise with the outline of their feet and other irregular shapes such as leaves, as outlined in Level C.1.

Name that Area!

Using square centimetre paper, pupils draw an outline of their name in block letters and find the area of their name. Pupils can design their name and record the area of their drawing in square centimetres. A further challenge would be pupils to draw a name/word but they must use a set area for example, 100 square centimetres.

Make a Square Metre[99]

Assessment

The purpose of this activity is to develop the concept of a square metre. Outline a square with sides of 1 metre on the floor with tape, or outside with chalk. Pupils discuss the length of each side and predict what the area of the square would be.

98 http://nzmaths.co.nz/resource/outlining-area

99 http://www.schools.nsw.edu.au/learning/7-12assessments/naplan/teachstrategies/yr2014/img/TM2-3.pdf p.58

To measure the square metre, several pupils place 10 cm square tiles inside one side of the shape. Class estimates, then counts how many tiles will be needed along adjoining sides. Discuss how to use the array structure to calculate the total number of tiles needed to cover the square metre.

What is the area of this shape? Can you think of other shapes or surfaces that have an area of 1 square metre? How could I measure the area of this square metre? Is there a quick way of drawing the tiles? What is this pattern called? (Array)

How many tiles will be needed for each row? How many rows will there be? How did you work that out? How many tiles altogether?

How Much Room?[100]

Using masking tape or chalk, draw a metre square on the floor/playground.

How many pupils do you think would be able to sit in this space? What about stand in this space? How many of these squares would we need to fit all the pupils in this class? Estimate and let's check.

Estimating in Square Metres

The purpose of this activity is to identify areas which are approximately 1 square metre. Small groups of pupils work together to make a square metre template using 4 metre strips of card stuck or stapled together. Pupils must find and record surfaces that have an area of about 1 square metre. These templates can be used for the following activity also.

It is an important learning point that one square metre of area can take many shapes. When we speak of a square metre, we are not referring to shape but to an area equal to that of a metre square (Deboys & Pitt, 1979, p. 171).

[100] http://nzmaths.co.nz/resource/outlining-area

The purpose of this activity to highlight that the term square metre refers to an area equal to one square metre or 10,000 square centimetres. Using the square metre templates, pupils can cut their templates in halves or quarters and rearrange the pieces in to a new shape. Pupils must measure and record the area of the new shape and record their findings.

CONSOLIDATION ACTIVITIES

Fixed Area [101]

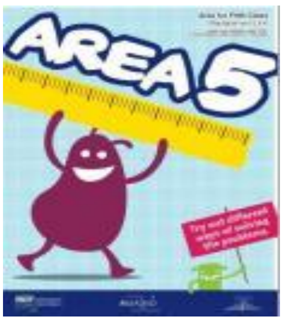
With 1 centimetre grid paper, pupils investigate, in pairs, how many rectangles they can make with an area of 36 square centimetres – that is to make filled in rectangles. Each new rectangle should be recorded on the 1cm grid paper. Pupils should be encouraged to record the dimensions of the rectangle also.

Hopscotch[102]

Using the poster from maths eyes [103] discuss the hopscotch grid with the pupils.

How could we accurately measure the area of this grid? If the length of each square is 50cm, what would the area of each square be? With your partner, find a strategy to find the area of the full grid? Find the area of the blue and red squares?

Pupils in groups make a 50 cm square from paper or cardboard and investigate how many tiles will be needed to make 1 m^2. Using the 50 cm x 50 cm tile, groups of pupils design a hopscotch grid that has a total area of 3 m^2. Record the design and the total area. Pupils could measure and draw their hopscotch grid in the yard with chalk.



Dr. Seán Delaney, in collaboration with PDST has published a resource for teaching area in fifth class. The resource may also be helpful for teaching area to pupils in fourth or sixth class. Throughout the unit, there are 16 tasks based on developing the concept of area. The teacher's handbook and pupils' book may be accessed through the image or through the hyperlink below. [104]

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[101] Van de Walle, p. 388
[102] http://www.schools.nsw.edu.au/learning/7-12assessments/naplan/teachstrategies/yr2014/img/TM2-3.pdf p.59
[103] http://www.haveyougotmathseyes.com/wp-content/gallery/resources/003-hopscotch.jpg
[104]http://www.pdst.ie/Measures

MEASURES - AREA

LEVEL D.1

COMPARE VISUALLY SQUARE METRES AND SQUARE CENTIMETRES, IDENTIFY THE RELATIONSHIP BETWEEN SQUARE METRES AND SQUARE CENTIMETRES AND CALCULATE AREA USING SQUARE CENTIMETRES AND SQUARE METRES

In Level C.2, pupils are introduced to the square metre and the square centimetre. It may be useful to revisit the activities from level C.2. The relationship between these two measurements will be further investigated at this level.

How Many Square Centimetres in a Square Metre?[105]

Mark out a square metre using chalk or tape on the floor. Using centimetre squared paper, cut out the squares 25cm by 25cm. Pupils can cover the square metre with these – 16 in all. Pupils should calculate the area of 1 sheet, then the area of 16. The 16 sheets altogether measure 10,000 square centimetres.

> It is an important learning point that one square metre of area can take many shapes. When we speak of a square metre, we are not referring to shape but to an area equal to that of a metre square (Deboys & Pitt, 171).

CONSOLIDATION ACTIVITY

Garden Challenge

The principal of the school wants to put a new flowerbed in the garden with an area of exactly 1 square metre. The flower bed must be rectangular or square. The children must work in groups to design the new flowerbed, ensuring the area is exactly 1 square metre. Pupils could begin this activity by measuring and cutting out pieces of paper that measure exactly 1 square metre. How many different variations can the pupils design in keeping with the principal's criteria?

[105] Deboys & Pitt, p.300

LEVEL D.2

DISCOVER THAT THE AREA OF A RECTANGLE IS LENGTH BY BREADTH

Make sure that children are aware that the rule for finding the area of a rectangle only applies to rectangles! Using the phrase 'area is length times width' is sloppy and wrong! Other two dimensional shapes have areas as well, but these are not found by applying that rule (Haylock, 2015, p.348).

Discovering a Formula for Finding the Area of a Rectangle[106]

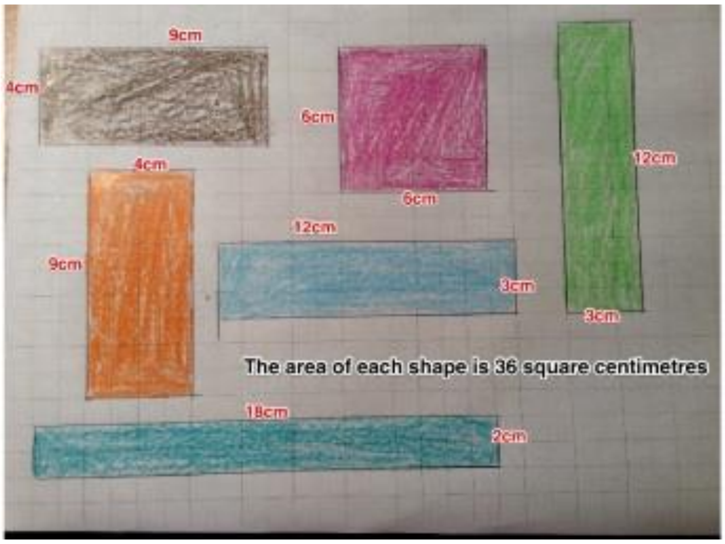
The purpose of this activity is to encourage pupils to discover a formula for finding the area of a rectangle. Distribute copies of 1 cm square paper to pairs of pupils. On each page, there should be three rectangles drawn in with an area of 24 square centimetres, as in the image below. The rectangles should be of the following dimensions:

- 24 squares x 1 square
- 8 square x 3 squares
- 3 squares x 8 squares

Pose the following questions to pupils and allow time for them to explain their reasoning.

How are these shapes similar? How are they different? Explain why you think so. How many more rectangles can you and your partner draw with an area of 24 square centimetres?

Allow pupils time to draw the rectangles and share their ideas which hopefully will include the relationship between the rows and the columns, eventually leading to the discovery of a formula. Pupils draw different rectangular shapes that have an area of 36 square centimetres. Some pupils may realise they do not have to draw all the shapes to identify which ones have an area of 36. While taking answers and feedback from the pupils, construct a table similar to the following which records the different rectangular shapes. Pupils could record their findings using ICT such as Skitch[107] like in this example.



106 http://nzmaths.co.nz/resource/you-can-count-squares
107 https://evernote.com/skitch/?var=2

Rows	Columns	Area
1		36 square cm
2		
3		
4		
6		
9		
12		
18		
36		

When this table is completed, have pupils work in pairs to complete a table to find all the different rectangles that have an area of 60 square centimetres.

From the work we have completed, could you find a rule that would help you find the area of a labelled rectangle efficiently?

CONSOLIDATION ACTIVITY

Rectangle Comparison (2)

At this level, pupils could revisit the 'Rectangle Comparison' activity from Level C.1. At this level, pupils should have sufficient practical experience to be able to apply the formula for finding the area of a rectangle. If pupils are still drawing in the squares and counting to find the area, they may need further experience to develop the understanding of the formula, for finding the area of a rectangle.

Assessment

A common error is to confuse the formulas for area and perimeter. Such results are largely due to an overemphasis on formulas with little or no conceptual background (Van de Walle et al., 2007, p.388).

MEASURES - AREA

LEVEL D.3

ESTIMATE, CALCULATE AND MEASURE THE AREA OF REGULAR AND IRREGULAR 2-D SHAPES

TEACHING NOTES

Activities from Level C.2 can be used at this level to develop and consolidate pupils' conceptual understanding of area. Pupils should have the opportunity to estimate, calculate and measure the area of a variety of objects and spaces. At this level it is appropriate for pupils to explore the area of other geometric shapes such as parallelograms, triangles and circles.

SAMPLE LEARNING EXPERIENCES

Find the Area of…

The purpose of this activity to provide pupils with opportunities to find the area of larger areas in the school. With the pupils, choose large areas to be measured, such as the cloakroom, classroom, secretary's office, playing field etc. Pupils must select an appropriate measuring device and calculate the area by taking the measurements to the nearest metres. Pupils may record their findings in a learning log.

Assessment

Area of Rectilinear Shapes

Children should learn how to find the area of a 'rectilinear' shape by dividing it into rectangles, finding the areas of these and adding them up (Haylock, p.348). A rectilinear shape is a shape all of whose edges meet at right angles.

Total Area[108]

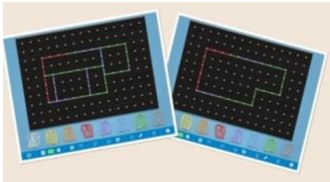
Teacher

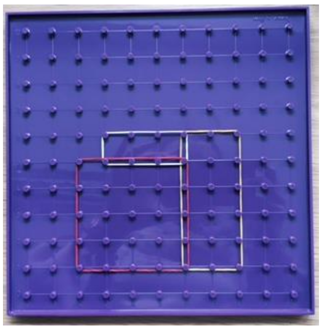
With your partner, calculate the total area of the shape you created? Explain your strategy.

[108] http://www.schools.nsw.edu.au/learning/7-12assessments/naplan/teachstrategies/yr2014/img/TM2-3.pdf

MEASURES - AREA

The purpose of the activity is to allow pupils to devise and share strategies for finding the area of an irregular shape. Pairs or small groups of pupils work together to design an irregular shape or shape comprised of five rectangles on a geoboard and transfer their design onto square cm paper. This activity could also be completed on an interactive geoboard[109].





ICT
[Geoboard App]

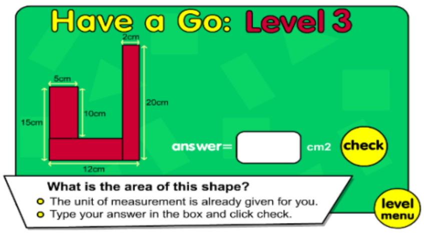
Elicit various strategies from the pupils about calculating the area of the irregular shape. Some pupils may have calculated the area of individual rectangles/squares while other may calculate the area of the 'missing bit' first to find the area of the shape. A further challenge would be to ask children to design and create irregular shapes for their classmates to calculate the area of.

Area of Compound Shapes[110]

Level 2 & 3 of this interactive site challenge pupils to calculate and find the area of compound shapes. Pupils are given shapes with all measurements provided in Level 2 to work with and in Level 3, pupils must calculate some missing dimensions before calculating the area of the whole shape.

Area Builder: [111]

This tablet friendly game is useful to further develop pupils' understanding of rectilinear shapes. This game also has 6 levels of difficulty.



ICT
[Area of Compound Shapes]

ICT
[Area Builder]

109 http://catalog.mathlearningcenter.org/apps
110 http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/maths/perimeter_and_area/index.html
111 http://phet.colorado.edu/sims/html/area-builder/latest/area-builder_en.html

Area of a Parallelogram

Pupils make a rectangle using split pins and cardboard strips. Push out the edges to form a parallelogram.

What do we call shapes like this? Which is greater, the area of the rectangle we started with or the area of the parallelogram? How can we be sure? Let's check.

In pairs, pupils construct parallelograms using cardboard strips and split pins. The pupils must measure the area of the parallelogram. Give the pupils time to discuss how they measured the area of the parallelogram. This activity[112] from the NCTM Illuminations website can be used to highlight and stimulate discussion about the area of parallelograms. The activity will demonstrate to pupils that the area measure will remain fixed when the rectangle is changed to a parallelogram of similar dimensions.

ICT
Area of a Parallelogram 1

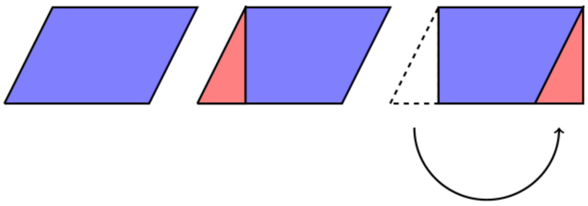
Area of a Parallelogram (2)[113][114]

Give pupils 2 or 3 parallelograms either drawn on grid paper or, for a more difficult challenge, drawn on plain paper with the dimensions (length of all four sides and height). Pupils must find the area of the parallelograms.

How can you find the area of the parallelograms? How can we apply what we know about the area of rectangles to the area of parallelograms? Using a scissors can you cut the parallelogram and rearrange it to make a rectangle?

Pupils can cut off a triangle from one end of the parallelogram and move it to the other side to form a rectangle. Pupils can use pattern blocks to explore this concept.

When you rearranged the parallelogram to form a rectangle, did you alter the area? What does this tell us about finding the area of a parallelogram? How can we record our findings?



ICT
Area of a Parallelogram 2

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[112] http://illuminations.nctm.org/Activity.aspx?id=4166
[113] Van de Walle et all, p.389
[114] http://illuminations.nctm.org/Activity.aspx?id=4158

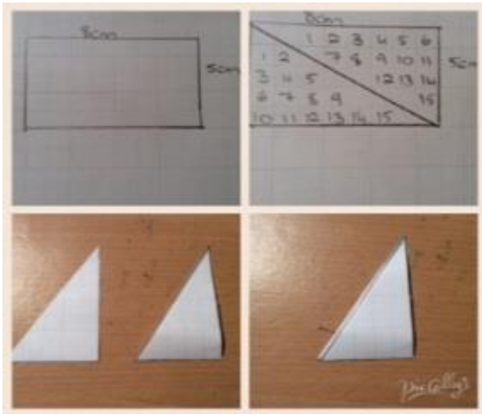
Area of a Right Angled Triangle[115]

Pupils draw a rectangle on square centimetre paper. It may be necessary to put limits on the size of the rectangles for example no more than 10 centimetres along each side. Pupils calculate the area of their rectangle in square centimetres and share the answer with a partner. Pupils should also be encouraged to share their method with their partner. Pupils can share these strategies with the class. Pupils draw a straight line from one corner of their rectangle to the diagonally opposite corner.

Teacher

What shapes have you created within your rectangle? What kind of triangles are these?

Pupils calculate the area of each triangle. Elicit solution strategies. Some pupils may count the full squares while others will recognise that the area of one triangle is half the area of the full rectangle. Pupils should notice that the two triangles have the same area. This can be reinforced by asking the pupils to cut along the diagonal and rotate one triangle to sit on top the other. The pupils should see the triangles have the same area and are identical triangles.



Teacher

Could we devise a rule for finding the area of a right angled triangle? How we prove this rule will work?

Areas of all Triangles'[116]

Provide pupils with at least two non-right angled triangles drawn on square centimetre paper. Challenge the pupils to use what they know about rectangles and parallelograms to find the area of each of the triangles and develop a method to find the area of any triangle.

Teacher

Can you find a parallelogram or a rectangle that is related to your triangle?

115 http://nzmaths.co.nz/resource/triangles

116 Van de Walle et al, 2007 p.390

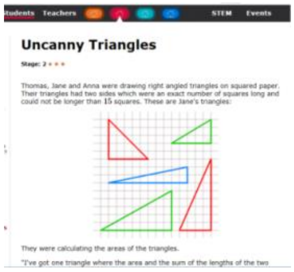
Pupils may benefit from experiences of folding a piece of paper in half and cutting it to make two identical triangles. Use the copies to fit the pieces together into a parallelogram. This will provide a useful visual of how a triangle is related to a parallelogram. Pupils should further explore triangles and parallelograms to discover that the area of a triangle will be one half as much of that as a parallelogram. [117]

ICT
Area of a Triangle

CONSOLIDATION ACTIVITIES

Uncanny Triangles[118]

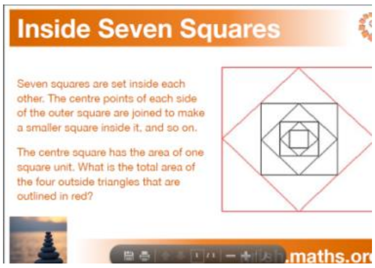
This activity from the Nrich website challenges pupils to find the two triangles which have the lengths of two sides numerically equal to their areas.



ICT
Uncanny Triangles

Inside 7 Squares[119]

This problem challenges pupils to find the total area of the four outside triangles which are outlined in red in this arrangement of squares inside each other. Pupils could work in pairs/groups to solve the problem.



ICT
Inside Seven Squares

Area and perimeter are continual source of confusion for students. Students may be surprised to find out that rectangles having the same areas do not necessarily have the same perimeters and vice versa. This fact is not restricted to rectangles (Van De Walle, 2007).

117 http://illuminations.nctm.org/Activity.aspx?id=4160
118 http://nrich. maths.org/2418
119 http://nrich.maths.org/2132

LEVEL D.4

RECOGNISE THAT THE LENGTH OF THE PERIMETER OF A RECTANGULAR SHAPE DOES NOT DETERMINE THE AREA OF THE SHAPE

Fixed Perimeters[120]

Give pairs or groups of pupils a piece of non-stretching string that is 24cm long and 1cm grid paper. Alternatively, they can work with the paper alone. Pupils investigate the various rectangles that can be made with a perimeter of 24cm. Each different rectangle can be recorded on grid paper and pupils record the area of the rectangle also.

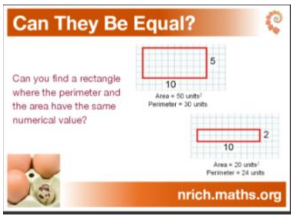
Fixed Areas

This activity follows on from the previous activity and challenges pupils to see how many rectangles can be made with an area of 36cm^2. Pupils can use square paper to record their rectangles. For each rectangle, pupils should determine and record the perimeter inside the figure.

Are all the rectangles we have recorded equal? In what way are they similar? In what way are they different?

CONSOLIDATION ACTIVITIES

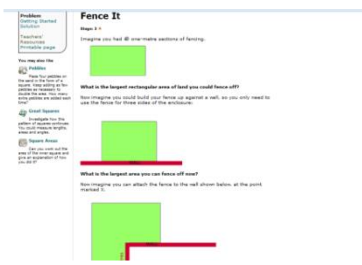
Can they be Equal?[121]



This poster based on a problem from the Nrich website challenges pupils to find a rectangle whose area and perimeter is of the same numerical value.

Fence it[122]

This problem is also from the Nrich website. Pupils must consider the greatest area of rectangular land that can be enclosed using 40 metres of fencing. This problem has varying levels of difficulty and can be differentiated to enable multiple access points for pupils.



[120] Van De Walle 2007 p.388
[121] http://nrich.maths.org/7996
[122] https://nrich.maths.org/2663

LEVEL D.5

MEASURE THE SURFACE AREA OF SPECIFIED 3-D SHAPES

TEACHING NOTES

The surface area of a solid object is the sum of the areas of all its surfaces measured in square units (Haylock, 2014 p.351).

Van de Walle et al.,[123] suggest that one of the best approaches to teaching the surface area of three dimensional figures is to create several prisms , cubes or cylinders so students can think about the components or the 'net' of the figure as they break the figure into pieces and calculate the surface area. PDST have produced a resource to assist teachers in teaching the strand Shape & Space. This manual is available free to download from www.pdst.ie or by clicking on the image of the manual. The activities at Level D.2 would be particularly useful at this level. These activities engage pupils in visualising, constructing and modelling 3-D shapes.

SAMPLE LEARNING EXPERIENCES

Surface Area of Cuboids

The purpose of this activity is to develop pupils' understanding of the surface area of a cuboid through practical exploration. Ask pupils to collect a sample of cuboids from home, preferably cardboard which can be deconstructed to show nets.

Look at this box. What shape is it? How many sides/vertices/faces has it got? How is it different to a cube? Look at this face. What shape is it? How could we calculate the area of this rectangle?

Take suggestions from the pupils as to how they could measure the surface area of the cuboid. Demonstrate opening the cuboid to reveal its net and taking its measurements.

[123] Van de Walle et al. (2007) Elementary and Middle School Mathematics Teaching Developmentally.

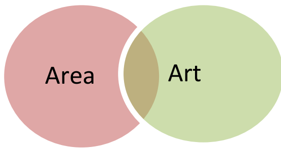
Which sides are equal? How many pairs of equal sides is there on this cuboid? How will this help us find the surface area of the cuboid? What measurements will you need to find the area of each face?

Pupils work in pairs or small groups to calculate the surface area of a cuboid. Pupils record their work by drawing the net of the cuboid and noting its measurements.

Now let's look at a cube. How many edges/corners/faces? How could we find the surface area of a cube?

Elicit ideas from the pupils .They may suggest finding the area of each face and adding them altogether. Extend pupils thinking through questioning to find a more efficient strategy for finding the surface area of a cube. Pupils should have the opportunity to practically discover the surface area of a cube.

Design a Cereal Box



Examine a number of cereal boxes with pupils. Tell the pupils that they have been tasked with designing a new cereal box and the company has given them strict instructions about the surface area of the cereal box for example $110cm^{2,}$ $120cm^2$. Pairs or small groups will calculate the dimensions of a box that will meet the requirements of the challenge, but the package design can be an individual or paired activity. Using dot or square centimetre paper, the pupils can plan their design. Encourage pupils to create at least two designs before settling on one to create. Using poster card or cardboard, children can create and design their cereal boxes. Pupils can evaluate their designs using a learning log, or describe the process in their maths journals.

CONSOLIDATION ACTIVITY

Wrapping Presents[124]

Using a variety of boxes, groups of pupils must select a box and work out the smallest rectangle of paper needed to wrap it so that it's completely covered. Pupils can then measure and cut out pieces of paper to wrap the present.

124 http://nrich.maths.org/163

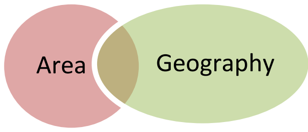
LEVEL D.6

FIND THE AREA OF A ROOM FROM A SCALE PLAN

TEACHING NOTES

Haylock[125] suggests that primary school children use only simple scales in scale drawings, such as 1:2, 1:5 or 1:10. Pupils should have experience of scaling in practical contexts. Examples of this could include using the photocopier to scale images up for display. Pupils could make scale drawings of the classroom or the car park also. Pupils should apply their knowledge of attributes and appropriate units of measure to convert, calculate and record measurements. Level D.4 of the Length Trajectory provides further examples of learning experiences to enable pupils to use and interpret scales on maps and plans.

Drawing to scale is the reverse of enlargement. Plans of rooms, houses etc. may be drawn on cm grid paper, the scale given as 1cm represents 1m (Pitt & Deboys, 1979 p.302).



Design a Bedroom Project

Show pupils a selection of simple apartment plans. Discuss the layout of the apartment.

How many bedrooms are in this apartment? How many bathrooms? Where is the living room? How many doors can you see? How many windows? What does the map not tell us? Could we estimate the area of this apartment? Why not? Who looks at house maps? Why are the measurements important? How can we show true measurements on small maps?

Tell the pupils that this is a bird's eye view of the apartment and it is not drawn to scale. Give pupils the dimension of one of the bedrooms for example 8 metres by 5 metres. Elicit ideas about drawing to scale. In pairs children will draw out a plan of the bedroom and also note the area of the room. Pupils make a drawing of a bedroom with the same dimensions. How would they design the bedroom? Pupils could make a model of the bedroom of the apartment. Extension activities could include finding out the cost of carpeting their bedroom. Pupils could work with a certain budget, for example €1,000, and use a catalogue to furnish the bedroom.

Assessment

Consolidation: Other Scale Investigations

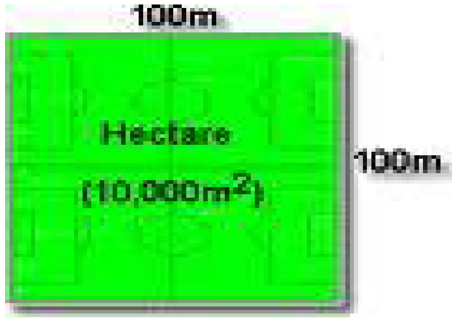
- Drawing the pupil's desk to scale
- Drawing the classroom/school playground/school garden to scale

[125] Haylock (2014) Mathematics explained for primary teachers.

LEVEL D.7

CALCULATE AREA USING ACRES & HECTARES

TEACHING NOTES



A hectare is the standard metric unit used to measure large areas. The size of farms, for example, is measured in hectares. Level C.2 of the Length section of the Measures manual explores the concept of the hectometre. A hectometre is a measurement of 100m. It may be useful to revise the concept of a hectometre, as a square with sides of length 100m would have an area of 1 hectare or 10,000 square metres. An acre is the standard imperial unit used to measure large areas. An acre is equal to 4840 square yards. An acre is equal to approximately 4,047 square metres.

SAMPLE LEARNING EXPERIENCES

Believe it or Not[126]

Introduce the hectare and square kilometre as units of area measure. Explain that the hectare is 10 000 square metres and a square kilometre is 100 hectares. Discuss strategies which pupils might use to calculate how many students will fit into a hectare and a square kilometre. Pupils will probably decide to work with 1 m^2, but other strategies may be suggested and evaluated. Have your pupils work in pairs or small groups to discuss and implement a chosen strategy that the student may:

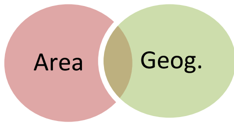
- calculate the number of students that could stand shoulder to shoulder in a square metre
- calculate the number of students that would fit in a hectare
- calculate the number of students that would fit in a square kilometre.

What units of measure are used to measure large areas? Do you know how many square metres are equal to a hectare? How many hectares are equal to a square kilometre? How could we calculate the number of pupils that will fit in an area of 1 square metre/1 hectare/1 sq km?

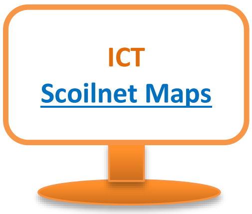
Groups report on their findings and suggest reasons why some groups may have different results. Discuss any difficulties encountered with calculations.

[126] http://www.schools.nsw.edu.au/learning/7-12assessments/naplan/teachstrategies/yr2014/img/TM2-3.pdf

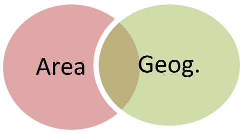
School in the Square[127]



Pupils use a locality map and the scale on the map to mark a square kilometre with the school in the centre. Pupils describe the interesting features included in the square kilometre. Square kilometres can also be marked out on Google Maps or Scoilnet Maps.



Province Investigation[128]



Using a map of Ireland, discuss with pupils the size of the provinces of Ireland. In groups have pupils pick a province. Groups can use the internet to answer the following questions, using pupil friendly search engines such as www.kidrex.org or www.duckduckduckgo.com. Irish primary schools also have free access to Encyclopedia Britannica through www.scoilnet.ie

- What is the area of your chosen province?
- What is the largest county in your chosen province?
- What is the difference in area between the largest and smallest county in your province?
- Find the area of the four provinces and order them from smallest and largest?
- Using this information, find the total area of Ireland.
- Approximately, how many times larger than Ireland is Australia?
- Name three counties whose combined area is smaller than County Cork.
- About how many times bigger in area is Co. Louth than Co. Tipperary?

Pupils should record and display their findings about the area of Ireland.

[127] http://www.schools.nsw.edu.au/learning/7-12assessments/naplan/teachstrategies/yr2014/img/TM2-3.pdf
[128] Area 5 Manual, PDST.

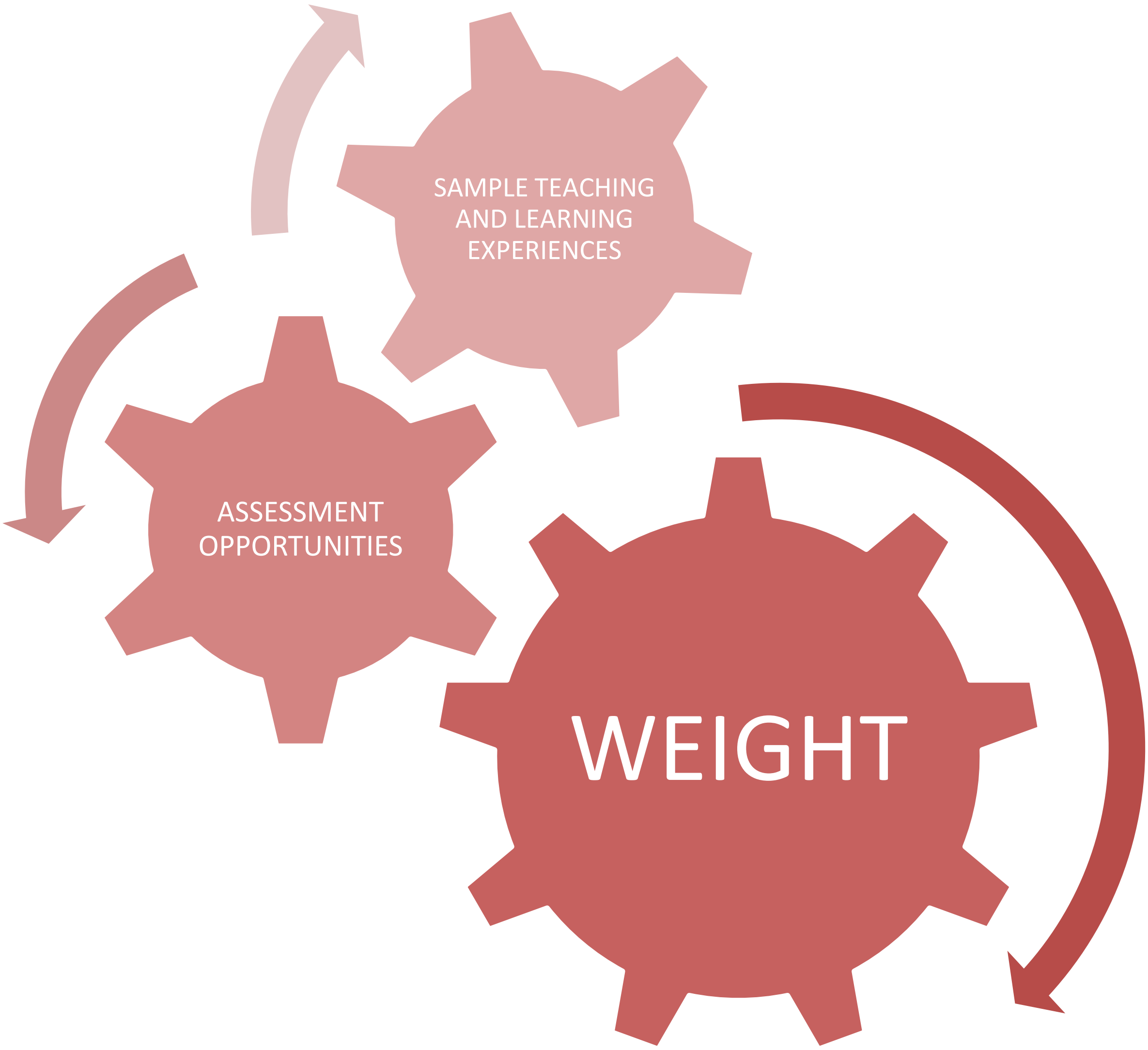
CONSOLIDATION ACTIVITIES

Design a Park[129]

Pupils design a new park and playground area. The total area of the park is 1 hectare. Discuss the scale chosen to design the park, for example1 cm^2 is equal to 25 m^2. The following features may be included in the plan:

- A car park measuring 1000 m^2
- A playing field of half a hectare
- A children's playground area of 500 m^2
- A sand pit of 50 m^2
- A toilet block of 250 m^2
- Paths for bike riding and walking that should be 2.5 metres wide
- Picnic and barbeque areas that take up 200 m^2 each. The rest of the park should be landscaped with lawn and creative designs for gardens. A water feature could be added!

[129] http://www.schools.nsw.edu.au/learning/7-12assessments/naplan/teachstrategies/yr2014/img/TM2-3.pdf

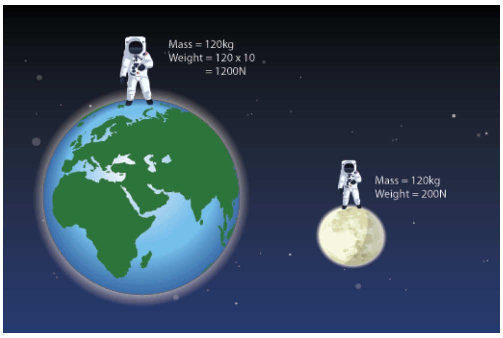

SAMPLE TEACHING
AND LEARNING
EXPERIENCES
ASSESSMENT
OPPORTUNITIES
WEIGHT

WEIGHT

WEIGHT - BACKGROUND KNOWLEDGE FOR TEACHERS -

What is weight?

There are some discrepancies between the correct scientific definition of weight and the meaning that is generally used in everyday life:



In scientific terms, the **mass** of an object is the amount of matter in an object; it is measured in kilogrammes and it does not vary according to position. However, the **weight** of an object is the force extended on it by gravity. It can vary according to its position, for example when an object is on top of a mountain or at sea level it is measured in newtons. In general terms when people talk about weight they expect it to be measured in kilogrammes not in newtons.[130]

Your mass stays the same on earth and the moon. Your weight changes because of the change in gravity.

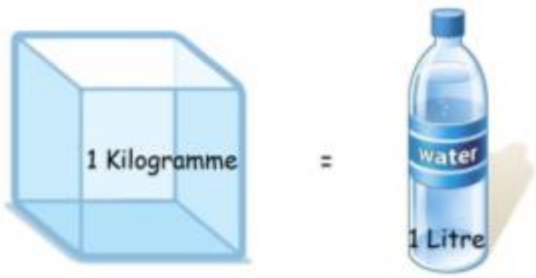
In simple terms, **weight** is the pull or force of gravity on an object. **Mass** is the amount of matter in an object and is a measure of the force needed to accelerate it.[131]

The term 'weight' rather than 'mass' will be used throughout this manual

Under normal conditions:

- 1 litre of water weighs 1 kilogramme
- 1 ml or 1cc weighs 1 gramme.

This is true for water but we cannot assume that it is true for other liquids.[132]



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[130] Suggate, Davis and Goulding (2010, p. 207)
[131] Van de Walle, Karp, Bay-Williams (2013, p. 395)
[132] Department of Western Australia (2005, p. 86)

MEASURES - WEIGHT

What tools are useful when measuring weight?

There are many useful tools for pupils to experience when measuring weight. These tools can be generally categorised into tools for measuring weight using non-standard units and those for measuring weight using standard units. The tools include but are not limited to:

Non-standard units

- marbles
- counters
- unifix cubes
- wooden blocks
- paint bottles
- pencils
- rubbers
- copies
- coins
- beads
- large paper clips
- cotton wool
- sponges
- buttons
- glue sticks

Variety of Scales

Beam Balance or Pan Scales

A beam balance is any balance where the beam is horizontal when equal weights are in both pans/sides.

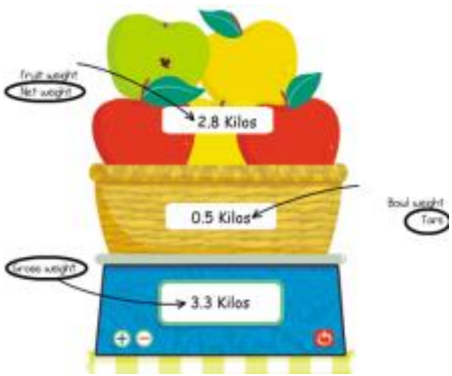
Spring Balance

The spring will be pulled or extended depending on the force (officially, this is measuring the weight because gravity is taken into account).

Kitchen Scales & Bathroom Scales

Luggage Scales

Tare Weight



Tare weight is the weight of an empty vehicle or container. By subtracting it from the gross weight (laden weight), the weight of the goods carried (the net weight) may be determined.

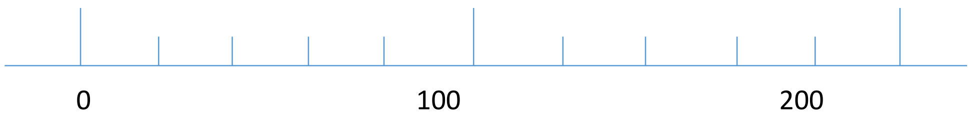
This can be useful in computing the cost of the goods carried for purposes of taxation (sometimes called a tariff) or for tolls related to barge, rail, road, or other traffic. This is important where the toll will vary with the value of the goods carried (*for example*, tolls on the Erie Canal). Tare weight is often published upon the sides of railway cars and transport vehicles to facilitate the computation of the load carried. Tare weight is also used in body composition assessment when doing underwater weighing. The word tare originates from the Middle French word tare "wastage in goods, deficiency, and imperfection". Tare weight is accounted for in kitchen scales, analytical (scientific) weighing scales and other scales. These scales include a button that resets to zero the scale of display when an empty container is placed on the weighing platform. Subsequently only the weight of the contents of the container is displayed.

Net Weight + Tare Weight= Gross Weight

WEIGHT - POSSIBLE PUPIL MISCONCEPTIONS

Common pupil misconceptions in relation to *Weight* include:

1. Using weighing scales: Some pupils find it difficult to understand and read gradations on kitchen scales[133] and bathroom scales. Some examples of this include:
   - Distinguishing between shorter and longer marks on scales;
   - Determining amounts indicated by blank marks on the scale; and
   - Weighing certain amounts may require pupils to estimate, for example, 75 grams may not be shown on the scale so estimation will be required.



2. Conservation of Weight: Some pupils may assume the larger the object, the greater the weight of that object. For example, a pupil may incorrectly assume that a large box filled with cotton wool is a greater

[133] Suggate, Davis & Goulding (2010, p. 209)

weight than a smaller box filled with beads. Similarly, pupils may assume that boxes of the same size have the same weight regardless of the contents or what material the boxes are made from. Pupils will need rich and varied experiences of holding, comparing, measuring and discussing objects to overcome and address this misconception.

3. Unit of measure: Some pupils may be surprised to discover that the weight of an object stays the same regardless of what unit is used to measure an object. For example, if firstly paper clips are used on a beam balance to find out the weight of an object using non-standard units; and then wooden blocks are used to find the weight of the same object, pupils may find it difficult to understand that the weight of the object remains the same. This may be due to the fact that more of the lighter units of measure (paper clips) will be required than the heavier units (wooden blocks) in order to balance the object. This greater numerical amount might confuse some pupils and result in them thinking that the object is heavier when compared with the paper clips. This possible misconception emphasises the importance of the unit of measure.
4. It is important to encourage pupils to handle and lift objects with both hands, the dominant hand and the helper hand. Our natural tendency to use our dominant hand for lifting items can cause things to 'seem' lighter in that hand, than when we lift items using our 'assistant or helper' hand.
5. Overcoming limitations[134]: pupils should be encouraged to overcome the limitations of weighing devices, for example, provide pupils with a kitchen scale that only weighs to 500 grams. Instruct pupils that you want them to weigh a bag of flour that weighs more than 500 grams.

WEIGHT - PARENTAL INVOVLEMENT

The following are a list of ideas that may be shared with parents to help develop pupils' conceptual understanding of weight. Some of the following ideas are taken from the NCCA's tip sheets for parents available through the NCCA website.[135]

Here you will also find some short videos of parents working with their children to develop their mathematical understanding.

Level A:

- When talking about weight use words like heavy, heavier and heaviest or light, lighter and lightest rather than talking about the size of the object (big or small).
- Ask your child to help you sort the shopping. Can you find the heaviest item, the lightest item, two items that weigh about the same amount?

[134] Department of Western Australia (2005, p. 55)
[135] http://www.ncca.ie/en/Curriculum_and_Assessment/Parents/

MEASURES - WEIGHT

Level B:

- Encourage your child to work out approximately how many kilograms a bag of rice weighs or how many litres in a bottle. Then check by weighing or measuring. Talk about the markings on the weighing scales or the measuring jug.
- Encourage your child to look at the weights of items in the cupboard. How many grams in a kilogram (1000)? So 500g is half a kilogram. How many of these packets would add up to a kilogram? If 1 kilogram costs €2, how much would 4 kilograms cost?

Level C:

- Discuss how important it is to be exact when measuring. Why do your measurements need to be more accurate when you are baking than when you are making a casserole?
- Look for items at home that are heavier/lighter than a certain weight
- Involve your child in baking at home and weighing out ingredients.

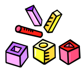
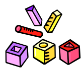
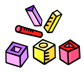
Level D:

- Look at recipes with your child. Discuss altering the measurements to make larger/smaller quantities. How could we change the ingredients to make 60 buns for the school cake-sale?
- Look at prices in shops. Explore items sold by weight for example flour/fruit/vegetables

WEIGHT - LEARNING TRAJECTORY

The learning trajectory is based on the objectives for Measures in the Primary School Mathematics Curriculum. In some instances, similar objectives at the same class level have been collapsed into one objective. Objectives that only refer to problem solving have not been included as discrete objectives because a problem solving approach is advocated throughout all of the teaching and learning experiences. Problem solving is viewed in this manual as a fundamental, integral part of mathematics teaching and learning that pupils should experience every day. The same colour coding from the curriculum is used – infants (green); first and second (red); third and fourth (blue); fifth and sixth (orange).

WEIGHT LEARNING TRAJECTORY LEVEL A[136]

Trajectory Levels	Concept	Developmental Experiences Concrete	 Pictorial	 Abstract
	Level A.1 Develop an understanding of the concept of weight through exploration, handling of objects, and use of appropriate vocabulary			
	Level A.2 Compare and order objects in terms of weight			
	Level A.3 Select and use appropriate non-standard units to weigh objects and estimate and measure weight in non-standard units.			

[136] This level is generally aligned with the objectives for Junior and Senior infants.

MEASURES - WEIGHT

WEIGHT LEARNING TRAJECTORY LEVEL B[137]

Trajectory Levels	Concept	Developmental Experiences: Concrete	Pictorial	Abstract
	Level B.1 Estimate, compare, measure and record weight using non-standard units and deepen understanding of conservation of weight	***As for Learning Experiences in Level A.1 Extension activities included***		
	Level B.2 Select and use appropriate non-standard measuring units and instruments and discuss reasons for choice	***As for Learning Experiences in Level A.2 Extension activities included***		
	Level B.3 Estimate, measure and record weight using standard unit (the kilogram), half kilogram and quarter kilogram and explore instances when objects or substances that weigh 1 kg vary greatly in size	***As for Learning Experiences in Level A.3 Extension activities included***		

[137] This level is generally aligned with the objectives for First and Second class.

MEASURES - WEIGHT

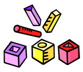
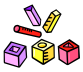
WEIGHT LEARNING TRAJECTORY LEVEL C[138]

Trajectory Levels	Concept	Developmental Experiences: Concrete	Pictorial	Abstract
	Level C.1 Estimate, compare, measure and record the weight of a wide variety of objects using appropriate metric units (kg, g) and select suitable instruments of measurement			
	Level C.2 Rename units of weight in kg and g and rename units of weight in decimal or fraction form			

[138] This level is generally aligned with the objectives for Third and Fourth class.

MEASURES - WEIGHT

WEIGHT LEARNING TRAJECTORY LEVEL D[139]

Trajectory Levels	Concept	Developmental Experiences		
		Concrete	Pictorial	Abstract
	Level D.1 Estimate, compare, measure and record the weight of a wide variety of objects using appropriate metric units and select suitable instruments of measurement			
	Level D.2 Rename units of weight in kg and g and decimal or fraction form			

[139] This level is generally aligned with the objectives for Fifth and Sixth class.

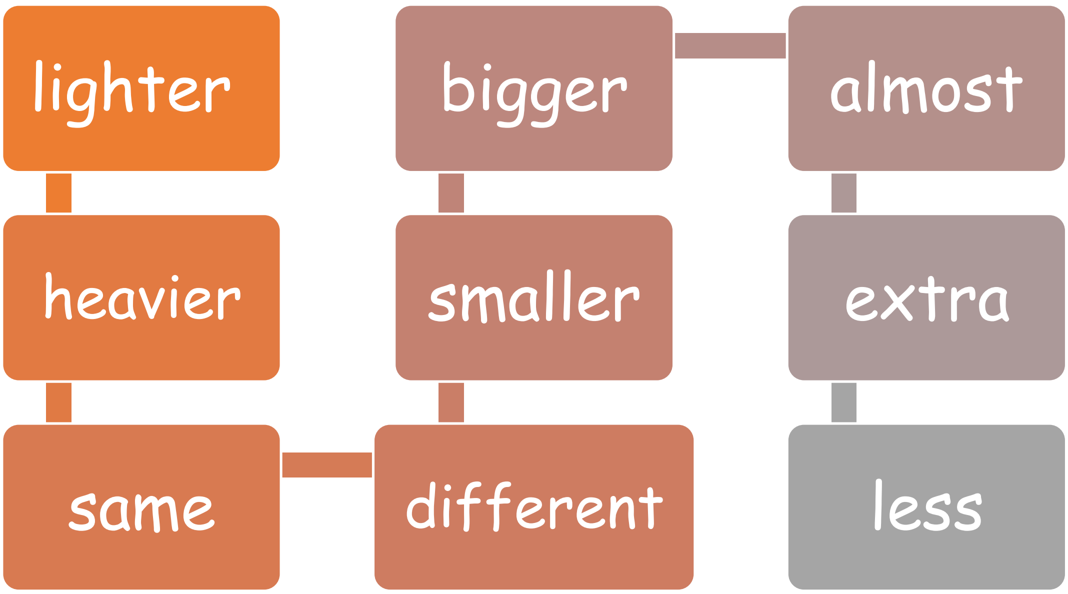
MEASURES - WEIGHT

LEVEL A.1

DEVELOP AN UNDERSTANDING OF THE CONCEPT OF WEIGHT THROUGH EXPLORATION, HANDLING OF OBJECTS, AND USE OF APPROPRIATE VOCABULARY

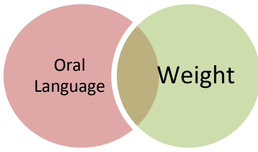
TEACHING NOTES

Early experiences should develop pupils' conceptual awareness of what weight actually is, and of the range of vocabulary that can be used to describe it. Weight is an attribute that cannot be seen. This needs to be brought to pupils' attention through exploration and handling of objects. Pupils should learn to pick up and pull objects to feel their heaviness. Initially young pupils describe objects as heavy or not heavy. They gradually learn to compare and use more meaningful terms such as lighter and heavier. It is important that experiences are provided so that the size of an object is not confused with its weight. Initial exaggerated differences between objects are necessary. Pupils must have some idea of heavy, light, heavier, lighter and so on before they can start to measure weight. Pupils who find these concepts difficult will benefit from spending longer on the use of real-life concrete objects. Pupils will need to engage in discussion with the teacher to enhance their experiences and the discussion should focus on the key vocabulary described below. All of the learning experiences described at each level, can be differentiated or extended to accommodate different pupil learning styles and pace of learning.



MEASURES - WEIGHT

Tricky Bags[140]

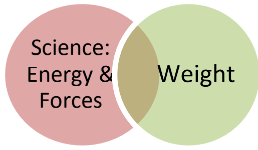


In this activity, pupils investigate two gift bags that look the same but one is empty and the other is filled with books. Display the two gift bags for the pupils to see. Elicit from the pupils what makes the bags the same or different and why. Invite two pupils to lift each bag and describe what they feel. Allow other pupils to lift the bags and share their descriptions. In the initial stages ensure consistency of items in the bags, for example groups of books/ copies/ lunchboxes/ Lego bricks. Extend pupil thinking by asking them to guess which group of objects are in the mystery bag.

Are these bags the same or different?
How do you know?
Are you sure? Can you guess by just looking which bag is heavy?

Push and Pull

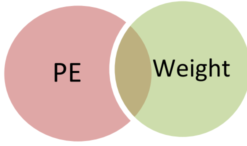
In this activity, pupils push and pull objects to see which feels heavier. Using two large boxes, fill one box with items from the classroom, for example copies or books or lunch boxes. Show pupils the two large boxes. Explain to pupils that the boxes are too large to be lifted. Ask if they could think of another way of comparing them. Elicit from pupils ways we could move the boxes. Pupils take turns pushing or pulling the boxes. Discuss with pupils items they observed being pushed or pulled rather than lifted and why. For example: beds, tables, television cabinet, couches.



Are these boxes the same or different? How do you know? Are you sure? How could you find out? The boxes are too large for you to lift safely. Can you think of another way of finding out how heavy they are? Why do you have to push instead of lift? What happened when you pulled the first box?

[140] http://nzmaths.co.nz/resource/tricky-bags

Tug of War

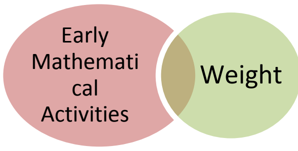


Move outside with the boxes from the previous activity. Once on the playground attach bungee cords to each box used in the previous activity. Ask pupils which box will be easier to pull. Once pupils have investigated, arrange pupils into groups and provide each with a long rope. Get pupils to show what an unfair Tug of War would look like. Next ask pupils to show what a fair game of Tug of war might look like.

How does tug of war work? Ann's side has two people and Jack's side has four people, which side will have to work harder to win? How could the Tug of War be fairer?

CONSOLIDATION ACTIVITY

Heavier than or Lighter than?[141]



Groups are given three small items, the same amount of each, for example, three full drink cartons, three plastic bears, three tubs of playdough. First pupils explore sorting the items. There are many suggestions for sorting in Ready Set Go Maths [142] Pupils then express their estimates regarding weight of each item, before ordering the objects by lifting. Pupils and teachers discuss and record using pictures.

Extension: Check by dropping each item or group of items into suspended, knee-high stockings or long socks.

How can I sort these items? What could I find out about these things? How could I find the heaviest? The lightest? What does it mean if I ask you to put them in order? How could you do that? What are the words you would use? Which one of these do you think is the lightest? Why? How would you draw what you are measuring? What will you see happening if I drop a heavy toy into this stocking, and a light toy into the next stocking? What are the stockings showing us? Why?

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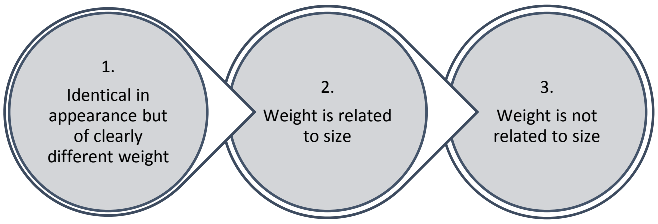
[141] Teaching Measurement: Early Stage 1 and Stage 1: PG.124
[142] fPitt, E. (2001) Ready Set Go Maths

LEVEL A.2

COMPARE AND ORDER OBJECTS IN TERMS OF WEIGHT

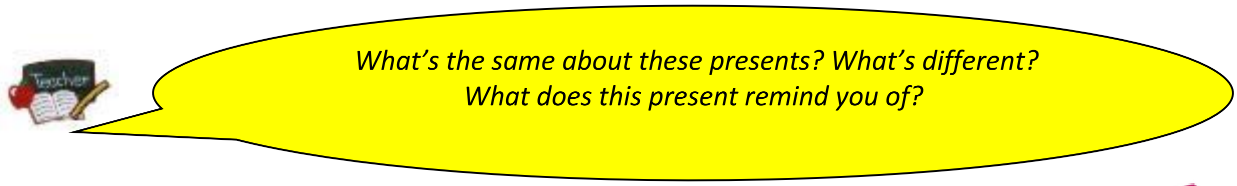
Compare and Order Objects in Terms of Weight

Comparing the weight of objects is the second stage in developing an understanding of weight. When comparing, pupils should have opportunities to compare items:



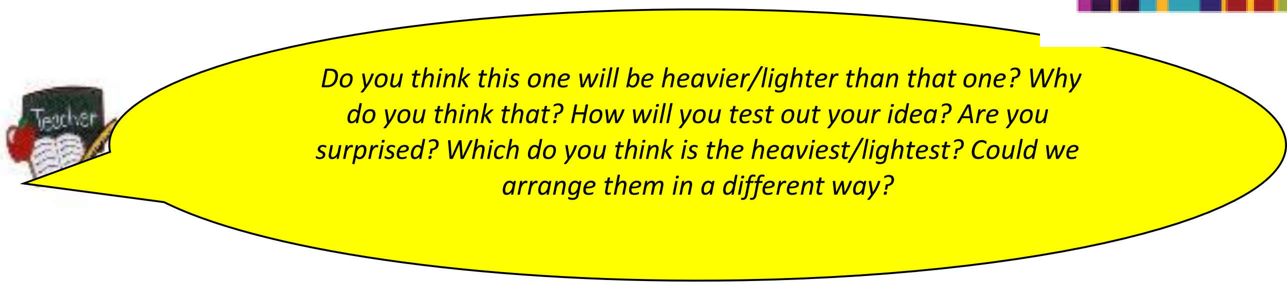
Mystery Parcels

Several parcels are prepared which are similar in size but of different weight. Use different coloured wrapping to distinguish between each parcel. Pupils put the parcels in order by handling and confirm their findings using the balance.



A more difficult version involves using three parcels of different size, where size is related to weight. Allow pupils to again describe the parcels, handle the parcels and order the parcels from heaviest to lightest – a larger parcel can be lighter than a smaller parcel.

Finally select a group of parcels where weight is not related to size. Allow pupils to select ways of ordering the parcels. Record the various ways pupils order the parcels using a camera. This will allow further discussion on the various ways of ordering the parcels. At this stage pupils have the opportunity to discover the important point that a larger parcel can be lighter than a smaller parcel. This consolidates their understanding that size and weight are independent attributes. Many pupils require rich and varied practical experiences to disassociate weight from size and to accept that small objects can be heavy and large objects light.



This interactive activity challenges pupils to discriminate between light, medium and heavy items as they pack up the moving truck.[143]

ICT
Moving Day

Sorting

Pupils sort heavy and light objects into two groups. Objects should be obviously light or obviously heavy, for example, a piece of string, paperclip, large stone, large bottle of liquid etc. Pupils report back on why objects were placed into different groups.

TEACHING NOTES

The use of pictorial representations such as Tree, Carroll and Venn diagrams can add a further dimension to pupils' sorting experiences. These provide a fresh perspective on pupils' sorting activities. For each of the representations pupils select one object at a time and decide where to place it on the diagram. The teacher engages the pupils in discussions about their arrangements.[144]

Tell me about these objects. Why do they go together?
Why does this one not belong?

[143] http://www.learningliftoff.com/wp-content/uploads/2014/02/Moving_Van.swf
[144] Pitt, E.,(2001). *Ready Set Go*. Pg. 44

MEASURES - WEIGHT

The following activities could be used for further sorting work and also to explore how to represent a set when it is sorted.

1. Random Collection of items: Limited collection of small items selected, so that pupils can focus on the property of weight

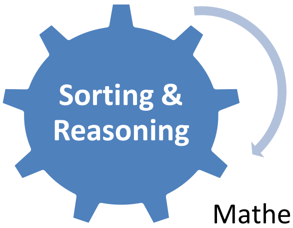
2. Collection sorted by weight

3. Random collection of big and small items

4. Sorting is represented using a Venn Diagram

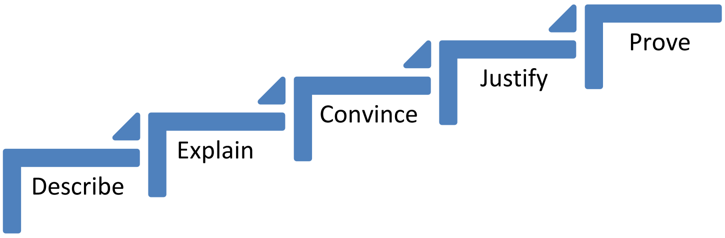
5. A Tree Diagram can also be used to scaffold the process of sorting the collection and to represent a sorted collection

TEACHING NOTES [145]



Pupils who can undertake these sorting activities with various random collections and can explain their decisions are growing in their ability to reason mathematically.

Mathematical reasoning involves pupils in the following mathematical practices



Blindfold[146]

In pairs, pupils take turns to be blindfolded. The second pupil places an object or a container in each hand of the blindfolded pupil. Objects should be obviously light or obviously heavy objects. Examples include, a piece of string, paperclip, a large stone, large bottle of liquid. Pupils state which hand is holding the heavier object or container.

Tell me about these two rocks, blocks or books? How can I work out which bag is heavier? What can you see when I hold a bag in each hand? How could I draw a picture that shows which bag is heavier?

Weight Hunt

Teacher invites pupils to find objects in the classroom heavier than a schoolbag, lighter than a lunchbox.

Who can find something heavier than a lunchbox, how would you check? Does everyone agree? Does anyone have any other suggestions? I wonder could we find something lighter than this storybook? Can you convince me it's lighter.

I Spy

Following teacher led activities such as 'Weight Hunt', pupils can engage in a game of 'I spy', in pairs/groups/whole class. The children 'spy' objects around the classroom, heavier than or lighter than a visible chosen item. For example, I spy with my little eye, something heavier than my pencil, book, plastic bear. Something lighter than my bag, my bottle of water etc.

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145 The NRICH, 'Reasoning: The Journey from Novice to Expert (article): Nrich.maths.org' (December 2014) http://nrich.maths.org/11336

146 Teaching Measurement: Early Stage andli Stage 1 pg. 121

MEASURES - WEIGHT

LEVEL A.3

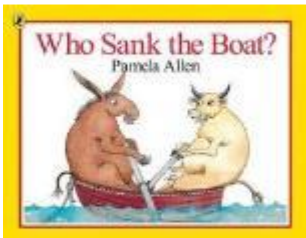
SELECT AND USE APPROPRIATE NON-STANDARD UNITS TO WEIGH OBJECTS AND ESTIMATE AND MEASURE WEIGHT IN NON-STANDARD UNITS.

TEACHING NOTES

Measuring the weight of objects using non-standard or informal units is the third level of Level A in the Learning Trajectory. Beginning with non-standard but familiar units allows the pupils to focus on the process of repeatedly using a unit as a measuring device. Pupils need opportunities to measure a wide range of objects using pan balances or "home-made" beam balances. The use of non-standard units such as blocks, plastic teddy bears, copybooks, tin cans, bags of beads, decks of cards or pencils is appropriate. Balance scales and see-saws can be used to directly compare the weight of two objects. Home-made beam balances can be constructed, see examples below. For further information on the various types of weighing devices please see *Background Knowledge for Teachers.* The Interactive link also shows the use of a pan balance.[147]

Storybooks

These storybooks provide further opportunities for consolidation of Level A activities 'Who sank the boat?' [148], 'Mr. Grumpy's Outing',[149] and 'Just a Little Bit'.[150]



The interactive activity[151] allows pupils to explore and investigate how to balance a see-saw using bunnies. Pupils could investigate this practically in the classroom using a long ruler as the lever, some cubes for the load and their pencil case as the fulcrum.

[147] http://pbskids.org/sid/fablab_panbalance.html
[148] Allen P., *Who Sank the Boat? (Picture Puffin)* (Puffin 2000)
[149] Burningham J, *Mr Gumpy's outing* (Red Fox 2001)
[150] Tompert A *Just a little bit* (Houghton Mifflin, Harcourt 1993)
[151] http://peepandthebigwideworld.com/en/kids/games/2/bunny-balance/

LEVEL B.1

ESTIMATE, COMPARE, MEASURE AND RECORD WEIGHT USING NON-STANDARD UNITS AND DEEPEN UNDERSTANDING OF CONSERVATION OF WEIGHT

TEACHING NOTES

At this level the focus is on the Kilogram. It may however make sense to begin with the ½ Kg, as 1Kg can be too heavy for pupils to work with[152]. *This will be discussed further in Level B3.*

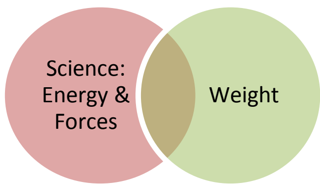
A child's schoolbag is a weight with which pupils are very familiar and can therefore be a useful benchmark during weight activities. Objects that are not usually picked up are not particularly good benchmarks as pupils will not be familiar with their weight. During the activities to discover and establish 1Kg benchmarks, pupils themselves may suggest reasons some benchmarks are more useful.

Lifting Closed Tubs[153]

Art and Craft materials are often stored in identical closed tubs for example matchsticks, clay, lollipop sticks. Invite pupils to lift each tub and say what they think each tub is holding because of its weight. Open tubs for pupils to talk about the contents and elicit from pupils reasons why one tub was heavier than another. Add an empty closed tub and invite pupils to lift it.

What do you think is inside this tub? Is this tub the lightest? Guide pupils to view and handle containers. Ask: How could we order the containers on a shelf from heaviest to lightest? What other ways could the containers be ordered?

See-Saws



Pupils can make a see-saw using a can, plasticine and a shoe box lid. Stop the soft drink can from rolling by fixing it to the table with tape or put plasticine rolls on each side. Pupils can investigate whether it is possible to balance the see-saw when both sides are empty. Pupils will then use the see-saw to find objects that are the same weight. Next pupils can use toy cars and animals to see if they can make the see-saw balance. Encourage pupils to record their findings by drawing a picture to show some of the things that balanced.

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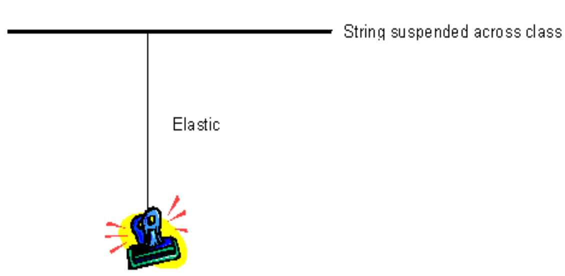
[152] Pitt, E., & Deboys, M. (1980). *Lines of Development in Primary Mathematics* (3rd ed.). Belfast: Blackstaff Press [for] the Queen's University of Belfast, Teachers' Centre.

[153] Willis, S., & Devlin, W. (2009). *First steps in mathematics: measurement: understand units, direct measure*. Ascot, W.A.: STEPS Professional Development

Bungees [154]

Pupils use a simple piece of elastic as a bungee and measure how far the elastic stretches to compare the weight of different objects. Pupils record the 'stretch of the elastic on a page behind the bungee. Set up a bungee by tying a piece of elastic onto a clothes peg/ bull clip. The top of the bungee will need to be attached to something it can hang from, a metre ruler suspended across two desks would be ideal. There also needs to be a piece of paper behind the bungee, which the pupils can use to mark how far down the wall the bungee extends. This record on paper will allow pupils to compare various items later on. Have pupils take one object at a time and attach it to the peg. They then let the objects go, wait till the elastic comes to rest and mark on the paper how far down the object falls.



Pupils repeat for all objects and then decide which is heaviest.

In this interactive activity pupils investigate combinations of weights to identify the Poddle's weight.[155]

[154] http://nzmaths.co.nz/resource/seesaws
[155] http://pbskids.org/cyberchase/math-games/poddle-weigh-in/

TEACHING NOTES

Conservation of Weight

Young pupils are influenced by what they see and can be easily deceived by the shape or the size of an object. For example, pupils who do not yet conserve the property of weight will think that if the shape of an object changes so does its weight.

To determine pupils' understanding of the conservation of weight, two pieces of plasticine can be compared and found to be the same weight. Allow pupils to then change the shape of one of the pieces. Many young pupils may now say that the longer object has a greater weight. Pupils who are able to conserve the property of weight can reason that because nothing has been added or taken away they both have the same weight.

To develop pupils' concept of conservation of weight, many practical experiences disassociating weight from size and accepting a small object can be heavy and a large object light, are required.

CONSOLIDATION ACTIVITIES

What's Your Prediction? [156]

Pairs of pupils compare three groups of items which have the same number, but different kinds of objects, such as five pencils, five cups and five interlocking blocks or three empty margarine containers, three blocks and three balls. Pupils estimate first, then find which group has the greater weight by using a beam/pan balance.

[156] Teaching Measurement Early Stage 1 and Stage 1 2003 pg. 128

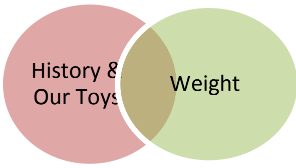
LEVEL B.2

SELECT AND USE APPROPRIATE NON-STANDARD MEASURING UNITS AND INSTRUMENTS AND DISCUSS REASONS FOR CHOICE

TEACHING NOTES

It is a good idea to set up a ‘weight table’ in the classroom with a selection of balances and items which children will enjoy investigating, comparing different weights and engaging in discussion.

Using Our Toys



Begin this activity by looking at two toys of similar size but different weights, for example a doll and a teddy.

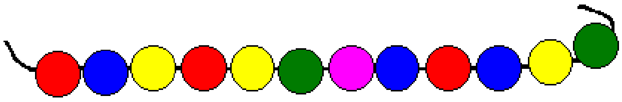
Which of these toys do you think is the heaviest? Why do you think that? How could you check? Introduce another soft toy. *Is this toy heavier or lighter or in-between?*

Pupils compare the weight of the toys by holding them. Introduce another soft toy. Allow pupils to compare the objects by handling them. Elicit from pupils ways we could order the toys. Photograph various ways pupils suggest to order the toys. Use photos to discuss scenarios where pupils ordered the toys by size and by weight. Place a balance scale in the centre of the group of pupils. Pupils share their ideas and use balance scales to check the weight of the toys. Encourage pupils to predict what will happen if one toy is placed into this pan on the scales. Pupils investigate with all three toys. Next give each pupil a string of 10 beads (alternatively pupils could make these prior to the activity).

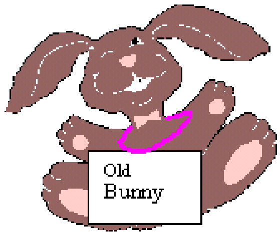
What will happen if a string of beads is put into one pan of the scales? Let’s see what happens if Aoife and Saoirse place their string of beads into each pan. Why are they balanced? Let’s investigate the toys and see if we can balance the scales.

Place a toy in one pan of the balance and invite pupils to add bead strings to the other pan until the toy is balanced. Take turns adding bead strings to the scales counting as they are added and record. Use skip-counting to count the bead strings. For the scale to balance, this may require using some individual beads at the end. Try this activity with the other toys. *How many beads do you think this toy weighs?*

If you have a sufficient quantity of balances, organise pupils to work in small groups to weigh other toys or classroom objects. Alternatively leave the balance set up on the maths table for the pupils to take turns using throughout the day. **Extension**: During the following lessons pupils can measure the weight of their own toys in non-standard units for example bead strings or cubes. As well as weighing the toys, pupils could measure their toys' height and waist using bead measuring strings. Demonstrate how to take and record the three measurements, see example below



Height	23 beads
Tummy	30 beads
Weight	27 beads



Mystery Toys Game

Pupils can record the three measurements of another toy on their mystery card. As the pupils work, assign each pupil a letter to go on their "mystery" card. Enter this letter next to the pupil's name and the toy's name on a class list. After the pupils have measured their toy collect the "Mystery" cards. In pairs, pupils select, at random, a mystery card from the pile. Pairs predict, reasoning from the measurements on the card which toy it might be. Next pupils measure to find a match. When the pairs are sure they have found the right toy, they tell the toy's name and code number to the teacher who checks against the class list. If they are correct they get another card to solve. If not they continue.

Sample: Mystery Card K

Height	Width	Weight
16 Beads	10 Beads	12 Beads

How did you go about finding your mystery toy? Which measurement on the card was the most useful? Discuss with the class the strategies used while finding the mystery toys. What did you do to work out the mystery toys?

Make Another Bag

The teacher displays a bag with some blocks in it. Pupils make a bag that has the same weight by filling with blocks and then lifting the two bags. Pupils find the weight of their bag by choosing appropriate units and measuring on a balance. The measuring process and results are recorded, including a description on the choice of units.

How would you compare the weights accurately? What kind of units would you use for this? Does the second bag need to be the same size as the first bag? Why? Any advice for choosing and using units? Do I have to pack the units or make sure they cover? Why? How could you record this?

LEVEL B.3

ESTIMATE, MEASURE AND RECORD WEIGHT USING STANDARD UNITS (THE KILOGRAM), 1/2 KILOGRAM AND THE 1/4 KILOGRAM AND EXPLORE INSTANCES WHEN OBJECTS OR SUBSTANCES THAT WEIGH 1KG VARY GREATLY IN SIZE

TEACHING NOTES

It is difficult to estimate the weight of individual items. Try picking up a schoolbag and estimating its weight. This is something pupils find challenging as they may not have had much practice in doing so. Also we don't have the same 'on-board' means in which to benchmark like length for example, fingertip to opposite shoulder= 1 metre[157]. Pupils need to develop personal benchmarks with which to measure various objects in their daily lives. Their personal benchmarks need to gradually relate more to standard measures such as 1Kg or ½ Kg.[158]

It is sensible to begin with the kilogramme as the gramme is too small to 'feel'. An appreciation for the feel of a kilogramme needs to be built up with lots of examples of 1Kg weight. Pupils should compare a standard 1Kg weight with other objects, first by holding them and then by using a balance. The following sequence may be helpful to divide the kilogramme into smaller parts for example a ½ Kg, a ¼ Kg in 2nd and in 3rd class $^{1}/_{10}$of a Kg or 100g.

School Bags

Begin by asking pupils to bring their school bags and allow pupils to compare the heaviness of their bags to their partner's bag by handling.

Which bag is heavier, yours or your classmates? What does your bag contain? What makes it heavier? Does it make a difference when you hold the bag in your other hand?

Next, select five or six bags from around the class (it is important to select bags of various sizes and shapes). The purpose of this activity is for the pupils to discover that the biggest bag does not necessarily mean the heaviest bag. This will also provide opportunities to consolidate the idea of conservation. Gather solution strategies, then trial strategies and elicit from pupils an effective way to order the school bags from lightest to heaviest.

[157] Haylock, D., *Mathematics Explained for Primary Teachers* (4th edn, SAGE Publications 2010)
[158] www.nzmaths.co.nz/resource/making-benchmarks-mass

Buckets for Benchmarking

The purpose of this activity is to provide pupils with experiences of comparing weights and of creating a benchmark of what a kilogramme feels like. Make available a 1Kg weight for pupils to use to give them the

'feel' of a kilogramme. Many food items come in 1Kg packages and may be useful for this activity. Seat the class in a circle around a variety of items from the classroom and from items found in a standard kitchen. Place three buckets on the floor with the following labels.

Pass a selection of items around the circle to pupils to give them an opportunity to feel the weight of each object. Once the group has got the opportunity to handle the items, they predict what bucket each item should go in. Instruct individual pupils to place the items into the buckets. This activity could be carried out in smaller groups if necessary to give pupils more hands-on experience. On large sheets of paper, pupils record their findings.

Elicit from pupils how they could get a more accurate idea of whether the items in each bucket are classified properly, why?, why not?, how could we check? Using a balance allow pupils to investigate their predictions. Establish what the differences were between pupils' estimates and the actual weight.

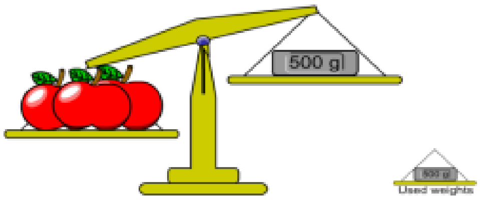
What benchmarks can we use for 1Kg? What strategies did you use to establish an estimate for an object's weight? What other possible items could be used as benchmarks for items that are less than 1Kg in weight? Can we make a list of our classroom benchmarks? What items from home could you use as benchmarks?

This activity could be repeated to establish benchmarks for the ½ Kg and the ¼ Kg.

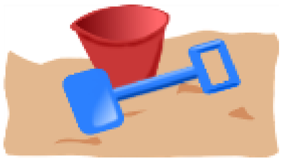
The Inequality Sign

The foundation of all aspects of measurement is direct comparison, putting two and then more than two objects in order according to the attribute in question. Recording the results of comparison and ordering can be an opportunity to develop the use of the inequality sign. So for example 'The tomatoes are heavier than 1Kg' can be recorded as illustrated below. This is a formal expression of the practical context of the principal of inequalities[159]. This ICT link will open a pan balance to explore this concept further.

ICT
[Interactive Pan Balance]



Sandbags



Pupils can use the sand tray and some plastic Ziplock bags for this activity. Either individually or in small groups, give pupils a bag and ask them to put ½ Kg of sand into it. Pupils could use the benchmarks established in the previous activity to refine their measurements. Next allow pupils to investigate whether their bags weigh ½ Kg.

A Cup of Rice[160]

Pairs of pupils choose suitable non-standard units to find the weight of a cup of rice. Pupils record the weight and state why they chose the units. The class discusses the results and compares the units which were chosen. Some units may have a greater or smaller volume than other units, for example, marbles, unifix cubes, etc.

Does it Balance?[161]

Pupils are given a choice of non-standard units. This may include cubes that they can use to find the weight of different objects using a balance. Such objects could include a large but light teddy bear or a small but heavy stone. Other examples of non-standard units would allow pupils to select lollipop sticks, marbles, blocks. Providing a variety of non-standard units allows pupils to engage in open-ended investigations identifying what quantities of non-standard units that can balance an item. Pupils could also combine varieties of non-standard units to balance the big dice, challenge each group to come up with a different way.

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[159] Haylock, D. *Mathematics Explained for Primary Teachers* (4th edn, SAGE Publications 2010)
[160] Teaching Measurement: Early Stage and Stage 1 2003 pg. 132
[161] Teaching Measurement: Early Stage and Stage 1 2003 pg. 131

It is important that pupils are given tasks which emphasises different volumes can have the same weight. Likewise the related concept of same volume, different weight also applies.

Heaviest Pencil Case[162]

Pupils work in groups of three or four to estimate, then measure whose pencil case is heaviest by measuring the weight of each pencil case with blocks (teddies, marbles etc.) and a balance. Ensure that the same units are selected for measuring. Record in order of weight.

How could I measure this pencil case? What units could I use for each measurement (length, area, volume, weight)? How can I compare the weight of two pencil cases, without putting both pencil cases on a balance at the same time? What could I use as units? Will all members of my group use the same units? Why? Could we use a mix of units to measure? Why? How would we write our results? What words could be used in my recording?

No More Gaps[163]

The purpose of this activity is to discuss and predict the weight of the same quantity of a specific object in two different structures. For example: Do ten loose lollipop sticks have the same weight as a bundle of ten lollipop sticks? Using Dienes Blocks investigate whether a flat has the same weight as 100 ones, ten loose interlocking blocks and a rectangular prism of ten blocks? Explain how you investigated this challenge. Were your predictions accurate? What did you discover?

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[162] Teaching Measurement: Early Stage and Stage 1 2003 pg. 136

[163] Teaching Measurement: Early Stage and Stage 1 2003 pg. 135

CONSOLIDATION ACTIVITIES

Work it Out[164]

Pupils measure the weight of an object using Dienes blocks as a non-standard unit (for example using 10 ones). Using this measure, pupils predict how many of another unit (for example how many rods?) would be needed to balance the object. Record the estimate and calculation before using a balance to check.

Solve the Mystery

Use a selection of objects whose weight pupils have explored and are familiar with. Place one of these items into the mystery box and balance it. Describe the following scenario to pupils: Can you balance my mystery object using the cubes. Great we can see that my mystery object can be balanced by __ blocks. What could my mystery object be? As pupils become more efficient, it may be possible to combine objects in the mystery box, leading to a variety of possibilities.

Extension: How about we use another non-standard unit? How many pencils would it take to balance it? Predict and check.

[164] Teaching Measurement: Early Stage and Stage 1 2003 pg.141

MEASURES - WEIGHT

LEVEL C.1

ESTIMATE, COMPARE, MEASURE AND RECORD THE WEIGHT OF A WIDE VARIETY OF OBJECTS USING APPROPRIATE METRIC UNITS (KG, G) AND SELECT SUITABLE INSTRUMENTS OF MEASUREMENT

TEACHING NOTES

Pupils should have experience of weighing a variety of different objects using various weighing devices. Some of these are outlined in Level 1 and 2, for example direct handling, beam balance, spring balance and home-made weighing devices using elastic. Pupils also need to have experience of weighing objects using the following devices:

- digital kitchen scales
- non-digital kitchen scales
- digital bathroom scales
- non-digital bathroom scales
- luggage spring scales

Many pupils find it difficult to ‘read’ the weight using these various devices. Pupils should be encouraged to estimate first, then weigh the item, read the weight and then compare this weight to their estimation. Through exploration pupils should be enabled to discriminate between the suitability of the devices, for example, the bathroom scales is more suited to weighing a full school bag than the kitchen scales.

Bags of Weight[165]

Provide pupils with pre-prepared labelled bags of various weights, for example 100g, 250g, 500g, 800g, 1000g. Invite the pupils to choose one of the bags of weight. Next, ask pupils to choose from a variety of substances such as rice, porridge, pasta, playdough, peas etc. and to place what they think will match their weight bag into a tub. Pupils can then compare their weight bag with the tubs on a scales.

How much of each will I need to make my weight bag?	Rice 	Porridge 	Pasta 	Playdough 	Peas 

[165] Department of Western Australia (2005, p. 86)

- Pupils can refine their estimation based on this comparison and add or take away from their bag if necessary
- Pupils can check their estimations using a scales[166]

ICT
Alien Cookbook

In this interactive activity, pupils consolidate their skills of identifying standard measuring units and using a measuring scale to help 'Hutch' bake a cake.

Pupils should be encouraged to work with one quantity of weight for a variety of substances in order to ensure a solid understating of that weight before moving onto other bags of weight.

Estimation Station- Fruit and Vegetables


[167]

For this activity it may be useful to assign pupils a vegetable type to bring in prior to the activity. Discuss with the pupils how fruit and vegetables are weighed in the supermarket. Show them three packets of potatoes of varying size for example 1kg, 500g, 1.5kg and ask them to decide which one weighs 1kg. Encourage pupils to consider the classroom benchmarks we have for one kilogramme. Afterwards, elicit from pupils what strategies they used to decide the weight of the bags.

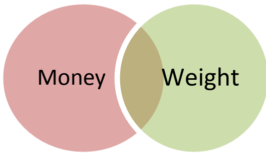
Explain to the pupils that they are going to do the fruit and vegetable shopping today. They will need to write a shopping list and estimate how many pieces would be close to 1 kilogramme. Explain to the pupils that it would be difficult to reach a kilogramme exactly, and so an amount that is just over or under is adequate.

Is it possible to decide how heavy something is just by looking at it? Are all big things heavy? Are all small things light? How do you know? Convince me. What is the difference between your benchmark and the potato bag you picked? Are they the same? Is the size of your benchmark different to the size of the potato bag?

[166] http://www.bbc.co.uk/schools/starship/maths/games/alien_cookbook/big_sound/full.shtml

Using the vegetables the groups have brought in, allow pupils to investigate how they can create a bag of groceries that weighs close to 1Kg. Provide the pupils with potatoes, onions, oranges, lemons, and carrots and have them bag them into 1kg amounts. Discuss their results with them. Were they close to 1Kg? Was there a particular fruit or vegetable that was lighter or heavier than they thought?

Post the Parcel[168]



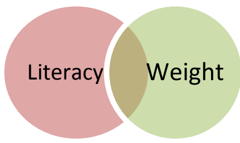
A selection of parcels, a metre stick and a weighing scales are required for this activity. In this activity, pupils estimate then measure the weight of different parcels to be posted. Choose one of the boxes to work with. Pick it up and estimate its weight. Record your estimate. Use the scales to weigh the box. Record the weight. Compare your estimate with the actual weight. How close were you? Use the table to work out how much it would cost to send your parcel to Belfast. Try the interactive to see if you can post the parcels according to the correct weight.

PostalRateTable

Zone 1 - Ireland & NI

Weight Not Over	Letter / Postcard	Large Envelope	Packet	Parcel
100g	€0.70	€1.25	€2.80	€7.00
250g		€1.70	€3.50	€7.00
500g		€2.30	€4.50	€7.00
1kg			€7.00	€7.00
1.5kg			€8.25	€8.25
2kg			€8.25	€8.25
2.5kg				€10.00

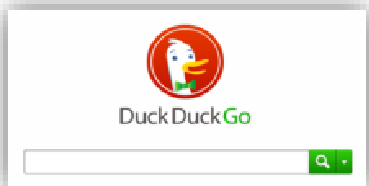
At the Zoo



Find out facts about the weight of very large animals and make a report about these for the class. To get you started here are some facts about the African elephant.

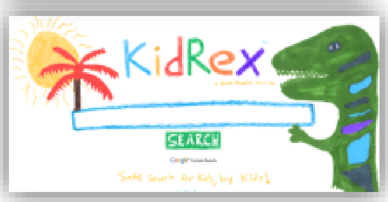
[168] Pitt, E., & Deboys, M. (1980). *Lines of Development in Primary Mathematics* (3rd ed.). Belfast: Blackstaff Press [for] the Queen's University of Belfast, Teachers' Centre.

The African elephant is the biggest animal on land. Fully grown the male can be 7 metres long, 3.2 metres tall at the shoulder and have a weight *of 6500 kg. Its tusks can weigh as much as 100 kg each. The largest pair of tusks on record are in the British Museum and weigh 133 kg each.*

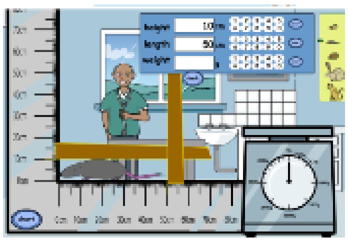


Ask pupils to research an animal on the site www.a-z-animals.com and see if they can find out the following:

- What combination of animals could be equal to the elephant's weight?
- If the zoo's scales could hold a tonne, what animals could go on them?



The following search engines www.kidrex.com and www.duckduckgo.com are tailored towards pupils and may be useful in researching animals for the 'At the Zoo' activity.



At the Vets[169]

In this interactive activity pupils use their measurement skills across the measurement strand units to identify the height, length and weight of animals. A high level of accuracy is required when measuring the animals! Pupils' findings are automatically recorded on a printable chart.

Extension: Pupils create questions for their partner based on the findings on their printed chart.

ICT
At the Vets

Investigation: A Kilo of Coins

You have won the competition and can select one of the prizes below. Which of the following would you choose? Why? Explain your choice. Convince me.

1 kilogramme of €1 coins

A metre long of €2 coins (lying flat and touching)

2 kilogrammes of 50c coins

This interactive requires pupils to create weights using smaller combinations of weights.
[170]

ICT
Make 1 Kg

[169] http://www.iboard.co.uk/iwb/At-the-Vets-Measuring-471

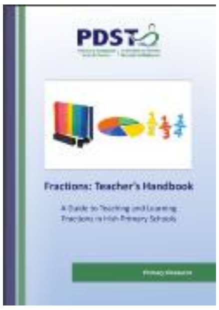
[170] http://www.iboard.co.uk/iwb/Measuring-Boxes-Make-1kg-260

MEASURES - WEIGHT

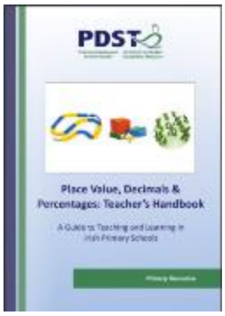
LEVEL C.2

RENAME UNITS OF WEIGHT IN KG AND G AND RENAME UNITS OF WEIGHT IN DECIMAL OR FRACTION FORM

These PDST handbooks on fractions, place value, decimals and percentages, provide a wide variety of learning experiences involving concrete, pictorial and symbolic stages. Such activities may be useful to revisit prior to engaging in renaming units of capacity.



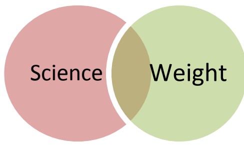
Click here to access PDST Manuals



TEACHING NOTES

Pupils need to be provided with opportunities and experiences to explore the connections between kilogrammes and grammes. The ultimate aim is for pupils to be able to choose appropriately from a range of strategies including estimation, knowledge of benchmarks and knowledge of standard measures in order to approach various tasks with confidence and accuracy.

Eat Like a Bird?



A bird eats about half of its body weight each day. That means that if you weighed 30 kg you would need to eat 15 kg.

Use the catalogue from the local shop to help you decide what would you choose if you had to eat a total of 15 kg?

Bird Watch Ireland has information on the diets of various species of birds. Pupils could use this site to research the diet of a particular bird. www.birdwatchireland.ie

BirdWatchIreland

Design and record your day's eating.

Design your day's eating		
Item	weight (in grams)	Fraction of a Kg

LEVEL D.1

ESTIMATE, COMPARE, MEASURE AND RECORD THE WEIGHT OF A WIDE VARIETY OF OBJECTS USING APPROPRIATE METRIC UNITS AND SELECT SUITABLE INSTRUMENTS OF MEASUREMENT

Consolidating Benchmarks using Dough[171]

You will need to make up blocks of clay / play dough for this activity. Where possible, the pieces should be of different colours and have weights of about 100g, 200 g, 300 g, 400 g, and 600 g. This allows pupils to apply ratios when they are answering the questions, for example, “The red piece is about half the weight of the blue piece.” This activity is aimed at pupils developing benchmarks from which they can estimate other weights. This is in keeping with the common practice of relating unfamiliar weight to known benchmarks, for example, “It’s slightly heavier than a 500 gram block of butter.” The pupils get better at estimating unknown weight as the activity progresses. Some pupils may need leading questions, such as, “Before you estimate, how heavy is that piece compared to others you know about?” Selecting objects of a weight similar to a selected piece opens up the concept of density. That is, the weight of an object is not just related to its size. For example, a piece of wood may be lighter than a smaller piece of steel. Aim for pupils to hypothetically balance the selected object and the piece of clay or play dough. Encourage them to check their decision by weighing the object on a set of scales. Pupils should explain differences between the predicted and actual weight, for example, “The stapler looked heavy because it seemed to be made of metal, but most of it turned out to be plastic.”

<u>Playdough Recipe</u>

Play dough is very easy to make, and provides hours of fun! It can be expensive to buy at stores, but it is cheap to make! The only materials you will need are:

$\frac{1}{2}$ cup salt

$\frac{1}{2}$ cup water

1 cup flour

food dye (any colour, be creative!)

newspaper to cover the surface you

Sharing Play Dough[172]

Arrange pupils into groups of 3. Give each group some dough and a balance. Allow groups to use a balance to share a lump of play dough evenly between 3 people. Next, allow the pupils to experiment with ratios to share the dough, for example:

[171] http://www.nzmaths.co.nz/sites/default/files/FIO/Measurement23.pdf page
[172] Department of Western Australia (2005, p. 108)

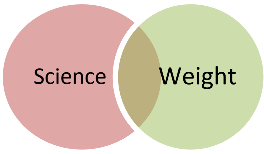
Share the dough so that the oldest person in the group gets double the quantity of the youngest person? How much of the dough will the third member of the group get? Decide on another way to share the dough. Record your investigation.

Tin Foil Boats

Give pupils a variety of weights for example coins/ marbles. Allow pupils to explore making boats for the coins using tin foil. A lunch box filled with water can be the 'lake' in which to investigate the ability of the boats to float. Discuss floating and sinking. When we compare our findings are the results accurate?

Many primary school pupils have serious misconceptions about the meaning of the equal sign. (Molina & Ambroce, 2006).

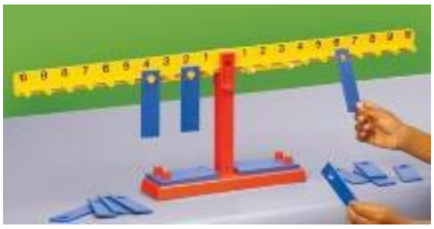
The following activity involving weights provides opportunities for pupils to develop and refine their understanding of the equals sign.[173]



Investigations Using a Number Balance

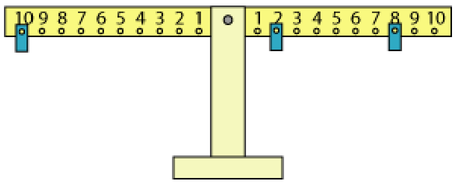
This is a number balance.

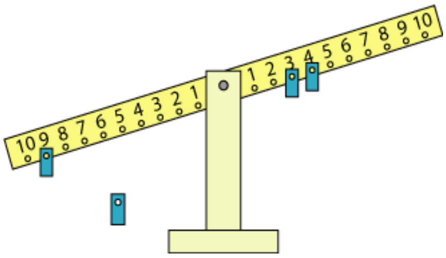
It is also called a 'balance bar' or an 'equaliser'. It has weights which are all the same size. These can be hung under the numbers on the bar. In the photo the weights are distributed in a way that makes each side balance. It is demonstrating that 4 + 2 = 6 Also this image shows us that 10 = 2 + 8

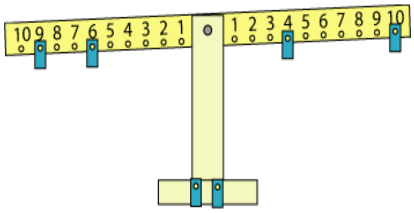


[173] Fostering Relational Thinking while Negotiating the Meaning of the Equals Sign, Molina & Ambrose, Teaching Children Mathematics, Sept. 2006

Investigate where to hang the weights on the right to stabilise this number balance? Find another possible solution







If you had to use two weights to stabilise this number balance, where could you put the weights? What have you discovered? Explain your answer. Can your partner revoice your solution?

This interactive is an online number balance which is used to solve different challenges. Using the balance pose the following questions to pupils. Also see can they create their own questions for their partner.[174]

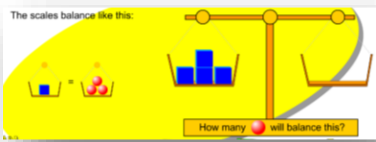
ICT
[NRICH Interactive Number Balance]

Tell me how the balance works. How can you tell which side we need to add a weight to? What is the total of the weights on this side? What is the total of the weights on the other side?

[Click the video icon] for a video of a Number Balance being used to investigate number bonds.[175]

CONSOLIDATION ACTIVITY

Investigation



ICT
[Reasoning & Weight it Up!]

Click the interactive link below to access the investigation. Each shape has a different value. See if you can work out the values of the shapes by adding them to the scales. Try balancing the scales with different shapes on either side- it can be done![176]

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[174] https://nrich.maths.org/4725
[175] https://www.youtube.com/watch?v=mXtkD22GlYk
[176] http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/maths/weigh/free.htm

MEASURES - WEIGHT

LEVEL D.2

RENAME UNITS OF WEIGHT IN KG AND G AND DECIMAL OR FRACTION FORM

TEACHING NOTES

There are many opportunities for pupils to further consolidate their conceptual understanding of weight. Pupils could be encouraged to keep a 'weight diary' including for example, the number of grammes/kgs of various foods, they/their family consumes in an average day or week. Pupils could examine what factors influence changes in the average consumption, for example Christmas time, a birthday party. Building on their earlier work on weight benchmarks, pupils could keep a 'personal home diary' of the weight of items of personal interest to them, for example the weight of a games device, a set of golf clubs, their football boots, a hurl, a musical instrument.

There are also many opportunities for collecting data on weight at classroom and school level. Problem solving, incorporating project based learning, could include gathering data on the average weight of a schoolbag from infants to sixth class. Further examples could include average consumption of different food groups, proteins, carbohydrates, fruit, in an infant classroom, in the second classroom, in the sixth classroom. Gathering and analysing data involves application of skills across several strands of mathematics and other areas of the curriculum, enabling integration and real-life problem solving.

Largest Lasagne[177]

The largest lasagne was made for the Dublin Spring Show in Ireland. It weighed 1637.3 kg and measured 15.24 m x 1.52 m

How many people do you think it could feed? How did you come up with the estimate? What fraction of the lasagne would we need to feed our class?

Balancing Sub-multiples[178]

In pairs/groups, pupils explore balancing various weights using the balance scales, for example, balancing a 1kg weight with 2 weights weighing 500g each. Pupils record this relationship in an as many ways as possible using symbols, for example:

[177] http://nzmaths.co.nz/resource/weighty-problems
[178] http://www.bbc.co.uk/bitesize/ks1/maths/length_and_weight/play/

1kg = 500g + 500g
1kg = ½ kg + ½ kg
½ kg = 500g
½ kg = 0.5kg

This activity can be repeated to explore various relationships, for example, 1 Kg will balance five 200g weights or 1 kg will balance ten 100g weights; 500 grammes will balance two 200g weight and one 100g weight or 500 grammes will also balance five 100g weights; etc. Pupils can record these relationships in their personal benchmarks notebook or diary.

Assessment

In this interactive pupils must weigh parcels before placing them on a boat. Pupils use their skills of reading linear and dial type measurement instruments of quantities up to three decimal places. The activity has three optional levels of difficulty.

ICT
Harbour Measurements

Target Board

The target board can be used to consolidate the concept of weight, for example:

- *Which weights are more than 1kg?*
- *Which weights are less than 500g?*
- *Which weights add exactly to make 2kg?*
- *Convert all the kilogramme weights to grammes*
- *Convert all the gramme weights to kilogrammes*
- *Which weights are equal to ¼ of a kilogramme?*

50g	1g	5000g
1.25g	100g	125g
1.5Kg	1.5g	5g

Provide opportunities for pupils to create their own questions.

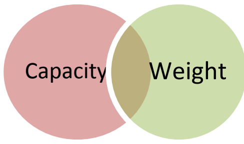
Assessment

Order and Find using Weight Cards

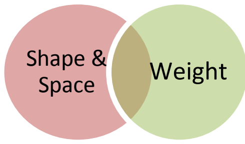
In pairs, order the weight cards from smallest to largest. During the week complete a project with your partner where you collect items and images of items that weigh these amounts. See resources in Appendix B for an example of weight cards).

1kg	150g
3.5kg	3400g
2kg	1250g
1kg,240g	3750g
2010g	$\frac{1}{2}$kg
760g	$\frac{1}{4}$kg

The Relationship between Weight and Capacity[179]



Fill a measuring jug with one litre of water. Pour this litre into a plastic bag and secure with an elastic band. Weigh the bag of water. How much does it weigh? The fact that 1 litre of water has a weight of 1 kilogramme is discovered in this activity. Similarly, more difficult questions can be posed using plastic household containers such as 500 millilitres, 750 millilitres, and 1.25 litre bottles. This is likely to lead to pupils discovering that the weight of 1 cubic centimetre (1 millilitre) of water is 1 gram. This means that 750 millilitres of water has a weight of 750 grams. There will also be scenarios when for example 1 litre of water in a plastic bottle weighs for example 1.02Kg encourage pupils to reason why there is a difference of 0.02 between the volume of water in the bottle and the weight of water in the plastic bag.



Pupils can also explore the relationship between capacity and volume. Your school may have cubic 1 litre capacity measures, which pupils can push the 1 litre bag of water into. This is the same size as a large place value block (10 centimetres x 10 centimetres x 10 centimetres or 1000 cubic centimetres). Alternatively, pupils can make an open cube of the same size from card, using what they know about nets.

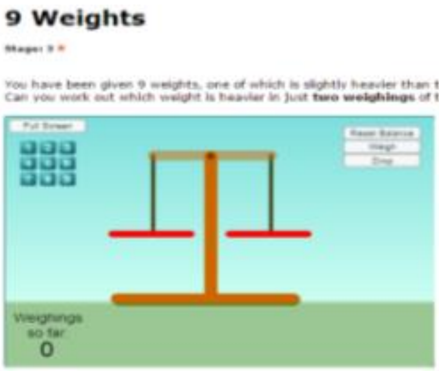
Marble in the Cup[180]`

Pupils fill 9 small drink cartons with the same amount of sand. Pupils use a balance to directly compare each of these cartons to make sure they are the same weight. While classes are out of the room the teacher places a marble in one of the sand-filled cartons. The teacher explains to the class*: I know something fell into one of these cartons at lunchtime but I don't know which carton. I need you to find out which carton is slightly heavier, but you are only allowed to use the balance twice.*

In the following interactive activity you have been given 9 weights, one of which is heavier than the rest. Can you work out which is heavier in just two weighing's of the balance? Provide pupils with time to discuss and share possible solution strategies with their partner.

[179] http://www.nzmaths.co.nz/sites/default/files/FIO/Measurement23.pdf (page 15)
[180] Department of Western Australia (2005, p. 98)

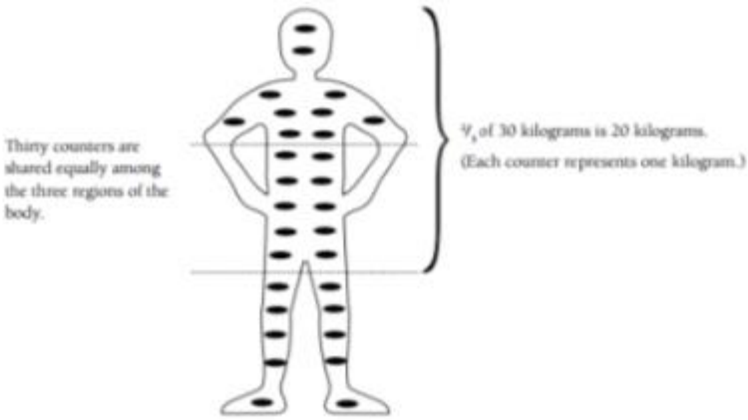
[181]



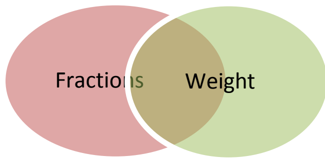
Tom's Challenge[182]

Tom's teacher told him that $^2/_3$ of his body is made up of water. Tom thinks "I weigh 30 Kilogrammes so I must have 20 Kilogrammes of water in my body".

Is Tom right? How do you know? Give a reason for your answer.



Tom's problem involves finding a fraction of a whole number. A pictorial model involving 30 counters may help: Thirty counters are shared equally among the three regions of the body, each counter representing 1 Kilogramme.



Supermarket Catalogues

Shopping catalogues provide a free, hands-on way for pupils to explore many aspects of mathematics that are part of every-day life. These include store opening & closing times, maps with store locations, dates of offers as well as prices and dimensions of items on-sale. Use of shopping catalogues can enhance a pupil's conceptual understandings by providing him/her with a real-life context in which to explore mathematics. Furthermore it can develop pupils' decision making skills and enable them to make thoughtful evidence-based decisions when interacting with the world around them. If a catalogue is being explored in class by pupils or in pairs the catalogue can also be displayed on the interactive whiteboard. Many stores have their catalogues available online.

<u>Always read through catalogues before providing them to pupils. There may be sensitive pictures and items inappropriate for classroom discussion and content.</u>

181 https://nrich.maths.org/5827
182 http://www.nzmaths.co.nz/sites/default/files/FIO/Measurement23.pdf pg 15

MEASURES - WEIGHT

Supermarket Measures

Pupils could use a page from a catalogue from a local shop to solve tasks such as the following:

Help the supermarket manager solve the following tasks:

1. Order the groceries from smallest weight to largest.

2. Which is the heaviest item?

3. Which is the lightest item?

4. What are the total weights of:

    - The Mini Cheddars and the bread?
    - The Frosties and the Muller Corner?
    - The Eggs and the Dairy Chocolate?

5. What item weighs more than 1kg?

6. What two items weigh the same?

7. Multiply the weights of these items:

    - Mini Cheddars x5
    - Eggs x3
    - Muller Corner x 2

Estimate what a slice of bread weighs taking into consideration the weight of the sliced pan and an estimate of the amount of slices in the pan. Explain your reasoning for this estimation. How could you check? Why do you think your estimation was accurate or inaccurate? What reasons can you come up with for any differences?

Similar investigations can be conducted for other items, for example:

- the weight of an individual page in a book/copy/notebook;
- the weight of an individual piece of pasta in a bag of pasta;
- the weight of a slice of ham in a packet of ham;
- the weight of an individual apple in a bag of apples;

Investigating Food Packaging & Contents

The purpose of this activity is to develop pupils' critical thinking skills and enable pupils to become competent when shopping for groceries. Identifying the weight of a product and its food content raises awareness of the need to be able to distinguish between value and price when purchasing food items.

Provide pupils with a variety of food packages for example cans, packets and tubs. Discuss with pupils the contents of the package. Pupils explore the label of each item.

What weights are stated on the package? Check the weight of each item on the scales. What did you discover? What do you think gross weight and net weight mean? Could you measure the weight of the contents? How? What percentage of this item's gross weight is packaging?

Pupils could create a graph to show the relationship between the meat content versus the weight of the package.

This site provides an interactive pupil friendly package for creating a variety of different types of graphs.[183]

ICT
[Kids' Zone Create a Graph]

[183] http://nces.ed.gov/nceskids/createagraph/default.aspx

Meat Content Investigations

Sausages produced in Ireland receive a quality assurance mark from Bord Bia if they have a meat content of 70% or over[184]. Pupils investigate a variety of packets of sausages to identify the meat content. Black and White pudding may also be suitable for this investigation.

Similar investigations can be carried out on cans of tinned foods.

Recipes

"...Charlie put the mug to his lips, and as the rich warm creamy chocolate ran down his throat into his empty tummy, his whole body from head to toe began to tingle with pleasure, and a feeling of intense happiness spread over him."

- *From* Charlie and the Chocolate Factory[185]

ICT
Roald Dahl's Revolting Recipes

Recipes provide a practical context to develop measurement skills. Roald Dahl's Revolting Recipes book will provide many suggestions for recipes that may be of particular interest to pupils. Click this interactive to visit some online 'Revolting Recipes'.

What will it cost to create this dish? Will there be enough for your family? Why do you think so? Can you halve the measurements on the recipe? Order the ingredients from smallest quantity to largest quantity. Can you convert the measurements from grammes to fraction of a kilogramme? Can you round each ingredient to the nearest ten?

[184] http://www.irishexaminer.com/lifestyle/healthandlife/dietandfitness/wheres-the-pork-223349.html
[185] http://www.roalddahl.com/create-and-learn/make/revolting-recipes

Guess my Receipt

Buy some items from a shop whose catalogue is being used in class. Place the receipt in an envelope. Create a set of differentiated clues so pupils can reason as to what the contents of the receipt were. This is based on the supermarket catalogue used in class that week. Pupils can use the catalogue from the shop to enable them to make guesses regarding the purchased products. The receipt could be stored in an envelope in order to build motivation and suspense around the activity during the week.

~***Mystery Receipt Clues***~

- None of the products I bought were wrapped in paper.
- There was a difference of just 20 grams between two of the items.
- Three of the items had a total weight of 450g.
- On Saturday I am having a birthday party and I want each of my 20 friends to get something crispy to eat.
- I placed the orange packet at the top of the pile as it was the lightest item in the bag.

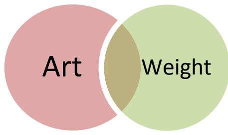
Catalogue Quiz

Can you make up questions for the catalogue page? For this activity you must include one of the following buzz words in your question.

This activity involves encouraging pupils to create questions based on a shopping catalogue. Present pupils with a page from a shopping catalogue on the IWB. Ask pupils in pairs to create questions based on this page. Provide the pupils with some question prompts.

What? → Estimate? → Measure? → Calculate? → How many? → Who? What? Where? When? Why?

Encourage pupils to use a variety of different question starters rather than just focusing on one. The page of the catalogue could be hung up in the maths area giving pupils the opportunity during the week to create questions for the page and publish these on post-its. These questions could then be investigated at the end of the week.



Follow this link to see 'Questioning' *Assessment in the Primary School Curriculum Guidelines for Schools Primary Assessment Guidelines pgs. 86 & 87*

186

The Perfect Suitcase Project

The need to be skilled in estimating the precise weight and dimensions of a piece of luggage have become more important in recent times. This activity involves pupils using their knowledge of measures to design the 'perfect sized suitcase'. Cardboard and various art and crafts materials will be required for this project. Pupils may also decide to bring in materials from home. Provide pupils with a copy of the table of baggage sizes. Discuss the table with pupils. Pose the following problem to the pupils

Marian and Emer were preparing for their first school tour abroad. They were both very excited about visiting Amsterdam. Emer was particularly interested in getting to see Anne Frank's house for the first time. The teacher warned the group to make sure they chose and packed their hand luggage carefully. The German airline carrier had a strict hand baggage policy. Working in groups, design a suitcase prototype for Marian and Emer to use as a benchmark, in choosing the correct suitcase.

[186] http://www.ncca.ie/uploadedfiles/publications/assess%20%20guide.pdf

Which airline would you like to travel with? Why? Which airlines have baggage restrictions? Where could we find out more about the airlines' baggage allowance? Can you and your partner come up with a reason for the different baggage restrictions?

What materials could you use for the case? Are there items from the environment you could use to create the required weight? How can you check to see if this is the correctly sized prototype? Record your findings.

[187]

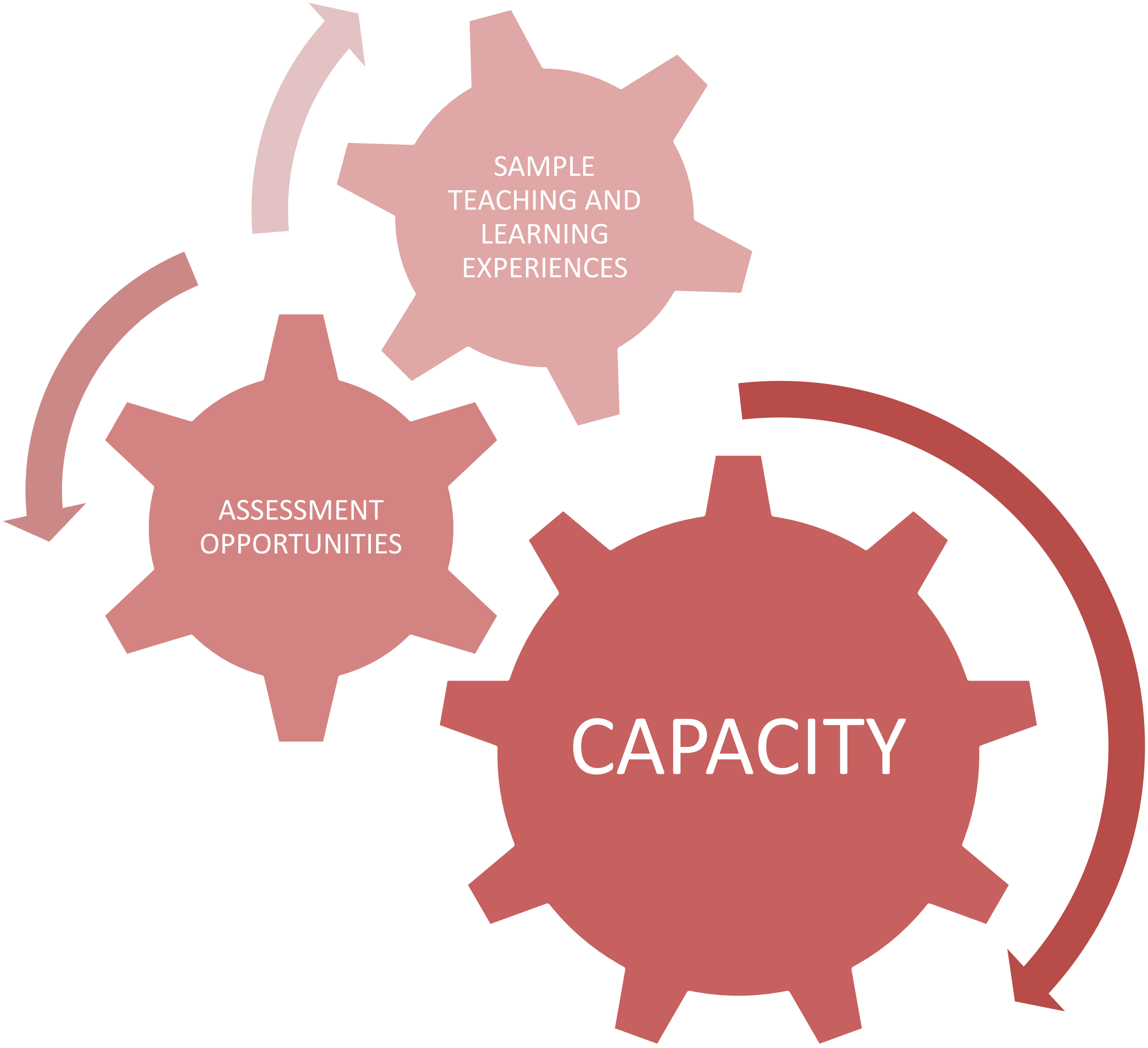
Baggage Sizes

Airline	Size restriction*	Weight allowance
Aeroflot	55x40x20cm	10kg
Air Berlin	55x40x20cm	8kg (10kg with laptop)
Air France	55x35x25cm	12kg
British Airways	56x45x25cm	23kg
Easyjet	50x40x20cm	Unlimited Weight
Flybe	55x40x23cm	10kg
German Wings	55x40x20cm	8kg
Iberia	56x45x25cm	Unlimited Weight
Jet2	56x45x25cm	10kg
KLM	55x35x25cm	12kg
Lufthansa	55x40x23cm	8kg
Norwegian	55x40x23cm	10kg
Ryanair	55x40x20cm	10kg
Scandinavian Airlines	55x40x20cm	10kg
Thomas Cook	55x40x20cm	5kg
Transaero	55x40x20cm	10kg
Volotea	55x40x20cm	10kg
Vueling	55x40x20cm	10kg

*A normal cabin bag measures 55x40x20cm. Some airlines will offer more space, allowing you to take a larger bag. These airlines have been marked in green. Those marked in orange are smaller than the average, and you should be careful with overpacking your bag.

ICT
[Airline Baggage Information]

[187] http://www.netflights.com/baggage-allowances.aspx


SAMPLE TEACHING AND LEARNING EXPERIENCES
ASSESSMENT OPPORTUNITIES
CAPACITY

CAPACITY

CAPACITY - BACKGROUND KNOWLEDGE FOR TEACHERS

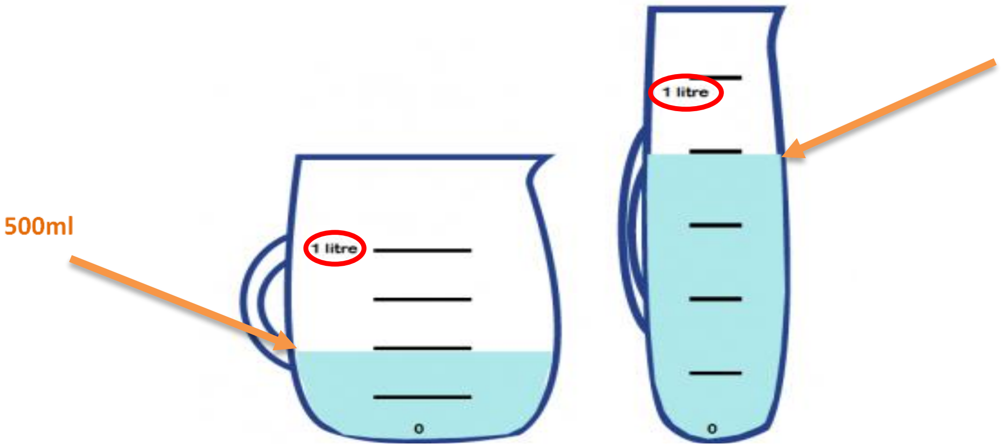
Capacity-The amount of space in a hollow container. Only containers have capacity.

Volume- The amount of three dimensional space occupied by a substance. Solids, liquids and gases have volume.

Each jug has a **capacity** of 1 litre

The first jug contains a **volume** of water measuring 500 millilitres

The second jug contains a **volume** of water measuring 800 millilitres



Solid Volume and Liquid Volume

There are two types of volume: solid volume and liquid volume. These two concepts are exactly the same, however by historical accident they are measured in different units[188].

The volume of a liquid is given in litres (L) or millilitres (ml). The volume of a solid shape is given in cubic metres (m^3) or cubic centimetres (cm^3)

Non-standard Units

Measurement involving the use of non-standard units is an important part of the development of measurement skills (see Introduction section).

Approximate Measurements

It is important to note that all measurement is approximate.

The **estimated sign** *is a mark required to be appended to the nominal mass or volume printed on pre-packaged goods for sale within the European Union. It certifies that the actual contents of the package comply with specified criteria for estimation*[189] ℮

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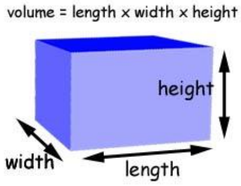
188 Haylock, D.(2010). *Mathematics Explained for Primary Teachers*, p.279. (4th edn, SAGE Publications).

189 https://en.wikipedia.org/wiki/Estimated_sign

Area, Perimeter & Volume

Area, perimeter and volume are related. For example, as the shapes of regions or three-dimensional objects change while maintaining the same areas or volumes, there is an associated effect on the perimeters and surface areas[190].



Cuboid

The general formula for finding the volume of a cuboid is length by breadth by height.

CAPACITY - POSSIBLE PUPIL MISCONCEPTIONS

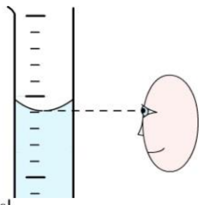
1. The principle of conservation is a fundamental aspect in learning about all types of measurement. Conservation of liquid volume is one that can be particularly challenging for pupils. When they empty water from one container into another differently shaped container as shown in picture c below, pupils can tend to focus their attention on the heights of water in the container and wrongly conclude that the taller container holds more water. This is incorrect because the volume of water has actually been unchanged by the transformation.


[191]

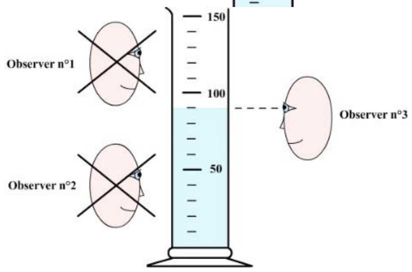
2. Pupils who have acquired mental benchmarks or reference points for measurements and have practised using them in class activities are much better estimators than pupils who have not learned to use benchmarks. [192]
3. Pupils may be able to use a measuring instrument but may not fully understand how it works[193].
4. Reading the scale accurately is an integral part to developing measurement skills. The following are some common stumbling blocks pupils may face when reading scales involving liquids.
    - Pupils pick the container up and fail to keep it vertical when reading the scale.

[190] John A. Van De Walle et al. (2010) Elementary and Middle School Mathematics Teaching Developmentally, p.369.
[191] http://homepage.ntlworld.com/gary.sturt/sambry.htm
[192] Joram, E. (2003). Benchmarks as tools for developing measurement sense. In D.H. Clements, & G. Bright (Eds.), *Learning and Teaching Measurement: 2003 year book* (pp. 57-67
[193] John A. Van De Walle et al,(2010). Elementary and Middle School Mathematics Teaching Developmentally, p.372.

- Pupils read the scale by looking at the value at the top of the meniscus (curve in the surface of a liquid, produced in response to the surface of the container or another object. It can be either concave or convex).



- Pupils read the scale from different heights so that parallax (difference of orientation of an object viewed along two different lines of sight) occurs.
- Pupils do not understand the measurement between marked divisions on a scale.



Due to the capillary action, water tends to be attracted to the walls of a container. This phenomenon is negligible for wide containers but in narrow containers it can be observed that the surface becomes curved. To correctly determine the volume of liquid, you should take as a reference the lowest point of the curved surface.

CAPACITY - PARENTAL INVOVLEMENT

The following are a list of ideas that may be shared with parents to help develop pupils' conceptual understanding of capacity. Some of the following ideas are taken from the NCCA's tip sheets for parents available through the NCCA website.[194]

Here you will also find some short videos of parents working with their children to develop their mathematical understanding.

Level A:

- If you are pouring water or milk, talk about full and empty; the glass holding less than the cup; the jug holding more than the eggcup; the glasses that hold the same amount.

Level B:

- Look at containers you have at home. Which bottles hold a litre? A half-litre? A quarter-litre?

Level C:

- Ask your children to compare containers. How many litres are in the big orange juice bottle? Is that cheaper than buying 1 litre boxes?

Level D:

- Ask your child to solve some problems. The 1.5 l bottles of coke come in boxes of 12. How many litres are in a box? How many litres do you think you need for your birthday party?

[194] http://www.ncca.ie/en/Curriculum_and_Assessment/Parents/

CAPACITY - LEARNING TRAJECTORY

The learning trajectory is based on the objectives for Measures in the Primary School Mathematics Curriculum. In some instances, similar objectives at the same class level have been collapsed into one objective. Objectives that only refer to problem solving have not been included as discrete objectives because a problem solving approach is advocated throughout all of the teaching and learning experiences. Problem solving is viewed in this manual as a fundamental, integral part of mathematics teaching and learning that pupils should experience every day. The same colour coding from the curriculum is used – infants (green); first and second (red); third and fourth (blue); fifth and sixth (orange).

CAPACITY LEARNING TRAJECTORY LEVEL A[195]

Trajectory Levels	Concept	Developmental Experiences: Concrete	Pictorial	Abstract
	Level A.1 Develop an understanding of the concept of capacity through exploration and the use of appropriate vocabulary			
	Level A.2 Compare and order containers according to capacity			
	Level A.3 Select and use appropriate non-standard units to measure capacity and estimate and measure capacity in non-standard units.			

[195] This level is generally aligned with the objectives for Junior and Senior infants.

CAPACITY LEARNING TRAJECTORY LEVEL B[196]

Trajectory Levels	Concept	Developmental Experiences: Concrete	Pictorial	Abstract
	Level B.1 Estimate, compare, measure and record capacity using non-standard units			
	Level B.2 Select and use appropriate non-standard measuring units and instruments and discuss reasons for choice			
	Level B.3 Estimate, measure and record capacity using standard unit (the litre) and half-litre and quarter-litre			

[196] This level is generally aligned with the objectives for First and Second class.

CAPACITY LEARNING TRAJECTORY LEVEL C[197]

Trajectory Levels	Concept	Developmental Experiences		
		Concrete	Pictorial	Abstract
	Level C.1 Estimate, compare, measure and record the capacity of a wide variety of objects using appropriate metric units (l, ml)			
	Level C.2 Select and use appropriate non-standard measuring units and instruments and discuss reasons for choice			
	Level C.3 Rename units of capacity in l and ml and decimal and fraction form			

[197] This level is generally aligned with the objectives for Third and Fourth class.

CAPACITY LEARNING TRAJECTORY LEVEL D[198]

Trajectory Levels	Concept	Developmental Experiences: Concrete	Pictorial	Abstract
	Level D.1 Estimate, compare, measure and record capacity using appropriate metric units and select suitable instruments of measurement			
	Level D.2 Rename units of capacity in l and ml and decimal and fraction form			
	Level D.3 Find the volume of a cuboid experimentally			

[198] This level is generally aligned with the objectives for Fifth and Sixth class.

LEVEL A.1

DEVELOP AN UNDERSTANDING OF THE CONCEPT OF CAPACITY THROUGH EXPLORATION AND THE USE OF APPROPRIATE VOCABULARY

TEACHING NOTES

Free Play

Undirected play with a wide variety of materials is essential for laying the foundations of measurement. Pupils should encounter both aspects of volume – the amount of 3-D space a solid occupies and the amount of space in a hollow container. Experiences of filling in 3-D space with sand and water will eventually lead pupils to the concepts of capacity and volume. The meaning of 'full' must be clearly demonstrated. Informal discussion should direct pupils' attention to the appearance, size and shape of containers, the shape the water makes when poured into a container, the experience of emptying a jug full of water into a container then returning the water to refill the jug again.

Informal Exploration of Volume

While packing away equipment, ask pupils to estimate capacity. For example, ask pupils to choose a box that all the blocks / balls will fit into.

Are you sure the blocks will fit in that box? What about the smaller box? Why do you think the long blocks won't fit? Which boxes do you think will definitely not be big enough? What helped you decide? Which bucket can hold the most sand? How can we check?

Give pupils the opportunity to fill a variety of large containers with water or sand using various small containers, for example, spoons, paper cups, egg cups, bottle caps. Elicit from pupils what item makes more sense and why when filling containers. Engagement in these activities and encouraging pupils to justify their answers will enhance development of reasoning skills.

Why did you choose to use the teaspoon to fill the egg cup? What could we use to put sand into the sand bucket? Will I fill the bucket using the small spoon or the cup? Will I fill the egg cup with a teaspoon or a large cup? How many blue cups of sand do I need to fill the red bucket? Do I need the same amount of white buckets?

Aistear

There are many ideas in the Aistear Early Childhood Curriculum Framework for enhancing pupils' understanding of capacity. Below are some suggestions.

- Fill water balloons and put them in the freezer or leave them outside overnight when it is freezing. The following day, remove the balloons to reveal balloon-shaped ice cubes! Add food colouring to the water to create different coloured balloons.
- Blow bubbles outdoors.
- Make books using magazines and pictures from the internet about water and water-related activities.[199]
-

Water Play

The following key ideas should emerge gradually from water play

- Water takes the shape of the container that holds it.
- Water occupies space.
- Pupils should use the words 'full', 'empty' appropriately in practical situations.
- When a container is part-full, pupils should be helped to distinguish between the actual amount of water and the total amount the container could hold.
- Pupils should confirm their intuitive idea that identical containers hold the same amount of water.

[199] http://ncca.ie/en/file/aistearsiolta/Aistear-LE-Water-Play-Screen.pdf

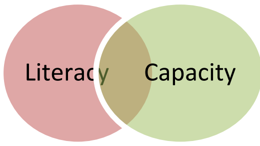
COMPARE AND ORDER CONTAINERS ACCORDING TO CAPACITY

TEACHING NOTES

Initially containers selected should be obviously different in size. Pupils are required to estimate first which container holds more and then check their estimations. Pupils should have the opportunity to check by

- filling one container and transferring to the other
- filling each container using a smaller container and counting
- filling both containers and tipping contents into two identical containers for comparison

Further experiences should involve less obvious comparisons, for example a tall narrow container and a wide short container.



The Three Bears[200]

The story of The Three Bears is an excellent starting point for discussion on comparison of different sized containers.

Which bowl do you think has the most porridge in it? Why do you think that one? How could we find out?

Show the pupils three different sized bowls and ask them to think which one belongs to Father Bear. Let a pupil demonstrate their idea for determining which bowl holds the most porridge. Elicit other strategies investigating which bowl belongs to Father Bear. Pouring from one bowl to another is a likely approach. It is also possible to pack the smaller bowl inside the larger one to demonstrate the difference.

Show the class a collection of plastic glasses and cups. Explain that these are for the bears' drinks. In pairs the pupils are to find 3 cups for the bears and put them in order for Father, Mother and Baby Bear. Explore the volumes of the cups using water and the glasses using orange squash. Using the different liquids will help to consolidate pupil learning by visiting the concept of capacity with various liquids and containers. As the pupils contribute to the discussion, ask questions to activate mathematical thinking: Give pupils the opportunity to draw the items for each bear. Elicit from pupils the language of capacity as you encourage them to explain which items hold the most/ least.

Popcorn Party Tub

Explain to pupils that the teddy bears want to celebrate Goldilocks' birthday. Discuss parties with the pupils and what they like to eat for their birthday party and the importance of sharing at the party. Elicit from pupils, words such as 'each' 'share' 'slice' 'same'. Show pupils the tub of popcorn. Pupils select which type of cup would be most suitable for the party so we can share the

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200 http://nzmaths.co.nz/resource/three-bears

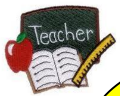
contents of the tub of popcorn with Goldilocks and the Three Bears. Provide containers such as mugs, jugs, glasses and paper cups so pupils have the opportunity to reason which type of container would be most suitable.



Next using the paper cups, ask pupils how much popcorn should you put in the cups so that each bear and Goldilocks will get some. Give pupils opportunities to fill the cups in a variety of ways for example all to the same height, all to different levels, two similar and two different etc. Elicit from pupils their reasoning for filling the set of four cups in each way. If possible, photograph for later discussion.[201]


Find a Partner[202]

Each pupil in the group is given a different container. Pupils explore what their container can hold, for example, pencils, marbles, bears, sand. Next using cubes, they must predict which of their classmates has a container that will hold about the same amount as their container. Pupils can investigate their predictions. Elicit from pupils how they could check; for example check by pouring from one to the other.



Show us what a full cup of water looks like. How do you know it is full? Guess which of these two cups will hold more water? How could we find out? What does it mean if all of the water in my cup won't fit into my friend's cup? How will we know if the cups hold about the same amount?

Who can hold the most?[203]

Pupils work in small groups to find who can hold the most dried beans in two hands (cupped together). Pupils compare the volume of their handfuls by putting their beans into clear plastic cups and looking at the height of the beans in the cup. Some pupils may be able to count the beans. Teachers may need to scaffold the pupils' use of appropriate language during these activities.

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[201] Image sourced: http://www.minilandeducational.com/en/set-4-baby-buckets-2/

[202] *Teaching Measurement: Early Stage 1 and Stage 1* 2003 pg. 88.

[203] *Teaching Measurement: Early Stage 1 and Stage 1* 2003 pg. 93.

LEVEL A.3

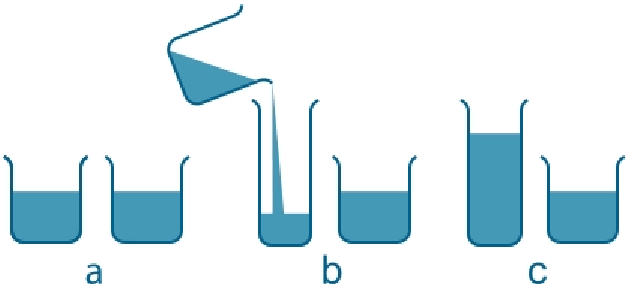
SELECT AND USE APPROPRIATE NON-STANDARD UNITS TO MEASURE CAPACITY AND ESTIMATE AND MEASURE CAPACITY IN NON-STANDARD UNITS

TEACHING NOTES

Conservation Experiences[204]

Pupils often believe that the amount of liquid has changed when a set amount has been poured from one container to another of a different size. They might believe that there is more liquid in the one that has the highest level, for example the left hand container in figure c.

Teachers need to provide pupils with lots of opportunities to take part in practical activities transferring liquids from one container to the other and encourage the practical application of mathematical language more than/less than/equal to.



CONSOLIDATION ACTIVITIES

Consolidating Conservation

Idea 1: Volume of water remains unchanged when poured into different jars

Present pupils with three or four differently shaped containers and ask them to find out which two hold the same amount of sand, water or blocks. It is important that pupils discover that the volume can be the same while the shape is different. Pupils can investigate the fact that the volume of water remains unchanged when poured into different jars. Using a jug of liquid and two identical jars, the teacher pours liquid into one of the jars and the child pours the same amount into the other jar.

Have the two jars got the same amount of liquid? Have the glass and the jar the same amount of liquid? Why? How can we check?

Further exploration: Volume of a shape made by cubes remains unchanged when cubes are rearranged.

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[204] NCETM, 'Maths to Share' [2010] *NCETM Primary Magazine Issue 20*
https://www.ncetm.org.uk/public/files/648833/ncetm_primary_magazine_issue_20.pdf

Measurement provides a context for the further development and reinforcement of number skills. Pupils can "measure" without the use of number up to the stage of indirect comparison. However as soon as they repeatedly use a unit to measure an object they need numbers to keep track of the repetitions.

How will I Pack?[205]

Pack clear plastic containers (for example take-away boxes) with different materials, for example, sand, rice, marbles and blocks. Discuss which material is best for filling the containers, without leaving gaps.

What's the Capacity?

Pupils choose a unit (cup, jug, egg cup, yoghurt container), to measure the capacity of a bucket to be filled with water or sand. Pupils predict and then investigate how many units were required to fill the bucket. Elicit from pupils why the unit they chose was the most suitable.

Don't be Tricked[206]

Give pupils four containers and a specific number of blocks. Pupils predict which of the containers would be filled by the blocks and then investigate, recording their results. Extension: Using the same four containers, but a smaller amount of cubes, can pupils estimate which container would be ½ full using the smaller amount of cubes. Probe pupil understanding by asking pupils if the way cubes are arranged in the container affects the volume of cubes that can be placed in the container.

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[205] *Teaching Measurement: Early Stage 1 and Stage 1* 2003 Pg. 98
[206] *Teaching Measurement: Early Stage 1 and Stage 1* 2003 Pg. 102

MEASURES - CAPACITY

LEVEL B.1

ESTIMATE, COMPARE, MEASURE AND RECORD CAPACITY USING NON-STANDARD UNITS

TEACHING NOTES

Comparisons of volume using non-standard units such as cups, egg cups, spoons etc. is an important forerunner to the introduction of standard units. Experiences in Level A involving comparison and conservation now allow for more meaningful activities involving non-standard units.

Videos of measurement activities can be found on the PDST Measurement page or click this link[207]

Homemade Measure

Pupils bring in a small empty container from home. Such containers may include a yogurt carton, a drink bottle, a small tub, a large lid etc. Pupils can then use this as their non-standard measure when filling/ measuring the capacity of various large containers in school. The capacity of the following containers could be estimated, measured and recorded by pupils using the non-standard measure from home, in school.

Lunchbox	Teacher's Cup	Empty yogurt container
Drink bottle	Paint pot	Basin of water
Toy container	Paint Tray	Sand bucket

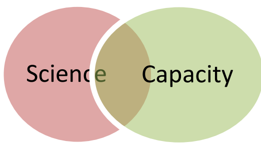
Investigation with a Non-standard Measure[208]

Further exploration of this could involve the class coming up with our classes' official 'non-standard unit'. Pupils select one of the containers from home for the class, for example Jack's Lid and use it to conduct an investigation like below:

1 of Jack's Lid fills_______ 10 of Jack's Lid fill_______ 5 of Jack's Lid fill______

Such exploration will also consolidate pupils' understanding of conservation.

Paintshop



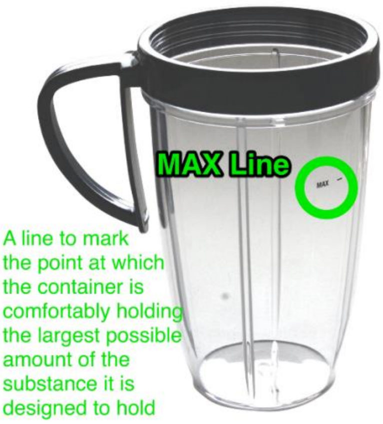
In this activity pupils use food dye (or powder paint) to discover how different volumes of dye can change the concentration of a colour. Working in groups, provide each group with a small jar and give the pupils 2 drops of food dye. Groups add an agreed quantity of spoonfuls of water and observe the changes. Now add more water. Ask the pupils to describe how the colour has changed. Through guided discovery pupils will become aware that the more water that is added the lighter the colour. Now start with fresh jars with a few drops of dye in them. On a tray place various containers of water; bottle tops, film canisters, small jars, jar lids for further exploration etc.

[207] http://www.pdst.ie/measures

[208] Pitt E and Deboys M, *Lines of Development in Primary Mathematics* (3rd edn, Blackstaff Press Queen's University of Belfast, Teachers' Centre 1980) 71.

Guess what the smallest or biggest container of water is? Which container of water when added to the first jar will make the dye lightest in shade? Which container when added to the next jar will make the darkest colour?

Pupils could use the food dye to paint on a page enabling them to record, explain and justify their discoveries.



Bean Baskets and the MAX Line

In this activity pupils make paper cones and use them to estimate, measure and record their capacity. Then pupils order the cones from smallest to largest capacity which leads to a discussion on 'MAX' line. You use maximum to describe the amount that is allowed, possible or required[209]. In groups, pupils use paper and tape to construct a cone that will hold some dried beans. It may be useful to model this first with pupils, giving them time to create large cones and smaller cones. Once pupils have created two different cones, encourage them to estimate, measure and record how many beans their cone will hold. They can write their findings on the side of the cone. When all pupils have finished they can order the cones their group has made according to their capacity. This activity may lead to discussion on what is a 'full' cone. Use this as an opportunity to introduce the concept of the 'MAX' line which indicates the maximum point of capacity. Pupils put a 'Max' Line on their cones. Elicit from pupils ideas on how we can create a class rule for applying the max line to our containers. Some pupils may suggest the max line cannot be more than one centimetre from the top. Challenge self-directed learners to see if they can make a container that holds exactly one cup of beans.

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[209] http://dictionary.reverso.net/english-cobuild/maximum%20capacity

Fill the Box[210]

Pupils pack boxes with blocks. Then they count the blocks and discuss, draw and write about the structure of their packing. Emphasise layers, rows and columns, as boxes may have been packed in horizontal or vertical layers.

If I want to measure the capacity of this container by using the blocks, how should I put them in? What pattern are the blocks in? Could we fit any more in for this layer? What does the pattern of this layer look like? How many blocks have we got now? How many more do you think we will need? Is there a quick way of counting the blocks we use?

What will Happen? [211]

In this activity pupils explore filling containers with two or more substances. Pupils explore what happens when you fill a plastic cup with unifix cubes and then add in some small cubes for example, Dienes blocks. As a whole class activity ask pupils to help you fill a glass with marbles. Elicit from pupils ideas on the 'fullness' of the glass, for example, are there any spaces. Elicit ideas on what else could be used to fit into the glass. Elicit discoveries from pupils when they get to see a half a cup of sand being added to the full glass of marbles.

In they Go[212]

Pupils have different materials for example, golf balls, marbles. They estimate and then investigate how many of their units can be added to a container of water so it will be just filled without overflowing.

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[210] *Teaching Measurement: Early Stage 1 and Stage 1* 2003 Pg.98
[211] *Teaching Measurement: Early Stage 1 and Stage 1* 2003 Pg. 98
[212] *Teaching Measurement: Early Stage 1 and Stage 1* 2003 Pg. 106

MEASURES - CAPACITY

LEVEL B.2

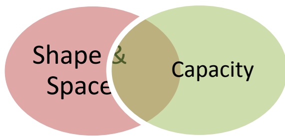
SELECT AND USE APPROPRIATE NON-STANDARD MEASURING UNITS AND INSTRUMENTS AND DISCUSS REASONS FOR CHOICE

Do we have Enough? [213]

A variety of differently sized plastic cups and water are required for this outdoor activity. Pose the following scenario to the pupils. There are five people arriving for a party and we need to have one cupful of juice for each person. Which of these bottles of juice should we buy?

Pupils need to measure how many cupfuls of drink are in each bottle to find out. Pupils can estimate and then measure the capacity of each bottle using cups of water. As they measure they need to keep a record of how much each one holds so they can compare at the end. Once all the measuring is complete, pupils compare the capacities of the bottles to see which bottle holds enough juice for five people. The pupil with the role of 'recorder' could record the predictions of the group prior to the investigation.

The Toy Factory



Show the class a cylinder made from a rectangle of lightweight card. The base of the cylinder is a piece of paper held in place with masking tape. How many medium plastic bears do you think it would hold **exactly**? Discuss what exactly means, that no more bears could fit into the cylinder. Some pupils may suggest putting a 'MAX' line on the cylinder. Count the bears one by one into the cylinder and compare the measurement with their estimations.

Explain to pupils that they are toy manufacturers and that they sell bears in packages of ten. Their task is to design and make a cylinder that holds exactly 10 medium bears. Make available a selection of different sizes of rectangles of card, paper, scissors and tape for the cylinders. Paper rectangles that can be used to make short wide cylinders and tall narrow cylinders allows for comparison opportunities. Ask pupils to work with a partner to first take 10 medium bears and then make a cylinder. When they have completed one cylinder they can be challenged to make another different cylinder that also holds exactly 10 bears. As the pupils construct their cylinders circulate asking questions:

[213] Image sourced: http://www.sweetcicely.ie/menus/taste/fresh-fruit-juice-bottles/

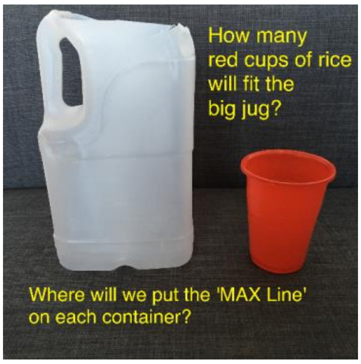
Does your cylinder fit exactly 10 bears? Can you fit any more bears in your cylinder? Are cylinders a good container for bears? Why or why not? Could you make a cylinder for 20 bears? What would it be like? What do you notice about the cylinder? Can you see any cylinders which are the exactly the same?

Discuss with pupils the features of the cylinders constructed. Challenge the pupils to think about how cylinders can look different but still hold the same amount. Elicit from pupils the reasons for this being possible. To extend this activity further use smaller sized bears for the next part of the activity. Ask the class to identify the numerals 1-20 as you drop the numerals into a hat. Next in pairs pupils select a numeral card from the "hat". Instruct pupils to create a cylinder to fit exactly that quantity of small bears. When the cylinders have been created, direct the pupils to write the numeral on the outside of the cylinder. Ask the pupils to place their cylinders, in order, at the front of the classroom. At the end of the session gather the pupils together to look at and compare the capacity of the cylinders. Discuss the different shapes and sizes of the cylinders.

Where does your cylinder belong? How do you know it comes after __? Which cylinder will come after your one? Which cylinder looks the biggest? Does that cylinder hold the most bears? Why?

Making a Measuring Container[214]

Pupils mark gradations on a large container as they place cupfuls of sand or water or rice into the container.



[214] *Teaching Measurement: Early Stage 1 and Stage 1* pg. 110

How could we compare the capacity of these different-sized containers? How many cupfuls do you think we will pour into the jug? If I use a larger cup, how many cupfuls will we now fit into the jug? How will the size of the cup affect what the jug can hold?

Pupils could then use the same amount of rice and pour cupfuls of it into another, differently shaped container. Estimate, then measure the capacities of containers in terms of the two different units. Explain the difference in measurements.

Thirsty[215]

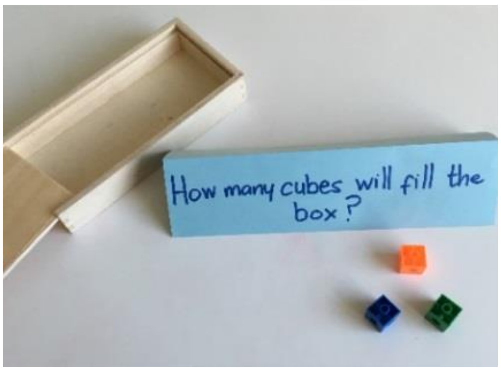
In this task pupils coordinate three pieces of information about each glass: what it contains (orange or blackcurrant juice); how much it contains (full, half full or empty) and its height (tall or short). Through productive mathematical talk, the group will be developing their logical reasoning skills. This task involves working collaboratively. By working together on this activity, the task is shared and therefore becomes more manageable than if working alone.

ICT
Nrich Thirsty Task

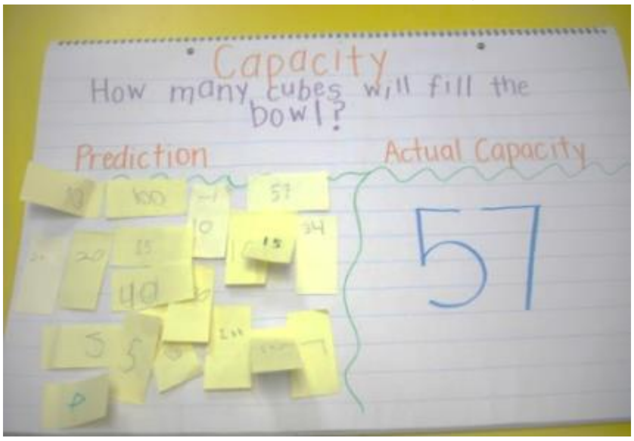
CONSOLIDATION ACTIVITIES

Capacity Games

Introduce the class to a game where they have to guess how many tablespoons of rice (or cubes) it will take to fill a cup. Play the game, first as a class, then in pairs to find out how many spoonfuls of rice will fit in a cup. Show the whole class a tablespoon and a cup. Ask pupils to predict how many spoons of rice it will take to fill the cup, which they can publish on a post-it.



Next select one pupil to hare their estimation. They roll a die, show the result to the class, and say what number they have rolled. Next they scoop that number of spoons of rice into the cup counting aloud. When completed ask pupils "Is the cup full yet?" Select another pupil to take a turn rolling the die. This time, once they have identified the number rolled, they should add that many spoons of rice to the cup, **continuing the count** from where the previous pupil finished. The count can be tracked on a number line or on a 100 square on the whiteboard, see the interactive link. Continue to select pupils until the cup is full.



215 http://nrich.maths.org/6971/note

How many spoonfuls are in the cup so far? What is the number after that? How many spoonfuls will there be if we put one more in? Is the cup full yet? How many spoons of rice fit in the cup? Were your predictions close?

To consolidate pupil learning, repeat this activity with different sized cups or spoons to allow more pupils the chance to participate. When all pupils understand how the game works put them into pairs (small groups will also work) and give each pair a die, a cup, a spoon, and a container of rice to play the game on their own. As they play, circulate around the room reinforcing reasonable predictions and accurate counting-on; supporting those pupils that require it.

The game can be repeated with these scenarios:

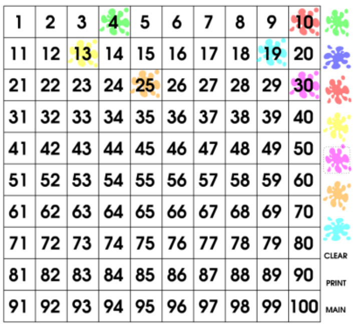
How Many Cups?

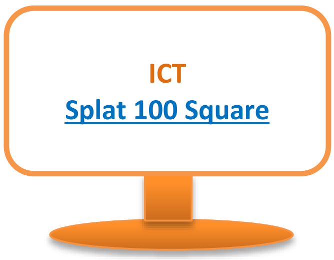
Pupils predict how many cups (small measuring cup) of rice will fit into a bowl/ bucket

How Many Bowls?

Pupils predict how many bowls of water will fit into a bucket. This activity will need to be done out outside. (A sandpit could also be an alternative if one is available).

The Splat Hundred Square is a useful visual tool for supporting pupils in counting on. Click the interactive link to open the Splat Hundred Square. [216]





[216] http://www.oswego.org/ocsd-web/games/SplatSquares/splatsq100.html

LEVEL B.3

ESTIMATE, MEASURE AND RECORD CAPACITY USING STANDARD UNIT (THE LITRE) AND HALF-LITRE AND QUARTER-LITRE

TEACHING NOTES

Using Standard Measurement Instruments

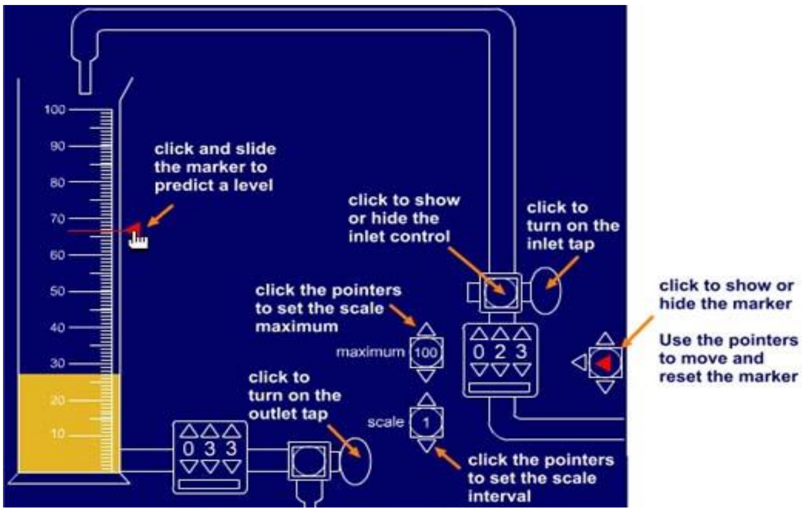
It is necessary to ensure that pupils have the opportunity to use and read a range of measuring scales on **real containers** with different scales in different orientations. Encourage pupils to make sure that they have the container on a flat surface and are looking at the liquid at the same level. They must ensure that they are looking at the base of the meniscus. Encouraging pupils to estimate liquid volume before measuring will support some of these activities. The following Interactive cylinder[217] is a useful interactive tool in consolidating measurement skills.

Hands-on experience is fundamental to the development of the skills-set required to read measurement. Many misconceptions can occur when pupils are reading scales. It is important to be aware of these in order to support pupil learning. See section on 'Pupil Misconceptions'.

Comparing and Ordering Containers

ICT
Interactive Measuring Cylinder

Pupils compare a variety of containers with a standard measuring cup for example a paper cup (250ml) estimating and lining them up from the smallest to largest capacity. Ask them to measure and investigate how many millilitres each one holds and order them again. Invite pupils to look at other containers and say whether they hold more or less than 250 millilitres or about how many millilitres they hold. Discuss strategies for making the comparison.

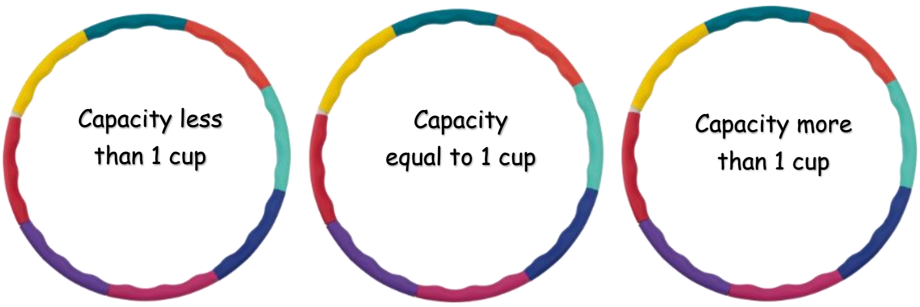


217 http://www.taw.org.uk/lic/itp/meas_cylinder.html

How does the shape of the container affect your estimate?
How can visualising changes in shape help you to compare the container to the measuring cup?

Though pupils can become very good at recognising particular sized millilitre and litre containers in common use, they do not consciously make comparisons between them, nor do they often have strategies for using known volumes to judge the capacity of less familiar containers. They need help in making such connections.

Sort the containers and place them into the correct hoop:



LEVEL C.1

ESTIMATE, COMPARE, MEASURE AND RECORD THE CAPACITY OF A WIDE VARIETY OF OBJECTS USING APPROPRIATE METRIC UNITS (L, ML)

Making a Graduated Container

This activity may work best outdoors.

Working in pairs, provide pupils with a selection of empty containers for example takeaway boxes, jars, paper cups, milk containers and a litre of water in a jug. Pupils choose an empty container, estimate and record in millilitres how much water will fit into the container up to an agreed max line. Pupils fill the empty container using the litre of water from the jug. Together pupils measure the capacity of the mystery container selecting a piece of the measuring equipment and record their findings.

To extend this activity further and create a visual record, pupils could use a marker and write intervals on their container. Large containers may work best for the example, ½ litre, ¼ litre, ¾ litre. Allowing pupils to use 1 litre of water in a jug for this activity consolidates their understanding of conservation of capacity.

Central to pupils' learning in capacity as with all measures is that pupils should be engaged in practical activities and nowhere is this more evident than in developing the concept of conservation.

Conservation of Capacity

To enhance pupils' conservation of capacity the same water is used to investigate the capacity of the variety of containers. For pupils to be successful with this next activity it will be vital that they carefully pour the water back into its original container.

Devices for Measuring Capacity

Pupils can try this activity in pairs. It involves making estimates of a container's capacity, recording the estimate, next measuring the capacity using a measuring device and finally comparing their estimate and actual measurement. Pupils select and use a variety of measuring tools when measuring the capacity of the various containers. Hands-on experience will create meaningful, memorable learning experiences for pupils. Repeat this with many containers.

Below is a table pupils could use to keep a record of their work.

Container	Player A's estimate	Player B's estimate	Measured capacity	Which estimate was closer?

PE Capacity Relay[218]

In a relay race, pupils will try to move as much water from one bowl to another using one sponge per team. Highlight to pupils that the relay will not be timed. This is a relay of accuracy: Which team can move the most water from one bowl to other, wasting the least amount of water.

Each team begins with one litre of water. Elicit from pupils strategies which may be helpful in moving the water accurately from one bowl to the other. Some pupils may suggest cradling hands under the sponge, not over saturating the sponge so water can be lost in the transfer etc. Once outside give each team two bowls (one containing a litre of water) and one sponge. The team lines up one behind the other with one bowl in front of the first member of each team and the other about 5 metres away. When teams have transferred all of their water they estimate how much they think they have transferred and then measure the actual amount using measuring cups with a graduated cylinder/ measuring jug. In their teams, pupils compare the original 1 litre of water with the value they have left, and calculate the amount of water they lost during the activity.

Who was able to transfer the most water? How much water was lost by each group? Compare findings and order the teams in places from 1st through to last. Discuss any strategies which helped in the transfer of water. Would you change how you transferred the water? What helped?

Pupils could also record their findings in a table and display their findings on a graph. Pupils can repeat this activity, varying the distance between the bowls, varying the item used to carry the water and varying the quantity of water in the bowl.

Competitive Estimates

Pupils work in small groups/ pairs for this activity. Each puts what they believe to be 100ml of water into an empty container. Elicit from pupils how they could check their estimate. Pupils record their findings and

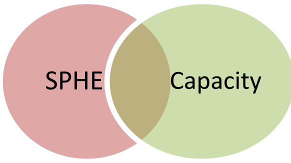
218 http://www.australiancurriculumlessons.com.au/2013/11/09/capacity-capers-water-relay-measuring-capacity-grade-34/

identify the differences between their estimates and what 100ml actually looks like in the container. On a table, pupils record their results and calculate the differences. Pupils repeat the activity using different quantities of water. Discuss with pupils whether their estimates became more accurate the more they did the activity.

Collaborative Estimates

Another option could be to combine the answers of each pair so that they must estimate 100ml together each taking a turn to complete the 100ml. The pupils will need to work collaboratively and actively listen to and observe their partner putting in the first part of the 100ml estimate. Repeat the activity with the focus on estimating 500 ml and 750 ml.

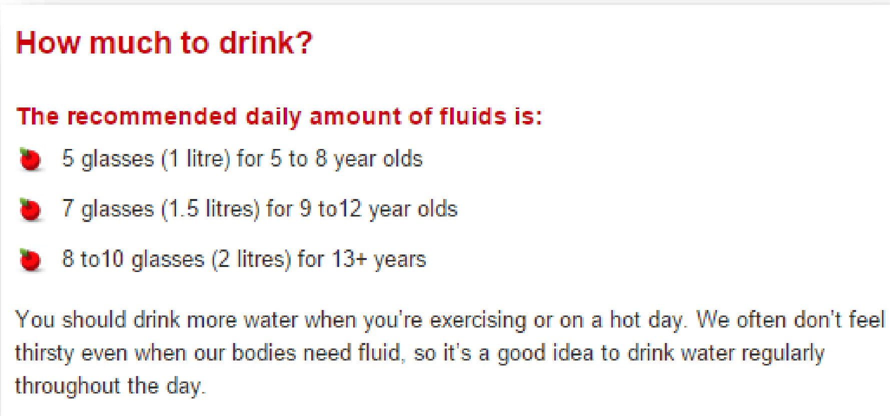
Class Lunch



Estimating quantities of water for a class lunch could involve posing the following scenario to pupils. For example, ask: How much water would we need for a class lunch?

Would 10 litres be a sensible estimate? How could you decide from what you know about the capacity of a cup and 1 litre? How much water would we need altogether so everyone in the class could *get their recommended daily intake?*

Use this as an opportunity to explore healthy eating and recommended daily intake of water for the age of the class. [219]



How much to drink?

The recommended daily amount of fluids is:

- 5 glasses (1 litre) for 5 to 8 year olds
- 7 glasses (1.5 litres) for 9 to12 year olds
- 8 to10 glasses (2 litres) for 13+ years

You should drink more water when you're exercising or on a hot day. We often don't feel thirsty even when our bodies need fluid, so it's a good idea to drink water regularly throughout the day.

ICT
[Measures: Volume & Capacity]

Reading Scales[220]

This interactive activity provides opportunities for pupils to consolidate and refine their skills of reading a scale.

[219] https://www.healthykids.nsw.gov.au/kids-teens/choose-water-as-a-drink-kids.aspx
[220] http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/maths/measures/index.htm

LEVEL C2.

SELECT AND USE APPROPRIATE NON-STANDARD MEASURING UNITS AND INSTRUMENTS AND DISCUSS REASONS FOR CHOICE.

TEACHING NOTES

When pupils can measure effectively using non-standard units, they are ready to move to the use of standard units.[221]

The motivation for moving to this stage, often follows from experiences where the pupils have used different non-standard units for the same volume. This allows them to appreciate that consistency in the units used allows for easier and more accurate communication.

Popcorn can be useful when teaching capacity and provides a purposeful and enjoyable measuring context, not to mention classroom-friendly! The following activities involve using popcorn to introduce pupils to the need for a standard unit for measuring volume.

Popcorn Project

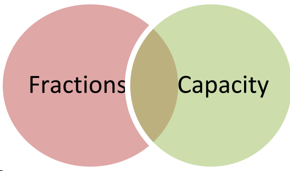
In this activity pupils investigate the volume of corn kernels before and after popping. Pupils use non-standard units for example paper cups or pupils' own non-standard measure from home (see B.1.)

Estimate the Corn: Present the pupils with a small container for example ¼ of a litre, a large bag of popcorn kernels and a variety of spoons (pupils could also bring in a spoon from home). Pupils select a spoon to collect kernels of popcorn to place into the small container. Encourage pupils to first estimate how many spoonfuls they may need and then investigate their estimate by placing their spoonfuls into the small container. Record the volume of the container on a chart with illustrations of the spoons used and how many it took to fill the container. Ask 'Why do we all have different results?' Record answers, encouraging pupils to make comparisons between the sizes of spoons.

Make the Corn: Make the popcorn from the small container–(popcorn makers are the easiest to use in the classroom setting). As the popcorn pops, pupils make predictions about the volume the popcorn will occupy. Pupils could compare this to the unpopped kernels. Pupils select from a variety of containers, which one will be most appropriate for the popped corn and record their predictions. Once the popcorn is popped, pupils check their predictions using the containers to see which is the best fit. Encourage pupils to consider the original small container used for the kernels and think about how many of these

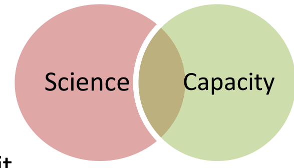
[221] Van de Walle J, Karp K and Bay-Williams J, *Elementary School Mathematics: Elementary & Middle School Mathematics Teaching Developmentally* (8th edn, Pearson Custom Publishing 2001)

containers would be filled by the popped corn. Elicit what volume of kernels would be required to make a '1 litre' of popcorn?



Sharing the Corn: To further develop this activity, pupils could investigate how many litres of kernels will be needed for everyone in the class to get a fair share of popcorn. Pupils should first determine how many kernels are needed. Encourage pupils to check the back of the packet to see the 'Recommended Daily Intake' in order to decide how much each person should ideally get. Pupils may consider making a menu of 'Popcorn Intake Recommendations' for various age groups.

Displacement[222]



A selection of various sized rocks are required for this activity.
Displacement involves putting an object into water and carefully recording how much the water level rises. The rise in the volume of water is equal to the volume of the object that it now contains. In this activity pupils investigate which rock has the greatest volume by immersing them one-by-one in a container of water and measuring the amount of water displaced. Different pupils could measure different rocks. Encourage pupils to estimate and mark the new height of water. Pupils could compare the rocks also.

What will happen if you gently drop this rock into the jug of water? Why? About how much do you think the water level will rise? Why? How could we measure how much the water level rises? What is your estimate for this rock? Will the water level be higher or lower? What does it mean if the water level is higher?

How Many Marbles?[223]

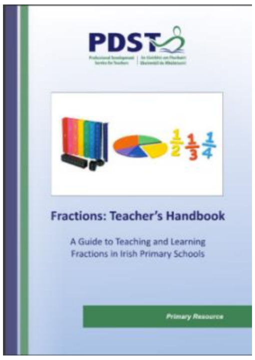
In this activity, pupils compare the volume of different units by using displacement. Half-fill two identical containers with water and mark the water levels. Put a golf ball in one container and then mark the displacement level. Estimate how many marbles would have the same volume as the golf ball and then measure by adding the marbles until the water levels are equal. (Estimates could be revised after five marbles have been added). Estimate and check how many marbles would displace the same volume of water as two golf balls.

[222] Teaching Measurement: Early Stage 1 and Stage 1 2003 pg. 106
[223] *Teaching Measurement: Early Stage 1 and Stage 1 2003* Pg. 110

LEVEL C.3

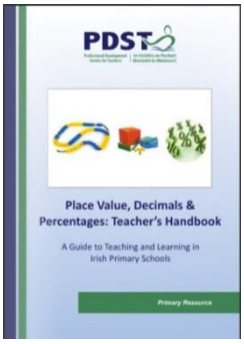
RENAME UNITS OF CAPACITY IN L AND ML AND IN DECIMAL AND FRACTION FORM

TEACHING NOTES



These PDST Handbooks provide a wide variety of learning experiences involving concrete, pictorial and symbolic stages. Such activities may be useful to revisit prior to engaging in renaming units of capacity. To access these resources just click these books.

Click here to access PDST Manuals

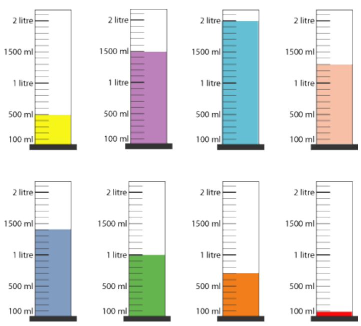


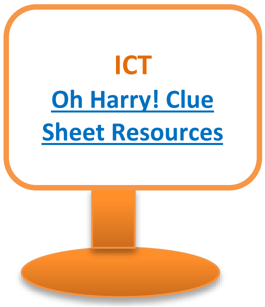
Is Tallest Wettest?

Pupils bring in an empty shampoo bottle from home. Arrange pupils in groups and provide each group with a selection of empty shampoo bottles from the variety that were brought in for example 2L bottle, 1L bottle, 500ml, 25ml, 5ml, 330ml. Discuss ways the bottles could be ordered. Pupils present various ways of ordering the bottles. Pupils use a camera to record and compare their orderings. Next pupils order them shortest to tallest. Pose the following scenario for pupils to investigate, is the tallest the wettest. i.e. does it contain the most liquid? Give pupils the opportunity to predict, then investigate the question, and finally present their findings.

Oh Harry![224]

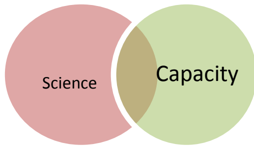
A group of eight pupils were measuring water using measuring cylinders. They coloured the water to make reading the scales easier. They lined up the cylinders in two neat rows, each labelled with a pupil's name and the amount they had measured out. Then Harry opened the window and the wind blew most of the labels onto the floor! "Oh! Harry!" they all wailed. Can you relabel them? This task comes from the Nrich site. Follow this interactive link for resources and further information on the task.



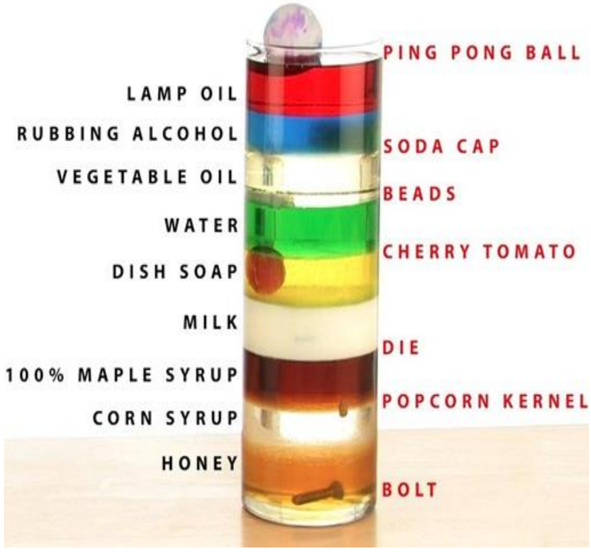


224 http://nrich.maths.org/5979/note

Nine Layer Density Tower[225]



Investigating a density tower can be a fun, interactive and practical activity providing opportunities for real life application of pupils' measurement skills. A more simplified density tower can be made using just some of the materials in the picture.



How does it work? The same volume of two different liquids will have different weights. The liquids that weigh more (have a higher density) will sink below the liquids that weigh less (have a lower density). Follow the interactive link for more information.

What is **density**? The **density** of a substance is the relationship between the mass of the substance and how much space it takes up, its volume[226].

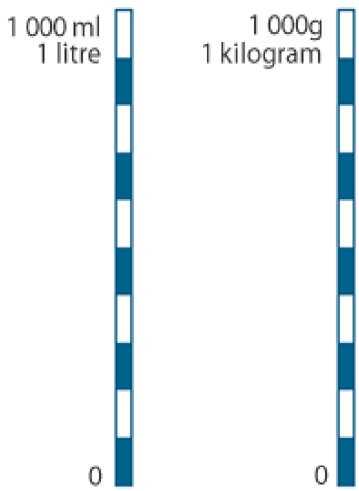
Mercury is the densest liquid. 1 litre of water weighs 1Kg, however 1 litre of Mercury would weigh 13.5 Kg.

ICT
Steve Spangler's 9 Layer Density Tower

Using the Counting Stick for Capacity[227]

For measures, hold the counting stick vertically. If the bottom is zero and the top is 1 litre (1 000ml)

- Where would 730 ml be?
- How many millilitres is this...and this...?
- How many millilitres to fill to the top?
- Use as a spring balance showing weight – this time the bottom is 0g, the top is 1 000g or 1 Kg.
- Use the stick to convert between millilitres and litres.



This activity could also be used with other measurement strands involving grammes and kilogrammes, centimetres and metres. Do the same for temperature, if this is 0, where is...? For time, set a start time at the beginning of the stick and specify the time interval of each block. So if the beginning of the stick is 10 o'clock, where is ½ past 11? Digital time can also be used.

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225 http://www.stevespanglerscience.com/lab/experiments/density-tower-magic-with-science/
226 http://science-facts.top5.com/the-top-5-densest-things-on-earth/
227 NCETM, 'Maths to Share- Using the Counting Stick' [2011] *NCETM Primary Magazine Issue 32* https://www.ncetm.org.uk/files/4525314/ncetm_primary_magazine_issue_32.pdf

LEVEL D.1

ESTIMATE, COMPARE, MEASURE AND RECORD CAPACITY USING APPROPRIATE METRIC UNITS AND SELECT SUITABLE INSTRUMENTS OF MEASUREMENT

TEACHING NOTES

It is not necessary to do much work on solid volume, measured in cubic centimetres in the primary age range. However, given the scope for practical experience with water and various containers, the measurement of liquid volume and capacity in litres and millilitres, is an important component of primary school mathematics.

Puddles[228]

In this activity, pupils visualise how different amounts of water look when displaced from containers. This activity will also lead to visualising actual quantities of liquids, for example "what do we think 5ml looks like when displaced from the container to the ground? This activity may work best on a flat area of the playground. In pairs, pupils choose a container from a selection of containers where capacity is familiar, and fill with water (for example milk jugs, yogurt tubs, empty drinks bottles). They need to think carefully about the size of the puddle they predict the water will make once it is poured onto the playground. Encourage the pupils to give reasons for their estimations. Pupiils then draw a chalk circle on the playground to record their estimation of the containers' puddle size. Pupils investigate their estimate by pouring the water out of their chosen container, into their chalk circle, and compare the size of the puddle drawn with the actual puddle made (hence the flat ground; a good discussion point with the pupils). Tip: Use a large tub / baby's bath tub with water to create a central location for the water). Use teacher questioning to activate pupils' prior knowledge of estimation, capacity and volume.

Take 'before' and 'after' pictures of the puddles. Pupils then choose another differently sized container and repeat the activity. It is worth eliciting from the pupils strategies to work systematically, for example line up the containers in order from smallest to largest and visualise the size of puddles. If they work through them in this order it should help to inform them of their next estimation. As the activity progresses onto new containers the pupils' estimations should be more accurate and their reasoning more coherent as they are able to reflect upon previous results of other containers, especially if they have worked systematically and can start to identify patterns.

Assessment

[228] https://www.tes.com/teaching-resource/teachers-tv-primary-maths--measures-6044811

If we can see the puddle that a small container made, what will the next puddle look like if the next container is only a little bit bigger than the last one? Is the puddle likely to be smaller or the same size? Can you justify your answer?
Would you change how you transferred the water? What helped?

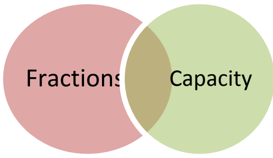
Weather Patterns[229]

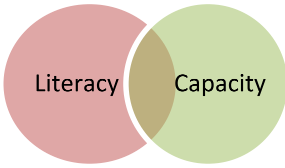
Weather conditions can also provide opportunities for investigating the relationship between time and volume. For example, pupils could investigate the volume or rainfall over the course of the month. Pupils could record their findings and present the data using different types of graphs.

This ICT link may be a useful resource to investigate weather patterns further.

ICT
Met Eireann

Magic Potion[230]





Pupils work as wizards and are tasked with inventing a magic potion for this activity. Display the cauldron image to stimulate discussion on possible potions. Suggestions could include invisibility potions, truth potions, flying potions, strength potions etc. The rule is the potion can only be 600 millilitres. They must think of 6 different ingredients to put into their potion. Unfortunately, if one of those ingredients is 100 millilitres the potion will explode. If pupils use more than 6 ingredients or if their ingredients don't add to 600 millilitres the potion will also explode. How many millilitres might each ingredient be if our potion is to be 600 millilitres?

Challenge pupils to identify different ways to get to 600 millilitres. They will then be given the measuring jug activity, where they are required to:

- Give their potion a name, write each ingredient down the bottom and write how many millilitres each one is.
- Upon completion they will write a recipe for their potion. They will list ingredients, method, what happens when the potion is taken and how it can be reversed.

At the end of the session, discuss strategies used to calculate quantities of ingredients as they added the 2nd, 3rd, 4th, 5th and 6th ingredient. Link this to addition and subtraction as being the inverse of each other and model how we can work out what is required by taking away what we have already added in. Pupils could practically investigate this if provided with a measuring device, a selection of food colourings and water to make 600ml.

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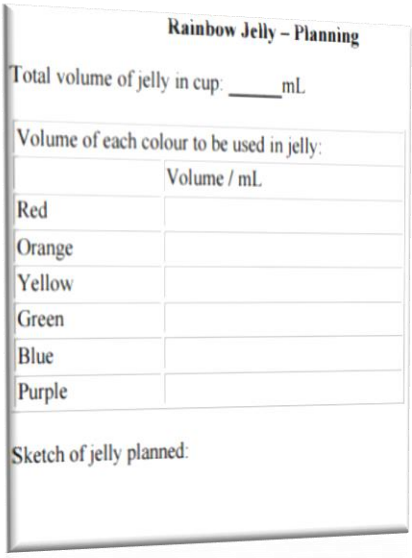
229 http://www.met.ie/
230 http://www.australiancurriculumlessons.com.au/2013/08/18/magic-potions-measuring-volume-and-capacity-lesson-234/

Click the icon [231]for a video of the puddles activity and the magic potion activity being taught with a class [232]

Rainbow Jelly[233]

In this activity pupils work with teaspoons, tablespoons and fractions of a cup to make their own rainbow jelly, converting between units of volume as required. The base units for these measurements are 1 tsp = 5 ml, 1 tbsp = 15 ml, 1 cup = 250 ml and ½ cup = 125 ml



Rainbow Jelly – Planning

Total volume of jelly in cup: _____mL

Volume of each colour to be used in jelly:

	Volume / mL
Red	
Orange	
Yellow	
Green	
Blue	
Purple	

Sketch of jelly planned:

Rainbow jellies are individual cups of jelly, striped different colours like a rainbow. Introduce pupils to the topic of rainbow jelly and show them an image of a pot of rainbow jelly. Ideally, bring in one that you have made at home and ask pupils how they think it has been made. Describe how the layers need to be made one at a time, the jelly being left to set between each layer. This activity is best done over the course of a week, thus allowing adequate time for each layer of jelly to set between the days.

Explain that this week they are going to work in groups to plan and make their own rainbow jelly. Divide pupils into groups and give out three different shaped transparent plastic containers to each group, for example a plastic cup, a plastic narrow glass and a clear yogurt pot (at least one item needs parallel sides). Note that the shape of the cup will affect the way the final jelly looks. For example, if the cup has parallel sides and equal volumes of jelly are used for all layers, the layers will be the same thickness. The task of making the stripes ‘even’, will be more complicated in containers where the sides are not parallel. The school’s science equipment might be worth looking at prior to this activity. Pupils experiment with measuring spoons and water in their cups to investigate the effects of the same volume in the different shaped containers. Encourage pupils to share their discoveries.

[231] https://www.tes.com/teaching-resource/teachers-tv-primary-maths-measures-6044811
[232] EdChat™ TV, ‘Teachers TV- Primary Maths - Measures’ https://www.youtube.com/watch?v=_5pjSg9SKa8
[233] http://nzmaths.co.nz/resource/rainbow-jelly

Making the Jelly:

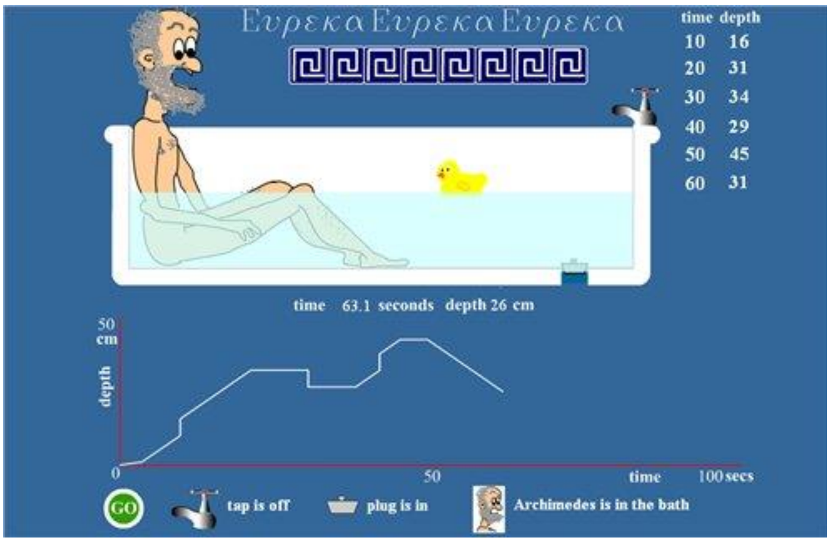
Give each group the opportunity to complete plans for several different rainbow pots. During the planning, pupils use water to fill pots to different levels. They may choose stripes of the same depth, alternating thick and thin stripes, stripes that get progressively narrower. The plans they make need to specify the colours and volumes for each stripe alongside a sketch of the design, see the 'Rainbow Planning Sheet' image. As different plans are drawn up, share these with the class and discuss the volumes of jelly used. Once planning is complete pupils choose which of their plans they will make. Tip: It would be simplest to mix the jelly required in bulk rather than have each group mix each colour individually.

Note that the hot water needed could present a safety issue. To overcome this, a small amount of very warm water could be used by the teacher to dissolve the crystals. Follow this by adding cold water before pupils use the jelly. Pupils to add to the layers of jelly over the course of the week.

How many tablespoons of red jelly have you used?
How many teaspoons would that be? How much of a cup? Which colour have you used most of? Which colour have you used least of
How much more red than yellow have you used?

Archimedes Bath Tub

Archimedes is sitting in the bath tub. The variables include the power of the water and the presence of Archimedes in the tub. Click the interactive link to explore the variables further. As teacher you could also generate a graph and allow pupils to investigate what the conditions were to lead to such a graph being created.



[234]

ICT
[Archimedes Bathtub Activity]

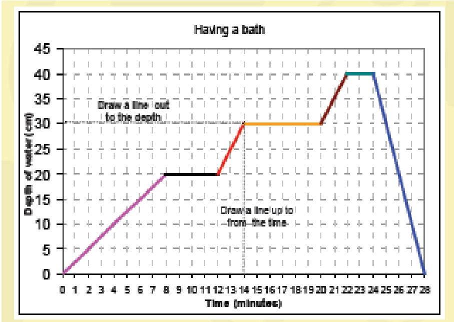
[234] http://www.colmanweb.co.uk/Assets/SWF/Archimedes.swf

My Bath Story- Introducing the Maths World

'Maths Eyes', a resource for developing pupils' mathematical skills also have a 'Bath Story' task on page 52. Click the interactive to access the Maths Eyes Handbook.

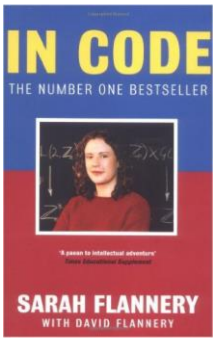
[235]

maths eyes



The true puzzle should be accessible to all; its solution should require no specialist knowledge other than at times the rudiments of arithmetic and algebra. It is perhaps the unconscious feeling that we all start out equal that gives puzzles their charm (Flannery, 2001)

TEACHING NOTES



Sarah Flannery was, at sixteen years old, the winner of the 1999 Esat Young Scientist Exhibition the EU Young Scientist of the Year Award in 1999. Her book 'In Code' , co-written with her father, mathematician David Flannery, retells the story of the making and breaking of her award winning algorithm and of the enjoyment that she got, as a child and throughout her life, from solving mathematical puzzles. She writes in detail about 'The Two Jar Puzzle' also known as 'The Decanter Puzzle'.

ICT
The Decanter Puzzle

The Decanter Puzzle

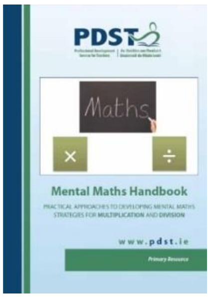
This interactive will engage pupils in exploring 'The Decanter Puzzle' to deeper levels. [236]

[235] http://www.haveyougotmathseyes.com/wp-content/uploads/resources/mathseyes_resource_pack.pdf

[236] http://www.interactive-maths.com/decanting-puzzle.html

LEVEL D.2

RENAME UNITS OF CAPACITY IN L AND ML AND DECIMAL AND FRACTION FORM



Converting millilitres to litres presents a challenge for pupils because it involves multiplying and dividing by 1000. Again, practical activities using a mixture of litres and millilitres should be encouraged. The PDST Mental Maths books also details mental strategies for multiplying and dividing by 1000.

Capacity Line Up

Pupils write a measure of liquid volume between 0 – 2 litres and pass it to a partner. Pupils can write their quantity in a variety of ways; a whole number, a decimal fraction etc. Write some examples on the board for example ½ L, 500ml, 1.75L. Pupils stand up and order their quantities largest to smallest, whole number-fractions-decimal fractions, odd-even quantities. The following questions will help pupils to develop their reasoning skills.

Stay standing if the capacity on your card is: greater than 0.4 litre, less than 1 500 ml, greater than ¾ litre, less than 1.2 litres, greater than 900 ml, less than 0.95 litres

When only a few pupils are left, ask for them to show their values and check with the group whether they should be standing. How can they justify that they are still standing?

TEACHING NOTES

The most common capacity of glasses is either 200 or 250 millilitres. Small jars, such as those that hold sandwich spreads, often contain 100 millilitres. Many pupils will at this stage be able to solve tasks involving finding the volume of such jars when they are not labelled. Pupils will be aware that they can solve such tasks by pouring water from a graduated container into an empty jar. However, the mechanical drudgery of such methods will leave pupils receptive to more efficient methods that use metric units. Understanding the connection between millilitres and litres involves some comprehension of place value because the metric system is based on ten. Encourage pupils to discover the relationship from their findings using open questions.

You think the 2 litre bottle holds 10 glasses. How many glasses would a 1 litre bottle hold? Does your answer match with the number of glasses in the 1.5 litre bottle? How many millilitres are there in 1 litre? I wonder what 'milli' means (1000)?

Once pupils have developed a feel for the capacity of glasses and bottles, this understanding can be applied by getting containers such as a 3 litre fruit juice bottle and asking how many glasses of juice it holds. Estimates can be revised as successive glasses of water are poured in.

Medicine Bottles

Using medicine bottles is a useful way of helping pupils to relate to capacity. Medicine bottles commonly come in 100 millilitre or 200 millilitre capacities and are available cheaply from chemist shops. Medicine bottles have their capacity written on the bottom, allow pupils to examine some empty medicine bottles. Challenge them to see if they can find out what quantity a person should take and what fraction of the contents this would represent. Pose the following scenario to pupils. Connor is sick and has to take 1 teaspoon (5 mL) of medicine three times a day from his 100 millilitre bottle of medicine. Would it be possible to find out how many days 100 ml of medicine will last?

How many millilitres of medicine would be in his bottle if it needed to last exactly 10 days? How many days would 100 ml last if he had 4 teaspoons each day? If his mum and dad get the bug, what size bottle will be required for the family?

This interactive activity is from the BBC Skillswise site. The activity requires you to match the graduated containers with quantities of liquids. Decimal volumes are included. [237]

ICT
Taking Measures: Capacity Activity

[237] http://www.bbc.co.uk/skillswise/game/ma23capa-game-taking-measures-capacity

LEVEL D.3

FIND THE VOLUME OF A CUBOID EXPERIMENTALLY

TEACHING NOTES

The volume of an object is the amount of 3-D space that it occupies. Liquid volume and solid volume are measured in different units, although the concepts are the same. Liquid volume is measured in millilitres and litres whilst solid volume is measured in cubic centimetres and metres. Pupils should use cubes to solve volume and capacity problems. Encourage pupils to 'discover' more efficient ways of calculating the total number of cubes in a cuboid, thus discovering the formula for calculating the volume of a cuboid.

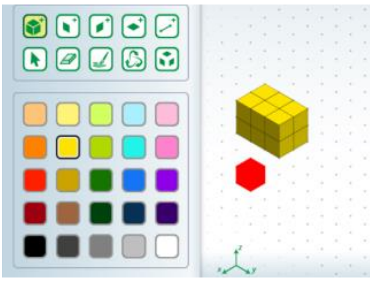
Volume with Sugar Cubes[238]

Using sugar cubes, allow pupils the opportunity to build cuboids. Encourage pupils to start small and work towards building bigger models. This will scaffold the development of the concept of finding volume, Pupils count and record the dimensions of their cuboids.[239] Challenge the pupils to build three cuboids which have the same volume, but different dimensions. Record and annotate the various models the pupils have created on the board.

How many sugar cubes long is it? How many sugar cubes is it? How many sugar cubes wide is it? How many cubes did you use altogether? Using the same amount of cubes could we create cuboids with different dimensions?

With repetition, the relationship between the volume and dimensions of the cuboid will be established. Through exploration and reflecting on their learning, pupils will discover for themselves the formula for volume.

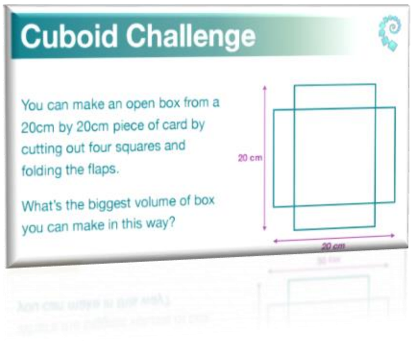
[238] http://www.education.com/activity/article/understand-volume-sugar-cubes/



This activity can be extended by making a building out of multiple cuboids. Support pupils in figuring out how to mentally break down the building into separate rectangular prisms. Pupils will need to find the volume of each section in order to calculate the total volume of the building. Finally pupils can physically break down the building and check their answer. This interactive may support pupils in recording their findings and also in visualising their designs[240].

ICT
Isometric Drawing Tool

Cuboid Challenge[241]



This problem solving task comes from the Nrich site. Simply click the interactive to access this task and the possible strategies and solutions linked to the task.

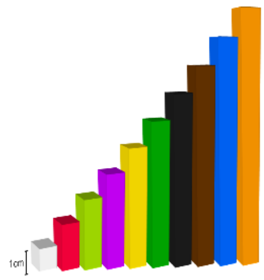
ICT
Cuboid Challenge

Lunch Boxes[242]

Pose the following scenario to the pupils and elicit from pupils their possible solution methods. Invite pupils to select concrete materials which they could use to work out the capacity of their lunch boxes. Encourage pupils to express and share their solution methods. This activity may work well with pupils working in pairs.

A school group were on an excursion when their bus broke down. They had to trek a distance and carry water back in their lunch boxes to fill the bus water container that had a capacity of one cubic metre. Would they be able to carry back enough water to fill it in one trip?

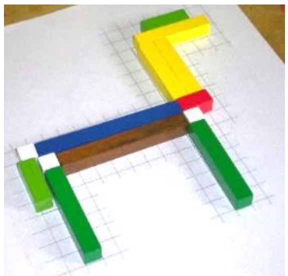
Solid Volume Using Cuisenaire Rods



Begin by letting pupils freely explore the set of 10 'Cuisenaire Rods'. Pupils could create 'Rod Challenges for their class, for example,

- -Using only one of each colour what is the longest line that can be created?
- -Make a Cuisenaire Staircase with only odd steps

[240] http://illuminations.nctm.org/Activity.aspx?id=4182
[241] http://nrich.maths.org/8680
[242] ***First Steps in Mathematics: Number***, Understand Operations, Key Understanding 3 Sample Lesson 1,page 26



Now use the rods to explore 'volume'. You could begin by using the white rod. Elicit from pupils what is the volume of this cube? How could you convince me? Some pupils may need to explore this using their rulers, or another strategy.

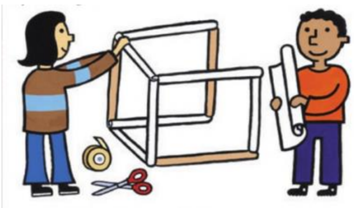
Encourage pupils to find the volume of each rod and the total volume of the set.

Pupils could record their findings and then look for patterns to consolidate the formula for finding the volume of a solid. Length X width X height = Volume

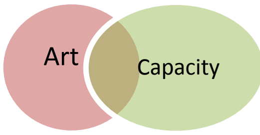
Next allow pupils to create an object with their rods. Encourage pupils to determine the volume of their creation.

Click this link to see pupils use Cuisenaire Rods to investigate litres and millilitres[243]

Make a Cubic Metre[244]



Provide pupils with newspapers in order to create their own cubic metre. Could we turn the construction into a cuboid? How?



Can you visualise 1 metre cubed? Would it be important to be able to imagine this? What furniture in the classroom takes up 1 cubic metre?

Maths Project: Design a Cereal Box

This potential of this project to develop mathematical skills can be best explored if pupils work in pairs or groups. Discuss with pupils the object (cuboid) and its features. Encourage pupils to identify cuboids in the classroom and also cuboids they touch on a daily basis. Using a standard cereal box discuss the dimensions of the box and how they can be determined. Model drawing this cuboid and annotating the dimensions of it. Next give the pupils their brief which they can work on in pairs or groups.

243 http://www.cuisenaire.co.uk/index.php/home/videos/cuisenaire-rods-in-the-classroom/video/year-3-use-the-rods-to-represent-scales-of-volume-and-length

244 http://nzmaths.co.nz/resource/would-wood-fit

You have been hired to design three different sized cereal boxes for your favourite cereal. You are to create the following:

A regular sized cereal box

A 'mini' cereal box that is half as tall, half as wide and half as long as the regular cereal box

A 'value' sized box that is three times as tall, three times as wide and three times as long as the regular box

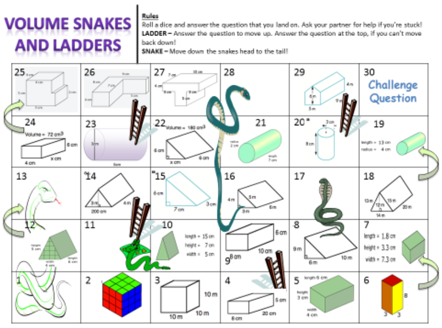
Give pupils time to draw out their designs and annotate the dimensions. Provide pupils with card, scissors and masking tape to create 3-D models. Give groups the opportunity to share their designs with the class. Finally discuss with pupils how we could find out the volume of the boxes created.

CONSOLIDATION ACTIVITIES

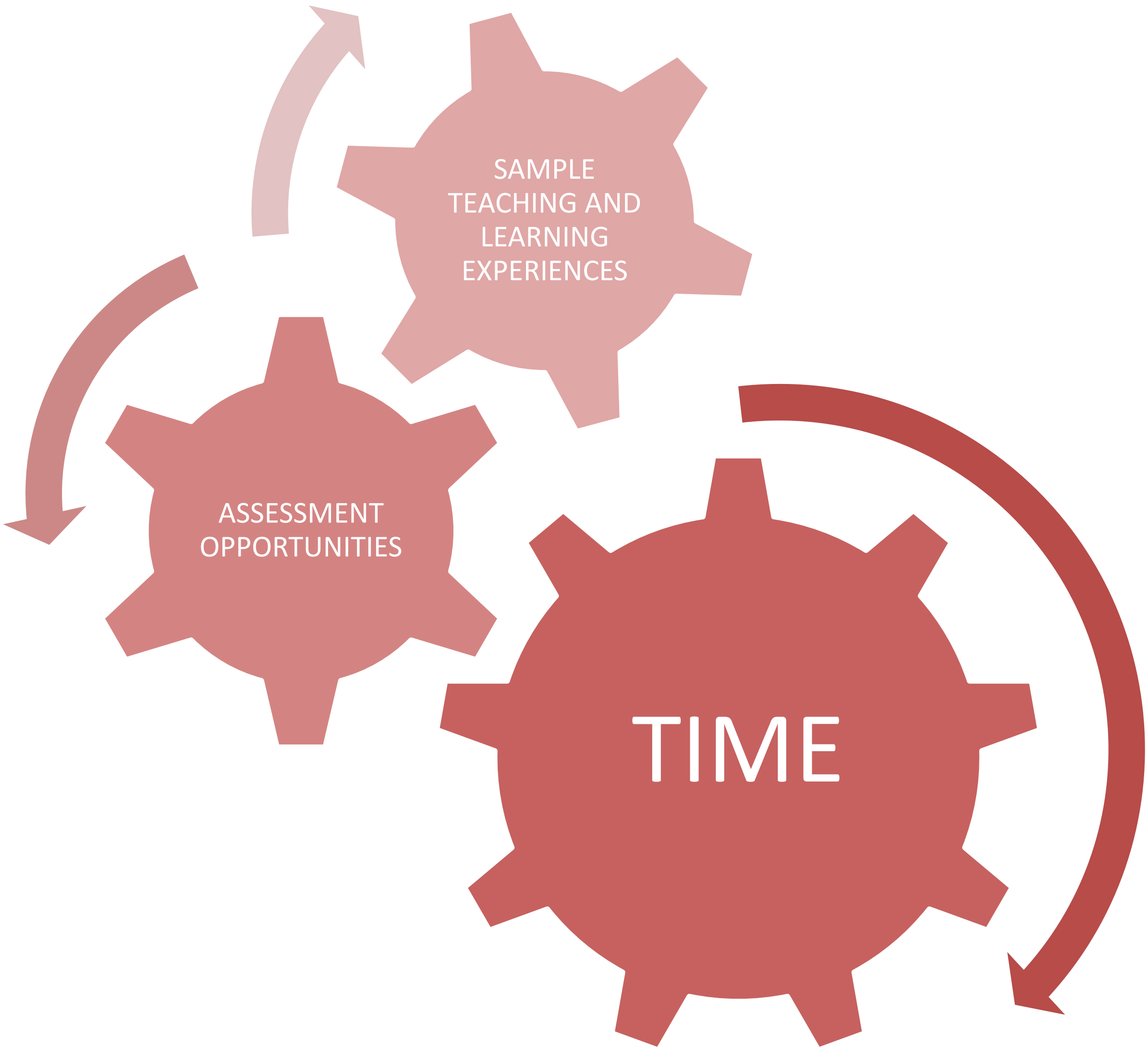
Volume Snakes and Ladders[245]

Click the interactive to access this 'Volume Snakes and Ladders' board Game

ICT
[Volume Snakes and Ladders Board]



[245] https://www.tes.com/teaching-resource/volume-of-prisms-snakes-and-ladders-6449596


SAMPLE TEACHING AND LEARNING EXPERIENCES
ASSESSMENT OPPORTUNITIES
TIME

TIME

TIME - BACKGROUND KNOWLEDGE FOR TEACHERS

Days & Years

The year and the day are the only 'natural' units of time. The day is the time the Earth takes to make one complete revolution on its own axis in relation to the Sun (i.e. the average time between one midday and the next). The year is the time the Earth takes to make one complete circuit around the Sun. This takes roughly 365 ¼ days.[246]

A.M. & P.M.

It may not be necessary for children to know the origin of a.m. and p.m., but it is an interesting sidelight on how measurements of time developed. The morning was 'ante meridiem' that is before the Sun passed the meridian (was the highest in the sky) or noon, and the afternoon was 'post meridiem' (after the Sun had been at its highest). In fact p.m. is just an abbreviation of the Latin for afternoon. It was possible, on a sunny day, to find the time the Sun was at its highest by observation (for example by the length of the shadow of a stick) and then the hours could be measured by other means, for example candle or water clocks, or later mechanical time keepers.[247]

Time Zones

The need for standard time zones emerged with the spread of high speed transportation systems – first trains and later airplanes. In 1884, delegates from twenty seven countries met in Washington, DC at the Meridian Conference and agreed on a system of time zones that is essentially the one we still use today.

Time zones are based on the fact that Earth moves through 15 degrees of longitude each hour. Therefore, there are 24 standard time zones (24 hours x 15º=360º). Time zones are counted from the Prime Meridian (0º longitude), which runs through Greenwich, England. Each time zone is based on a central meridian, counted at 15º intervals from the Prime Meridian, and extends 7½º to either side of the central meridian. For example, New York City lies in the zone of the 75ºW central meridian, and the time zone includes all locations between 67½ºW and 82½ºW.[248]

TIME - POSSIBLE PUPIL MISCONCEPTIONS

Time v Elapsed Time

Many pupils confuse 'the time' with 'how much time'. A teacher of 7-year-olds asked them to draw pictures of themselves going to bed and getting up and to write their names and the times underneath. They then sorted their pictures into groups by time. Asked to work out who was in bed the longest, most responded that

[246] Suggate et al, Mathematical Knowledge for Primary Teachers, 4th Edition, 2010 p.220
[247] Suggate et al, Mathematical Knowledge for Primary Teachers, 4th Edition, 2010 p.222
[248] http://www.nationalgeographic.com/geobee/study-corner/activity-10/

it was those who went to bed at 7 p.m. In vain, the teacher pointed out that one child who went to bed at 7 p.m. was up at 6 a.m., while another who went to bed at 7:30 p.m. wasn't up until 7:30 a.m. The pupils were convinced that those who went to bed first were in bed longest.

The teacher thought that probably the pupils could not work out how much time they were in bed, so she helped them to mark a long strip with hours of the day a centimetre apart. They then coloured the strip between when they went to bed and when they woke and cut off the uncoloured parts. The teacher then made a column graph by using the paper strips and lining up the bases. She suggested that the graph would help the pupils to find out who was in bed longest. To her surprise, the pupils rejected her approach. *You've made a mistake — you've made us all go to bed at the same time.* The pupils have not yet understood the difference between time and elapsed time. [249]

Intangible Time

Time is a different from the other attributes that are commonly measured in school because it cannot be seen or felt. It is therefore more difficult for pupils to comprehend units of time or how they are matched against a given time period or duration. [250]

Continuous Time

The concept of time is a very difficult one for young children living, as they do, in the present. Activities must be provided to give experiences in the passing of time and leading to the idea that time is continuous.[251]

Time Passes at a Constant Rate

One of the problems with measuring time intervals is that the scientists insist that time passes at a constant rate, yet it does not feel like that in direct experience. Five minutes in the dentist's waiting room can feel longer that an hour at the cinema (if the film is good). [252]

Recording the Date

Most children learn how to record the date without too many problems. There may be some confusion (for example in some computer software) between the American form of month, day, year and the English day, month, year. [253]

[249] First Steps Measure Book 1 p.21
[250] John A. Van De Walle et al, Elementary and Middle School Mathematics Teaching Developmentally, 7th Edition, 2010 p. 383
[251] Deboys & Pitt, Lines of Development in Primary Maths 1979 p. 84
[252] Suggate et al, Mathematical Knowledge for Primary Teachers, 4th Edition, 2010 p.224
[253] Suggate et al, Mathematical Knowledge for Primary Teachers, 4th Edition, 2010 p.221

Recording Time

There are several different ways in which time within the day is recorded. Children need to understand them all and be able to move from one to another. The first challenge is to understand the analogue clock face. By convention, the clock face only shows 12 hours and has 12 at the top with the other numbers spaced equally around. Most clock have two hands, so in effect there are two different scales around the clock, even if only one is shown. [254]

Reading the Time

Learning to tell the time has little to do with time measurement and more to do with the skills of learning to read a dial-type instrument. Clock reading can be a difficult skill to teach.[255]

Base 60

Because of the 60-base, it is quite difficult to find the time interval between two times of day, especially as the standard way of writing the time appears to suggest it might be treated as numbers in the base ten decimal system. For example, if a train leaves one station at 9.46 and arrives at the next at 10.12, how long does the journey between the two take? Probably the best way to find the time taken is to use a counting-on method. So from 9.46 to 10.00 is 14 minutes, from 10.00 to 10.12 is 12 minutes. 14 and 12 is 26 minutes.

The same method can be used even if the time goes over midnight. For example, how long does it take to travel from Edinburgh to London, if the train leaves Edinburgh at 19.30 and arrives at London at 00.45? 19.30 to 20.00 is 30 minutes. 20.00 to 00.00 is 4 hours. 00.00 to 00.45 is 45 minutes. So the total time take is 30 minutes + 4 hours + 45 minutes = 5 hours 15 minutes.

TIME - PARENTAL INVOVLEMENT

The following are a list of ideas that may be shared with parents to help develop pupils' conceptual understanding of time. Some of the following ideas are taken from the NCCA's tip sheets for parents available through the NCCA website.[256]

Here you will also find some short videos of parents working with their children to develop their mathematical understanding.

Level A:

- Ask questions such as: What do you do before you go to bed? After swimming? What do you think will happen next in the story? What day is it today?

[254] Suggate et al, Mathematical Knowledge for Primary Teachers, 4th Edition, 2010 p.222
[255] John A. Van De Walle et al, Elementary and Middle School Mathematics Teaching Developmentally, 7th Edition, 2010 p. 383
[256] http://www.ncca.ie/en/Curriculum_and_Assessment/Parents/

- Talk about times and the clock. Put times up on the fridge door or notice board, for example 'Football 3pm Tuesday, Training 9am Saturday'. Talk about things we do in Summer, in Winter, during the day, at night.

Level B:

- Read times and use a calendar Ask your child to find family birthdays on a calendar. Work out how many weeks or days it will be until the holidays or Grandad's birthday.
- Ask your child to mark the Tuesdays for swimming. How many swim days are in the month? What dates will we be swimming?
- Your child can check the 'use by' date on groceries and work out how many days are left before the item goes out of date.

Level C:

- Encourage your child to keep a calendar and track important dates.
- Look at the TV guide with your child. What time is your favourite programme on?
- How long is it on for?
- If waiting for a bus ask children to read the schedule and tell you what time the next bus comes, what time does the last one come, is there a pattern for the bus times.

Level D:

- Look at different time zones. Let's skype Auntie Mary in Australia. What time is it over there?
- Encourage your child to look up times for the cinema. What time will the film start? What time will it be over?
- Plan short trips with your child. The match starts at half 2, what time do you think we should leave her at? What time is the train leaving Limerick Junction? How long will it take to get to Cork?

TIME - TEACHING NOTES

There are two distinct aspects in studying time. The first is 'telling the time'. The second aspect is the measuring of time intervals.[257]

Time is a measure but it is a different from the other measures in that it cannot be seen or touched. However, we are surrounded by the effect of time passing, for example, day to night and one season to another. There are two aspects of time that pupils must develop:

- Time as an instant which can be named, for example, 6:15;
- Time as a duration which describes an amount of time that has passed, for example, a minute, the afternoon, the year.[258]

[257] Suggate et al, Mathematical Knowledge for Primary Teachers, 4th Edition, 2010 p.220
[258] http://nzmaths.co.nz/time-units-work

MEASURES - TIME

The following is an extract from http://nzmaths.co.nz/time-units-work

Pupils' experiences with time throughout the learning sequence has two aspects: ***- duration*** and ***telling time***.

Telling time must enable them to:

1. Develop an understanding of the size of the units of time. This includes being able to estimate and measure using units of time;

2. Read and tell the time using both analogue and digital displays.

Stage One: Identifying the Attribute

Duration

It is important that pupils develop an understanding of the duration of time in addition to being able to identify moments of time. Unlike the other time measures, pupils are frequently introduced to standard units of time, for example, days of the week, and hour times, before they have grasped the concept of the duration of time.

It is important that the concept of time as ***duration*** is emphasised from the start. Pupils need to have lots of experiences to establish that the duration of an event requires noting the starting and finishing points of time. Arranging pictures of events in the correct sequence helps develop the concept of duration. The use of words such as before, after, soon, now, later, bedtime and lunchtime, helps develop understanding of the attribute of time.

Looking at standard cycles of time follows from the sequencing of daily events. Pupils learn the sequence of the days of the week, but initially may not fully understand the way we use the names repeatedly. The terms today, tomorrow, yesterday and weekend, can be learnt in relation to the cycle of days. The sequence of months can also be developed as well as the grouping of the months into seasons.

Stage Two: Comparing and Ordering

Duration

Comparing the duration of two events is the second stage in developing an understanding of time passing. This can be done by directly comparing two activities that have common starting points, for example, a song on a tape or running around the building.

You can indirectly compare the duration of events by using a sand timer or a candle timer.

Telling time

At this stage time telling skills will focus on hour and half-hour.

Stage 3: Non-Standard Units

Duration

Measuring the duration of events using non-standard or informal units is the third stage in the learning sequence. Beginning with non-standard but familiar units allows the pupils to focus on the process of repeatedly using a unit as a measuring device. Parts of the body provide interesting units for introductory use.

Pupils can compare and order events using heartbeats, hand clapping and hopping. Pupils can also make their own timing devices and calibrate these arbitrarily, for example, by making marks at regular intervals on a burning candle, or by letting sand run through a small hole into a calibrated container.

From the earliest of these experiences, pupils should be encouraged to estimate. Initially these estimations may be no more than guesses, but estimating involves the pupils in developing a sense of the size of the time unit. As everyday life involves estimating at least as frequently as finding exact measures, the skill of estimating is important.

Although non-standard units reinforce most of the basic measuring principles, pupils need to realise that they are limited as a means of communication. This can be highlighted through activities that involve the pupils measuring the duration of a single event using, for example, counting rhythms (one rhinoceros, two rhinoceros, three rhinoceros etc).

Telling time

Pupils' skills in telling time can be extended to match their numeracy understanding. For example, the pupils may count in fives to tell time in five-minute intervals.

Stage 4: Standard Units

When pupils can measure the duration of events using non-standard units, they are ready to move to the use of standard units. The motivation for moving to this stage often follows from experiences where the pupils have used different non-standard units for the same event. They can then appreciate that consistency in the units used would allow for the easier and more accurate communication of duration.

Measuring with standard units involves the introduction of minutes, hours and seconds in addition to reading time on analogue and digital clocks.

Duration

The minute is often introduced first because it is small enough to measure common events. The duration of a minute can be established by watching the second hand on a clock or by constructing a minute sand-timer. An appreciation for the size of a minute can be built up through lots of experience in measuring everyday events. For example, how many minutes does the song play for? How long is morning break? How long does it take to walk around the school building? How many times can you hop in a minute? How many linked cubes can you join in a minute?

As the pupils become familiar with the size of a minute they should be given opportunities to estimate before measuring. Minutes need to be linked to the movement of the minute hand on the analogue clock and the digits on digital displays.

An understanding of the size of a second can be developed by investigating the relationship between seconds and minutes. This can be done by watching the digital displays on some watches, on stopwatches and on video-recorders. The pupils should be encouraged to develop their own reference for a second, for example, a counting pattern "one – banana – two – banana – three etc".

It is more difficult to give pupils a concept for one hour but references can be established for events that last an hour by setting an alarm to ring after an hour has passed.

Reading and telling time

The underlying number skills should be mastered before teaching pupils the skill of telling and reading time. Pupils are likely to be able to read the time to the hour and half-hour prior to fully developing a conceptual understanding of the size of time units. The usual progression for teaching the skill of telling and reading time is:

- read time on the hour by identifying where the hands point for time on the hour;
- read time to the ½ and ¼ hour;
- read clock time after the hour by counting the minutes after the hour (in fives);
- use digital notation to record the number of minutes after the hour;
- read clock time before the hour by counting the minutes to the hour (in fives);
- refine time-telling skills by associating numerals 1, 2, 3, 4, and 5 with time 5, 10, 15, 20 and 25 after the hour;
- associate numerals 7, 8, 9, 10 and 11 with time before the hour;
- 24 hour notation.

Stage 5: Applying and Interpreting

Once the pupils have an appreciation of the size of seconds, minutes and hours and the relationship between them, they can be introduced to time zones and time-lines. An understanding of time zones can be developed from contexts such as travel and holidays.

Phone or e-mail interactions with people in other time zones can be used to highlight the differences in time throughout the world. Also, historical projects bring to life year time-lines.

TIME - LEARNING TRAJECTORY

The learning trajectory is based on the objectives for Measures in the Primary School Mathematics Curriculum. In some instances, similar objectives at the same class level have been collapsed into one objective. Objectives that only refer to problem solving have not been included as discrete objectives because a problem solving approach is advocated throughout all of the teaching and learning experiences. Problem solving is viewed in this manual as a fundamental, integral part of mathematics teaching and learning that pupils should experience every day. The same colour coding from the curriculum is used – infants (green); first and second (red); third and fourth (blue); fifth and sixth (orange).

TIME LEARNING TRAJECTORY LEVEL A[259]

Trajectory Levels	Concept	Developmental Experiences		
		Concrete	Pictorial	Abstract
	Level A.1 Develop an understanding of the concept of time and use appropriate vocabulary			
	Level A.2 Sequence daily and weekly events or stages in a story			
	Level A.3 Read time in one-hour intervals			

[259] This level is generally aligned with the objectives for Junior and Senior Infants

TIME LEARNING TRAJECTORY LEVEL B[260]

Trajectory Levels	Concept	Developmental Experiences: Concrete	Pictorial	Abstract
	Level B.1 Use the vocabulary of time to sequence events	As for learning experiences in Levels A.1 and A.2. Extension activities included		
	Level B.2 Read and record time using simple devices	As for the comparison and duration activities in Level A.1		
	Level B.3 Read, record and calculate time in hours, half-hours and quarter hours on 12-hour analogue clock and on digital clock	As for learning experiences in Level A.3 Extension activities included		
	Level B.4 Read day, date and month using calendar and identify the season			

[260] This level is generally aligned with the objectives for First class and Second Class

MEASURES - TIME

TIME LEARNING TRAJECTORY LEVEL C[261]

Trajectory Levels	Concept	Developmental Experiences: Concrete	Pictorial	Abstract
	Level C.1 Consolidate and develop further a sense of time passing, *sequencing events*	As for learning experiences in Levels A.1, A.2 and B.1 Extension activities included		
	Level C.2 Read and record time in five-minute and one-minute intervals on analogue and digital clock (12-hour) and rename digital time as analogue time and vice versa	As for learning experiences in Levels A.3 and B.3 Extension activities included		
	Level C.3 Rename minutes as hours and hours as minutes			
	Level C.4 Read and interpret simple timetables			
	Level C.5 Read dates from calendars and express weeks as days and vice versa	As for learning experiences in Level B.4 Extension activities included		

[261] This level is generally aligned with the objectives for Third and Fourth class

TIME LEARNING TRAJECTORY LEVEL D[262]

Trajectory Levels	Concept	Developmental Experiences: Concrete	Pictorial	Abstract
	Level D.1 Read and interpret timetables and the 24-hour clock (digital and analogue) and interpret and convert between times in 12-hour and 24-hour format	As per the analogue, digital and timetable activities in Levels A.3, B.3, C.2, C.3 and C.4 Extension activities included		
	Level D.2 Explore international time zones			
	Level D.3 Explore the relationship between time, distance and average speed			

[262] This level is generally aligned with the objectives for Fifth and Sixth class

MEASURES - TIME

LEVELS A.1

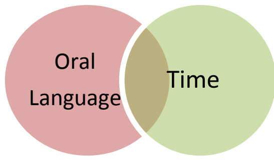
DEVELOP AN UNDERSTANDING OF THE CONCEPT OF TIME AND USE APPROPRIATE VOCABULARY

TEACHING NOTES

The concept of time is a very difficult one for young children living, as they do, in the present. Activities must be provided to give experiences in the passing of time and leading to the idea that time is continuous. [263] Time is different from the other attributes that are commonly measured in school. It cannot be seen or felt. It is more difficult for pupils to comprehend units of time or how they are matched against a given time period or duration[264].

One of the first 'time words' to become meaningful to the child is NOW with its opposite NOT NOW. Certain events familiar to the child are associated with time: lunch time, dinner time, time to get up, time for bed etc. General discussion of these topics provides the first stage in an understanding of time.

There is a natural opportunity for integration with the Oral Language strand of the English curriculum at this level.



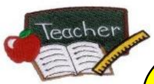
SAMPLE LEARNING EXPERIENCES

The following teaching and learning experiences are designed to support oral language skills and in particular, the vocabulary of time. Guided discussion and a hands-on approach are essential to enhancing pupils' learning.

Discussing Events Associated with Time

Pupils will need opportunities to try out language for themselves rather than only simply listening to it. The following are some sample questions to stimulate discussion.

Talk about Time-related Events



*Do you get up **early** in the morning? How do you know it's **early?** Did you go to bed **late** last night? How did you know it was **late**? It's almost break time so what will we do **now**? What happens **before** we go out to the yard? **Later**, it will be time to go home. What happens **before** we go home?*

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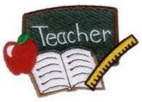
[263] Deboys & Pitt, Lines of Development in Primary Maths 1979 p. 84

[264] John A. Van De Walle et al, Elementary and Middle School Mathematics Teaching Developmentally, 7th Edition, 2010 p. 383

Realise that Events are Separated by Time



*What happens **between** little break and lunch? What happens **between** waking up and leaving the house to come to school? It's **nearly** home time. What have we done since lunch time?*

Comparing Intervals of Time and Exploring the Concept of Duration[265]

As the span of attention of the average young pupil is small the time intervals involved must be very brief. All the activities will be measured and compared purely by the children paying attention. In this way, pupils begin to become aware of longer and shorter intervals.

Who Finishes First? [266]

Begin by asking pupils which they think takes longer, *making a tower with 5 cubes or hopping 5 times on each foot?* Write the 2 events on a chart. Get 2 volunteers to complete the activity. Tell the pupils that today they are going to work with a partner comparing things they do to find out which takes longer. Ask the pupils for their ideas and add these to the chart. Possible ideas may include: completing a jigsaw and saying the alphabet, counting to 20 and putting on a pair of shoes, writing your name and doing 5 jumping jacks, saying a nursery rhyme and connecting 10 links. Get the pupils to work on the activities in pairs with a third pupil acting as referee. Pupil can share findings.

*Which did you think would take **longer**? Who did you think would finish **first**? Why? Were you correct? Which took a **shorter** time? Who was **quicker**? Who was **slower**? Did you both start the activity at **the same time**? Why was that important? Would it be fair if one of you started **before** the other? Why?*

A further comparison activity could involve comparing two pupils doing the *same* job, for example, building towers, filling containers, making jigsaws, jumping, hopping etc.

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265 Deboys & Pitt, Lines of Development in Primary Maths 1979 p. 84
266 http://nzmaths.co.nz/resource/how-long-now

Estimation

From the earliest of these experiences, pupils should be encouraged to estimate. Initially these estimations may be no more than guesses, but estimating involves the pupils in developing a sense of the size of the time unit. As everyday life involves estimating, at least as frequently as finding exact measures, the skill of estimating is important.

Clapping Time [267] [268]

Begin by asking the pupils which they think would take them longer, writing their name or walking to the board and back to their desk. Select a volunteer to complete the two events while the rest of the class time the events by clapping. Help the class keep a steady beat. Record the results:

Writing my name 9 claps

Walking to the board 11 claps

Ask for other ideas for timing events, for example, clicking fingers, stamping, linking cubes, drips from a tap, swings of a pendulum, beats on a metronome etc. List some events that could be timed. Ask the pupils to add their ideas. Some activities might include stacking ten blocks one at a time and them removing them one at a time, writing the alphabet, walking slowly around a designated path, making a tower of 15 connecting cubes [269]

Ask pupils to predict how long they think each activity will take using the measure selected. Record their predictions. Carry out the activities, record the results and compare the various durations taken to complete the activities.

*Which do you think will take **longer**? How many claps do you think writing your name will take? Does anyone think it will take **more/less**? How many counts of the metronome will it take complete a jigsaw? Will stacking ten blocks take **more/less** counts of the metronome, do you think? How many swings of the counting pendulum to tidy your table?*

Metronome[270]

Use of a metronome provides a consistent gauge of the duration of an activity.

ICT

[Online Metronome](http://www.metronomeonline.com/)

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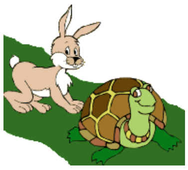
[267] http://nzmaths.co.nz/resource/how-long-now

[268] http://nzmaths.co.nz/time-units-work

[269] John A. Van De Walle et al, Elementary and Middle School Mathematics Teaching Developmentally, 7th Edition, 2010 p. 383

[270] http://www.metronomeonline.com/

The Hare and the Tortoise



Prior to reading the story discuss what the children know about hares and tortoises. *What do they look like? Where do they live? How do they move? Who do you think is faster, a hare or a tortoise? If they were to race each other, who do you think would win the race? Why?* Following the story, ask pupils questions to focus on the aspects of time and speed in the story. *Who did you expect to win the race? Who won the race in the end? Why didn't the hare win? If the hare hadn't stopped for a nap, do you think he would have beaten the tortoise? Why did the tortoise end up winning?*

Fast and Slow [271]

Begin the activity by exploring fast and slow actions. *Let's wave our hands quickly…now slowly. Let's clap quickly…slowly. Let's blink quickly…slowly.* Ask pupils to share their ideas for other fast and slow actions. *What other things can we do quickly and then slowly?* Discuss things that pupils know that go fast or slow. List these ideas on a chart of slow and fast things. Ask the pupils to think of their favourite fast thing and their favourite slow thing. Draw these onto a piece of paper. Share the pictures of fast and slow things.

Ages [272]

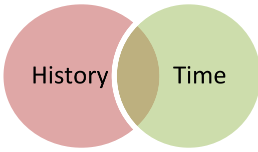
Collect pictures of people of varying ages, images from the internet, photos from magazine, family photos or pictures of the teacher since childhood. Show the pupils the pictures. Begin with the picture of a baby. (If it is a photo of you get the pupils to guess who they think it is.) *How old do you think the baby is? Do you know any babies? Who?* Show two more pictures to the pupils. *Who do you think is older? How can you tell? How old do you think that child might be? Is that older or younger than you?* Before you show the next picture ask the pupils to guess who it might be a picture of (mother, grandmother). *What picture do you think I am going to show you next? Why did you guess that?* As you discuss the pictures display them on a line in order of age. Ask the pupils to either cut from magazines or draw pictures of people of different ages. Give the sets to other pupils to order from youngest to oldest. Share the strips of pictures.

[271] http://nzmaths.co.nz/resource/passing-time
[272] http://nzmaths.co.nz/resource/passing-time

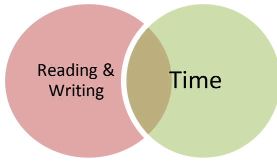
LEVEL A.2

SEQUENCE DAILY AND WEEKLY EVENTS OR STAGES IN A STORY

TEACHING NOTES



Sequencing activities in Mathematics naturally lend themselves to objectives, skills and concepts from the History curriculum in both Infants and First and Second classes.



There is a natural opportunity for integration with the Reading and Writing strands of the English curriculum at this level.

SAMPLE LEARNING EXPERIENCES

Time on a Line [273]

Brainstorm with the pupils all the things that they do in a school day, for example, reading, play, maths, break time, lunchtime, writing, home time. Ask the pupils to draw a picture of something that they do every day at school. Work with the pupils to write captions for their pictures. Bring the pupils together to peg their events on a clothes line. As each pupil pegs their picture to the line ask them to explain where it goes. If more than one pupil has drawn the same event tape them together. When everyone has pegged their picture to the line discuss the order of events and ask them to decide where new events belong.

Where does your picture go? What happens before your picture?
Where should I put playtime?
Where should I put your parents collecting you?
Which things happen in the morning?
Which things happen in the afternoon? So what happens first/next/last on our timeline?

My Day [274]

Distribute strips of paper divided into 5 sections to the pupils. Ask the pupils to tell you about the first thing they do when they wake up. Get them to draw the first thing that they do on the first segment of the strip. Share the drawings.

Ask the pupils to think about the last thing that they do each day (In bed asleep). Draw this on the last segment. Now ask the pupils to think about the other things that they do during the day. In pairs, pupils share ideas of things they do at different times of the day and fill in other pictures on their day chart. Pupils join the ends of the strip to make a wrap-around-book. Pupils share their "My day books".

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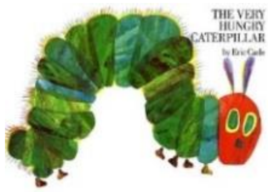
[273] http://nzmaths.co.nz/resource/passing-time
[274] http://nzmaths.co.nz/resource/passing-time

ICT
Sequencing Times of Day

Sequencing Times of Day [275]

The vocabulary associated with various times of day such as ***before, after, earlier, later, morning, afternoon, evening*** etc. can be developed through sequencing activities. The activity at the link opposite will support the acquisition and application of this vocabulary.

Sequencing Events in a Story



There are limitless possibilities for sequencing events in stories. Lots of picture books lend themselves to activities involving ordering events. One such example is *The Very Hungry Caterpillar* [276]. This book not only provides opportunities for sequencing and retelling, it also explores the days of the week and provides plenty of counting opportunities requiring one-to-one correspondence. As children engage in sequencing activities, use of language is important: *first, last, second, third, next, before, after that, then.* This language should be elicited from the pupils through teacher questioning during activities. If the vocabulary is not forthcoming, then it should be modelled by the teacher.

What days of the week do you come to school? Why do you not come to school on Saturday or Sunday? What special name do we have for Saturday and Sunday? What do you do at the weekend?
What did the caterpillar eat on Monday/Tuesday…? What day will come next? How many pears/plums … did he eat? Let's count them. Did he eat more on Friday or Thursday? On what day did he eat the most? What happened then? What happened on Sunday?
Which picture did you put ***first****? What happened* ***next****? And* ***after*** *that? And* ***then*** *what happened? What day was that? Retell the story to your partner*

Sequencing cards for *The Very Hungry Caterpillar* can be downloaded from this link.[277]

ICT
The Very Hungry Caterpillar Sequencing Cards

The following stories also provide ample stimulus for sequencing and ordering activities *We're Going on a Bear Hunt, Dear Zoo, Goldilocks and the Three Bears, The Gingerbread Man, The Three Little Pigs, Farmer Duck, Little Red Riding Hood, The Little Red Hen, Chicken Licken.*

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275 http://nrich.maths.org/6609/note
276 Carle, E., (1979). The very hungry caterpillar.
277 http://www.dltk-teach.com/t.asp?b=m&t=http://www.dltk-teach.com/books/hungrycaterpillar/csequencing.gif

Procedural Writing

Procedural writing in Literacy naturally lends itself to sequencing. Procedural texts list a sequence of actions or steps needed to make or do something. Typical examples of procedural texts include recipes, science experiments, assembly manuals or instructions for playing games. Involving pupils in activities like baking bread, making lemonade, cooking pancakes etc. and sequencing the procedure using pictures afterwards affords pupils the opportunity to apply their sequencing skills and to use appropriate mathematical vocabulary in a variety of contexts.

Sequencing Days of the Week [278]

This activity works best if it can be developed over a week, taking a couple of minutes a day. Prepare paper strips with the days of the week written on each strip. Use 2 colours, one colour for weekdays and another colour for the weekend. On Monday give each pupil a strip of paper with Monday written on it. Join the ends of the paper strip to form a circle. Ask the pupils to tell you events that happen on Monday – list these on a chart.

On Monday I ….go to ballet, visit Grandpa etc. On Tuesday repeat the process, adding the Tuesday loop to Monday's loop. Record events that happen on Tuesday on the class chart. Repeat this for Wednesday, Thursday and Friday.

On Friday ask the pupils: *On what day does the chain start? How many days are in the chain? Can you say them? What day was it yesterday? What day is it today? What day is it tomorrow?* Add loops for Saturday and Sunday. In the following weeks loops can be added to a class chain to develop the idea of repetition of days of the week.

Days of the Week Snap [279]

This card game provides opportunities for pupils to sequence the days of the week.

ICT
[Days of the Week Snap]

Weekly Class Timetable

Use of a weekly class timetable which is displayed and referred to on a daily basis helps to reinforce the sequence of days of the week and allows pupils to associate special events with particular days. If there is a school rota for areas of the yard, for example, these can be included on the timetable. *What day is today? On Mondays, where do Senior Infants play in the yard? That's right. On Mondays you play on the equipment. What days do you play in the field?* Further examples might include P.E days, Art days, Assembly, School Library, Computers, Best Class Line award etc.

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[278] http://nzmaths.co.nz/resource/passing-time
[279] http://nrich.maths.org/6082/note

LEVEL A.3

READ TIME IN ONE-HOUR INTERVALS

TEACHING NOTES

Learning to tell the time has little to do with time measurement and more to do with the skills of learning to read a dial-type instrument. Clock reading can be a difficult skill to teach (Van De Walle et al, 2010).

SAMPLE LEARNING EXPERIENCES

One-handed Clocks [280]

Begin with a one-handed clock. A clock with only an hour hand can be read with reasonable accuracy. Place the hand exactly on the hour, but also between the hours. Use lots of approximate language. *It's about 7 o'clock. It's a little past 9 o'clock. It's halfway between 2 o'clock and 3 o'clock.* Next, introduce the minute hand. Discuss what happens to the big hand as the little hand goes from one hour to the next. When the big hand is at 12, the hour hand is pointing exactly to a number. *If the hour hand is about halfway between numbers, about where would the minute hand be? If the hour hand is a little past or before an hour (10 to 15 minutes), about where would the minute hand be?* Use two real clocks, with only an hour hand and one with two hands. (Break off the minute hand from an old clock.) Cover the two-handed clock. Periodically during the day, draw the pupils' attention to the one-handed clock. Discuss the time in approximate language. Ask pupils to predict where the minute hand should be. Reveal the other clock and check.

Making Clocks [281]

In this activity the pupils create their own clocks using paper plates, cardboard hands and split pins. They then use the clock to show times during the school day. Ask pupils what they know about clocks. Look at a variety of analogue clocks and discuss their features (a large and a small hand fixed at the centre, digits 1 to 12). Discuss ideas for positioning the numbers evenly around the clock. *How are the numbers arranged? Is there the same amount of space between each number? How could we make sure our numbers are spaced evenly around our clocks?* Construct clocks fixing the hands in place with a split pin. Now use the clocks to show the hour times. Throughout the day ask the pupils to change their clocks to show the "real" time. Do this several times on the hour.

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[280] John A. Van De Walle et al, Elementary and Middle School Mathematics Teaching Developmentally, 7th Edition, 2010 p. 384

[281] http://nzmaths.co.nz/resource/how-long-now

The Best Times of the Day[282]

Ask pupils to draw pictures of different clocks they see at home, in school, outside. This could be done as a homework activity. Let the pupils share the pictures that they have drawn. Discuss the different types of clocks, for example, watches, clock radios, clocks on appliances, Grandfather clocks, novelty clocks, alarm clocks. Discuss why most of us have so many clocks and when it is important to know the time - so we won't be late for school, football, music lessons etc., so we won't miss our favourite TV programme, so we know when our food is cooked, so we don't sleep in

Ask the pupils to show their favourite time of the day on their paper clocks. Get the pupils to draw a picture of their favourite time. The pictures should include a clock showing the time. Share and display pictures.

Assessment

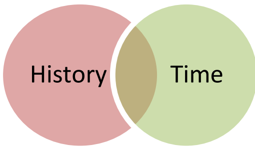
[282] http://nzmaths.co.nz/resource/how-long-now)

MEASURES - TIME

LEVEL B.1

USE THE VOCABULARY OF TIME TO SEQUENCE EVENTS

TEACHING NOTES



The work carried out in levels A.1 and A.2 may be repeated at this stage. This level lends itself to natural integration with many of the skills, concepts and objectives from the 1st and 2nd class History curriculum.

SAMPLE LEARNING EXPERIENCES

Class Calendar

Early in the school year, it is worth filling in a basic class calendar with just the months of the year on each page. A flip-chart would be ideal for this. Begin with September and record the important events for each month. Include individual pupil's birthdays, school events, holidays (Hallowe'en, Christmas, New Year, Valentine's Day, St. Patrick's Day, Easter, First Communion, Mothers'/Fathers' Day, school trips, (photos and pictures can be used). Colour-code the months to correspond to the seasons (orange for autumn months, blue for winter months, green for spring months and yellow for summer months). Display the page for the current month and discuss the events documented, as they arise.

We're going to fill in a class calendar for the year. Can anyone tell me what month it is? Let's write it on this page. How many months are there in a year? Can you say the months of the year? Let's fill in the rest of the months on each page. So what month will come after/before September/October…? What season does December fall in?

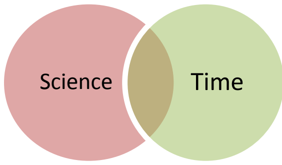
Assessment

MEASURES - TIME

LEVEL B.2

READ AND RECORD TIME USING SIMPLE DEVICES

TEACHING NOTES



The time comparison and duration activities carried out in levels A.1 may be repeated at this stage. This level provides natural opportunities to address many of the skills from the Science curriculum.

SAMPLE LEARNING EXPERIENCES

Candle Clock Activities

A candle clock is a candle marked with regular time intervals. Candles burn at a constant rate so as the candle melts, you can calculate how much time has elapsed. The thicker the candle, the longer it will burn, so to measure shorter time periods, use a thinner candle. To calibrate a candle, take two identical candles and a stop watch. Line up the candles exactly. Light the first candle for one minute. Mark the second candle at the level of the first candle with a permanent marker. Continue timing one-minute intervals and marking the second candle each time. You can then measure the distance between each minute and mark the remainder on the second candle. Write numbers at each marking to identify each minute that passes. Involve pupils in calibrating a candle to make a candle clock. The candle clock can then be used, with teacher supervision, to record the time taken to complete various class activities. Encourage pupils to predict how much time will pass before carrying out the activity. Some suitable activities might include:

- Calling the roll
- Shared reading of a book
- Tidying up
- Lining up
- Eating lunch

Record the time as measured on the candle clock and compare the results to the pupils' predictions. A candle clock can also be used as a timer. Place the candle on a hard surface, preferably metal or ceramic. Drill a small nail into each marking on the candle. As the candle burns down to each marking, the nail will drop to the surface below, drawing attention with the accompanying noise.

There are further candle clock activities in Level C.1 of this handbook.

Sand Timer Activities Similar activities to those with the candle clock can be carried out with sand timers. Again, encouraging pupils to estimate and predict the duration of the activity is important for developing their sense of time passing.

LEVEL B.3

READ, RECORD AND CALCULATE TIME IN HOURS, HALF-HOURS AND QUARTER HOURS ON 12-HOUR ANALOGUE CLOCK AND ON DIGITAL CLOCK

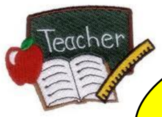
TEACHING NOTES

The work carried out in level A.3 may be repeated at this stage.

SAMPLE LEARNING EXPERIENCES

Clockwise [283]

Begin by brainstorming the words we use to talk about time and listing these. For example, *20 past/to, quarter or half past/to, o'clock, four fifty, hours, minutes and seconds*. Explain to the pupils that there are several different ways to describe time and today they are going to explore two of these. Show an analogue clock and discuss the face, drawing out the pupils' experiences with clocks and time.



What can you tell me about the face?
What numbers are there? Why (is there nothing above 12)?
How are the numbers arranged? Why?
Tell me about the clocks that you have at home?
What is the biggest clock you have ever seen?
When do you get up?

Have each pupil construct an analogue clock. Draw around an object to make a circle of light card or give each pupil a cut circle of card. Fold the circle in half and colour each half a different colour – this helps reinforce the concept of half. Make a big and a small hand and attach these with a split pin. Draw numbers onto the clock-face.

Exploring Analogue Time [284]

As a group, discuss times of the day when well-known events happen and have the pupils model these times on the analogue clocks they have made. At first focus on hours, for example, waking up at 8 o'clock, morning break at 11 o'clock, school finishing at 3 o'clock. Then move on to half-hours, for example, half past 12 is lunch time, half past 8 is bed-time etc. As pupils work at modelling times, focus on the position of the hands (big hand shows minutes and little hand shows hours) and the terms *half past* and *o'clock.* Pupils can record some of their favourite times of the day and draw clocks showing those times.

It may help young children to have a clock face with both the numbers for the hours (say in black for the hour hand) and those for the minutes (say in red for the minute hand) (Suggate et al, 2010 p.222).

[283] http://nzmaths.co.nz/resource/clock-wise
[284] http://nzmaths.co.nz/resource/clock-wise

Analogue Time Problems [285]

Pose several problems involving time (to hours, half-hours and quarter hours as required). Pupils work with a buddy and their analogue clock to solve them and record their answers. As a group, discuss the problems and their solutions.

Possible problems include:

- Sally went swimming at 4 o'clock and got out of the pool at half past five. How long did she swim for?
- If it takes half an hour to walk to school from home, how long will it take to walk to school and back home again?
- Laura had an appointment at the dentist at half past three. She didn't arrive until 4 o'clock. How late was she?
- The movie started at 6 o'clock and finished at half past seven. What was the running time of the movie?
- Mark went to bed at 8 o'clock and woke up 11 hours later. What time did he wake up?
- Muffins take half an hour to cook. If a batch of muffins goes into the oven at half past five what time will they be ready?

How did you solve the problem? Sophie, do you agree with Darragh's answer? Aoibhe, can you revoice what Saoirse said? Did anyone solve the problem in another way? Record your answer on a blank clock. This evening, fill in what you know about clocks in your learning log for homework. Include a picture of your favourite time of day.

Clock it [286]

Provide instructions to pupils to make a clock.

Fold a circle in half and then in half again. Now try to fold it so there are 12 even spaces around the circle. Put the numbers 1 to 12 evenly around the edge of your circle. Cut two hands (one long and one short). Attach these hands in the centre of your clock with a split pin.

Following the instructions to fold a circle into 12 equal parts is valuable in that it reinforces the need for the numbers on a clock to be evenly spaced. Some card is difficult to flatten after folding, so pupils may need to stick the circle to firm card in order to make a clock they can use. A hole-punch helps to make holes for the split pin in both the hands and the clock face.

[285] http://nzmaths.co.nz/resource/clock-wise
[286] http://nzmaths.co.nz/resource/clock-it)

Using their clocks, ask pupils to show 5 past, 10 past, 15 past. At 15 past, stop and ask the pupils, *Does anyone know another way of saying '15 past'? Why do we say 'quarter past'? So how many minutes are in a quarter of an hour?*

Ask pupils to shade in the four different quarter segments on their clocks using four different colours.

Continue showing 20 past, 25 past, 30 past. Stop again and ask, *what's another way of saying '30 past'? Why do we say 'half past'? How many quarters are in a half? How many minutes are in two quarters of an hour?*

Once 30 minutes (half past) is reached, pupils must learn to count down to the hour, initially by fives. So the count progresses: half past 1, 25 to 2, 20 to 2, quarter to 2….

ICT
Interactive Analogue Clock

The clock at this link [287] can be used with the whole class to consolidate their clock reading skills. It can also be viewed in digital format.

Telling the Time [288]

ICT
Telling the Time

The interactive problem-solving activities at the link opposite provides an enjoyable and engaging context for applying and refining clock-reading skills. They also challenge pupils to perform calculations with time. The levels can be adjusted to suit abilities.

Hickory Clock [289]

ICT
Hickory Clock

A further clock-reading game involving the hour, half-hour and quarter hour can be accessed at this link.

Digital Time [290]

Have the pupils carefully examine a digital alarm clock and count the number of minutes in each hour. Establish that there are 60 minutes in each hour, and therefore 30 minutes in half an hour and 15 minutes in the quarter hour. This is also a good opportunity to practice skip-counting with fives. Establish with the pupils that the first number of the digital clock shows the hour and the second number shows the number of minutes past that hour. Challenge the pupils by asking them how they think the digital clock would show times that they are familiar with on an analogue clock for example eleven o'clock (break time), and half past twelve (lunch time). Reinforce the fact that half an hour is thirty minutes, therefore digital times ending in 00 are the same as *o'clock* and times ending in 30, 15 and 45 are the same as *half past, quarter past* and *quarter to* respectively.

[287] http://resources.oswego.org/games/ClassClock/clockres.html
[288] http://www.bbc.co.uk/bitesize/ks1/maths/telling_the_time/play/popup.shtml
[289] http://www.ictgames.com/hickory4.html
[290] http://nzmaths.co.nz/resource/clock-wise

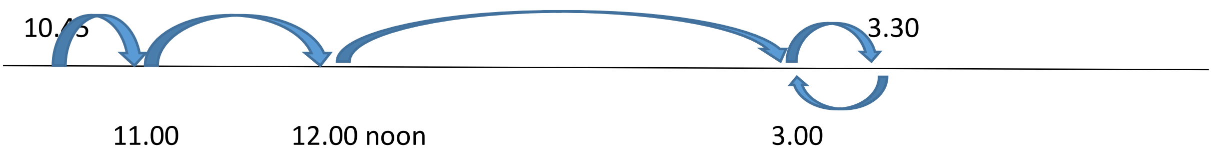
What do you know about digital clocks? Where have you seen or used a digital clock? How are they different from analogue clocks? How are they the same? Is the way we read the digital clock different from how we read the analogue clock? How would you say 'ten past 2' in digital format? Let's use our digital clock to count minutes. How many minutes did we count? 59, well done. What happened after we reached 59? Why? So how many minutes in an hour? How many minutes in half an hour? How would that be shown on a digital clock?

List some of the pupils' favourite TV programmes and have them record the digital times these programmes come on. Look at a TV guide to confirm the times that they have recorded.

Empty Number Lines

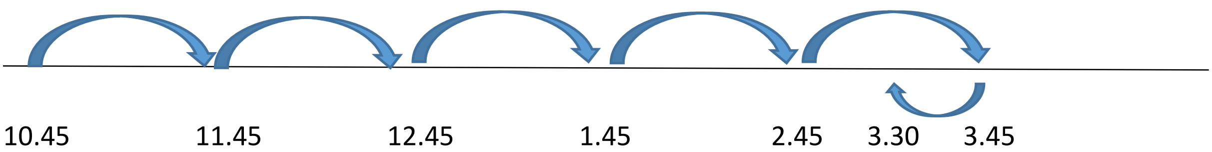
Use of an empty number line as a strategy for solving elapsed time problems can be very effective. Suggest that pupils sketch an empty line to help solve the problem. It is important not to be overly prescriptive in telling pupils how to use the time line since there are various alternatives as to how it may be used. See example below.

If school began at 10.45 am and finished at 3.30 pm, how much time would you have spent in school? [291]



'15 minutes to 11 am. Then 4 hours until 3 pm and 30 minutes until 3.30 pm. That's 4 hours 45 minutes in total'

OR

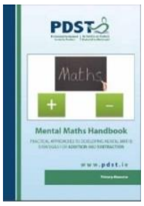


'10.45, 11.45, 12.45, 1.45, 2.45, 3.45.

That's 5 hours. And go back 15 minutes to 3.30. That's 4 hours 45 minutes in total.' Provide pupils with the opportunity to share their empty number lines with the class. 'Show-me' boards are a useful resource for this.

[291] John A. Van De Walle et al, Elementary and Middle School Mathematics Teaching Developmentally, 7th Edition, 2010 p. 385

The PDST Mental Maths Handbook has further information on the use of empty number lines. Click the image to access this handbook[292].



Mixed Time[293]

Divide the class into two groups. One of these is the analogue group and the other the digital group. Give each group an appropriate clock. Let the analogue group choose a time (involving hours or half-hours). The digital group shows that time on their clock. Then the digital group choose a time and ask the analogue group to show it on their clock. Repeat this a few times. The groups make up their own problems like these and attempt solutions. Pupils draw their answers using drawings of both an analogue clock and a digital clock. Ask the pupils to tell you the main times of the day (when school starts, play time, lunchtime, etc.). Get pupils to draw these in both analogue and digital formats.

Linking Analogue & Digital [294]

The link opposite provides opportunities for pupils to consolidate their understanding of analogue and digital time.

ICT
Linking Analogue & Digital

[292] http://www.pdst.ie/sites/default/files/Mental%20Maths%20Workshop%201%20Handbook.pdf
[293] http://nzmaths.co.nz/resource/clock-wise
[294] http://www.teachingtime.co.uk/draggames/sthec2.html

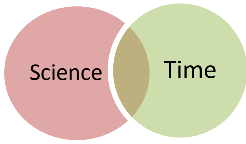
LEVEL B.4

READ DAY, DATE AND MONTH USING CALENDAR AND IDENTIFY THE SEASON

TEACHING NOTES

The Class Calendar activity in Level B.1 can be revisited here with a particular focus on the seasons in which the months fall. See also the activities in Level C.5.

> Learning experiences involving the calendar provide ideal opportunities to revise and use ordinal number. Pupils should be encouraged to read dates aloud and record dates in written form using 1^{st}, 2^{nd}…30^{th}, 31^{st}.



Investigating the seasons through calendar activities allows for natural integration with the Plants and Animals strand unit of the Science curriculum at 1^{st} and 2^{nd} class levels. Videos highlighting the changes in nature as each season evolves can accessed at this link:

Seasons

There can often be confusion regarding which months fall in which season. Is the first day of spring February 1^{st} or March 1^{st}? It is agreed that each season last three months. In Ireland, for meteorological purposes[295], the seasons are categorised according to air temperature. So spring officially commences on March 1^{st}.

> For Climatological purposes, on the basis of air temperature, seasons are regarded as three-month periods as follows: December to February - winter, March to May - spring, June to August- summer and September to November - autumn. This is a common grouping in the meteorological practice of many countries in the middle and northern latitudes.

SAMPLE LEARNING EXPERIENCES

Introducing the Calendar [296]

1^{st}	
2^{nd}	
3^{rd}	
4^{th}	March
5^{th}	
6^{th}	
7^{th}	
8th	

A simple calendar, as illustrated, is a good way to first introduce pupils to the calendar before the more common arrangement in weeks is used. Referring to this calendar daily allows valuable opportunities to link with ordinal number.

[295] www.met.ie
[296] Deboys & Pitt, Lines of Development in Primary Maths 1979 p. 86

Patterns of 7s in Calendars [297]

Using calendars (with both a horizontal and vertical layout) discuss a calendar month with the pupils. Emphasise the pattern of repeated days. Record the dates of consecutive Sundays. Repeat the activity for other days of the week encouraging pupils to spot the pattern of 7s.

Days in the Calendar Month [298]

Pupils need to become aware that while there is a standard length for weeks, i.e. 7 days, there is no standard length for our calendar months.

To establish the length of each month, examine a calendar and focus on the number of days in the following: months when birthdays occur, Christmas/Hallowe'en/Summer months etc.

Days in the Months Rhyme

To assist pupils in recalling the number of days in each month, the following rhyme could be useful:

30 days has September,
April, June and November,
All the rest have 31,
Except for February all alone,
Which has 28 days clear,
And 29 in each leap year.

Calendar Challenge [299]

A variety of questions based on the calendar in both horizontal and vertical format can be generated to provide opportunities for pupils to engage with the calendar.

June						
Sun	Mon	Tues	Wed	Thur	Fri	Sat
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

June					
Sun		4	11	18	25
Mon		5	12	19	26
Tues		6	13	20	27
Wed		7	14	21	28
Thur	1	8	15	22	29
Fri	2	9	16	23	30
Sat	3	10	17	24	

[297] Deboys & Pitt, Lines of Development in Primary Maths 1979 p. 87
[298] Deboys & Pitt, Lines of Development in Primary Maths 1979 p. 182
[299] Deboys & Pitt, Lines of Development in Primary Maths 1979 p. 183

How many days in the month of June?
Tom's birthday is on the 10th of June. What day is that?
Which date is 6 days after June 17th?
What day of the week is this?
Give the date of the last Monday in June.
Give the dates of all the Thursdays in June. Do you notice anything about these numbers?
Count forwards/backwards one week from the following dates....

Calendar Templates [300]

A variety of calendar templates and resources can be downloaded from the link opposite

ICT
Calendar Templates

Calendar Trail [301]

Using a yearly calendar for a given year, pupils can work in pairs to complete the following grid:

Month	**No. of days**	**No. of Sundays**	**No. of Wednesdays**	**No. of completed weeks**	**Season**
January	31				
February					
March					
April	30			3	
				4	
June					
July					
August					
	30				
			4		
November					
December	31				

Visiting Grandma [302]

Jane is going to see her Grandma in 6 days from now. If today is 23rd April what day will she see her Grandma?

Using a calendar ask pupils to identify the current day and date. Give each of the pupils (or pair of pupils) a copy of the calendar for April (or month to suit). Ask them to point to certain dates to reinforce their use of the calendar. Using their calendars, pupils could work in pairs to solve Jane's problem.

300 http://www.abcteach.com/directory/teaching-extras-calendars-903-2-1
301 Deboys & Pitt, Lines of Development in Primary Maths 1979 p. 84

302 http://nzmaths.co.nz/resource/visiting-grandma

How many days are there in April? What day is the 23rd? Show me how you found that out. How many Mondays are there in April? Lucy, how did you work out your answer? Isabelle, can you revoice that? Did anyone solve the problem in another way?

Extension Activities

Jane is going to see her Grandma in 8 days from now. If today is Monday 23rd April what day will she see her Grandma?

Variation 1: *Jane is going to see her Grandma in 7 days from now. If Jane sees her Grandma on 18th April what day is it now?*

Variation 2: *Jane is going to see her Grandma on 26th April. If today is 23rd April, how many sleeps is it before she will she see her Grandma?*

Calendar Patterns [303] and Calendar Sorting [304]

These investigations from Nrich.org provides opportunities for pupils to explore number patterns in the calendar.

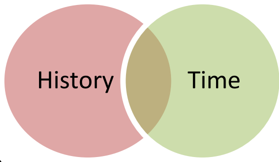
ICT
Calendar Patterns

ICT
Calendar Sorting

[303] http://nrich.maths.org/164
[304] http://nrich.maths.org/10322

LEVEL C.1

CONSOLIDATE AND DEVELOP FURTHER A SENSE OF TIME PASSING



TEACHING NOTES

The work carried out in levels A.1, A.2 and B.1may be repeated at this stage. Natural integration possibilities with the History curriculum at 3rd and 4th class exist at this level. The skills and concepts associated with time and chronology and objectives from a number of strand units allow for meaningful connections to be made with Time in Mathematics.

SAMPLE LEARNING EXPERIENCES

Personal Timelines [305]

Sequencing events from the pupils' lives on a timeline is meaningful and provides opportunities to further develop a sense of time passing. Discuss the dates of important events in their lives such as their birth, birth of younger siblings, starting play school, starting school, First Communion, moving house, or significant holidays or trips. Pupils can draw pictures or use photographs to capture these events and plot these in order on a timeline, outlining the years and months in which they occurred. Pupils should try to plot the events allowing for the elapsed time in between each event, allocating different space between these events accordingly. Provide opportunities to share their timelines with each other, encouraging use of sequential language *first, then, next, before, after, finally, now.*

Sequencing Events in Stories

Ordering the chronology of events from Irish myths and legends and the lives of historical figures, provides an ideal opportunity for integration with History. Tales such as *The Salmon of Knowledge, The Children of Lir* and the lives of St. Patrick/St Brigid allow pupils to develop both mathematical and historical skills and concepts.

History Time Lines [306]

There are further opportunities for integration with History through the use of historical timelines where pupils can order historical events according to chronology. An excellent resource which provides an overview of the history of Ireland since the Stone Age can be found at the link opposite.

ICT
[History Timelines]

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[305] John A. Van De Walle et al, Elementary and Middle School Mathematics Teaching Developmentally, 7th Edition, 2010 p. 480

[306] http://www.askaboutireland.ie/learning-zone/primary-students/3rd-+-4th-class/history/history-the-full-story/history-timeline/

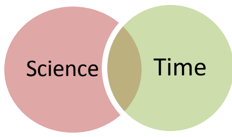
Minute Timers [307]

In this activity pupils develop an understanding of the duration of a minute by making their own salt timers. Begin with the pupils standing. Ask them to sit down when they think a minute has passed.

How did you decide when to sit down? What could we use to check?

Look at a minute passing using the second hand of an analogue clock/ a cooking clock timer/watching the digits change on a digital clock. Show the pupils the resources (empty 250 ml drink bottles - cut the base off the bottle and pierce a small hole in the bottle top, dry sand or salt and paper cups). Ask the pupils for their ideas about what they could be making. Tell them that they are going to work in pairs to make their own minute timer. Demonstrate how putting sand or salt in the upside down drink bottle allows the salt to "drip" into the empty paper cup. *How could we use this to measure a minute?* Let the pupils work in pairs to make their own minute timer. Share and check the timers.

Just a Minute [308]



In this activity the pupils explore all the different activities that can be completed in a minute. Begin by asking the pupils how many times they think they could write their name in a minute. Share some estimates. Get two volunteers to write their names on the board while the rest measure a minute using one of the sand timers constructed in the previous activity. When the minute is up count the number of names. Brainstorm together for other things that they could try to complete in a minute - counting by ones, joining multi-link cubes, hopping, heart beats, eye blinks, jumping jacks etc. List these on the board for the pupils to refer to. Let the pupils work in pairs doing the things on the list, using their salt timers to measure a minute each time. Ask them to make an estimate before each activity and to keep a record of what could be achieved in a minute for each activity. The results of the activities could be recorded in the pupils' Maths journals.

The following activities provide a natural opportunity to develop skills from the Science curriculum at 3rd and 4th class level as pupils engage in the designing and making of various time measurement devices.

[307] http://nzmaths.co.nz/resource/just-minute
[308] http://nzmaths.co.nz/resource/just-minute

Calibrating Clocks [309]

Brainstorm with pupils the different kinds of clocks they know: watches, wall clocks, oven timers etc. Explain that they will be making and experimenting with a variety of different clocks.

Candle Clocks

Show the pupils a candle. Ask them to estimate how long it would take for the whole candle to melt. How much would be melted in 5 minutes? Group or pair pupils and give each group a candle. Pupils record the length of their candle, either by drawing a line to show how long it is, or measuring it in cm. They also draw a mark on the candle to show how much they think will be melted in 5 minutes.

Collect candles together (maybe in a dish of sand so they stand up easily) and burn all candles for five minutes. Pupils observe and watch to see the accuracy of their five minute marks. Blow out the candles after five minutes and return them to the pupils for them to check the accuracy of their mark and record how long the candle is now (either by drawing a line to show how long it is, or measuring it in cm).

Pupils put three more marks on their candles: showing how much they think will be melted in 5, 10 and 15 minutes. Collect all candles together and burn for 15 minutes, observing the accuracy of the marks as they burn. At the conclusion of the activity return candles to pupils and discuss:

How accurate was your candle clock?
How could you make your candle clock more accurate?
What are the limitations of a candle clock?

Water Clocks [310]

Note: Due to the nature of the tools used, this activity should be led by the teacher.

To make a water clock you will need an empty 2 litre plastic soft drink bottle, a narrow-headed screwdriver or some other pointed-headed tool and a sharp scissors.

Cut the bottle in half using the scissors. The top should ideally be only slightly shorter than the bottom. Use the screwdriver to puncture the lid, creating a small hole with a diameter of roughly 1/8 inch (3.175 mm). The hole should not be much larger than this, since a bottle with a hole that is too large will lose water too fast to be effective as a water clock. (Alternatively, if you do not have a tool that can create a hole in the hard plastic lid, you can use a large plastic cup instead of the top part of the bottle. Pierce the bottom of the cup with an embroidery needle or sewing needle to create a small hole.)

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[309] http://nzmaths.co.nz/resource/calibrating-clocks

[310] http://www.wikihow.com/Make-a-Water-Clock-%28Clepsydra%29

Invert the upper part inside the lower part. Flip the top part of the bottle over so that the cap faces down. Slip this top part into the bottom portion of the bottle so that the cap rests 4 inches (10 cm) or so above the bottom of the lower half. Make sure that the cap does not touch the bottom of the bottle. The higher up it is, the better, as long as you can create a snug fit. If using a plastic cup instead of the top portion of the bottle, rest the cup inside the lower half of the bottle with the bottom of the cup facing down. Pour water into your water clock and start timing the flow of water using the timer on a mobile phone or some other stop watch. Use a permanent marker to mark, on the lower half of the bottle, how much water falls within one minute. Repeat this for the second minute, third minute, and so on, until all the water originally poured into the upper half has dripped out. Depending on the speed of the flow of the water, you may prefer to calibrate the clock in increments of 30 seconds instead.

Sand Timers [311]

To make a sand timer you will need pairs of plastic bottles with the same size openings and same capacity (soft drink bottles, water bottles), small squares of cardboard, a hole punch, sand or salt and rice and masking tape. Mix 2 cups of rice with 5 cups of sand. Pour the cups into one of the bottles. Add a tablespoon of glitter if desired. (Use less sand if your bottles are smaller). Use the bottle neck to trace a circle onto the cardboard so it is the exact circumference of the opening. Cut it out and use a standard hole punch to punch a hole in the middle of the cardboard circle. Tape the circle over the opening of one of the bottles using the masking tape, but don’t cover the hole you punched in the cardboard. Place the bottles one on top of the other, neck to neck, with the cardboard piece in between and use the tape to firmly attach them. Ask pupils to estimate how long it will take all the sand to drain through. Use a timer to record how long it takes. Repeat this again but beforehand ask pupils to mark on their bottles where they think the 30 second/1 minute mark will be. Mark the exact spot afterwards with a permanent marker. If further 30 second or minute intervals can be measured, repeat the activity until these have all been identified. Again, encourage pupils to estimate the markings in advance.

Pendulums

To make pendulums, you will need string of various lengths and a variety of weights. Attribute bears, threading beads, unifix cubes, erasers, wine corks, small jewellery boxes would all work well as weights as they can easily be attached to the string and vary in weight. Attach a weight to a piece of string and start the pendulum swinging to measure time. To help your pendulum measure the same each time you use it, you will need to make sure you hold the pendulum in the same place each time you release it. Make sure you let the weight go rather than push it and count the number of swings carefully from the highest place in the swing each time. Encourage pupils to estimate the time it will take for the pendulum to swing 5, 10, 15 and 20 times and to record their estimates on a table. Have pupils measure how long each set of swings takes using a timer and record these on their table.

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[311] http://www.wikihow.com/Make-a-Sand-Timer-from-Recycled-Plastic-Bottles

Number of swings	Estimated time taken	Measured time taken
5		
10		
15		
20		

Provide time for pupils to experiment with their pendulums and the way they measure time.

What happens if you make the string longer? What happens if you make the string shorter? What happens if you use a heavier weight? What happens if you use a lighter weight? Can you make your pendulum measure 1 minute accurately? Can you make your pendulum measure 5 minutes accurately?

Comparing Clocks [312]

In this activity pupils will use a variety of timing devices to estimate and measure the time taken for the one activity and then compare the results. Distribute timing devices amongst pairs of pupils. Use a variety of devices including stop watches, clocks, watches, candle clocks and the water clocks, sand timers and pendulums designed in the previous activities. In pairs, pupils will take turns either timing the activity or carrying out the activity. Pupils can record equivalent measures on a chart.

Task	Estimate	Equivalent measures for time taken
Recite the alphabet	_______ seconds ________candle marks _________water clock marks ________sand clock marks _________pendulum swings	_______ seconds ________candle marks _________water clock marks ________sand clock marks _________pendulum swings
Count to 500	_______ seconds ________candle marks _________water clock marks ________sand clock marks _________pendulum swings	_______ seconds ________candle marks _________water clock marks ________sand clock marks _________pendulum swings

312 http://nzmaths.co.nz/resource/calibrating-clocks

LEVEL C.2

READ AND RECORD TIME IN FIVE-MINUTE AND ONE-MINUTE INTERVALS ON ANALOGUE AND DIGITAL CLOCK (12-HOUR) AND RENAME DIGITAL TIME AS ANALOGUE TIME AND VICE VERSA

TEACHING NOTES

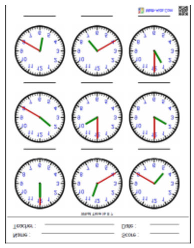
The work carried out in level A.3 and B.3 may be repeated at this stage.

> The move to using minutes as well as hours needs practice. It is probably best to concentrate on telling the time to the nearest 5 minutes at first, but there are some difficulties, especially as there are basically two ways of stating the time. The traditional way associated with the analogue clock face is, for example, 25 past 4 (i.e. 25 minutes after 4 o'clock) or 25 to 10 (i.e.25 minutes before 10 o'clock). The digital notation is entirely numbers. The number before the dot to the minutes, so 4.25 (read as four twenty five) is the same as 25 past 4. However, 25 to 10 would be 9.35 (i.e. 35 minutes after 9). To understand this equivalence, children need to know that there are 60 minutes in an hour.
> (Suggate et al, 2010, p.222).

SAMPLE LEARNING EXPERIENCES

Make a Clock

Fold a circle in half and then in half again. Now try to fold it so there are 12 even spaces around the circle. Put the numbers 1 to 12 evenly around the edge of your circle. Cut two hands (one long and one short). Attach these hands in the centre of your clock with a split pin. Pupils will work with a partner to make a range of times on their clocks as set by the teacher and to compare their recorded times with their partner's. Include a combination of hour, half-hour, quarter past/to, five-minute times (5/10/20/25 past/to). Have pupils list various times and to challenge their partner to record these times on their clocks.



What's the Time?

Provide pupils with a worksheet containing clock faces showing various times. (A clock stamp is useful for generating the worksheet) or go to the ICT lin [313] for clock templates. Pupils identify the times recorded on the clocks.

Ten Green Bottles Problem[314]

This problem involves calculating time in 5 minute intervals.

ICT
Clock Templates

ICT
[Ten Green Bottles](http://nrich.maths.org/242)

313 http://www.math-aids.com/Time/
314 http://nrich.maths.org/242

MEASURES - TIME

Stop the Clock [315]

ICT
Stop the Clock

In this online game, pupils match the written time in words with an interactive clock face.

Minutes after the Hour [316]

Begin by looking at the minute hand on the analogue clock.

How long does it take the minute hand to get around the clock face once? Ask the pupils a number of questions about the time it takes for the minute hand to move between positions on the clock. Check by counting (in ones until the pupils notice that you can count in fives). Move the hands on the analogue clock to 20 minutes after 9. Ask: *What time is on the clock? How do you know? Can anyone tell me what 20 minutes after 9 would look like on the digital clock?* Repeat with other examples (initially limit this to minutes after the hour). List times on the board for pupils to draw clocks. Include both digital and analogue times. For example: Half past 3, 7:15, 25 minutes past 7, 2:40, 18 minutes past 10.

ICT
On Time

On Time [317]

This online game highlights the movement of the small hand on the clock as the big hand moves around. It requires accuracy in where the small hand is placed.

Wait a Second [318]

In this activity the pupils are encouraged to develop their own reference for a second, for example, a counting pattern "one – banana – two – banana – three etc".

Look at the second hand on a clock. *How long does it take for the second hand to travel once around the clock?* Ask for a couple of volunteers to stand at the front of the class (facing away from the clock). Ask them to put their hand up when they think 10 seconds has passed. Repeat with another two volunteers. Brainstorm ideas for estimating seconds. Let the pupils work in pairs to find their own method for estimating seconds. Ask them to record their method on a piece of paper for sharing. Share and check the accuracy of methods.

ICT
Flashing Lights

Flashing Lights [319]

This problem requires pupils to apply their knowledge of seconds and minutes.

Time Tools [320]

A variety of interactive games involving analogue, digital, a.m, p.m and 24-hour clock can be accessed via this link.

ICT
Time Tools

[315] http://www.teachingtime.co.uk/clock2/clockwordsres.html
[316] http://nzmaths.co.nz/resource/just-minute
[317] http://www.sheppardsoftware.com/mathgames/earlymath/on_time_game4.swf
[318] http://nzmaths.co.nz/resource/just-minute
[319] http://nrich.maths.org/1014
[320] http://splash.abc.net.au/res/i/L9642/index.html

Timing Activities [321]

Begin by posing the question. *How long does it take to write your full name on the board?* Ask for a volunteer to give their estimate. As the volunteer writes her/his name ask the rest of the class to use their checking methods from the 'Wait a Second' activity. Brainstorm a list of activities that will take a short time to complete (but longer than a minute). For example: writing the digits to 100, eating an apple, reading a picture book, writing down homework, Ask the pupils to work in pairs to complete some of the activities listed using digital watches to record the time taken. Encourage them to estimate the length of time each activity will take first. They are then to record the start and end time for each activity on both a digital and analogue display. Ask them also to record how many minutes the activity took.

Assessment

Activity	**Estimate**	**Start Analogue**	**Start Digital**	**End Analogue**	**End Digital**	**Time Taken**
Digits to 100						
Apple						
Reading						
Homework						

Two Clocks Problem [322]

This problem works best if pupils collaborate with others to solve it.

ICT
Two Clocks

Time & Fractions Problem [323]

This problem provides a good opportunity for pupils to link their fractions and time knowledge.

ICT
Time & Fractions

Wonky Watches [324]

Another ideal problem for pupils to engage in collaboratively.

Various strategies may be adopted and shared by pupils as they attempt to solve the problem.

ICT
Wonky Watches

Five on the Clock [325]

This investigation provides opportunities for pupils to link their numbers and time knowledge.

ICT
Five on the Clock

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321 http://nzmaths.co.nz/resource/just-minute
322 http://nrich.maths.org/4806
323 http://nrich.maths.org/1100
324 http://nrich.maths.org/1002
325 http://nrich.maths.org/1981

5:55 Investigation [326]

A problem that requires pupils to calculate how many minutes will pass before a digital clock next shows a time for which all the digits are the same.

Approaching Midnight[327]

This interactive game requires strategic thinking and has endless possibilities for extension.

Getting to the Office [328]

This problem will require reasoning and may work best if done collaboratively.

[326] http://nrich.maths.org/2533
[327] http://nrich.maths.org/10775
[328] http://nrich.maths.org/630

MEASURES - TIME

LEVEL C.3

RENAME MINUTES AS HOURS AND HOURS AS MINUTES

TEACHING NOTES

Pupils often struggle with time calculations because the base ten place value system is not at play when calculating durations of time. Providing frequent opportunities for pupils to develop a 'Bridging through 60' strategy will allow them to mentally calculate with hours and minutes.

SAMPLE LEARNING EXPERIENCES

A digital clock displaying 9:59 will, in two minutes time, read 10:01 not 9:61. When children use minutes and hours to calculate time intervals, they have to bridge through 60. So to find the time 20 minutes after 8:50am, for example, children might say 8:50 plus 10 minutes takes us to 9:00am, then add another 10 minutes (Crown, 2010 p. 45).

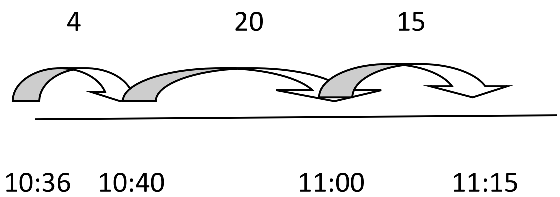
Mental Strategy: Bridging Through 60 to Calculate a Time Interval [329]

Bridging Through 60 with a Digital Clock [330]

10:36

Get the class to look at the digital clock at various times of the day and ask: *'How many minutes is it to the next hour (or next o'clock)'.* Encourage children to count on from 36 to 40, then to 50, then to 60, to give a total of 24 minutes. Then ask questions such as: *'How long will it be to 11:15?'* Get them to count on to 11:00 and then add on the extra 15 minutes. The calculation can be modelled on a number line labelled in hours and minutes.

Assessment



Some children may think that minutes on a digital clock behave like ordinary numbers, so that they might count on 59, 60, 61 and so on, not realising that at 60 the numbers revert to zero as the hour is reached. It helps if you draw attention to what happens to the clock soon after, say, 9:58 and stress the difference between this and other digital meters such as electricity meters or car odometer
(Crown, 2010 p.46).

[329] Crown, 2010, Teaching Children to Calculate Mentally.p.45
[330] Crown, 2010 Teaching Children to Calculate Mentally.p.45

Applying Bridging Through 60 in Problem Solving [331]

Give groups of pupils statements such as:

'Jane leaves home at 8:35am. She arrives at school at 9:10am. How long is her journey?'

Elicit from the pupils their methods of finding the answer, writing each on the board. Some may say: '8:35, 8:40, 8:50, 9:00, 9:10,' counting 5 and 10 and 10 and 10 to give the total time. Others may say '8:35 and 25 minutes takes us to 9:00, so add on another 10 minutes'.

> Children need to remember that, for minutes, they need to count up to 60 before getting to the next hour. Some children might be tempted to say 8:35, 8:40, 8:50, 8:60, and so on, expecting to go on until they get to 100. Referring to a clock face should help them to see why this is incorrect (Crown, 2010 p.46).

Local Bus & Train Timetables [332]

Use local bus and train timetables, asking questions such as, '*How long does the 8:30 train take to get to Dublin etc.?'* Encourage pupils to build the starting times up to the next hour, and then add on the remaining minutes.

Plan a Journey

Plan a journey using information from a bus or train timetable. Discuss the strategies that pupils use to find the times. Encourage them to use empty number lines to model their calculations. If pupils can mentally rename minutes as hours and hours as minutes, they are well-equipped to deal with pen and paper calculations involving time. For example:

Maths starts at 10.25 and lasts for 45 minutes. At what time does Maths finish?

$$\begin{array}{r} 10.25 \\ \underline{+\ \ 0.45} \end{array}$$

Pupils may reason out this problem in different ways. For example:

[331] Crown, 2010, Teaching Children to Calculate Mentally.p.46

[332] Crown, 2010 Teaching Children to Calculate Mentally.p.47

So how would you work out the answer to this problem? If we add 45 minutes and 25 minutes, how many minutes is that altogether? Well done. It's 70 minutes. So what would our answer be then? 10.70. Does that seem right? Why not? Can you rename 70 minutes? Don't forget how many minutes there are in an hour. Good. It's the same as 1 hour and 10 minutes. So where will we record the 10 minutes? And what about the hour? So what's our final answer? Well done. 11.10.

Did anyone think about this problem in a different way? Okay Darragh, you counted on. Can you explain that to us? So, you counted on from 10.25 until 11.00 and that was 35 minutes. What did you do then? Why did you add on another 10 minutes? Okay, because you had already added on 35 minutes and you needed to add on 10 more to add 45 minutes in total. So that got you to 11.10. Sophie, can you revoice Darragh's strategy that for us please?

Loop Game for Addition

To improve pupils' flexibility in renaming hours and minutes for addition a loop game can be useful and enjoyable. It can be replayed on numerous occasions to build pupils' mastery and consolidation of renaming. (See the resources in Appendix C for an example). Encourage different pupils to share the strategies they use to rename time during the game.

For problems involving subtraction, being able to bridge through 60 will also be useful:

School starts at 8.50. How much time passes before morning break at 10.35?

$$\begin{array}{r} 10.35 \\ -\ 8.50 \\ \hline \end{array}$$

So how would you work out the answer to this problem? Can we take 50 minutes from 35 minutes? Why not? So what will we do? Can you rename 10 hours 35 minutes so we have enough minutes? So it's the same as 9 hours 95 minutes. Now can we take 50 minutes away now? How many minutes will we have left? Wells done. 45 minutes. So now we take 8 hours from 9 hours and get one hour, yes? So our final answer is 1 hour 45 minutes.

Did anyone think about this problem in a different way? Did anyone count on to figure out the answer? Can you explain that to us? So, you counted on from 8.50 until 10.35? Could you show us that on an empty number line please? So 8.50 to 9.00 is 10 minutes. 9.00 to 10.00 is 1 hour. So that's 1 hour 10 minutes so far. And 10 to 10.35 is 35 minutes so if we add that to our 1 hour 10 minutes we get 1 hour 45 minutes?

When different strategies are suggested for solving the same problem, it is important that pupils are given adequate opportunities to share these strategies with each other and to critically evaluate the strategies of others. The teacher should facilitate the sharing of these strategies, supporting pupils to clarify their thinking and to support their expression through questioning. Recording the strategy on the whiteboard or flip chart for all pupils to see will further enable pupils to gain an understanding of the strategy in question.

Loop Game for Subtraction

To improve pupils' flexibility in renaming hours and minutes for subtraction a loop game can be useful and enjoyable. It can be replayed on numerous occasions to build pupils' mastery and consolidation of renaming. (See the resources in Appendix C for an example). Encourage different pupils to share the strategies they use to rename time during the game.

Assessment

LEVELS C.4

READ AND INTERPRET SIMPLE TIMETABLES

SAMPLE LEARNING EXPERIENCES

Class Timetables [333]

Pupils should examine the class timetable to extract information. *What day do we do Art? On what days do PE lessons happen? At what time? How long do we spend on Literacy/Numeracy each week? Calculate the total amount of time spent on breaks each day and each week.*

Fill in the following table to show how much time is used weekly for each subject:

Subject	**Less than 1 hour**	**1 hour**	**1 ½ hours**	**2 hours**	**More than 2 hours**
Literacy					
Numeracy					
History					
Geography					
Science					
Music					
Art					
PE					
Religion					
SPHE					

[333] Deboys & Pitt, (1979), Lines of Development in Primary Maths, p. 188

My Weekday Timetable

Provide pupils with a blank weekday timetable for before and after school. The pupils fill in the corresponding times.

Activity	**Mon**	**Tues**	**Wed**	**Thurs**	**Fri**
Wake up					
Leave for school					
Arrive at school					
School starts					
School ends					
Get home					
Homework					
Dinner					
Bedtime					

A similar timetable could be generated for the weekend

Train Timetable Problem [334]

This problem requires logical thinking. The teacher could provide a blank timetable for pupils to complete to support them in solving the problem.

ICT
Train Timetable

Timetable Resources

A variety of timetable resources and activities can be downloaded from this website.[335]

[334] http://nrich.maths.org/958

[335] http://www.11plusforparents.co.uk/Maths/measure6.html

MEASURES - TIME

LEVEL C.5

READ DATES FROM CALENDARS AND EXPRESS WEEKS AS DAYS AND VICE VERSA

TEACHING NOTES

The work carried out in level B.4 may be repeated at this stage.

SAMPLE LEARNING EXPERIENCES

Calendars [336]

Examine and compare different calendars to find the various ways in which the months, the days and weeks are shown and written. Discuss the reasons for the different formats.

Why do you think some calendars have larger spaces for Mondays to Fridays? Which of these calendars are easiest to read?

How quickly can you find your birthday on this calendar (compared to another)?

Look at calendars from different years. *What do you notice about your birthday in each of these three years? Does this happen for all dates? Are any years different?*

Blank Calendar

Provide pupils with a blank grid laid out in the form of a monthly calendar. Provide them with clues to help them fill the calendar and identify the month. Use clues like the following:

The first day of the month is a Sunday.

The last day of the month is a Tuesday.

There are 31 days in this month.

There are 4 weekends in this month.

There are 5 Tuesdays in this month.

St Patrick's Day falls in this month.

Yearly Calendar Activities [337]

This activity can be repeated at various stages throughout the year. Provide pairs or small groups of pupils with a yearly calendar for the current year. Prepare cards with statements like the following:

Next Monday

First/last day of next month

First/last day of last month

Third Tuesday in this month

Number of days in this month / next month

Number of school days since start of term / year

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[336] First Steps Measures Book 1 p.156

[337] http://topnotchteaching.com/wp-content/uploads/2014/10/CalendarCardsV1.0.pdf

Days left in term / year

Number of Fridays in this month

Number of days until your birthday/Christmas etc.

Based on the current date, pupils identify the dates that correspond to the statements on the cards and record these in their copies or on a worksheet. Pupils can compare their results to check their answers.

Sample prompt cards for this activity can be downloaded from this link. [338]

True or False?

Take a month from the class calendar and display it. Google calendar on the whiteboard could be used for this. Review the layout of the calendar: the number of days, where the days are recorded (horizontally or vertically), location of weekends. Count the number of days using ordinal number and the days. *Wednesday 1st, Thursday 2nd etc.*

Involve the pupils in filling in some information on the calendar for the current month, for example, pupils' birthdays, school holidays, school or class events, P.E days, special occasions. Now play a game of true or false. Initially the teacher calls out some statements: *Seán's birthday is on a Wednesday this month. The first day of the month is a Wednesday. This is the fourth month of the year. The second Thursday in the month is the 13th.* The pupils identify whether the statement is true or false. They can record this on their own whiteboards or simply put their thumbs up for true or make an X with their arms for false. Pupils themselves can then take turns coming up with their own true/false statements for the rest of the class.

Calendar Calculations [339]

Activities involving the calculator provide an ideal opportunity for pupils to make links with their number knowledge and to apply their number skills. The problem at the link opposite is one such example.

[338] http://topnotchteaching.com/wp-content/uploads/2014/10/CalendarCardsV1.0.pdf

[339] http://nrich.maths.org/1037

LEVEL D.1

READ AND INTERPRET TIMETABLES AND THE 24-HOUR CLOCK (DIGITAL AND ANALOGUE) AND INTERPRET AND CONVERT BETWEEN TIMES IN 12-HOUR AND 24-HOUR FORMAT

TEACHING NOTES

The analogue, digital and timetable activities in levels A.3, B.3, C.2, C.3 and C.4 may be repeated at this stage.

The 24-hour clock is related to the digital form of telling the time and is used, for example, in railway timetables. Problems can occur with the 24-hour clock times. Midnight is taken as the starting point and is written as 00:00 or 00.00. The times in the morning are just like the digital form, so 10.40 means 20 to 11. In the 24-hour clock midday has no special status and the hours are counted from midnight, so 1.00 p.m. is 13.00, 2.00 p.m. is 14.00 and so on. It is very easy to make mistakes and read, say, 16.00 as 6 o'clock and not 4.00 pm. Children will need much practice in giving times in several forms and moving from one to another
(Suggate et al p.223).

SAMPLE LEARNING EXPERIENCES

Bus Timetable[340]

Examine a timetable that uses the 24-hour clock with pupils. A bus timetable of a local bus route might be most meaningful for pupils.

Pose some questions to prompt pupils to read and interpret the timetable.

ICT
[Sample Timetable]

What kind of timetable is this? (Bus, train, boat, aeroplane…) Where does each bus journey begin? Where does each bus journey end? How many stops does the bus make between Portlaoise and Dublin Airport? Will the driver automatically stop at every stop? Why/Why not? Let's look at a map of Ireland/Google Maps to plot the route taken by the bus. Has anyone been to any of these places? Can you work out how many buses travel each day? Do you notice any patterns in the times the bus departs and arrives? Does each journey take the same amount of time?

Next move the discussion on to focus on 24-hour time.

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340 http://www.dublincoach.ie/timetables-fares/N7-bus-dublin-airport-to-portlaoise.php

Do you notice anything unusual about the way the time is written for some of the bus journeys? How would you read '16:30'. Half past 16? Sixteen thirty? But we never see 16 o'clock on an analogue clock. What time do you think it might be? Share your idea with your partner. Do you notice any other unusual times on the timetable? Let's try and figure out what these times might mean.

SAMPLE LEARNING EXPERIENCES

Making a 24-Hour Analogue Clock

Leave the timetable from the previous activity on display. See the 'Make a Clock' activity in Level C.2 for guidance on making a clock. When the pupils are filling in their hours 1 – 12, ask them to leave space around the edge for more numbers to be added.

Can anyone tell me how many hours are in a day? But why do we only record 12 hours on our analogue clocks when there are 24 hours in a full day and night? Let's look back at the bus timetable. Find the time 13.30. What time do you think this is? Show it on your clock. Is 13.30 the same as half past one in the afternoon/ p.m.? Or is it the same as half past one in the morning/ a.m.? So where will we write 13 on our clock? How about 14? 15?

Pupils continue filling in the corresponding 24-hour times on their clocks until all hours have been identified. Explain to the pupils that when time is recorded in this way, we call it 24-hour time. Have a discussion about when pupils have encountered 24-hour time and elicit from pupils how it is read.

Assessment

In the 24-hour system we always use four digits to express time; the first two for the hours and the last two for the minutes. Children need practice, not only in writing the time but in saying what they have written, for example 1.00 pm is 13.00 hours *(thirteen hundred hours)*, 1.30 p.m. is 13.30 hours *(thirteen thirty hours),* midnight is 24.00 hours *(twenty-four hundred hours)* and midday is 12.00 hours *(twelve hundred hours)*
(Deboys & Pitt p.313).

Fill the Grid [341]

Using their clocks to assist them, the pupils could then complete a grid such as the following:

12-hour time	24-hour time
1.30 a.m.	
	06.40
12.00 p.m.	
	24.00
10.20 a.m.	
	15.50
12.15 a.m.	
	00.01
11.59 p.m.	
	20.20

ICT
Loop
Game

24-Hour Clock Loop Game

To build pupils' efficiency at renaming time in both 12-hour and 24-hour format, the following loop game will allow pupils to practice and consolidate this concept. Encourage different pupils to share the strategies they use to rename time during the game.

ICT
Difference
between Two
Times

Difference between Two Times [342]

An interactive that allows pupils to make timetable calculations whilst using an interactive timeline.

ICT
Matching 24-
Hour Time

Matching 24-Hour Time Digital and Analogue [343]

This online game reinforces pupils' 24-hour clock-reading skills.

[341] Deboys & Pitt, Lines of Development in Primary Maths 1979 p. 313
[342] http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/maths/timetables/index.htm
[343] http://www.teachingtime.co.uk/draggames/sthec5.html

The Time Is…[344]

A problem-solving ordering activity that involves pupils applying their understanding of 12 and 24-hour clock-reading, in both analogue and digital formats.

ICT
The Time Is…

The 24-Hour Clock [345]

Another interactive that explores the concept of clock-reading, the difference between a.m

24-hour clock.

ICT
24-Hour Clock

Interactive Clock-Reading [346]

A variety of activities including a time challenge and a time match game incorporating all aspects of clock-reading – analogue, digital, a.m., p.m. and 24-hour time.

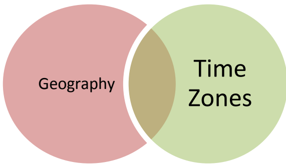
ICT
Interactive Clock-Reading

[344] http://nrich.maths.org/7384/note
[345] http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/maths/time/
[346] http://splash.abc.net.au/res/i/L9642/index.html

MEASURES - TIME

LEVEL D.2

EXPLORE INTERNATIONAL TIME ZONES



TEACHING NOTES

The concept of international time zones is inextricably linked to global Geography. Therefore there are numerous possibilities for integration with the skills, concepts and objectives of the Geography curriculum, particularly those pertaining to 5th and 6th classes.

SAMPLE LEARNING EXPERIENCES

Time zones are related to the rotation of the Earth and the fact that the sun is 'overhead' at places on the same meridian line at the same time. Because the Earth rotates through 360 degrees in 24 hours, it will move 15 degrees in one hour. This means that the time zones are roughly 15 degrees (of longitude) wide. The International Date Line roughly follows the 180 degree meridian through the Pacific Ocean. So, for example, if you travel from Japan (140 degrees East) to Samoa (170 degrees West) leaving Japan on Sunday evening, you would arrive in Samoa on Sunday morning. However, if you left Samoa late on Sunday evening and travelled to Japan, you would arrive on Tuesday morning (Suggate et al p.223).

Exploring Time Zones [347]

Note for this activity that, if possible, having a pre-arranged Skype phone call arranged to someone in a different time zone, USA, Australia, New Zealand; this will maximise the learning potential of this activity. If it possible to darken the room then do so. Have someone hold a globe and another shine the torch on it. Discuss what you can see. Remind pupils that the sun rises in the east and sets in the west so we are moving the globe in a clock-wise direction from east to west.

Pupils should come to the idea that when we are in daylight there are some places in the dark.

What does this mean for the time in other places? If it is 6.30 am here in Ireland and the sun is just rising, will it be bright or dark in America? About what time do you think it will be in New York? Do you think it will be earlier or later in San Francisco? How about in Poland? China? Australia? If it's 22:00 here about what time will it be in France? India? Brazil? Alaska?

[347] http://nzmaths.co.nz/resource/time-zones

Approximate answers will do so long as the pupils get times in the right ballpark. The important thing is that they realise that different places on Earth have different times.

Explain that this difference is recognised by what are called ***time zones***. Two places in the same time zone have the same time. Ireland has one time zone. So everywhere in Ireland has the same time. Explain that you're going to call someone in a different time zone.

Do you have any relations or do you know anyone living in Australia? Have you ever tried phoning someone in Australia? Let's do it.

If possible, using a phone or Skype, ring someone in another time zone. No matter what else you say, be sure to ask them the time. After the phone call ask, *What was the time? So what is the difference between their time and ours?* Depending on where you called, discuss the continent where it is located. *How many time zones do you think America/Europe/Australia has?* Elicit pupils' guesses. Ask them to explain why they guessed the way they did. Use the map to check their guesses. Pose questions about the continent in question to get pupils to calculate the time difference between various cities or countries.

ICT
[Time Zone Maps]

Time & Place Game [348]

First Time	First Place	Second Time	Second Place

Ireland	Peru	Kenya	Algeria
Argentina	Iceland	East America	Bolivia
Madagascar	Greenland	Poland	Italy
East Australia	Saudi Arabia	South Africa	Russia

Explore a time zone map of the world.

Discuss the time differences going east from Ireland.

Has anyone travelled to a foreign country and had to change the time on their watch when they got there? Tell us about it. If I were to travel east to France/Egypt/China, when I get off the plane will I have to put my watch forward or backwards? By how much? Why is there a time difference between Ireland and these places?

[348] http://nzmaths.co.nz/resource/time-zones

Now discuss the time differences going west from Ireland.

If I travel west to Brazil/Mexico/Alaska, when I get off the plane will I have to put my watch forward or backwards? By how much? Why is there a time difference between Ireland and these places?

Explain that the pupils are now going to play a game called Time & Place in pairs. Distribute packs of game cards and a recording sheet to each pair. (See the resources in Appendix C for game cards and recording sheet). Display the time zone map of the world on the whiteboard. Pupils place the time cards face down in one pile and the place cards are placed face down in another. Pupils draw a Time card, put the time in the left-hand column of the table and place the card on the bottom of the deck. They then draw a Place card and put the name of the city in the next (First Place) column and place the card on the bottom of the deck. They then draw another place card, put this in the 'Second place' column of the table and place the card on the bottom of the deck. They then calculate what time it is at the second place when it is the first time at the first place. If the second place is in the same time zone as the first place, they time will remain unchanged. They put their answer in the last column of the table. When the table has been filled, get the class to report back to each other on what they discovered.

Extension Activity. Get pupils to compare their completed tables with each other but to hide the name of the second place with their ruler. The challenge is for the other pair of pupils to identify the hidden country based on the corresponding second time.

Online World Clock [349]

This is an excellent online resource for identifying times in cities all over the world.

ICT
World Clock

Planning a Trip to Belize[350]

The following link contains lesson plans that require pupils to plan a trip to Belize. 'Preparing for the Trip' and 'What Time is it in Belize?' are most relevant to time zone exploration.

ICT
Trip to Belize

[349] http://www.timeanddate.com/worldclock/
[350] http://illuminations.nctm.org/Unit.aspx?id=6539

Interactive Mapping Quizzes

There are three quizzes located at the following links that test pupils' understanding of international time zones and require them to calculate time changes.

http://www.harcourtschool.com/ss1/skills/g6_u4_ch9.html

http://www.harcourtschool.com/ss1/skills/g5_u6ch1_timezones.html

http://www.harcourtschool.com/ss1/skills/g4_timezones.html

Time Zone Quiz [351]

A child-friendly animated video with accompanying quiz questions to allow pupils to calculate global time differences.

ICT
Time Zones

Time Zone Clocks Activity [352]

Along with a child-friendly video that explains time zones, a suggested international clocks activity is also outlined at this link: http://playtivities.com/teaching-time-zones-simple-way/

[351] http://learnenglishkids.britishcouncil.org/en/short-stories/one-moment-around-the-world
[352] http://playtivities.com/teaching-time-zones-simple-way/

LEVEL D.3

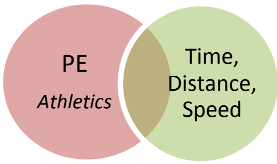
EXPLORE THE RELATIONSHIP BETWEEN TIME, DISTANCE AND AVERAGE SPEED

SAMPLE LEARNING EXPERIENCES

To measure speed we must know two things: (a) the distance travelled and (b) the time taken. Speed is expressed in metres per second or kilometres per hour (Deboys & Pitt (1979) p.315).

The concept of speed is closely related to time. Speed is a measure of the rate of travel. The idea of a rate involving two different units is quite difficult for some children, especially if average speeds are included (Suggate, Davis & Goulding, p.221).

The activities at this level provide natural opportunities for integration with the PE curriculum for 5th and 6th classes. The Athletics strand particularly allows for the application of Maths knowledge and skills associated with distance, speed and time.



Trundle Wheel Measures [353]

Involve pupils in carrying out activities involving the distance covered over a particular time, for example 20 seconds. Possible activities may include: walking, running, hopping, skipping or jumping in the yard. The activities will need to be carried out in groups of four or five. One pupil will be responsible for timing the activity, another for carrying out the activity and two or three others for measuring the distance covered.

Encourage pupils to estimate the distance that will be covered in 20 seconds for each activity before carrying it out. Provide them with a grid to record their findings. The next stage involves calculating the speed taken.

So if Cillian walked 25 metres in 20 seconds, how could we calculate how far he walked in 1 second? So Cillian walked at a speed of 1.25 metres per second. Does anyone know how we write that? (1.25 m/s) Could we work out the speed he would probably walk in a minute? Well done. 75 metres per minute. How about per hour? If Cillian were to run for 20 seconds, do you think he would cover the same distance? Why not?

[353] Deboys & Pitt, Lines of Development in Primary Maths 1979 p. 315

How far do you think Cillian will get in 20 seconds if he walks from here? How could we check if we are close? What will we use to time the activity? How will we measure the distance? How does a trundle wheel work? Why might it be a better measurement tool than a measuring tape or a metre stick? We will need to mark the start and end of Cillian's walk. How will we do that?

20 second Activity	Estimate Distance	Distance Covered	Speed per Second	Speed per Minute	Speed per Hour
Walking					
Running					
Skipping					
Jumping					

A further extension to this activity would be to compare the distances and speeds covered by a number of children completing the same activity.

The speed of light is around 300,000,000 metres per second and the speed of sound is around 340 metres per second (in air).

Discussing Speed [354]

Begin by asking the pupils the key question: *How fast is fast?* Record some answers on the board. Expect responses to come in a range of forms, for example, 100 km an hour, 100 miles an hour, a cheetah running, the speed of light, 10 metres per second, etc. Ask pupils to tell you how they would measure the speed of something. Discuss their suggestions. Refer back to the list of speeds that pupils said were fast. Ask: *Which of these are actual speeds? What do all the actual speeds have in common?* Support and guide the discussion to reach the conclusion that all have both a distance unit and a time unit involved in them.

Explain to pupils that speed is usually described by how far an object travels in a given amount of time. We usually use kilometres per hour or metres per second. Pose the question:

How fast am I travelling if I travel 100 kilometres in 2 hours? As the pupils give their answers encourage them

[354] http://nzmaths.co.nz/resource/how-fast-fast-0

to explain the strategy they used to find them.

I went half of 100 which is 50. So I was travelling 50 kilometres per hour.

I divided 100 by 2 to work out how far for each hour. 100 divided by 2 is 50.

Pose further questions of the same form: *How fast am I travelling if I travel 40 kilometres in half an hour? How fast am I travelling if I travel 10 metres in 2 seconds?* Again encourage pupils to explain the strategies they used to solve the problems.

ICT
[Bike Ride Problem]

Bike Ride Problem [355]

This problem involves calculating the speeds of two bike journeys.

Encourage pupils to share the strategies they used to solve the problem.

Running a Kilometre [356]

Ask pupils to estimate how fast they think they can run for 1 kilometre. Record pupils' predictions. Ask them how far they think a kilometre is. Use a unit like 'lengths of the field', or 'laps of the basketball court'. Explain that you are going to find out how fast they can actually run, by running a kilometre and timing how long it takes. Move outside and measure either the length of the field or the distance round the tennis court and agree on a course which is 1 kilometre long. Time pupils running around the course. Give each pupil their time in minutes and seconds. Back in the classroom challenge pupils to work out their speed in kilometres per hour. First give them a couple of minutes to try to work out their speed on their own, then return together as a class to discuss strategies used.

I took over 6 minutes, and there are 60 minutes in an hour. I know 6 x 10 is 60, so if I ran that speed for an hour I would go 10 kilometres.

I took 4 minutes and 52 seconds to run 1 kilometre, and I know that that is nearly 5 minutes. There are 60 minutes in an hour, which is 12 lots of 5, so I ran at about 12 kilometres per hour.

Help pupils work out some 'markers' like the ones above. 1km in 5 min = 12km per hour, 1km in 6 min = 10km per hour. This will make it easier for them to approximate their own speeds. Discuss how pupils could work out their speeds if they are between the 'markers'. Depending on the ability of your pupils you may want them to be more or less accurate. If they are using mental strategies the nearest half or quarter kilometre per hour is probably reasonable. If they are using calculators then you will need to discuss how many decimal places are appropriate.

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[355] http://nrich.maths.org/1010

[356] http://nzmaths.co.nz/resource/how-fast-fast-0

Ensure that all pupils have found their speed in kilometres per hour, then compare with their predictions from the start of the lesson. Ask pupils how fast they think the fastest runners in the world can run 1 kilometre.

ICT
World Records

Ask the pupils to work out the approximate speeds of the world record holders in kilometres per hour.

See Length section of this manual for further development of this concept

Running 100 Metres [357]

In this activity pupils find their speed over 100m, both in metres per second and in kilometres per hour. Ask pupils to estimate how fast they think they can run for 100 metres. Record their predictions. Discuss whether they think they can run faster for 100 metres than they did for a kilometre. Why or why not?

How does your speed for 1km compare to the world record holder? How close were you? How could we work out their speed per hour? What do the .96 and .98 mean in these times? Could we make these numbers easier to work with? How?

Ask pupils to describe how far they think 100 metres is. They should be able to use their experience from running 1km to make a reasonable estimate. Explain to them that today you are going to find out how fast they can actually run, by running 100 metres and timing how long it takes. Move outside and agree on a course which is 100 metres long. Time pupils running around the course. Give each pupil their time in seconds. Back in the classroom challenge pupils to work out their speed in metres per second. First give them a couple of minutes to try to work out their speed on their own, then return together as a class to discuss strategies used.

I took 20 seconds, and went 100 metres. I know that 20 goes into 100 5 times so I must have gone 5 metres each second. I took 16 seconds to run 100 metres. 16 x 2 = 32 and 32 + 32 + 32 = 96 so there are six and a bit lots of 16 in 100. I must have been going over 6 metres per second.

Discuss how pupils could work out their speeds if the numbers don't work out evenly. Some pupils may be more accurate than others. If they are using mental strategies the nearest half or quarter metre per second is probably reasonable. If they are using calculators then you will need to discuss how many decimal places are appropriate.

How many seconds did you take? How many metres would that be for each second? If you share 100 metres among … seconds, how far did you go each second?

Ensure that all pupils have found their speed in metres per second, then challenge them to work it out in kilometres per hour. They may want to start again from their time for 100 metres, or work it out from their

[357] http://nzmaths.co.nz/resource/how-fast-fast-0

speed in metres per second. Discuss the different methods:

There are 60 seconds in a minute, so 6 metres per second is the same as 360 metres per minute, and there are 60 minutes per hour so 360 metres per minute is 60 x 360 metres per hour. 60 x 360 is the same as 30 x 720 (doubling and halving) and I can work that out by 3 x 72 x 100 = 21600. 21600 metres per hour is 21.6 kilometres per hour. I was running more than 6 metres per second, so that is more than 21.6 kilometres per hour. 16 seconds for 100 metres is the same as 160 seconds for 1 kilometre. That's 2 minutes and 40 seconds. Two and a half minutes goes into an hour 24 times, so I was running less than 24 kilometres per hour.

Compare pupils' speeds with their predictions from the start of the lesson. Ask them how fast they think the fastest runners in the world can run 100 metres.

Assessment

The current world records for the 100m are:

Men:	9.58 seconds	Usain Bolt (2009)
Women:	10.49 seconds	Florence Griffith-Joyner (1998)

ICT
World Records

Ask pupils to work out the approximate speeds of the world record holders in kilometres per hour.

The Hare and the Tortoise [358]

This problem provides opportunities for pupils to apply their understanding of distance, speed and time. If pupils work in pairs, they will be enabled to communicate strategies with each other.

ICT
The Hare & the Tortoise

Timmy, Tammy & Tommy [359]

This is a problem involving average speed, distance and time.

ICT
Timmy, Tammy & Tommy

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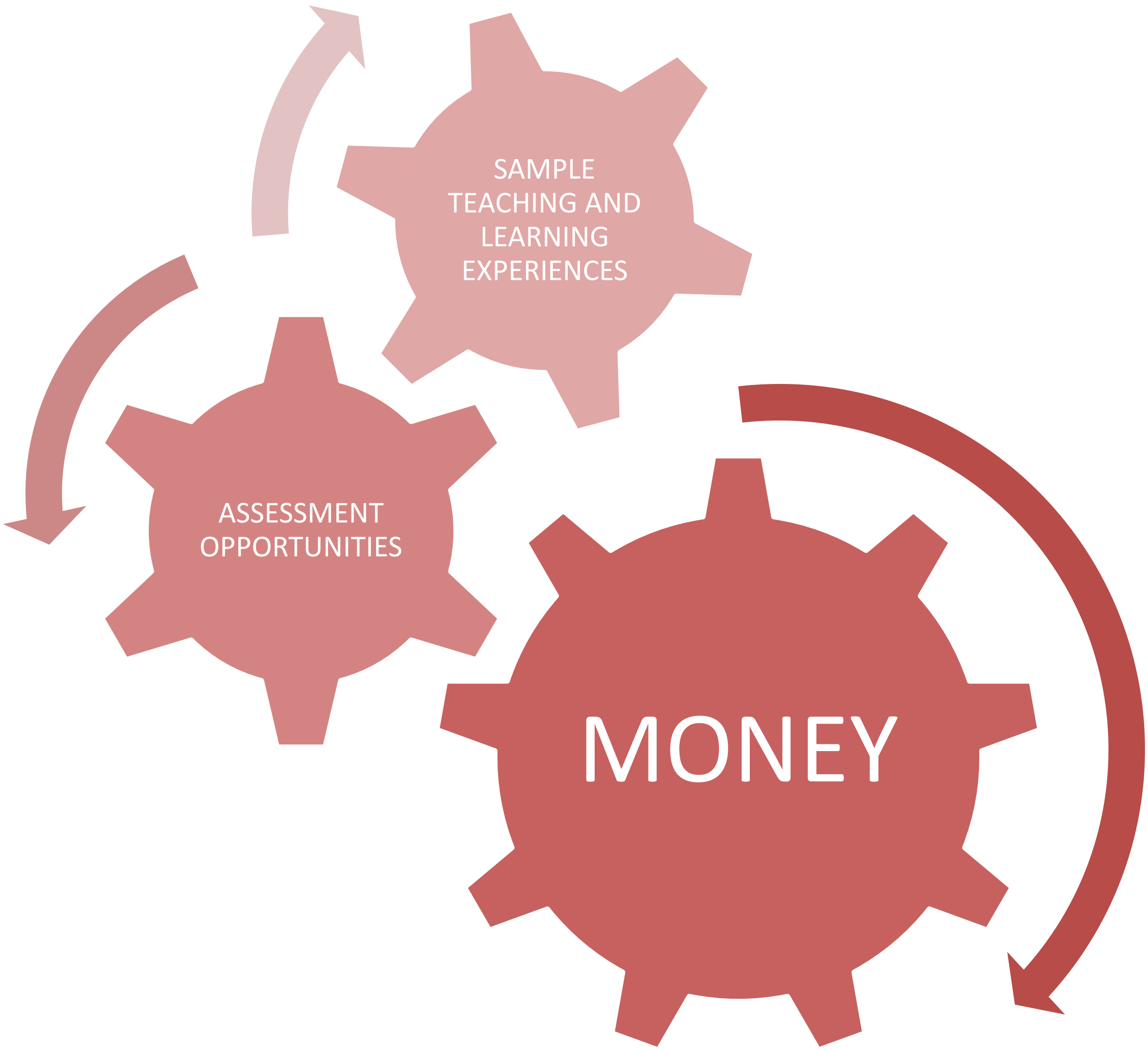
[358] http://nrich.maths.org/1082
[359] http://nrich.maths.org/2408


SAMPLE TEACHING AND LEARNING EXPERIENCES
ASSESSMENT OPPORTUNITIES
MONEY

MONEY

MONEY - BACKGROUND KNOWLEDGE FOR TEACHERS

What is Money?[360]

For something to be considered as money it must meet the following criteria. It must be:

1. A medium of Exchange
2. A unit of Account
3. Store of Value

By medium of exchange we mean that everyone has to accept that "it" is money. Gold is money because it is widely accepted in trade for goods and services. In some ancient cultures, shells were recognised as money. Unit of account necessitates that money can be broken down parts i.e. 1c, 2c, 50c, €1, €20 etc.so that it can be traded fairly. It also means that other goods can be priced in terms of money.
Money must also be able to hold its value. If we are to hold money, we must know that it will be worth something tomorrow. For this reason, most money tends to be made from metal or paper.

The Eurozone[361]

Of the 27 EU member states, 17 countries are now using the euro, namely, Austria, Belgium, Cyprus, Estonia, France, Finland, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Malta, Portugal, Slovakia, Slovenia and Spain. These are known collectively as the eurozone.
The eurozone is a monetary union; all member states abandon their former currencies and transfer their power of setting national monetary policy to a transnational European Central Bank (ECB). The ECB is located in Frankfurt, Germany.

Euro or Euros[362]

The European Commission adopted Euro as the plural of Euro. However, the Directorate-General for Translation advises the use of the plural form Euros for any publication intended for general publication. The English Style Guide of the European Commission Translation Service states:
12.12 … Guidelines on the use of the euro, issued via the Secretariat-General, state that the plurals of both 'euro' and 'cent' are to be written without 's' in English. Do this when amending or referring to legal texts that themselves observe this rule. Elsewhere, and especially in documents intended for the general public, use the natural plural with 's' for both terms.

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[360] Council for Economic Education (2015) The Story of Jack and the Bank Stalk. Accessed at http://www.econedlink.org/lesson/66
[361] Taylor, H. (2012) The Euro: Creation and Current Crisis. Accessed at http://economics.fundamentalfinance.com/euro-creation-and-current-crisis.php
[362] Pettinger, T (2008) What is the Plural of Euro of Euro? Accessed at http://www.economicshelp.org/blog/506/euro/what-is-the-plural-of-euro/

Rounding[363]

Rounding is already in place in Australia, Belgium, Canada, Denmark, Finland, Hungary, New Zealand, Norway and Sweden. In Ireland, it means the reduction in the use of 1c and 2c coins.

Key points to remember in relation to rounding

- rounding is voluntary for both businesses and consumers; consumers have the right to demand exact change
- it only applies to cash payments
- It only applies to the change given on the final till amount, not on individual prices
- 1c and 2c coins will remain legal tender.
- change will be rounded up or down to the nearest five cent
  - if the total ends in **1** or **2** round to **0**
  - if it ends in **3** or **4** round to **5**
  - if it ends in **6** or **7** round to **5**
  - if it ends in **8** or **9** round to **10**.

MONEY - POSSIBLE PUPIL MISCONCEPTIONS

1. Although coins are concrete models of themselves, they are non-proportional in relation to their value. This can cause difficulty for young children whose early experiences with number will have been based on the concept of one-to-one correspondence; matching one number to one object. Pupils who are tied to counting objects will find it difficult to grasp the idea that one coin does not always equate to one, that it may mean 2 or 5 etc.
2. The physical size of coins may also present problems for young children. Some children may think that the bigger the coin, the greater its value. This is not always the case; our five cent coin is bigger in size than the ten cent coin.
3. Many pupils see paper money and coins as separate types of currency. When an amount is presented using euro and the decimal point, pupils see the figure before the decimal point as paper money and the figure after the decimal as coins or cents. They do not see that cents are a fractional part of a euro.
4. Problems involving money calculations presented to pupils frequently do not relate to their life experiences. For example, some problems require pupils to decide which coins to use when buying a given item, like a lollipop for ten cent. In many cases, pupils select a coin that is greater than the value of the item, perhaps 20 cent. While this is considered wrong in a school setting, using a larger coin and expecting change is acceptable in any shop.[364]
5. Problems presented to pupils also can also be unrealistic in terms of purchasing power. For example a problem might include a question about an ice cream for five cent. This can cause difficulties for pupils as the quoted price is so different from actual prices. This disconnect can cause pupils to lose confidence in their own common sense.

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[363] Accessed at http://www.betterallround.ie/

[364] Chen *et al.* (2010) Money Concepts p.17. Accessed at https://wiki.eee.uci.edu/index.php/Money_Concepts

6. Many problems given to pupils relating to money involve finding the percentage of a given quantity. Because 10% is one of the easiest percentages to work with, we usually begin with this. However, it needs to be clearly explained to pupils that 10% being the same as a tenth is a special case and this does not apply to other percentages i.e. 5% is not a fifth. Pupils often become confused and presume this.[365]
7. Problems involving discounts and VAT can also cause difficulties for students, in particular the case where the final price is given and the original price has to be calculated. In the following example, a suite was sold at €1,275 in the sale with a 15% reduction, pupils are asked to calculate the original price? Pupils may incorrectly find 15% of €1,275 and add it on, but the reduction was 15% of the original price. The final price is 85% of the original, so 85% of the original price = €1,275.

   Correct method:

   85% = €1,275

   1% = €$\frac{1275}{85}$ = €15

   100% = €15 x 100

   = €1,500[366]
8. In problems where a percentage increase is applied and then the same percentage decrease is applied, many pupils erroneously expect to arrive back at the original price. A typical problem given to pupils might look something like this:

   The price of an item is €200. The price is increased one month by 10%. The next month the price is decreased by 10%. What is the final price?

   After the 10% increase the price has gone up to €220. If we apply the 10% decrease to this we get €22, not €20, because the percentage change always applies to existing value. The item now costs €198.[367]

MONEY - PARENTAL INVOVLEMENT

The following are a list of ideas that may be shared with parents to help develop pupils' conceptual understanding of money. Some of the following ideas are taken from the NCCA's tip sheets for parents available through http://www.ncca.ie/en/Curriculum_and_Assessment/Parents/. Here you will also find some short videos of parents working with their children to develop their mathematical understanding.

Level A:

- Play shop. Price items up to 5 cents. Give your child a purse with some coins and talk about what you could buy.
- Which coin is worth the most? The least? What can you buy with 5 cents?

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[365] Haylock, D. (2014) Mathematics Explained for Primary Teachers p.268

[366] Suggate, J., Davis, A. and Goulding, M. (2010) Mathematical Knowledge for Primary Teachers p.147

[367] Haylock, D. (2014) Mathematics Explained for Primary Teachers p.270

MEASURES - MONEY

● Recognise coins up to 20 cents and use coins up to 10 cents. Let your child handle money and work out change. Talk about the value of coins when you are shopping: Which coin do you need to pay for the sweet? Why can we not use this one? Can you swap me some coins for this 20 cent coin?

Level B:

● Ask your child to put items less than 50 cents in order from the cheapest to the dearest. Ask: How many 10 cent coins can I change my 50 cent coin for? The orange costs 45 cents and the apple is 40 cents. Which is cheaper? How much change will I get if I buy an orange?

● Encourage your child to add up the coins in your purse or to work out what change you will get when buying things: I have €1.70 in my pocket. What is the least number of coins I could have?

● We're coming to the toll plaza. The toll is €2.40. Can you get the coins ready for the machine please?

● Notice all the signs that have euros on them: for example, petrol stations, supermarkets etc.

Level C:

● Give your child receipts and bills with totals removed and ask them to estimate the cost of the bill by rounding.

● Encourage your child to keep a running total while shopping for a few items.

● If you pass a petrol station watch the prices change over the course of a few weeks and ask the children to tell you if it is increasing or decreasing and by how much?

Level D:

● Create opportunities for your child to handle money and work within budgets.

● Encourage your child to look at value for money. Is it better value to get 6 bananas for €1.50 or 10 bananas for €2?

● Use the internet or catalogues to compare the cost of certain items. for example books, CDs. etc.

● If going on holidays, discuss currency conversions with your child. If you have €100, how much Sterling will you get?'

Suggested Resources for Teaching Money	
Coins	Price tags
Notes	Play cash register
Money stamps	Items for play shop
Pre-money tokens (counters and dot stickers)	Blank hundred square
Store catalogues	Blank bingo grids
Brochures	Magnifying glass
Menus	Uni-fix cubes
Shopping receipts/bills	Dice

MONEY - LEARNING TRAJECTORY

The learning trajectory is based on the objectives for Measures in the Primary School Mathematics Curriculum. In some instances, similar objectives at the same class level have been collapsed into one objective. Objectives that only refer to problem solving have not been included as discrete objectives because a problem solving approach is advocated throughout all of the teaching and learning experiences. Problem solving is viewed in this manual as a fundamental, integ ral part of mathematics teaching and learning that pupils should experience every day. The same colour coding from the curriculum is used – infants (green); first and second (red); third and fourth (blue); fifth and sixth (orange).

MONEY LEARNING TRAJECTORY LEVEL A[368]

Trajectory Levels	Concept	Developmental Experiences: Concrete	Pictorial	Abstract
	Level A.1 Identify, sort, select and use coins up to 5 cents, 10 cents and 20 cents			
	Level A.2 Exchange and use coins up to 20 cents and calculate change			

[368] This level is generally aligned with the objectives for Junior and Senior infants.

MONEY LEARNING TRAJECTORY LEVEL B[369]

Trajectory Levels	Concept	Developmental Experiences: Concrete	Pictorial	Abstract
	Level B.1 Recognise, use, exchange and calculate using coins up to the value of 50 cents and calculate change			
	Level B.2 Calculate how many items may be bought with a given sum			
	Level B.3 Recognise, use, exchange and calculate using coins up to the value of €2 and calculate change			
	Level B.4 Calculate and record the value of a group of coins in cents and rename as euro			

[369] This level is generally aligned with the objectives for First and Second class.

MONEY LEARNING TRAJECTORY LEVEL C[370]

Trajectory Levels	Concept	Developmental Experiences: Concrete	Pictorial	Abstract
	Level C.1 Solve and complete practical problems and tasks involving money			

MONEY LEARNING TRAJECTORY LEVEL D[371]

Trajectory Levels	Concept	Developmental Experiences: Concrete	Pictorial	Abstract
	Level D.1 Compare and calculate 'value for money' using unitary method and percentages			
	Level D.2 Convert other currencies to euro and vice versa			

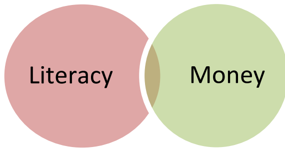
[370] This level is generally aligned with the objectives for Third and Fourth class.
[371] This level is generally aligned with the objectives for Fifth and Sixth class.

LEVEL A.1

IDENTIFY, SORT AND SELECT COINS UP TO 5C, 10C AND 20C

TEACHING NOTES

Due to the increased use of debit and credit cards for monetary transactions, pupils may have limited experience of cash and coins prior to coming to school. Their vocabulary may also be limited. It is imperative that they have opportunities to engage in play activities that involve handling money and provide opportunities for vocabulary development before they engage in formal money concepts. The following sequence of suggested activities intends to build on pupils' experiences of early mathematical activities and number concepts.



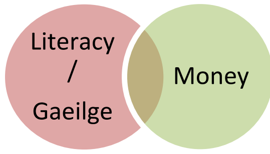
SAMPLE LEARNING EXPERIENCES

Nursery Rhymes

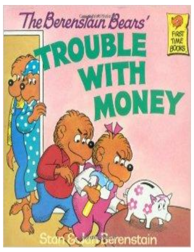
Nursery rhymes containing money references provide an excellent starting point when introducing the concept of money to young pupils. Rhymes such as Simple Simon, 5 Currant Buns, How much is that Doggie in the Window and Micilín Muc are particularly useful. In many cases, the word penny can be replaced by money or cent in order to make the rhymes more suitable for an Irish context.

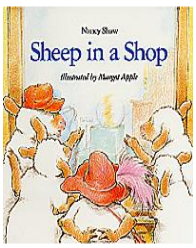
Who did Simple Simon meet? What did the pieman want from Simple Simon? Why do you think he wanted money? Have you ever paid money for something in the shop?

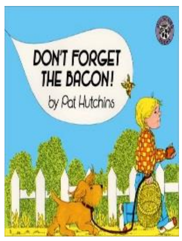
Using Stories



Words like 'coin' and 'change' are rarely used or understood by young pupils.[372] Books about money and shopping are a great way to generate initial discussion and to introduce the correct vocabulary in context. For example Stan and Jan Berenstain's 'The Berenstain Bears' Trouble with Money' exposes pupils to words associated with money like price, spend, expensive, too much, cent. In 'Don't forget the Bacon' by Pat Hutchins, the pupils encounter different types of shops like the junk shop and the cake shop. In Nancy Shaw's 'Sheep in a Shop', the sheep don't have enough money in their piggy bank to pay for their presents and need to come with a solution to pay their bill. Sa leabhar 'Ag siopadóireacht le Mam' téann Róisín go dtí an ollmhargadh le Mamaí.



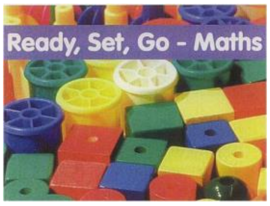




[372] Bond, M (2011): Money problems. Accessed at https://nrich.maths.org/2586

For values to make sense, pupils must understand 2, 5, 10, and think of these quantities without seeing countable objects. Where else do we say "this is 5" while pointing to a single item?
Van de Walle, Karp and Bay-Williams (2013) p.399.

Pre-Money Tokens

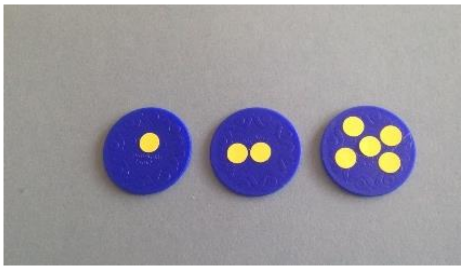


In their everyday lives, pupils will usually have experience of one-to-one correspondence; one lid for each jar, one shoe for each foot, one glove for each hand, etc. In *Ready Set Go - Maths*, Eunice Pitt suggests that this skill often develops slowly in children who have a difficulty with mathematics. Carefully planned matching activities can aid pupils' progress in developing this skill.

However, the concept of a monetary value of an object is not one with which most young pupils will be familiar. To understand exactly what a 2c coin represents, a pupil needs to have mastered connecting numbers with the value they represent and understanding that value may be independent of physical properties. They have to understand that the '2' on the coin means two cents and that this is the same as having two single one cent coins or two coins with '1' on them. Starting pupils off using 'pre-money' tokens that indicate their value is a more suitable way to introduce coins to young pupils than standard coins with digits on them.[373]

Introducing Tokens:

1. Use tokens/counters that are marked with paper dots to illustrate their value in a clear way; one dot stands for a value of one, two dots for a value of two and five dots for a value of five. The only way to tell the value of a particular counter should be to count the number of dots on it. Also, ensure that counters are of uniform size, shape and colour. This prevents pupils from being distracted by the properties of the counter and allows them to concentrate on its value. The dots on the counter should also be the same size, shape and colour and fixed to one side of the coin only.



2. Introduce the 'pre-money' coins to the pupils. Spend plenty of time counting a number of tokens and also counting the value of the tokens. Although this may seem trivial it is vital that the pupils understand the difference between the two.

Single Exchange Game

[373] Bond, M. (2011): Money problems, accessed at https://nrich.maths.org/2586

Pupils will need a die numbered 1, 1, 1, 1, 1, and 2 and a selection of ‘pre-money’ one dot and two dot coins.

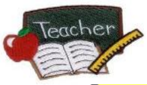
1. Pupils take turns throwing the die and collects the corresponding number of one dot coins.
2. When they have two one dot coins, they can exchange them for a two dot coin.
3. The winner is the pupil with the most two dot coins at the end of the game.

How many one dot coins do you have? Can you exchange them? How many more one dot coins do you need so you can exchange for a two dot coin?

Double Exchange Game

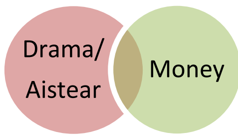
Pupils will need a die numbered 1, 1, 1, 1, 1, 2 and a selection of ‘pre-money’ one dot, two dot and five dot coins.

1. Pupils take turns throwing the die and collecting the corresponding number of one dot coins.
2. When they have two one dot coins, they can exchange them for a two dot coin or they can continue collecting until they have five one dot coins and then exchange them for a five dot coin.
3. They can also exchange two dot coins and a one dot token for a five dot coin
4. The winner is the pupils with the most five dot coins at the end of the game.



Can you exchange any of your coins? How many more one dot tokens do you need so you can exchange them for a five dot coin? Can anyone think of another way to exchange for a five dot coin?

At the Shop[374] (Part 1)



The majority of pupils come to school with an understanding that money and shopping go hand in hand.[375] Setting up a shop in the classroom provides a natural way to use money vocabulary such as ‘change’ ‘buy’, etc. It connects learning to everyday life and provides excellent opportunities for pupils to engage in buying and selling, exchanging coins and calculating change.

Price tags should be added to the items for sale in the shop. Price tags should only use dots like those dots used on the ‘pre-money’ coins i.e. they should be of uniform size, shape and colour. Pupils take turns to ‘buy’ desired goods from the shop by simply exchanging their counters. As the pupils’ ability grows, the counters can be used in shopping situations where change has to be given and pupils assume the role of the shopkeeper, while the teacher supervises the activity. As a final step before the real coins are introduced, introduce a second set of counters which have the dots on one side

[374] Bond, M. (2011): Money Problems accessed at https://nrich.maths.org/2586
[375] Baxter, N. (2002): The Fun and Creative Maths Classroom for 6-year-olds. Volume 6 p.109

and the appropriate number 1, 2 or 5 denoted on the back. This means the value of the counter may be seen whichever way the coin is facing, and provides an opportunity for the pupils to begin to link the digit symbol to its value.

Readiness Check
Before introducing coins to young pupils they need to understand or have developed the following:

- The names of numbers, in the correct order
- Count objects accurately
- The connection between digits and the values they represent
- The concept of addition
- That objects can have a value, which is irrespective of their colour, shape, size, mass, et
  Adapted from Bond, M. (2011): Money Problems accessed https://nrich.maths.org/2586

Magnifiers

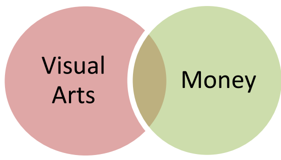
Magnifying glasses can be used as a fun tool to examine coins and their markings.

Describe what you can see on both sides of your coin?

What do you think this coin might be called?

Rubbings

Show pupils how to make rubbings of both sides of 1c, 2c and 5c coins using crayons. Ensure to draw pupils' attention to the fact that each coin has its name on it. When pupils have experience of making rubbings, they can be encouraged to make 'coin creatures' or 'money monsters' using the side denoting the value of the coin.



How much is your 'money monster' worth? Of what coin would you need to make a rubbing to have 5c?

The names of our coins are conventions of our social system. Pupils learn these names the same way that they learn the names of physical objects in their daily environment. The value of each coin is also a convention that pupils must simply be told
(Van de Walle, Karp and Bay-Williams, 2013, Elementary and Middle School Mathematics p.399).

Coins' size do not correlate to their value. This disparity in size to value causes much confusion for pupils and poses a major stumbling block in learning coin values
Drum and Petty (1999) Teaching the Values of Coins p.264

Guess my Coin[376]

[377]Slowly slide up an enlarged picture of a coin from a bag and stop when part of the coin is showing. Ask the pupils what coin it might be. Discuss the possibilities and encourage them to give reasons for their answers. Show a little more of the coin and repeat the process.

ICT
[Recognising and using coins]

Buried Treasure

Pupils dig to find coins that are buried in the sand tray and sort coins using sorting trays into piles of coins that are the same. Pupils order them; from the coins with the least value to the coins with the most value.

How do you know which coins to put together? Can you see any numbers on the coins? Which coin is worth the most/the least? How do you know?

Lucky Dip Game

A number of coins are placed in a "feely bag". The bag is passed around the group. Each pupil removes a coin from the bag. The pupils then find their partner; the child/pupils who has the same coin. The group of pupils must then arrange themselves in a line in order of the value of the coins from the smallest to the greatest.

Dominoes[378]

Money dominoes can be used as a fun way to reinforce coin recognition and for matching activities. You can make your own dominoes by cutting card into rectangular pieces and gluing on different coins. These printable dominoes may be downloaded from the site marked below.

ICT
[Printable Money Dominoes]

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[376] Adapted from Pitt, E. Ready Set Go-maths p. 102
[377]http://www.ncca.ie/en/Curriculum_and_Assessment/Parents/Primary/Infant_classes_Maths_resources/Infant_Classes_Maths_Resources.html
Link to Printable Money Dominoes: assessed at http://www.senteacher.org/worksheet/22/Dominoes.html

Match the Coin[379]

One pupil closes his/her eyes while a second pupil places a coin in their hands behind their back. The pupil opens their eyes, carefully feels the coin behind their back without looking at it and with the other hand explores the coins on the table in order to choose a coin that is the same. The pupil checks the two coins together.

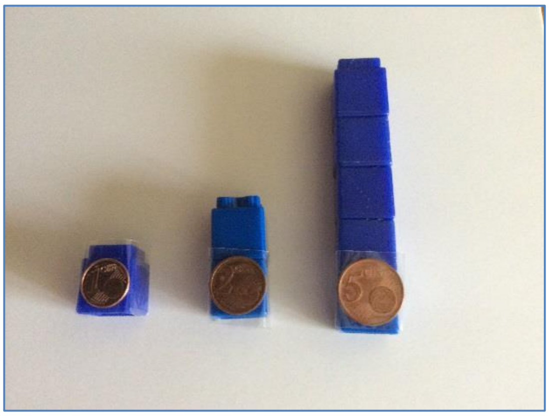
[379] Adapted from Pitt, E. Ready Set Go-maths p. 88

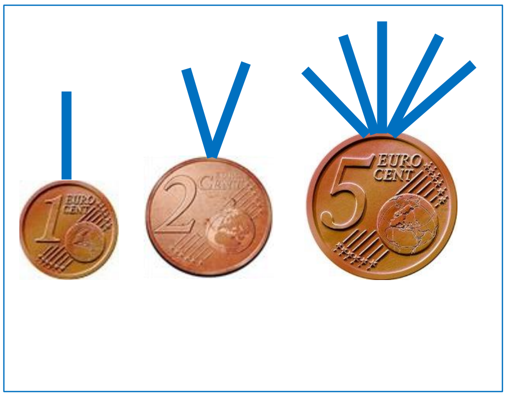
LEVEL A.2

EXCHANGE AND USE COINS UP TO 10C AND CALCULATE CHANGE

TEACHING NOTES

Until pupils have developed the understanding that a single coin may represent more than one, unifix cubes may be used. The cubes are used to highlight the fact that different coins have different values.[380] The addition of antennas to coins could also be used to help develop this understanding.



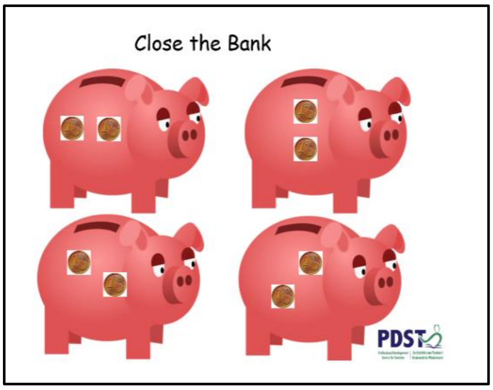


SAMPLE LEARNING EXPERIENCES

Buy a Letter[381]

Pupils make a name bracelet with letter beads. Pupils "buy" the letters to spell their name with cents. Each bead costs one cent. They count the letters in their name, then count out the amount of cents they will need. You can also use letter stickers to stick on paper instead of making a bracelet.

Close the Bank[382]



The game aims to develop pupils' understanding of exchange; to see that (i) two 1c coins make 2c and (ii) five 1c coins make 5c.

To play the game you will need a collection of 1c coins, a base board per pupil (see the resources in Appendix D), lids to close the piggy bank when it is full; either denoting 2c or 5c depending on the objective of the game. You will also need a die. Five of the faces of the die have the number 1 displayed on them and the remaining face is left blank and means 'miss a turn'. Pupils take turns to roll the die and collect the coins to fill their piggy bank. When their bank is full, they place the lid on top. The first pupil to fill all of their piggy banks, wins the game.

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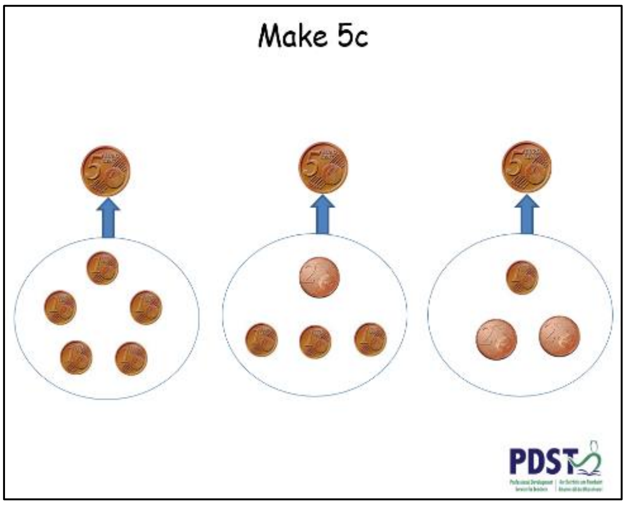
380 Adapted from Bamberger, H.J, Oberdorf, C and Schultz-Ferrell, K. (2010) Math Misconceptions.
381 Adapted from http://www.prekinders.com/preschool-money/
382 Adapted from Pitt, E.: Ready, Set Go-Maths p.128

How much do you have in your piggy bank? How much more do you need to make 2c/5c? How much money do you have altogether?

Make 5c

The objective of this game is to enable pupils to exchange 1c coins for 2c and 5c coins.



To play the game you will need a collection of coins; initially just 1c coins but as the pupils become familiar with the game you will also need 2c and 5c coins. You will also need a playing board per pupil and a die numbered 1, 1, 1, 2 and 2. The pupils take turns rolling the die and collecting the corresponding number of coins.

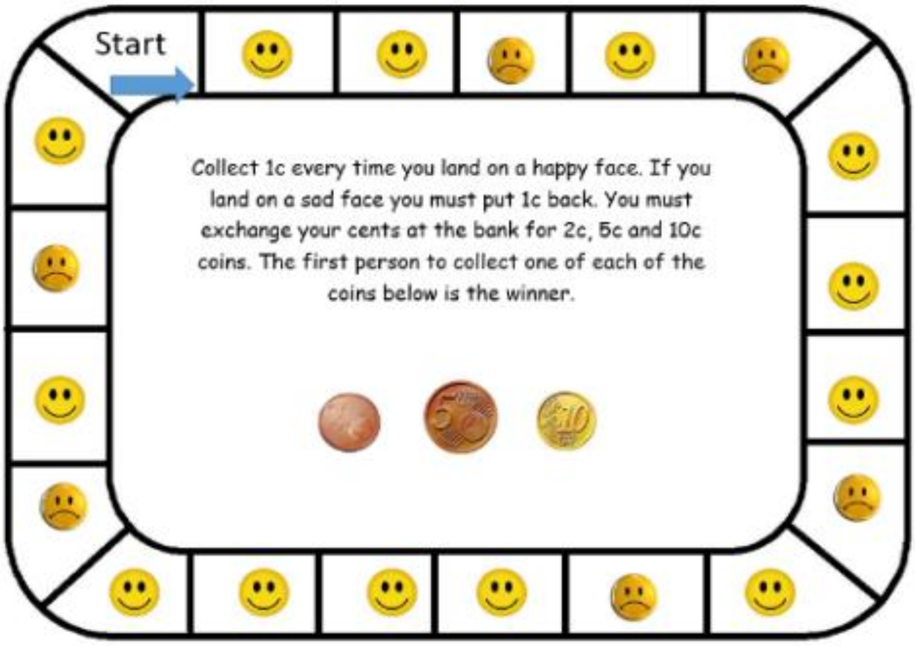
In the beginners' version of the game, to cover the 2c, the pupils place two 1c coins on it. They move their five 1c to cover the 5c coin.

As the pupils' understanding develops, 2c and 5c coins can be introduced to facilitate exchanging.

Instead of placing two 1c coins on the 2c, could you use anything else? Can anyone think of another way to make 5c?

Make 10c

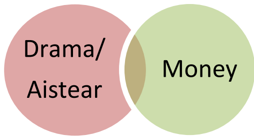
The objective of this game is to enable pupils to exchange 1c and 2c for 5c coins and 5c coins for 10c.



To play the game you will need a collection of coins; 1c, 2c, 5c and 10c coins. You will also need a playing board per pupil and a die numbered 1, 1, 1, 2 and 2. The pupils take turns rolling the die and collecting the corresponding number of coins. The first player to exchange their coins for a 10c coin is the winner.

Coin exchange Game[383]

The aim of this game is to allow pupils to exchange 1c coins for 2c, 5c and 10c coins. To play the game you will need 1 game board per group, a die numbered 1, 1, 1, 1, 2, and 2, counters and a collection of coins. The players take it turns to throw die to move their counter round the board, collecting a cent each time they land on a smiley face. If they land on a sad face, they must put a cent back. They must exchange their 1 c coins for 2c, 5c and 10c coins. Winner is the first one to collect a 2c, 5c and 10c coin.



> Because adding on to find differences is such a valuable skill, it makes sense to give students experiences with adding on to find differences before asking them to make change. As students become more skilful at adding on, they can see the process of making change as an extension of a skill already acquired.
> Van de Walle, Karp and Bay-Williams (2013) Elementary and Middle School Mathematics p.399

At the Shop (Part II)

This activity can be seen as an extension of the activity outlined above (p. 4). However, this time real coins will replace the 'pre-money counters'. Creating a variety of different shops can maintain pupils' interest for longer for example sweet shop, toy shop, cake shop etc. A bank or a post office could also be set up as these also provide opportunities for pupils to handle money.

The prices used in the classroom shop cannot be realistic because of the pupils' limited understanding of number. It is a good idea to explain this to the pupils; the prices in the classroom shop are not real and items would cost more in an actual shop. More able pupils can act as the shopkeeper at the initial stages. Calculating change provides them with excellent opportunities to consolidate their experiences of "counting on". For example you gave me 5c. The sweets cost 3c. That's 4, 5… you get 2c change.

Sale

Explore pupil's understanding of sales. Decide there is going to be a sale in the classroom shop for example 1c off everything in the shop today! This gives pupils opportunities to consolidate their "counting back"

Price Increase

Explain to pupils that sometimes shopkeepers increase their prices. Decide that everything in the shop will be increased by 1c. In this activity, pupils get further opportunities to practice their "counting on" skills.

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[383] Adapted from: https://www.tes.co.uk/teaching-resource/coin-exchange-6048253

CONSOLIDATION ACTIVITIES

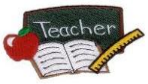
Gob Stopper[384]

Jade bought a gob-stopper. It cost 6c.

She paid for it exactly. Which coins did she use?

There are 5 different ways to do it. Find as many as you can.

What if the gob-stopper cost 7c?

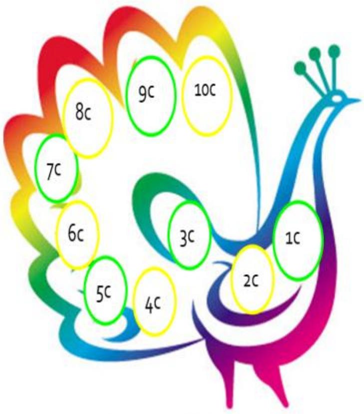


Mary, what coins did you use? Did anyone do it a different way? Tom, what was the least amount of coins you used? Could you have used less coins? Sarah can you revoice what Tom said?

Cents for the Peacock[385]

The objective of this game is help pupils to work out the coins to be used to make 2– 10c. It will also help with coin recognition and values.

To play the game you will need a die – marked with 1c, 2c, 3c, 4c, 5c, 6c, coins – 1c, 2c, 5c, 10c and a Peacock baseboard for each player. The players take it in turns to throw the die and then take that amount from the pile of coins for example if a pupil throws a 6c, they can take six 1c coins or a 5c coin and 1c coin etc. They put these coins in the places marked in circles on their peacock. Extra coins can be kept to use on later turns. More than one coin can be placed on any circle to make the total, for example 2c + 2c +1c could cover 5c. The first player to cover all his/her circles is the winner.



Do You Agree?

Statements which ask pupils to agree, disagree and provide justifications promote reasoning.

1. My friend thinks that 5c + 1c =7c. Use numbers, words or pictures to show if you think this is correct.
2. My friend also thinks that 3c + 5c is the same as 5c + 3c. Is she right? Draw a picture to explain your answer.

ICT
[Money Match (Amounts 2c-9c)](#)

Assessment

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[384] Crown, (2000): Mathematical Challenges for Able Pupils p.4
[385] Adapted from: https://www.tes.co.uk/teaching-resource/pennies-for-the-peacock-3010672

LEVEL B.1

RECOGNISE, USE, EXCHANGE AND CALCULATE CHANGE USING COINS UP TO THE VALUE OF 50C

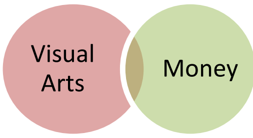
TEACHING NOTES

Learning experiences from level A.1 can be used for the recognition of coins up to the value of 50c.

Games provide excellent opportunities for pupils to practice exchanging and calculating totals in fun and engaging ways. They can be differentiated to cater for all abilities.

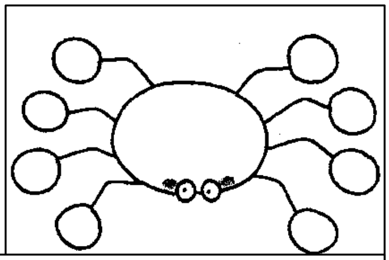
> For a child, money has no connection with measuring. The idea of money as measuring the exchange of goods will be beyond him/her for a long time….What he/she needs is practice in finding equivalences within coins and in matching coins to price. Exchanging and shopping activities give wide and useful experiences of numbers and can follow the child's growing ability to deal with numbers
> (Deboys, M. & Pitt E., (2007) Lines of Development in Primary Mathematics p.88).

SAMPLE LEARNING EXPERIENCES



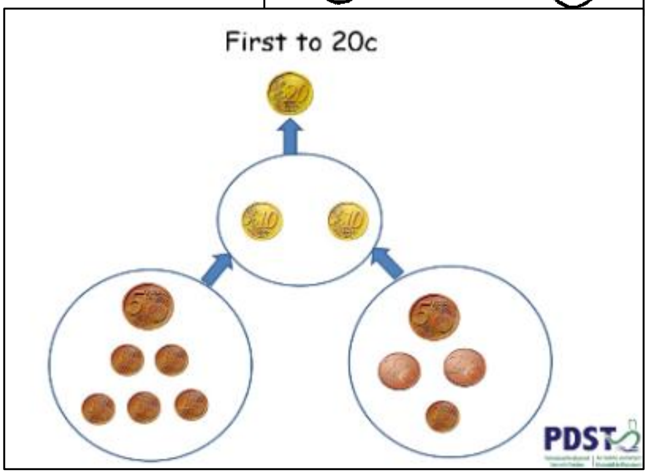
Money Spiders[386]

Using coin rubbings to fill in the legs of the spider. For example, the picture illustrates a 24c Money Spider, select coins for each leg so that the total of the legs equals **24c.**



First to 20c

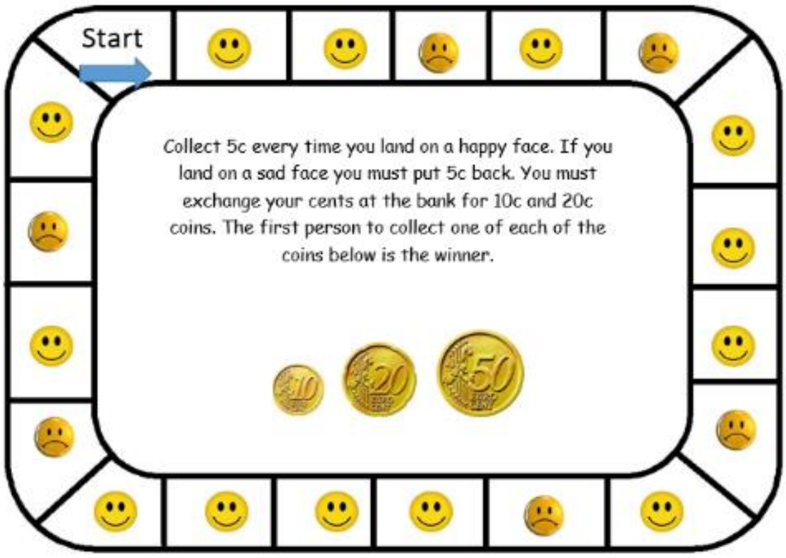
The purpose of this game is to allow pupils to practice exchanging 1c, 2c and 5c coins for 10c and 10c for 20c. To play the game you will need a collection of coins; 1c, 2c, 5c, 10c and 20c coins. You will also need a playing board per pupil and a die numbered 1, 1, 1, 2 and 2. The pupils take turns rolling the die and collecting the corresponding number of coins. The first player to exchange their coins for a 20c coin is the winner.



[386] Accessed at http://www.infant-resources.co.uk/Money.htm

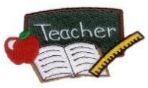
Coin exchange game (version 2)[387]

The aim of this game is to facilitate exchanging 2cs for 10c and 20c.



To play the game you require a collection of 2c, 10c and 20c coins, a game board per pair/group and a die. Collect 2c every time you land on a happy face. If you land on a sad face you must put 2c back. You must exchange your cents at the bank for 10c and 20c coins. Keep moving around the board until a player collects a 10c and 20c coin.

To extend this game (version 3), change the value of the happy/sad faces to 5c and encourage players to practice exchanging 5c for 10c, 20c and 50c coins.



Can you total your 5cs? How many more 5cs do you need to make 20c? Could you do it a different way using less coins?

Calendar Money[388]

Time | Money

Encourage pupils to represent their birthday and other significant dates using coins. For example, a pupil whose birthday is on the 13th could use a 10c, a 2c and a 1c to indicate the date. Then each day, have pupils represent the date with coins. As pupils become more accomplished with the activity, emphasise using the smallest amount of coins to illustrate the date.

What date is St. Patrick's Day? How could you make 17 using coins? Could anyone do it a different way using less coins?

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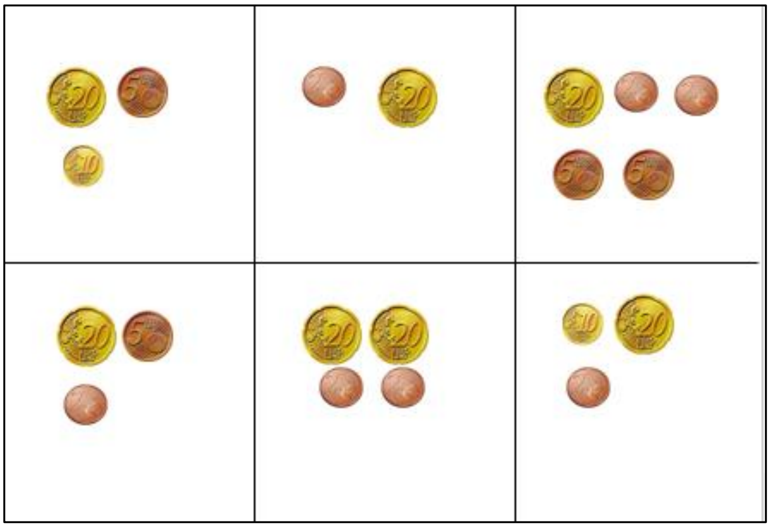
[387] Adapted from: https://www.tes.co.uk/teaching-resource/coin-exchange-6048253

[388] Adapted from http://www.busyteacherscafe.com/themes/money.html

The Money Match website below requires pupils to match the coins to the correct amount.

ICT
Money Match (Amounts 10c-20c)[389]

Money Bingo



Distribute a blank bingo grid to each pupil. Decide on the range of numbers that pupils will be working between for example totals between 20c and 50c. Pupils draw different coin combinations between these amounts in each square. Pupils swap their board with a partner. Each pupil must then total the values of the different sets of coin combinations displayed on their new bingo board and record the answer in each square. The teacher or a pupil calls out different amounts. If the pupil has this amount on their board, they can cover that square with a counter. The game continues until one or more players calls 'Bingo' by either covering 3 in a row or an entire card, as determined by the teacher.

CONSOLIDATION ACTIVITIES

Ride at the Fair[390]

Lucy had a ride at the fair. Her mum asked Lucy to pay less than 20c towards the ride.

Lucy paid exactly three coins towards the ride. How much did Lucy pay her mum?

Find different ways to do it?

[389] http://www.teachingmoney.co.uk/eurosite/games/EURO20match.html
[390]Crown, (2000): Mathematical Challenges for Able Pupils p.8

Money Bags[391]

ICT
Money Bags

Tom divided 15 1c coins among four small bags. He labelled each bag with the number of cents inside it. He could then pay any sum of money from 1c to 15c without opening any bag.

How many 1cs did Tom put in each bag?

Rows of Coins[392]

Take five coins: 1c, 2c, 5c, 10c and 20c.

Put them in rows using these clues.

The total of the first three coins is 27c.

The total of the last three coins is 31c.

The last coin is double the value of the first coin

Coins in my Pocket[393]

I have 5 coins in my pocket.

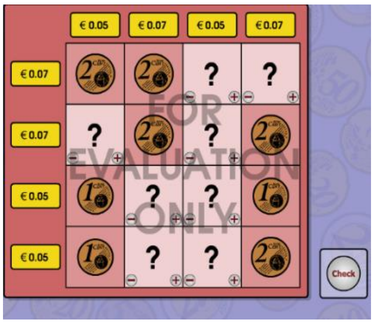
Two coins are the same.

How much could I have?

What's the least/most amount of money that I could have?

Coins Breaker[394]

Pupils work out what the missing coins are to make the totals displayed in the rows and columns correct.



ICT
Coin Breaker

[391] Adapted from https://nrich.maths.org/1116
[392] Crown, (2000): Mathematical Challenges for Able Pupils p.26
[393] Emry, K.; Lewis, L. & Morfett, C. (2006) Open-Ended maths Tasks, p.95
[394] http://www.teachingmoney.co.uk/eurosite/games/coinbreakerEURO.html

LEVEL B.2

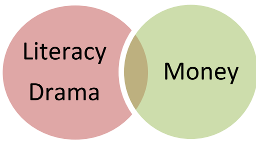
CALCULATE HOW MANY ITEMS MAY BE BOUGHT WITH A GIVEN SUM. CALCULATE CHANGE

TEACHING NOTES

Estimation is an essential skill for real-life mathematics. Pupils should be encouraged to estimate from a very young age. They should be encouraged to make sensible guesses, to test their guesses and revise where necessary. Shopping provides an excellent context for pupils to develop their estimation skills; they can estimate how many items they can buy with a fixed sum of money, what the total of their bill might be and what change they should receive.

Estimation is the process of taking an existing problem and changing it into a new form that is easier to compute mentally and gives an approximate answer. Estimation is a help towards finding a solution but need not in itself be the solution.
(Primary School Mathematics Curriculum (1999) p.32).

SAMPLE LEARNING EXPERIENCES



The Story of Jack and the Beanstalk

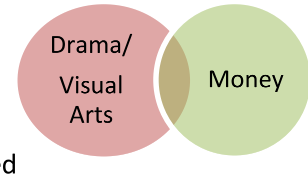
Teaching mathematics through stories plays a key role in enabling pupils to comprehend and generate ideas and imagine situations that reflect the processes and products of mathematics.[395] Fairy tales are one such example and are often used to teach lessons about life. The story of Jack and the Beanstalk teaches important lessons about the value of money and the importance of fair exchange.

Why did Jack's mother not like the trade which Jack made? Was it a fair exchange? Why? Can you think of any reason why beans might not be suitable?

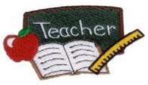
Drama activities such as Conscience Alley or Pupils in Role could be used to explore Jack's actions in exchanging the cow for the beans with an emphasis on price and value. This story could be used as a preliminary activity before setting up a shop in the classroom. It would allow pupils to consider the relationship between price and value before being asked to assign prices in their own shop.

[395] Wilburne, J.M; Keat, J. B; Dile, K; Trout, M; & Decker, S. (2007). Journey into mathematics through storybooks: A Kindergarten story. *Teaching Pupils Mathematics, 14 (4),* pp.232-237.

Snowman Shop[396]



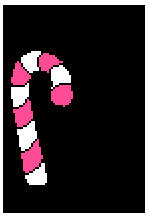
Consider setting up an art shop where pupils have to buy their materials in order to create art work for example making a snowman. This allows real world pricing to be used while working within numbers that pupils are comfortable with. Involve the pupils in fixing realistic prices for the materials and create a price list to be displayed at the shop. Give each pupil a fixed amount of money, depending on the values you working within 20c, 50c etc. and invite them to take turns purchasing items for their snowman. More able pupils can assume the role of the shopkeeper or each pupil can total their own items and calculate the change. The teacher can provide support as needed. When the pupils are finished shopping, they place their items in a Zip-lock bag labelled with their name. These can then be stored until the next Art Lesson.



Can you estimate what the total cost of you items will be? How much change do you need to get back? How many different items could you buy if you spend all of your 20c?

ICT
Change Please

Christmas Shopping[397]





3c	49c	45c	14c	52c

[396]Adapted from http://www.teacherspayteachers.com/Product/KFUN-Shopping-Receipts-Common-Core-concepts-through-FUN-shopping-activities-500531

[397] Adapted from https://nrich.maths.org/162

Vera is shopping at a market with these coins in her purse. Which things could she give exactly the right amount for?

Granny had 50c and She Went to the Sweet Shop

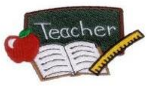
Organise pupils into groups of 6-8. Play the ‘Granny went Shopping’ memory game with the pupils to familiarise/remind them of the rules. The first pupil begins ‘Granny went shopping and she bought for example an apple. The next pupil says ‘Granny went shopping and she bought an apple and adds something else to the list. The game continues with each pupil taking their turn and adding something to the list. If a pupil can’t remember the list in the correct order, they are out. The winner is the pupil who can remember the entire list in the correct order.


Sweet Shop Price List

Fizzy Laces	25c
Sour Dummy	8c
Lollipop	15c
Giant Cola Bottle	12c
Jelly Baby	2c
Starmix	30c
Twisty Marshmallow	5c
Fried Egg	3c
Chocolate Mouse	6c
Catherine wheel	9c
Teeth	7c
Gobstopper	28c
Snake	20c
Strawberry	10c

To make the game more challenging, add in a price list and an amount that Granny can spend. For example, Granny had 50c and she went to the sweet shop. Create a ‘Sweet Shop Price List’ with the pupils and display it in the classroom. Pupils play the game as before but this time they must also take account of the prices of the different items as Granny only has 50c to spend.

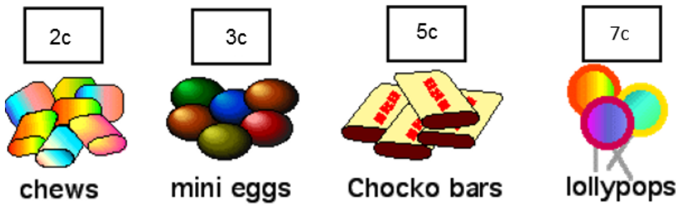
For differentiation purposes, the groups that pupils play the game in can be made smaller or bigger. The amount that Granny has to spend and prices in the ‘Sweet Shop’ can also be increased/decreased.



What’s the most amount of items, Granny can buy to spend her 50c?
If Granny bought Fizzy Laces first, how many more items could she buy? Jack, do you agree with Emma? Why?

The Puzzling Sweet Shop[398]

Rosie went into the sweet shop with 10c to spend. There were chews for 2c, mini eggs for 3c, mini Chocko bars for 5c and lollypops for 7c.



What could she buy if she wanted to spend all her money?

Alice, James, Katie and Henry went into the shop too. They each had 20c to spend and they all spent all of their money.

Alice bought at least one of each kind of sweet. Which one did she have two of?

James spent his money on just one kind of sweet, but he does not like chews. Which sweets did he buy?

Katie bought the same number of sweets as James but she had 3 different kinds. Which sweets did she buy?

Henry chose 8 sweets. What could he have bought?

[398] Adapted from https://nrich.maths.org/223

LEVEL B.3

RECOGNISE, USE, EXCHANGE AND CALCULATE USING COINS UP TO THE VALUE OF €2 AND CALCULATE CHANGE

TEACHING NOTES

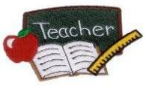
Learning experiences from level A.1 and B. 1 can be used for the recognition of coins up to the value of €2.

SAMPLE LEARNING EXPERIENCES

Hundred Square Cents[399]

The objective of this game is reinforce that there are 100 1cs in a €1

To play the game you need a blank 100 square per pupil and a die per pair/group. Pupils take turns rolling the die. The number they roll, denotes the number of 1c coins that they can draw on their 100 square. The first player to fill each box on their 100 square with a 1c coin, wins the game.



How many 1c coins did it take to fill your 100 square? How much money do you have? Is there another name for a 100c? How could we write it?

(Version 2)

This version of the game reinforces that there are 50 2cs in a €1.

Players need a crayon and a blank 100 square each, a collection of 2c coins and a die per pair/group. The numbers on the die represent jumps of two. For example if a pupil throws a 1, this means one jump of 2. They colour two boxes on their 100 square with their crayon and place a 2c coin on top. The first player to fill their 100 square with 2c coins wins the game.

ICT
Count Up

The game could also be played to highlight that there are 20 5cs in a €1.

[399] Adapted from:
http://www.coreknowledge.org/mimik/mimik_uploads/lesson_plans/1064/Money%20Money%20Money.pdf

Coin Exchange Game (version 4)[400]

The aim of this game is to facilitate exchanging 10cs for 20c, 50c, €1 and €2.

To play the game you require a collection of 10c, 20c, 50c, €1 and €2 coins, a game board per pair/group and a die. Collect 10c every time you land on a happy face. If you land on a sad face you must put 10c back. You must exchange your cents at the bank for 20c, 50c, €1 and €2 coins. Keep moving around the board until one player collects a 20c, 50c, €1 and €2 coin.

ICT
[Printable Ingredient Playing cards]

Checkout Challenge[401]

This game allows pupils to calculate using coins up to €1

The following resources are required for each group: a set of Ingredient Playing Cards for each group of 6 pupils, at least €2 for the shopkeeper in each group, made up of 1c, 2c, 5c, 10c and 20c coins. Each of the shoppers in the group needs 88c, comprising a 1c, a 2c, a 5c, a 10c, a 20c and a 50c coin.

Each group sits with a pile of Ingredient cards shuffled faced down in the centre of the group. The first shopper in the group turns over the top card from the pile, for example a '1 slice of cheese' card. If the shopper chooses to buy this card, he/she must pay the shopkeeper the amount stated on the card. The shopkeeper takes the money, gives change if necessary, and keeps the card. The next shopper turns over a card and also has the opportunity to buy it if desired. The game continues in this manner. If a shopper turns over a card that he/she already owns, he/she should choose not to buy the unnecessary card and should say 'Pass' instead. The winner is the first pupil to have bought each of the five correct Ingredient Cards to make the sandwich i.e. 2 slice of bread cards, 1 slice of cheese card, 1 tomato card and 1 lettuce card.

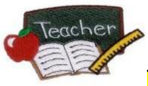
At the Pizzeria

[400] Adapted from: https://www.tes.co.uk/teaching-resource/coin-exchange-6048253

[401] Accessed at http://www.scoilnet.ie/uploads/resources/13163/12800.pdf

Invite pupils to talk about their favourite pizza and toppings. Make a list of the toppings mentioned. Then get the pupils to illustrate their favourite pizzas, labelling the various toppings they have chosen. Return to the toppings list and encourage the pupils to assign prices to the various toppings. Ask the pupils to estimate and work out the actual cost of the toppings on their pizza. Encourage them to swap their pizza with a friend and work out the cost of that pizza.

Assessment



If you only have €2 to spend can you afford all your toppings?

What's the most number of different toppings you can buy if you only have €2 to spend? Did anyone do it a different way?

CONSOLIDATION ACTIVITIES

Race to €1 [402]

Each pair/group will need a die, paper and pencil. The aim of the game is to make €1 first. Pupils takes turns rolling the die and can choose to either take the value shown on the dice for example 3c or to take ten times the value for example 30. Pupils keep adding their own scores until one of them reaches exactly €1. Alternatively, pupils could start at €1 and subtract their scores to see who can lose their €1 and reach 0 first. As pupils' understanding develops, the game could be extended to €2

More Stamps[403]

Rosie spent €2 on 10c and 20c stamps.

She bought three times as many 10c stamps as 20c stamps.

How many of each stamp did she buy?

The Broken Calculator

Mary's calculator is broken. It doesn't always add correctly. She added 57c and 25c. Her calculator showed 81c as the answer. Did the calculator get the correct answer? Why or why not? Explain your answer using pictures, words or numbers.

Assessment

[402] Adapted from PDST: Place Value, Decimals & Percentages: Teacher's Manual p.48
[403] Crown, (2000): Mathematical Challenges for Able Pupils p.60

LEVEL B.4

CALCULATE AND RECORD THE VALUE OF A GROUP OF COINS IN CENTS AND RENAME AS EURO USING € AND DECIMAL POINT.

TEACHING NOTES

Many of the activities outlined above at Level B.3 provide excellent in context examples for pupils to calculate the value of a group of coins and record in cents and euro.

CONSOLIDATION ACTIVITIES

Silent Money

Generate a set of cards that includes amounts in cents and also their equivalent in euro as shown in the example. Cut out the cards and distribute them, giving one to each pupil. Pupils must find their partner without talking. For example, a pupil whose card shows 28c must find the pupil whose card displays €0.28. When the pairs find each other, they can then work together to show the smallest number of coins to make their total.

€0.28	28c	€1.20	120c
€1.43	143c	€2	200c
€1.45	145c	€1.20	120c
€1.07	107c	€1.15	115c
€1.61	161c	€1.21	121c
€0.01	1c	€1.18	118c
€0.65	65c	€1.79	179c
€1.66	166c	€1.13	113c

Euro Cent Pairs

Use a set of cards similar to the above example. Shuffle the cards and spread them out between the players. Player one turns over two cards. If they match i.e. if they show the same amount 89c and €0.89, the player gets to keep the cards. If not, they are returned to the middle of the group, face down. The next player takes their turn. The game continues like this. The winner is the player with the most pairs at the end of the game.

Human Number Line

Distribute some cards with various amounts of money written in cents and euro for example €0.77, 89c, €0.02, €1.14 etc. to a number of pupils who display them in a human number line. Give instructions to the other pupils such as:

- Swap with the person whose card is the same as 2c
- Swap with the person whose card is 20c more 59c
- Swap with the person whose cards is 11c less than €1

Encourage pupils to come up with the questions for the number line.

Money Machine[404]

This interactive money machine demonstrates recording coins in cents and euro.

[404] http://www.teachingmoney.co.uk/eurosite/wb/calceuro.html

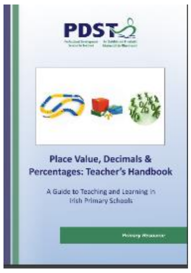
MEASURES - MONEY

LEVEL C.1

SOLVE AND COMPLETE PRACTICAL PROBLEMS AND TASKS INVOLVING MONEY

TEACHING NOTES

The following activities are included to provide pupils with genuine and meaningful contexts to practice addition, subtraction, multiplication and division strategies in relation to money. They are included to allow pupils apply their mathematical thinking to real life contexts. They also intend pupils to develop and use their own strategies and foster their higher order thinking skills.



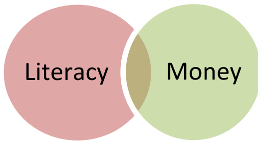
SAMPLE LEARNING EXPERIENCES

Place Value, Decimals & Percentages: Teacher's Manual[405]

Level B.6 of the Place Value, Decimals & Percentages manual has many activities that could be used to compliment pupils' learning experiences at this level.

Menus

Menus from local restaurants can provide excellent opportunities for pupils to work with money using real world situations. Questions can be generated to cater for different learning objectives and also different abilities. Some examples might include



- Which starter is the most expensive? Which is the least expensive? What is the difference between the most expensive and the least expensive?
- Look at the chicken dishes on the menu. Arrange them in order, of cost, starting with the cheapest.
- Mom spent €6.70 on Tom's meal and €11.25 on Rachel's. What could Tom and Rachel have eaten for dinner?
- Your friend Seán is a vegetarian. Which starters and main courses could he choose? Pick one starter and one main course for him and estimate the cost of his dinner. Now work out the actual cost. Pupils could also work in groups to design menus and generate their own questions for other pupils in the class to answer.

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405 Place Value, Decimals and Percentages: Teacher's Manual, A Guide to Teaching and Learning in Irish Primary Schools. Available from: http://cmsnew.pdst.ie/sites/default/files/A%20Guide%20to%20Teaching%20Place%20Value%20Percentages%20and%20Decimals_printing%20version_1.pdf

Franco's Fast Food Menu[406]

This is what food costs at Franco's café.

1 curry and 1 tea cost €4.

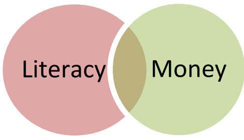
2 curries and 2 puddings cost €9.

1 pudding and 2 teas cost €2.

What do you have to pay in total for 1 curry and 1 pudding and 1 tea?

What does each item cost on its own?

Recipes



The class decides on a recipe that they would like to make. Pupils are then divided into groups. Each group is assigned a supermarket and must work out the cost of purchasing the ingredients needed to make the recipe from their supermarket using the store leaflet or online shop. Groups compare their answers to work out which is cheaper, by how much and what reasons they think there might be for this. Pupils could then make the recipe using ingredients from their assigned supermarket. They could compare the end product and decide if the cost had any influence on the taste.

Catalogues

Store catalogues are another useful resource when teaching money. They tend to be free and stores will usually provide multiple copies. They can form the basis of many open- ended problem solving activities. The following example is based on the Argos catalogue.

Dad hates shopping and has left his Christmas list to the last minute. He has enlisted your help. He has a budget of €400. You decide to look through the shopping catalogue to help him find presents for the rest of the family. His shopping list is as follows:

- Mom wants an electric toothbrush and a bracelet
- Grandad wants a new printer and ink
- Granny wants a golf bag and some golf balls
- Aoife (7) wants Lego and a scooter
- Jack (10) wants k'nex and a racing car

In order to make comparisons easier, it is a good idea if pupils cut out the relevant items and their price from the catalogue and include them as part of their answer.

[406] Crown, (2000): Mathematical Challenges for Able Pupils p.67

Can you estimate how much you will have to spend on each person? How can you check that your solution is correct?

Why did you select these particular gifts, tell me about your reasoning?

Book Order[407]

ICT
Scholastic Online Catalogue

Pupils work in groups and are given the task of compiling a proposal for a book order for the class library. They are given a fixed budget for example €75 and a catalogue from which to choose the books for example Scholastic Catalogue. When compiling the proposal they need to consider that a variety of books to cater for the different interests and reading abilities of the class will be required.

They must also consider the books currently available in the classroom library.
Pupils can choose any computation strategies to come up with their solution. They must record their strategy and check their answers. They submit their solution to the rest of the class. This allows pupils to share their strategies and explain their reasoning in relation to their choices. The class can then vote on the best proposal. This order could then be used to purchase books for classroom library. Pupils could be encouraged to think of fundraising ideas to raise the money to purchase the books for example a class bake sale.

What method did you use to calculate you findings? Could you do it another way? Is there another way you could represent the problem? Why did you select these books, tell me about your reasoning?

George's Marvellous Medicine

Literacy | Money

In this activity, pupils work in groups to find the cheapest way to purchase the ingredients required to make George's Marvellous Medicine. First, pupils devise the list of ingredients needed to make George's Marvellous Medicine from the Roald Dahl book. The task can be broken down into the ingredients George acquired in the different rooms for example bathroom, kitchen, bedroom, laundry room etc. Using store catalogues or online websites, the pupils must work out the cost of buying the ingredients George found in each room. Pupils will need to use their problem solving skills to find equivalents

[407] Accessed at http://www.australiancurriculumlessons.com.au/2014/05/29/create-book-order-authentic-maths-lesson-plan-addition-subtraction/

for the ingredients George found in the shed for example the bottle with the label 'for horses with hoarse throats' and be able to justify their replacement to the rest of the class. Pupils then add the cost of the ingredients from each room to find the total cost of the ingredients needed. Each group will record their solution including the price of the ingredient, where it can be purchased for this price etc. and present their findings to the class.

Pupils may find it useful to pupil-friendly search engines for this task such as www.kidrex.com or www.duckduckgo.com

CONSOLIDATION ACTIVITIES

A Bit Fishy[408]

A goldfish costs €1.80. An Angel fish costs €1.40.

Nóirín paid exactly €20 for some fish.

How many of each kind did she buy?

Picking Fruit[409]

Tom can earn 20c for every apple he pick, 5c for every strawberry and 15c for every banana. If he earned €34, what combination of fruit could he have picked?

[408] Adapted from Crown, (2000): Mathematical Challenges for Able Pupils, p.70

[409] Adapted from Emry, K.; Lewis, L. & Morfett, C. (2006) Open-Ended maths Tasks, p.141

LEVEL D.1

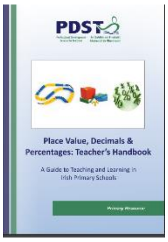
COMPARE AND CALCULATE 'VALUE FOR MONEY' USING UNITARY METHOD AND PERCENTAGES

TEACHING NOTES

The unitary method has many applications in the real world, chiefly in our dealings with money. It is an important concept for pupils to understand as it allows them to solve problems mentally and quickly.[410] It is of particular use when we are shopping and want to save money by comparing prices.

The unitary method involves finding the value of a unit and then using this value to calculate the value of a whole.

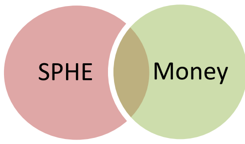
SAMPLE LEARNING EXPERIENCES



Place Value, Decimals & Percentages: Teacher's Manual[411]

Level D of the Place Value, Decimals & Percentages manual has many activities that could be used to compliment pupils' learning experiences at this level.

Good Deal or Bad Deal



Provide pupils with examples of offers and then ask them to decide which the better deal is. It is important that pupils can give a rationale for their answer.

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[410] Australian Mathematical Sciences Institute (2015) Accessed from http:Ll//aLmsi.org.au/teacher_modules/Unitary_Method.html

[411] Place Value, Decimals and Percentages: Teacher's Manual, A Guide to Teaching and Learning in Irish Primary Schools. Available from: http://cmsnew.pdst.ie/sites/default/files/A%20Guide%20to%20Teaching%20Place%20Value%20Percentages%20and%20Decimals_printing%20version_1.pdf

Which is better value? Give a reason for your answer

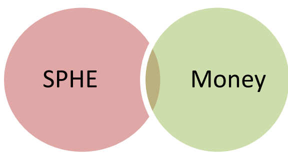
8 oranges for €1.36 or 6 oranges for €1.08
6 pack of crisps for €2.40 or 10 pack for €3.00
3 Danishes for €2.04 or 6 Danishes for €3.90
9 mini ice lollies for €4.41 or 6 mini ice lollies for €2.88
5 scones for €2.35 or 3 scones for €1.35
7 chocolate bars for €2.52 or 4 chocolate bars for €1.44

Loop Cards[412]

Loop cards can provide pupils with opportunity to practice the unitary method using mental calculations.

Product Labels[413]

Ask pupils if they had noticed that many price labels in supermarkets include unit pricing information the cost per unit. Ask them to compare the prices of similar products and find out which is better value for money. They can visit a supermarket to do this or visit online supermarkets. Photographs of price labels could also be used.



412 https://www.tes.com/teaching-resource/template-loop-cards-blank-6140458
413 Accessed from www.consumer.vic.gov.au

Item: ______________________ **Brand:** ______________________

Net amount: ________ Price: ________

Unit cost: ______________________

Compare your product's unit costs with at least two other similar products but a different brand.

Item: ______________________	**Item:** ______________________
Brand: ______________________	**Brand:** ______________________
Net amount: ________ Price: ________	Net amount: ________ Price: ________
Unit cost: ______________________	Unit cost: ______________________

Which product had the lowest unit cost?

....

....

Which product would you buy? Why?

....

....

Using Photographs

Photographs of signs outlining offers from supermarkets and other retail outlets can also be a useful stimulus for practicing the unitary method when considering value for money. Pupils can be encouraged to capture their own examples when they are shopping.

[414]

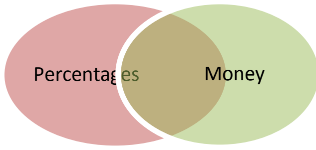
ICT
[Maths Eyes](http://www.haveyougotmathseyes.com/)

Liam, how did you calculate you findings? Tomás, do you agree with Liam? Why? Is there another way you could represent the problem?

[414] http://www.haveyougotmathseyes.com/

MEASURES - MONEY

Store Leaflets

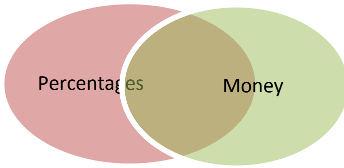


Most supermarkets issue a leaflet on a weekly basis outlining their in store deals. These provide many opportunities for pupils to calculate prices before discounts were applied and to calculate savings being made. Typical questions could include

- What does it cost to buy five Inis vale Fresh Irish Fillet Steaks (200g)? How much money do you save altogether with the 25% off special offer?
- Glenmór Sirloin steak costs €10.49 per kg. This includes a 40% saving. What did the steak cost before the saving was applied?
- Glorious! Chilled soups cost €3.75 per kg after a 25% saving has been applied. What was the cost per kg prior to the discount?

Pupils could work in pairs or groups to create their own questions for other pupils to answer.

Shopping Receipts/Bills



Receipts also allow pupils to examine prices for discounts and savings. They enable pupils to encounter real world examples of VAT; to calculate the percentage of VAT being added, the price before VAT was added etc. The receipt shown here is a copy of the one resulting from Beyonce's trip to Nando's. (See the resources in Appendix D). Questions based on this receipt might include

Nando's
Chicken Restaurants
www.nandos.co.uk

Dublin - Mary St
Telephone: +35318720011
Your Order Number Is -

BEYONCE

Host: Derek	12/05/2013
BEYONCE	10:28 PM
REPRINT# 7	10062
Order Type: Eat In	
Jumbo Platter (7 @56.95)	398.65
(7) Medium	
(7) Hot	
(7) Large Chips	
(7) Large Rice	
(7) Lrg Garlic Bread	
(7) Lrg Corn	
(14) Plain	
(7) Large Coleslaw	
Jumbo Platter (7 @57.25)	400.75
(7) Medium	
(7) Hot	
(7) Large Peri Chips	
(7) Large Rice	
(7) Large Coleslaw	
(7) Lrg Garlic Bread	
(7) Lrg Corn	
(14) Plain	
10 Wing Platter (7 @12.75)	89.25
(7) Roulette	
(7) No Sides	
Managers' Discretion	-12.75
Name: Derek Swan 90006	
Subtotal	803.58
Tax	72.32
Eat In Total	875.90
C Card	€ 870.00
CASH	€ 5.90

9% : 803.58 VAT: 72.32

******

Welcome to Nando's
VAT No : 662 8275 13
--- Check Closed ---

- The manager's discretion was deducted from the bill. What would the bill have cost if he had not done this?
- What percentage of Beyonce's bill was actually tax?
- Beyonce paid for her bill with her credit card, which charges 12.5% interest, how much will her bill actually cost her?
- What profit did the Nando's make if the cost price of the items sold was 75% of the total?

Pay

Pay is an excellent example of how money is used in the real world. It is important for pupils to know that employees may be paid for their work in a variety of ways. The two main ways being wages or salary.

> A wage is based on a fixed rate per hour. Hours outside the normal work period are paid at a higher rate. A salary is a fixed annual (yearly) amount, usually paid fortnightly or monthly. A person who receives a salary is paid to do a job, regardless of the number of hours worked.

MEASURES - MONEY

Different types of problems can be given to pupils to encourage them to consider how one is paid and how this affects the amount of pay one receives. Pupils could be asked to consider the following examples

1. Barbara is a second-year apprentice chef. She receives €12.08 per hour. Joanne, a fourth-year apprentice, earns €17.65 per hour.

   a. How much does Barbara earn in a 40-hour week?

   b. How much more does Joanne earn in the period of time?

2. Who earns more money each week: David who receives €38.55 an hour for 38 hours work, or Tom, who receives €41.87 an hour for 36 hours work?

3. When Shona was offered job as a trainee accountant, she was offered the following choice of 4 salary packages. Which should she choose? Give a reason for your answer

   a. €456 per week

   b. €915 per fortnight

   c. €1980 per calendar month

   d. €23, 700 per year

4. In her job as a receptionist, Elaine works 38 hours per week and is paid €32.26 per hour. Oisín, who works 38 hours per week in a similar job, is paid a salary of €55,280 per year.

   Who has the highest paying job? Give a reason for your answer.

A Mathematics Fantasy [415]

<u>You're sitting in Maths class, when a local businesswoman walks in. She wants to offer you a job. She is a bit sketchy about the details but she will need your services for 30 days and you won't be able to attend school.</u>

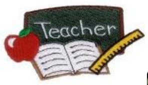
<u>She gives you your choice of two payment options:</u>

1. One cent on the first day, two cents on the second day and double your salary every day thereafter for the thirty days; or

2. Exactly €1,000,000

Which option will you choose and why?

[415] Adapted from http://math.rice.edu/~lanius/pro/rich.html



Clár, can you predict which the better option might be? Why do you think this? Does anyone disagree with Clár? Why? Fionn, how did you work out which the better option is? Is there another way you could represent the problem?

CONSOLIDATION ACTIVITIES

Pet Shop[416]

1. Jim bought a cat and a dog for €60 each. Later he sold them. He made a profit of 20% on the dog. He made a loss of 20% on the cat. How much did he get altogether when he sold the cat and the dog?
2. Jim bought another cat and dog. He sold them for €60 each. He made a profit of 20% on the dog. He made a loss of 20% on the cat. Did he make a profit or a loss on the whole deal?

Fred Says[417]

ICT
[Fred says](#)

Fred is very confused! He's been looking at some trainers that are on sale.

Before the sale they cost €25.

The shop says there is a sale on all footwear today.

BUT on the red price label of the pair he likes, it says there is an extra saving of 5% on all trainers with a red label.

Fred thinks these are a good buy!

BUT...how does he work out how much they are going to cost him?

Does he:

- take off 10% first, then 5% after that?
- take off 5% first then 10% after that?
- take off 15% straight away?

Does it make any difference? How do you know?

What would you tell Fred to tell the shop assistant?

What if...the trainers were €50, €100?

Sometimes shop bills show VAT, a tax paid to the government, separately.

What if … the trainers were reduced by 10% but there was a tax of 5% added?

Would it matter which was calculated first, the VAT or the sale price

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[416] Adapted from Crown, (2000): Mathematical Challenges for Able Pupils, p.71
[417] Adapted from http://nrich.maths.org/9702

LEVEL D.2

CONVERT OTHER CURRENCIES TO EURO AND VICE VERSA

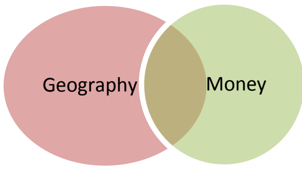
TEACHING NOTES

Currency conversion is the procedure of changing one currency into another currency. The exchange rate is the ratio by which one currency is converted into another. It is the price of one currency expressed in another currency. Exchange rates are necessary because currencies have different values relative to one another.
(Gorman, T. ,2003, The Complete Idiots Guide to Economics.

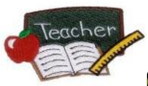
As we live in a globalised economy, it is important that pupils are able to convert one from currency to another. Engaging them in tasks that are based on real life experiences allows them to develop these skills in a meaningful way.

SAMPLE LEARNING EXPERIENCES

One Thousand Euro[418]



You have €1000 saved up. What is its value in different currencies? Investigate which are the 5 best and 5 worst exchange rates. Based on this information, where would you travel on a holiday?



Clár, can you predict which the better option might be? Why do you think this? Does anyone disagree with Clár? Why? Fionn, how did you work out which the better option is? Is there another way you could represent the problem?

[419][420]

ICT
Currencies of the world

ICT
Currency Converter

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[418] Adapted from Emry, K.; Lewis, L. & Morfett, C. (2006) Open-Ended maths Tasks, p.143
[419] http://www.xe.com/currencyconverter/
[420] http://fx.sauder.ubc.ca/currency_table.html

Holiday Brochures

Holiday brochures provide excellent opportunities for pupils to see first-hand the everyday relevance of maths. Pupils could work in groups to plan a family holiday. They could compare prices in different brochures; taking account of discounts given by different companies and value for money that they offer. Online brochures could also be used. The following task, based on planning a family holiday to Disney World, highlights the type of questions pupils could be asked to solve.

Assessment

[421]

Mrs Gibbons recently got a large bonus at work (x amount) and has decided to book a holiday for the whole family to Disney World, Florida. The holiday needs to be budgeted carefully to ensure that the bonus will be sufficient to pay for the trip and the activities they wish to undertake.

They plan to go during the Easter holidays (Mon 2nd to Sunday 15th April) for 7 nights. They will need a family room if possible, if not two adjoining rooms would suffice – one for Mr & Mrs Gibbons, and one for the two pupils: Dennis (aged 8) and Aoife (aged 11). They will need to fly from Dublin to Orlando, then they would prefer to stay in a hotel at the theme park for the week. However, they would be willing to stay outside the park if it is too expensive. In this case they would need to rent a car. The cost of this will need to be factored in budget (car hire, fuel, parking etc.). They will need to purchase park tickets and would like to have the option to visit more than one park. Mrs Gibbons would like to include the cost of food for the week in the budget.

Your task is to work out the costs involved in this trip-of-a-lifetime for the Gibbons family. You will need to select the most appropriate flights, and the best value hotel to ensure the best price. You must then work out:

- What is the cost of the holiday, i.e. flights and accommodation?
- How much will it cost to hire the car for 7 days during their stay if they don't stay at the park?
- What are best park ticket options available?
- How much can they expect to spend on food for the week?
- What will be the total cost of the dream trip for the Gibbons family?

  (Don't forget that the prices quoted will be in dollars and will need to be converted to euro).

ICT
Hotel Sumview

[421] Image accessed at http://matterhorn1959.blogspot.ie/2008/01/delayed-souvenir-friday-post-fly-twa-to.html

Pupils could also be asked to assume the role of a travel agent. They would still have to organise a holiday with the best deal for the family or group but with the added challenge of trying to make a profit for themselves.

ICT
Mathsopoly Resources

Mathsopoly[422]

The family board game Monopoly can be adapted to take account of currency conversions. Prices of hotels, properties, rents etc. are quoted in different currencies so that pupils will need to convert prices to their home currency in order to play the game. Pupils may use newspapers or search online for daily exchange rates.

Currency Traders[423]

ICT
Currency Conversion Challenge

This activity can be completed over a number of weeks or even over a term to monitor currency fluctuations. The teacher can decide how long the timeframe should be. Pupils work in groups and are given an imaginary starting portfolio of €5 million. They use the currency markets to trade their balance to make their €5 million as large as possible by transferring it between different currencies. At the end of the time frame decided, groups will have to transfer their portfolio back to euro and the group whose portfolio is worth the most, wins the game.

The following rules can be included to make the game more challenging

- the portfolio should consist of at least 4 different currencies at any one time
- currencies can only be traded from the given list (see table below)
- the portfolio needs to be checked on a daily basis
- a short summary of the decisions made by the group and the reasoning behind them needs to documented daily (see grid below). This needs to be done even if the group has decided not to change the portfolio on that day.

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[422] Accessed at https://www.tes.com/teaching-resource/currency-exchange-mathsopoly-6333984
[423] Adapted from http://www.bized.co.uk/educators/games/currency/student.htm

Pupils can be encouraged to examine the reasons for changes to exchange rates and to predict how these might influence their decisions regarding their portfolios. In History, pupils could study the effects of the stock market crashes such as during the Great Depression in 1930's America.

Date	Change (Currency and Amount)	To (Currency and Amount)	Exchange Rate

Reason for changes:

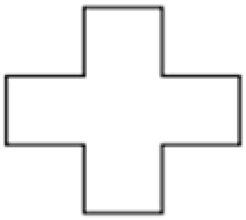
-
-
-
-
-

Country	Currency
Australia	Australian Dollar
Canada	Canadian Dollar
China	Renminbi (Yuan)
Eurozone	Euro
Hong Kong	Hong Kong Dollar
India	Rupee
Japan	Yen
Mexico	Peso
Russia	Rouble
Switzerland	Swiss Franc
Thailand	Baht
Turkey	Lire
United Kingdom	Pound Sterling
United States	Dollar

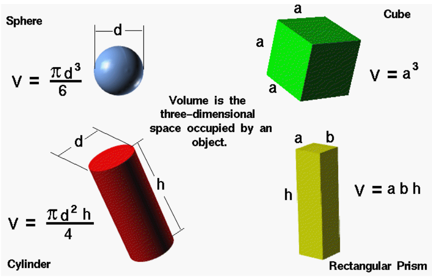
GLOSSARY OF MEASURES TERMS

TERM	DEFINITION
Area	The amount of a plane enclosed by a 2-D shape measured in square units.
Capacity 	Capacity is a measure of the amount of liquid that a container can hold. Only containers have capacity. Example: The capacity of the bucket is twenty litres so it takes a volume of twenty litres of water to fill it.
Centi	A prefix) denoting one hundredth; for example, a centilitre (cl) is one hundredth of a litre
Centimetre	A metric unit for measuring length, equal to one hundredth of a metre.
Currency	A system of money in general use in a particular country.
Decametre	very rarely used unit of length in the metric system, equal to ten metres or one hundredth of a kilometre.
Deci	A prefix (d) denoting one tenth for example a decilitre (dl) is one tenth of a litre.
Decimetre	A decimetre is one tenth of a metre or 10 centimetres.
Density	The ***density*** of a substance is the relationship between the mass of the substance and how much space it takes up, its volume. *Mercury is the densest liquid. 1 litre of water weighs 1Kg, however 1 litre of Mercury would weigh 13.5 Kg.*
Discount	This is a reduction (usually a percentage). This is associated with money.
Displacement	This involves putting an object into water and carefully recording how much the water level rises. The rise in the volume of water is equal to the volume of the object that it now contains.
Foreign Exchange Rate	This is the value one currency has in relation to another. Example: Foreign exchange rate. €1.00 = \$ 1.39 thus €100 = \$ 139. \$ 2085 = 2085 ÷ 1.39 = € 1500
GMT	The time in Greenwich, England, that is used as the basis of standard time throughout the world
Hectare	A hectare is the standard metric unit used to measure large area. A hectare is equal to 10,000 square metres.
Hectometre	A metric unit of length equal to one hundred metres.
Height	The measurement of length from top to bottom.

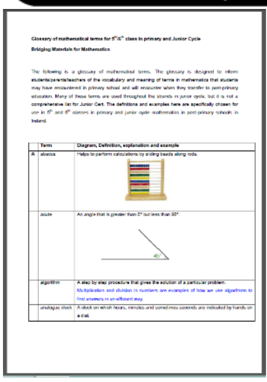
MEASURES - GLOSSARY

Imperial System	The system of **imperial units** first defined in the British Weights and Measures Act of 1824, which was later refined and reduced. By the late 20th century, most nations of the former empire had officially adopted the metric system as their main system of measurement; however some imperial units are still used in countries formerly part of the British Empire.
Interest Rate	This is the percentage of total earned on an investment or paid on a loan. Example: €100 invested in a bank for 1 year at an interest rate of 10% will accumulate to €110.
Kilogram	Unit of mass (1000 grams) 1 kg = 1000 g
Kilometre	Unit of length (1000 metres) 1kg = 1000m
Length	The **length** of the object is the number of standard units (such as centimetres) which can be laid in a straight line along or beside the object
Litre	Unit of capacity for measuring liquids. 1 litre =1000ml
Mass	A measurement of the quantity of matter in an object measured in grams and kilograms, technically not the same thing as weight
Metre (m)	The SI unit of length; about the distance from my nose to my fingertip when my arm is outstretched
Metronome	a device that makes a regular, repeated sound to show a musician how fast a piece of music should be played
Millilitre	One thousandth of a litre, written as 1 ml. 
Millimetre	This is one thousandth of a metre, written 1 mm.
Perimeter	This is the sum of the length of the sides of a figure or shape.
Profit	This is the measure of gain in a financial transaction.
Ratio	Is a comparison of two or more quantities. Example: When making concrete you mix 9 parts of gravel with 2 parts cement. The ratio of gravel to cement is 9:2
Rectilinear Shape	A rectilinear shape is a shape all of whose edges meet at right angles. 

MEASURES - GLOSSARY

Square centimetre	The area equal to a square that is 1 centimetre on each side.
Square metre	The area equal to a square that is 1 metre on each side.
Square kilometre	The area equal to a square that is 1 kilometre on each side.
SI Units	An agreed international system of units for measurement, based on one standard unit for each aspect of measurement.
Surface Area	Total area of the surface of a three-dimensional object. Surface area is measured in square units.
VAT	A **value added tax** (**VAT**) is a consumption tax added to a product's sales price. It represents a tax on the "value added" to the product throughout its production process
Volume	The three dimensional space taken up by an object. Measured in cubic units such as cubic centimetres(cm^3) or cubic metres ($m^{3)}$ 
Weight	The gravitational force or pull exerted on an object. Weight is measured in newtons; colloquially it is used incorrectly as a synonym for mass

www.amathsdictionaryforkids.com provides highly visual, interactive definitions on over 650 common mathematical terms.



Click the image to access the draft primary mathematical glossary designed by the NCCA to provide clarity of mathematical terms used in the Mathematics Curriculum. It is not a mathematical dictionary, but is written in 'plain English’

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APPENDICES

Appendices including loop cards and game boards are available online at www.pdst.ie/numeracy

Appendix A- Length
Appendix B - Weight
Appendix C – Time
Appendix D – Money
Appendix E – Sample Learning Log