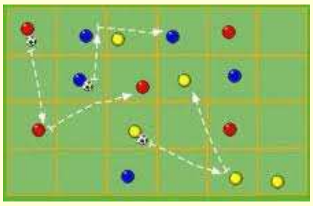
Professional Development Service for Teachers



Shape and Space Manual

A Guide to Teaching and Learning in Irish Primary Schools

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CONTENTS

AIM

The aim of this resource is to assist teachers in teaching the strand of Shape and Space (infants to 6th class). The resource is intended to complement and support the implementation of the Primary School Mathematics Curriculum (PSMC) rather than replace it. By providing additional guidance in the teaching and learning of Shape and Space, this resource attempts to illuminate an instructional framework for enhancing mathematical thinking. This instructional framework advocates methods of eliciting, supporting and extending higher-order mathematics skills such as reasoning; communicating and expressing; integrating and connecting; and applying and problem solving. Although, this resource highlights the Shape and Space strand, this instructional framework can be used for all strands and strand units of the PSMC.

INSTRUCTIONAL FRAMEWORK

INSTRUCTIONAL STRATEGIES

Table 1.1 on the following page illustrates a framework for advancing mathematical thinking. Although it does not explicitly refer to concrete materials or manipulatives, the use of these are often a prerequisite for developing mathematical thinking and can be used as a stimulus for this type of classroom discourse.

Table 1.1 Strategies for Supporting and Developing Mathematical Thinking

Eliciting	Supporting	Extending
Facilitates pupils' responding Elicits many solution methods for one problem from the entire class *e.g. "Who did it another way?; did anyone do it differently?; did someone do it in a different way to X?; is there another way of doing it?"* Waits for pupils' descriptions of solution methods and encourages elaboration Creates a safe environment for mathematical thinking *e.g. all efforts are valued and errors are used as learning points* Promotes collaborative problem solving *Orchestrates classroom discussions* Uses pupils explanations for lesson's content Identifies ideas and methods that need to be shared publicly *e.g. "John could you share your method with all of us; Mary has an interesting idea which I think would be useful for us to hear."*	*Supports describer's thinking* Reminds pupils of conceptually similar problem situations Directs group help for an individual student through collective group responsibility Assists individual pupils in clarifying their own solution methods *Supports listeners' thinking* Provides teacher-led instant replays *e.g. "Harry suggests that ...; So what you did was ...; So you think that ...".* Demonstrates teacher-selected solution methods without endorsing the adoption of a particular method *e.g. "I have an idea ...; How about ...?; Would it work if we ...?; Could we ...?".* *Supports describer's and listeners' thinking* Records representation of each solution method on the board Asks a different student to explain a peer's method *e.g. revoicing (see footnote on page 8)*	*Maintains high standards and expectations for all pupils* Asks all pupils to attempt to solve difficult problems and to try various solution methods *Encourages mathematical reflection* Facilitates development of mathematical skills as outlined in the PSMC for each class level *e.g. reasoning, hypothesising, justifying, etc.* Promotes use of learning logs by all pupils *e.g. see Appendix A for a sample learning log* *Goes beyond initial solution methods* Pushes individual pupils to try alternative solution methods for one problem situation Encourages pupils to critically analyse and evaluate solution methods *e.g. by asking themselves "are there other ways of solving this?; which is the most efficient way?; which way is easiest to understand and why?".* Encourages pupils to articulate, justify and refine mathematical thinking *Revoicing can also be used here* Uses pupils' responses, questions, and problems as core lesson including student-generated problems *Cultivates love of challenge*

This is adapted from Fraivillig, Murphy and Fuson's (1999) Advancing Pupils' Mathematical Thinking (ACT) framework.

CLASSROOM CULTURE

Creating and maintaining the correct classroom culture is a pre-requisite for developing and enhancing mathematical thinking. This requires the teacher to:

- cultivate a 'have ago' attitude where all contributions are valued;
- emphasise the importance of the process and experimenting with various methods;
- facilitate collaborative learning through whole-class, pair and group work;
- praise effort;
- encourage pupils to share their ideas and solutions with others;
- recognise that he/she is not the sole validator of knowledge in the mathematics lesson;
- ask probing questions (see Appendix B for a list of sample questions and sample teacher language);
- expect pupils to grapple with deep mathematical content;
- value understanding over 'quick-fix' answers; and
- use revoicing[1] (reformulation of ideas) as a tool for clarifying and extending thinking.

In this type of classroom pupils are expected to:

- share ideas and solutions but also be willing to listen to those of others; and
- take responsibility for their own understanding but also that of others.

[1] Revoicing is 'the reporting, repeating, expanding or reformulating a student's contribution so as to articulate presupposed information, emphasise particular aspects of the explanation, disambiguate terminology, align pupils with positions in an argument or attribute motivational states to pupils' (Forman & Larreamendy-Joerns, 1998, p. 106).

BACKGROUND KNOWLEDGE FOR TEACHERS

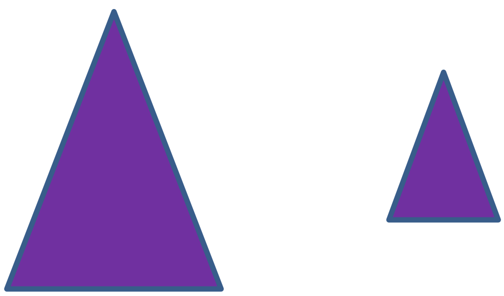
FUNDAMENTAL FACTS REGARDING SHAPE AND SPACE

These fundamental facts have been divided into four sections:

- fundamental facts associated with 2-D shapes;
- fundamental facts associated with 3-D shapes;
- fundamental facts associated with symmetry; and
- fundamental facts associated with spatial awareness.

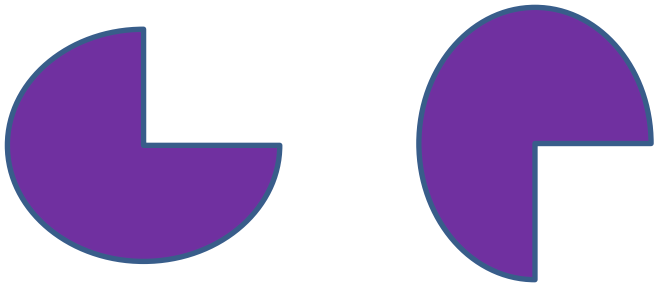
2-D SHAPES: FUNDAMENTAL FACTS

1. 2-D shapes are strictly speaking the surfaces of 3-D shapes.[2]
2. Shapes can be described and categorised by their geometric properties. Shapes:
    - have sides that are parallel, perpendicular, or neither;
    - have line symmetry, rotational symmetry, or neither; and
    - are similar, congruent, or neither.[3]
3. The word 'similar' is used to describe 2-D shapes that have the same angles, even if the size of the shapes is different. So proportionality is not important for shapes to be 'similar' but angles are. One can be a scaled version of another; however, all matching angles must be the same size and all matching edges must be in the same proportion.



[2] Suggate, Davis & Goulding (2010, p. 229)
[3] Van de Walle, Karp & Bay-Williams (2013, p. 402)

4. However, shapes are 'congruent' if they are identical in shape and size – even if this requires one shape being rotated or reflected. [4]



5. Tessellation means 'tiling' – shapes tessellate if they can be used to cover a surface without gaps between them or without overlapping. Equilateral triangles, squares and regular hexagons tessellate. Any triangle or any quadrilateral can be used to form a tessellation. Shapes that will tessellate can also be made into irregular shapes that also tessellate.

3-D SHAPES: FUNDAMENTAL FACTS

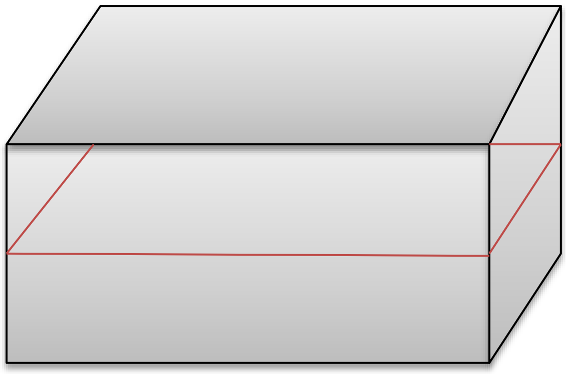
1. 'Nets' are 'flat' shape patterns that can be folded into 3-D shapes. The arrangement of these figures on the flat configuration does not necessarily correspond with the arrangement of these figures on the object to be made because some of these sides may be some distance from each other.
2. 2-D shapes are the surfaces of 3-D objects. It is incorrect to refer to 2-D shapes as being 'flat' shapes because these could in fact be very thin 3-D objects. Therefore, using a 'net' of a 3-D shape may be helpful when considering the construction of that shape; however, the spreading out flat of the net is only a model, strictly speaking it is not 2-D.
3. Some 3-D shapes exhibit a particular rule between the number of vertices, edges and faces. Euler's rule does not apply to all 3-D shapes; however, it applies to most of the ones that primary school pupils encounter (see table 1.1 below). Euler's rule states that if you add the number of faces and the number of vertices together, the total is two more than the number of edges. [5]

[4] Suggate, Davis & Goulding (2010, p. 242)
[5] Suggate, Davis & Goulding (2010, p. 258-259)

3-D Shape	*Number of edges*	*Number of vertices*	*Number of faces*
cube	12	8	6
triangular prism	9	6	5
tetrahedron	6	4	4
octahedron	12	6	8
square pyramid	8	5	5

V + F = E + 2

4. Prisms and pyramids gain their names from their polygon 'bases'.
5. Planes of symmetry are ways in which 3-D shapes can sliced to leave two identical 'halves



SYMMETRY: FUNDAMENTAL FACTS

1. 'Line symmetry' is when one side of a line in a shape mirrors the other side, that is, one side is an exact reflection of the other. There are two simple ways to check for line symmetry: a) if it is a piece of paper, by folding it along the line and if both sides match the shape has line symmetry and b) by holding a mirror along the line, the part visible plus its reflection would look the very same as the original shape. It is incorrect to suggest that a picture is symmetrical if it is the 'same on both sides' because if it is the same on both sides then the reflection may be different. So, it is

possible for both sides to be different yet for the picture to be symmetrical, provided that each side is reflection of the other. (FS, p.148) The reflected image is the result of points on either side of the line of symmetry being equidistant.

2. 'Rotational symmetry' is when a shape fits its original outline when rotated about its centre. To calculate the angle that a shape must be turned for rotational symmetry, you must find out in how many positions the shape fits its original outline, for example, in a square this would be four positions. So a square has rotational symmetry of order 4. So 4 is then divided into 360° (a full rotation) – the turn required for the shape to fit its original outline is 90°.
3. There are forms of symmetry associated with each of the transformations – translational (slide) symmetry, reflectional (mirror or flip) symmetry and rotational (turn) symmetry.[6]
4. If a shape has 2 lines of symmetry then it will have a rotational order of 2. If it has 3 lines of symmetry it will have a rotational order of 3. If it has n lines of symmetry, and n is greater than 1, it will have rotational symmetry of order n.[7]
5. A shape can have rotational symmetry *without* having line symmetry.

SPATIAL AWARENESS: FUNDAMENTAL FACTS

1. All maps have one common feature – they represent what comes 'between' what, that is, the arrangement or order of objects within an environment.
2. Plans show the placement and relative size of things from a top view. Plans have many common features with the types of maps that are found in road directories, for example, they show a top view and are drawn to scale. This allows for direction and relative distances between objects to be preserved. However, in a map objects are not usually drawn to scale, instead they are represented by symbols and labels. In plans, how objects are positioned to each other are important, so objects are drawn to scale and are angled correctly in relation to each other. [8]

[6] Department for Education and Training for Western Australia (2005, p. 140)
[7] Suggate, Davis & Goulding (2012, p. 239)
[8] Department for Education and Training for Western Australia (2005, p. 38)

3. Furthermore, there are many different types of maps and plans. The characteristics of these maps and plans are illustrated in the following table which is found in First Steps in Mathematics: Shape (p.54):

Map Type	Representations	Distances between Objects (proximity)	Relative Position of Objects
Mud maps	Key features relevant to a particular journey drawn informally	Not drawn to scale but symbols or verbal instructions may be used	Landmarks usually drawn in order; informal indication of direction may be used
Network diagrams	Dots (nodes) represent the feature of interest; lines indicate connections between items	Not shown	Order preserved by lines that indicate connections between objects; direction not shown
Scale maps	Key features within certain boundaries are represented (mainly with symbols)	Drawn to scale	Bird's eye view showing relative position
Plans	Key features within certain boundaries are drawn to scale	Drawn to scale	Bird's eye view showing relative position and orientation

PUPIL MISCONCEPTIONS INVOLVING SHAPE AND SPACE

1. Many pupils initially relate direction to their own bodies, therefore directions such as ‘turn to your left’ can be easier to learn that directions associated with a compass (north, south, east, west) or fixed external reference points, for example, ‘turn towards Dublin’.[9]
2. Many pupils have difficulty understanding the conventional ‘bird’s eye’ view that is used in plans and maps, in particular they may have difficulty with the interpretation of scale.[10]
3. Orienting oneself to directions and location on a map or a plan is quite difficult because it requires an understanding of relationships between size, shape and angle.
4. Many pupils come to associate a particular shape with a mental image, for example, all triangles being equilateral and sitting on its base. In order to classify a shape, pupils compare a shape to their mental image. This may not be helpful as a pupil’s mental image of a triangle may be limited.

   Therefore, using mental images to classify shapes is mathematically unsatisfactory. [11] Pupils need to progress from thinking of shapes as whole images to thinking of the properties of shapes or parts of shapes.

5. Using the word ‘shape’ to describe 2-D shapes and 3-D shapes can cause confusion for pupils. It may be helpful for teachers to use the following convention:
   - call 2-D things ‘figures’
   - call 3-D things ‘objects’

   so a circle is a figure but a sphere is an object. Similarly, a tree is an object but a surface of a leaf is a figure.[12]

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[9] Department for Education and Training for Western Australia (2005, p. 19)
[10] Department for Education and Training for Western Australia (2005, p. 49)
[11] Suggate, Davis & Goulding (2010, p. 232)

6. Making prisms from pre-drawn nets does not help pupils to focus on the number of shapes (figures), the types of shapes (figures) and the way that these shapes (figures) are connected to each other.

[12] Department for Education and Training for Western Australia (2005, p. 71)

LEARNING TRAJECTORY

The learning trajectory is based on the objectives for Shape and Space in the Primary School Mathematics Curriculum. In some instances, similar objectives at the same class level have been collapsed into one objective. Objectives that only refer to problem solving have not been included as discrete objectives because a problem solving approach is advocated throughout all of the teaching and learning experiences. Problem solving is viewed in this manual as a fundamental, integral part of mathematics teaching and learning that pupils should experience every day. The same colour coding from the curriculum is used – infants (green); first and second (red); third and fourth (blue); fifth and sixth (orange).

SHAPE AND SPACE LEARNING TRAJECTORY LEVEL A[13]

Trajectory Levels	Concept	Developmental Experiences		
		Concrete	Pictorial	Abstract
	Level A.1 Explore, discuss, develop and use the vocabulary of spatial relations (positional and directional)			
	Level A.2 Sort, describe and name 3-D shapes including cube, cuboid, sphere and cylinder			
	Level A.3 Sort, describe and name 2-D shapes including square, circle, triangle and rectangle			
	Level A.4 Combine and divide 3-D and 2-D shapes to make larger or smaller shapes			
	Level A.5 Use suitable 3-D and 2-D structured materials to create pictures			

[13] This level is generally aligned with the objectives for junior and senior infants.

SHAPE AND SPACE LEARNING TRAJECTORY LEVEL B[14]

Trajectory Levels	Concept	Developmental Experiences: Concrete	Pictorial	Abstract
	Level B.1 Explore, discuss, develop and use the vocabulary of spatial relations (positional and directional)	***As for Learning Experiences in Level A.1***		
	Level B.2 Describe, compare and name 3-D shapes including cone	***As for Learning Experiences in Level A.2***		
	Level B.3 Sort, describe, compare and name 2-D shapes including square, rectangle, triangle, circle, semicircle, oval	***As for Learning Experiences in Level A.3***		
	Level B.4 Construct, draw, combine and partition 2-D shapes			
	Level B.5 Identify halves and quarters of 2-D shapes			
	Level B.6 Identify line symmetry in shape and in the environment			
	Level B.7 Explore and recognise angles in the environment			
	Level B.8 Identify and discuss the use of 2-D and 3-D shapes in the environment			

[14] This level is generally aligned with the objectives for first and second class.

SHAPE AND SPACE LEARNING TRAJECTORY LEVEL C[15]

Trajectory Levels	Concept	Developmental Experiences: Concrete	Pictorial	Abstract
	Level C.1 Identify, describe and classify 3-D shapes by comparing their properties including triangular prism and pyramid	***As for Learning Experiences in Level A.2 and B.2 Extension activities included***		
	Level C.2 Explore, describe, compare and classify the properties of 2-D shapes including oval and irregular shapes	***As for Learning Experiences in Level A.3 and B.3 Extension activities included***		
	Level C.3 Construct and draw 2-D shapes; construct 3-D shapes			
	Level C.4 Identify, describe and classify 2-D shapes including equilateral, isosceles and scalene triangle; parallelogram; rhombus; pentagon; octagon			
	Level C.5 Combine, tessellate and make patterns with 2-D shapes			
	Level C.6 Identify, draw and recognise line symmetry in the environment and in shapes			
	Level C.7 Identify, describe and classify parallel, perpendicular, vertical, horizontal and oblique lines			
	Level C.8 Classify angles as greater than, less than or equal to a right angle			
	Level C.9 Recognise an angle in terms of a rotation			
	Level C.10 Draw, discuss and describe intersecting lines and their angles			

[15] This level is generally aligned with the objectives for third and fourth class.

SHAPE AND SPACE LEARNING TRAJECTORY LEVEL D

Trajectory Levels	Concept	Developmental Experiences: Concrete	Pictorial	Abstract
	Level D.1 Identify and examine 3-D shapes and explore relationships including tetrahedron octahedron	***As for Learning Experiences in Level C.1 Extension activities included***		
	Level D.2 Draw the nets of simple 3-D shapes and construct the shapes			
	Level D.3 Tessellate combinations of 2-D shapes			
	Level D.4 Classify 2-D shapes according to their lines of symmetry			
	Level D.5 Make informal deductions about 2-D shapes and their properties			
	Level D.6 Identify the properties of the circle and construct a circle of given radius or diameter			
	Level D.7 Construct triangles from given sides or angles			
	Level D.8 Plot simple co-ordinates and apply where appropriate			
	Level D.9 Recognise, classify and describe angles and relate angles to shape and the environment			
	Level D.10 Recognise angles in terms of a rotation	***As for Learning Experiences in Level C.11 Extension activities included***		

	Level D.11 Estimate, measure and construct angles in degrees			
	Level D.12 Explore the sum of the angles in a triangle and quadrilateral			

LEVEL A TEACHING AND LEARNING EXPERIENCES

LEVEL A.1: EXPLORE, DISCUSS, DEVELOP AND USE THE VOCABULARY OF SPATIAL RELATIONS (POSITIONAL AND DIRECTIONAL)

TEACHING NOTES

> 'Pupils need to hear the language of position and have an opportunity to respond to the language before they can begin to use the language for themselves…unless pupils are given a real purpose for using the appropriate language, they are unlikely to make it a useful part of their vocabulary.' (First Steps, p.23)

Young children initially begin to indicate position by pointing, then by using simple phrases like 'over there'. Subsequently, the concept of position progresses being about the position of things in relation to other things, for example, "above"; "behind"; "in between"; "next to"; etc. It is important that the teacher models specific positional language at this stage. These ideas quickly develop into concepts of direction and movement, often in the context of giving instructions to get from one place to another. Pupils will need many varied experiences in arranging and rearranging familiar objects. This should be undertaken a) freely and b) by following and giving oral instructions. Pupils will need opportunities to try out the language for themselves rather than simply listening to it.

> To develop confidence and proficiency in the vocabulary of spatial awareness, the pupil's experience must be practical and be applied to real-life situations.

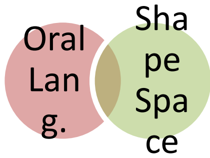
Pupils also need to develop spatial awareness where they can visualise shapes, spaces and objects in their 'mind's eye'. This can be developed by encouraging pupils to visualise things familiar to them.

We can't see it but close your eyes and picture the PE hall,
what shape are the tiles on the wall?
What is underneath the big window?
What is beside the stage?

> Many people believe that you either naturally have spatial sense or you don't. This is not true! Meaningful and consistent experiences with shape and spatial relations over time will develop spatial sense.

The following teaching and learning experiences are designed to support oral language skills and in particular, the vocabulary of spatial relations. Guided discussion and a hands-on approach are essential to enhancing pupils' learning.

SAMPLE LEARNING EXPERIENCES



Incidental use of Language

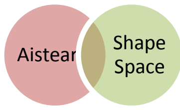
It is important to model the use of positional language in context throughout the school day. It is also important to ask questions that require answers that use the language of position and direction.

The crayons are in between the blocks and the play dough, can you please get them?
Where is the whiteboard marker?
Should we put those paintings underneath the shelf?

It is also important that pupils do not point to indicate position or direction but that they use language instead. If pupils do not have the language then the teacher can model it and then ask the pupils to repeat it.

Where is it Isobel?
Try to tell me with words please.
Do you mean it is underneath the sink? Yes, I can see it now.
Now you tell me where it is.

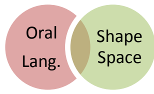
Play



Informal play experiences provide ideal opportunities for pupils to apply positional and directional language. Undirected play including water and sand play; use of construction materials; and model making will give pupils the opportunity to handle objects in a variety of situations.

Tell me how you made your building.
What is happening to the water?
Tell me about your collage.

Living Charts



1. The topic of position can be introduced by asking a series of questions that require answers that use the language of position and direction.

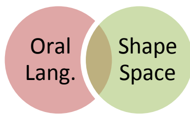
Where are the crayons?
Where is Sarah sitting?
Where would I have to walk to get outside? (

2. The teacher writes down the words that pupils use to describe position or direction on a 'living chart' (a large piece of card or paper on which the teacher acts as a scribe to record the pupils' ideas and language.)

3. The teacher asks pupils to suggest why these words have been chosen for the chart (these are words that we use to describe where things are or where they are going).
4. The teacher adds to the list by including any other words that pupils suggest.
5. The teacher asks pupils to use the words in other contexts to ensure that all pupils understand the meaning of the words on the list.
6. The living chart is displayed and can be added to over time as new words arise in stories, videos or through conversation.
7. Teachers can ask questions to reinforce the language and also as a form of assessment.

I am thinking of something that is under Sam's desk – what is it? Here is a book, can you put it in-between Mary and Tom? I am thinking of something that is on the shelf beside the dictionaries – what is it?

It is important to have high expectations regarding pupils' use of language. When pupils use limited positional and directional language *'it's over there/ it's near the door'*, they should be encouraged to be more precise, for example, *'it's underneath the window, it's to the left of the door', etc.*



Deir O'Grádaigh

1. Play a game of Deir O'Grádaigh using the language of position and direction.

Deir O'Grádaigh: stand on your chair, crouch under your desk, stand behind your chair, turn to your left/right, step forward, step backward, put your left/right hand up.

2. Pupils can take turns to pose the questions to the rest of the class.
3. Pupils can play in groups with one child in each group asking the questions.

Giving Instructions

After playing games like 'Deir O'Grádaigh where pupils follow instructions:

1. invite pupils to give the instructions for a partner to follow, for example, how to set the table; how to make a car from blocks; etc.
2. extend this by having pupils direct the movement and positioning of a third object, for example, *put teddy in the car next to the toy box*, etc. Encourage pupils to use positional language to clarify what their partner means, for example, *Do you want the teddy in, on or next to the toy box?*, etc.

Eye Spy

Play Eye Spy games with the pupils and let them take turns asking the questions also.

Eye Spy something that is above the window, something that is underneath the tables, something that is between the coat racks…

Magnetic Whiteboards

In the absence of magnetic whiteboards, this activity can be completed by placing objects on a table instead. Simple instructions are given to pupils. Once pupils are comfortable with this activity they can give instructions to a partner.

Put the red aeroplane above the boat, put the van below the green aeroplane, put the car beside the van...

Geog. Shape Space

Play Mats[16]

Play mats showing a street or farm layout can provide opportunities for the development of spatial and directional language. They introduce an aerial perspective to pupils in a play setting. Directions and distances can be discussed as toy vehicles and figures are moved around the mat. Railways and model farms can also be used in this way.

Where should we put the shops in our town? Ok, right beside the school is a great idea. Let us drive past the school and then turn left to go to the park.
Tell me about the route you took.

Geog. Shape Space

Google Maps[17]

Google maps can be used to display a large aerial photo of the local area. Pupils can practise describing routes using 'left' and 'right' in relation to the streets or roads to go from one place to another. For example, *to go from Stephen's house to school, turn left at the first corner, and then left at the next corner and then right*. A simple neighbourhood walk could

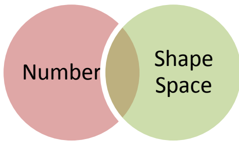
ICT Opportunities

[Google Maps](https://maps.google.co.uk/)

[16] Primary School Curriculum: Geography Teacher Guidelines
[17] Google Maps: accessed at https://maps.google.co.uk/

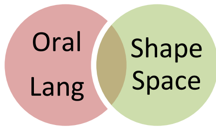
also be used where the route can be followed on Google Maps. Furthermore, local maps can often be sourced in tourist information centres.

Human Number Line



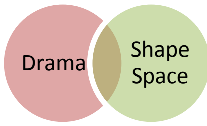
1. A4 sized number cards are distributed to a number of pupils, for example, numbers from 2 to 12.
2. Pupils randomly line up at the front of the class, holding up these numbers at chest height.
3. The rest of the class instruct pupils to move in the line in order to put the numbers into the correct sequence. The teacher decides whether the numbers are to go forwards or backwards. *'Number five move beside number four, no not that side, the right hand side, number ten please stand in between number nine and eleven.'*
4. The pupils sitting at desks can have their own set of small number cards in front of them so that they can also follow the instructions given.
5. Once the numbers are in the correct order they can be used for counting forwards and backwards. Making a line on the floor with masking tape can help young pupils to organise themselves.

Mixed-up Pictures



Pupils are shown "mixed-up" pictures, in which some items are upside-down or in the wrong position. Pupils have to identify the proper orientation or position for each item. *'The baby shouldn't be on top of the fridge, the baby should be in the cot.' 'The chair is upside down'. 'The bowls shouldn't be under the table, they should be on top of the table'.*

Using Stories



Pupil's knowledge of relative position in space can be extended through conversations, demonstrations, and stories. Pupils should be encouraged to use familiar stories as the basis for short dramas that incorporate positional language. Following this, pupils can draw simple maps based on the story. For example, when pupils act out the story of The Three Billy Goats and illustrate *over* and *under*, *up* and *down*, *near* and *far*, *beside* and *between*, they are learning about location, space, and shape. Similarly, Little Red Riding Hood, The Gingerbread Man, or any number of other stories could be used as a means of putting positional language into context. There are also stories, where positional language is a direct feature of the text. For example, in Rosie's Walk by Pat Hutchins, Rosie the hen walks across the yard, around the pond and so on as she cleverly evades the fox. In Michael Rosen's We're Going on a Bear Hunt, language such as 'over, under and through' is used throughout.

Story Maps

Geog. | Shape Space

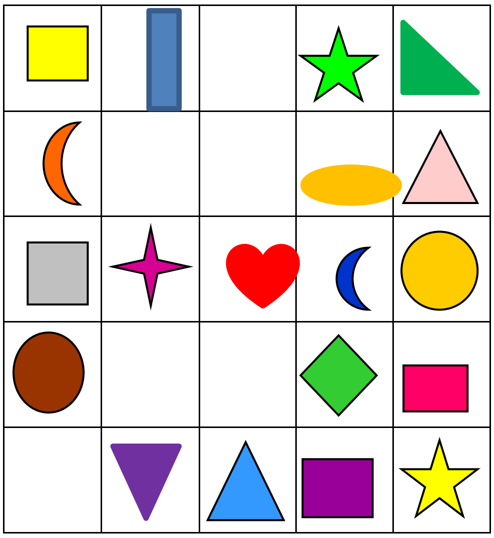
1. After reading a familiar story, such as Little Red Riding Hood, pupils can suggest events and landmarks that describe the pathway taken by the characters. The teacher can draw the events and connect them thus making a story map. For example, the teacher can ask Where *was Little Red Riding Hood at the start of the story? What happened next? What do I need to draw to show that she walked through the woods to get to Grandma's house? Where did she go after she got to the front door? What happened next?*
2. When the story map is complete, questions can be posed about the order and position of landmarks and events in the story, in relation to other landmarks and events. For example, *Did Little Red Riding Hood see the wolf before or after she got to Grandma's house? How did we show that on the map?*

Hot and Cold

Oral Lang | Shape Space

A pupil is chosen to hide an object in the classroom. Other pupils take turns in asking questions to find it, for example, *is it behind the computer*, *is it under the nature table*, *is it in between the two bookshelves*, *etc*. The pupil replies *'hot'* or '*cold'* depending on how close the guess is to the hidden object. Once familiar with the game, pupils can play in pairs or small groups.

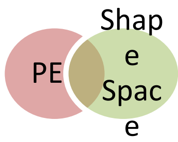
Describing Position



1. Pairs of pupils are given a grid containing many shapes (see example below).
2. The teacher models asking questions about the position of shapes in the grid.
3. Pupils take turns in asking questions with their partner describing the position of the shapes.
4. As an extension activity, pupils can draw in their own shapes on a blank grid and the repeat the activity in step 3.

Which shape is: above the orange oval, below the green star, to the right of the green diamond, in between the blue triangle and the yellow star…
Mary, please ask that question a different way.

PE Games



PE Games that involve positional and directional language are a very good way of teaching and reinforcing the language. A selection of these opportunities is outlined below although this is not an exhaustive list.

Instructions

- Pupils follow oral instructions that involve directional language during PE activities using terms such as 'in front of', 'behind', 'next to' in relation to themselves and to others. For example, during warm-up activities pupils can be asked to wave their hands behind, above, at their sides, etc. They can also be asked to jump backwards, sideways forwards, etc.
- Pupils each stand in a hula hoop which becomes their base. PE equipment is placed around the hall or outdoor space. The teacher gives instructions which pupils must complete before returning to their base, for example, *crawl under the benches twice, jump over the bean bags four times, hop between the cones* and so on. For a more competitive version, the last pupil to make it back to base is eliminated until there is a winner.
- Pupils give directions to each other through an obstacle course in the hall or outdoor space. One pupil in each pair is blindfolded. The second pupil guides the blindfolded pupil through the obstacle course using only language. Careful instructions might include *take five small steps forward, stop, turn to your left, take 2 small steps, stop, now turn to your right*, etc.

Orienteering[18]

- The ICT link is for an orienteering lesson designed for the infant classes. Outdoor activities are a useful way for pupils to practice and refine their spatial abilities. The control cards provided for this activity have animals on them but these could easily be replaced by pictures of 2-D shapes.

[18] Link to Orienteering lesson: accessed at http://www.ppds.ie/pcsparchive/pe/pssi/pdf/out/out_a_1.pdf

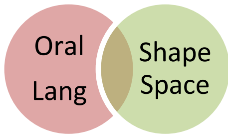
Class Book [19]

Free online software such as Photostory 3 or Photopeach, can be used to create an online slideshow or book. Pupils can take photographs of a class teddy bear positioned in different places, for example, under the swings, behind the Principal's car, between the football goals, in front of the school door, etc. The pupils should have a discussion first about where to position the bear. Each photograph can be annotated with an appropriate caption, for example, *Now Teddy is in-between the football goals!* Photostory 3 also allows audio recordings so pupils can record themselves describing the bear's location. These presentations can be shared with parents, families and other classes.

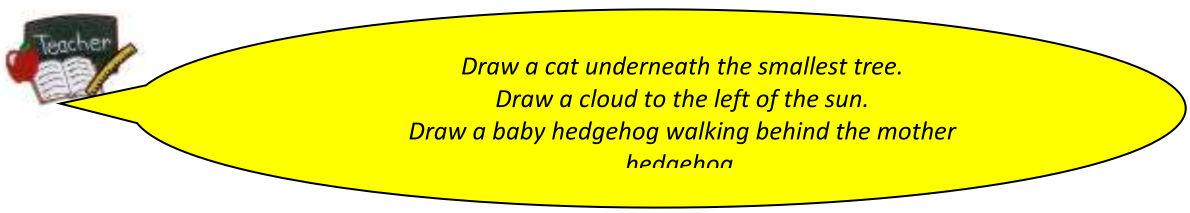
ICT Opportunities

Photopeach
Photostory 3

Following Instructions



1. Pupils have an A4 sheet with various pictures on it.
2. The teacher instructs pupils to draw other objects in relation to the objects already on the page.
3. Once they are comfortable with this, pupils can re-do the activity in pairs.
4. Alternatively, a blank sheet of paper can be used where pupils fill in items themselves and then describe the location to their partner.
5. This activity can be further extended into a barrier game where one pupil has page with pictures and one has a blank page. One pupil gives explicit instructions about location whilst the other pupil draws items on the blank page. The objective is to have similar pictures at the end of the activity without the illustrator seeing the initial picture (see consolidation activities for more information on barrier games).

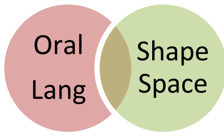


Using Every Day Examples for Directions

A meaningful way to improve pupil's ability to clearly give and follow directions is to do it in a natural context. For example, if two pupils are taking the roll to the office, other pupils could give them directions of how to get there, etc.

[19] Link to Photopeach: accessed at http://photopeach.com/ and Link to Photostory: download details accessed at http://www.microsoft.com/en-gb/download/details.aspx?id=11132

CONSOLIDATION ACTIVITIES



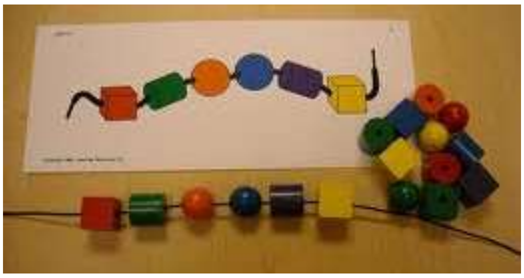
Barrier Games [20]

Barrier games are an effective and enjoyable way of developing oral language and receptive language skills. Barrier games require pupils to give and receive simple instructions and procedures. They provide an ideal opportunity to practice vocabulary, turn taking, giving directions and responding to directions. Pupils can work in groups of three where one acts as the ‘teacher’ and holds a barrier, such as a large book, piece of cardboard, individual whiteboard, etc. in between the other two pupils. This pupil is also responsible for ensuring that the others in their group stay on task. Alternatively, two pupils can simply place a barrier between them and work as a pair. The following activities provide opportunities for pupils to put into practice the positional and directional vocabulary they have encountered and used in the teaching and learning experiences outlined above.

1. Construction

Both pupils are given the same objects, for example, four differently coloured or shaped blocks. Pupil A provides directions to pupil B regarding how to place the blocks so that they replicate pupil A’s pattern, for example, *first, put your blue block on the left hand side, next put your yellow block on top of it, now put your red block to the right of the blue block, finally put the orange block beside the red block.* The barrier can then be lifted to reveal whether both sides are the same. Instead of using objects, pupils could be asked to draw, for example, *draw a brown ball in the middle of your page, and now draw a green ball on top of it, next draw a tiny ball under the red ball*, etc. The fact that the instructions come from other pupils rather than the teacher enriches the learning in these activities. If pupils struggle to find the language, the teacher can play the barrier game for the class, modelling suitable language.

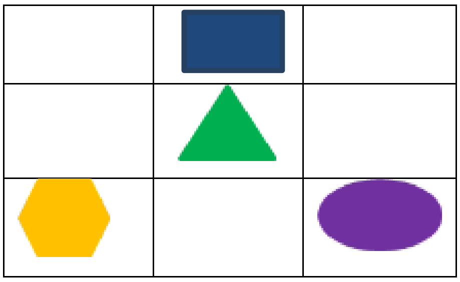
2. Simple sequences or patterns (can be 3-D or 2-D)



Pupils describe to their partner successive items to be threaded on a string or simply placed in a line. Any commercial or non-commercial 3-D or 2-D objects could be used including 3-D beads, attribute blocks, or materials gathered by the children. When the sequence is completed the barrier is lifted to reveal whether the sequences are the same.

[20] First Steps Speaking and Listening

3. Grids



Pupils each need an A4 sized game board, marked with a 3 x 3 grid. These can be made on coloured card and laminated. Pupils use positional language to instruct their partner to place their shapes to match the layout of their own shapes on the grid. It is a good idea to model playing this game initially for pupils in order to demonstrate the range of language that can be used. The task can be extended by using different manipulatives, for example, if using small plastic bears, the size and colour of each bear would have to be specified.

Place the oval in the third box in the last row.
Place the purple oval in the third row in the box on the right.
Now put the green triangle in the box in the middle of the grid.
Put a middle-sized green bear …

4. Assembly

Pupils create a picture using 2-D shape pieces. First, pupil A constructs a simple picture using shapes, for example, they might choose to make a clown using two circles for the eyes, two skinny triangles above the eyes as eyebrows, a big red circle for the nose, and five small triangles to form the mouth. Pupil A must then give precise instructions to pupil B including how many of each shape is needed, as well as the size, colour and position. As with all barrier games, some pupils may need the teacher to model the game, in particular the precise use of language. The teacher can also model giving inaccurate and vague instructions to highlight the need for precise instructions.

5. Location

Pupils need a simple map or picture, for example, a simple picture of a park. Google maps of the local area can be used for this activity. They will also need a bank of small objects or toys such as those commonly used in infant maths activities (small plastic teddy bears, transport vehicles, farm animals, etc.). Pupil A instructs their partner regarding where to place the toys on their board, for example, *put a big blue teddy bear sitting on top of the slide in the park, put a cow beside the biggest tree*, etc. Alternatively, if using a map, pupils can instruct their partner regarding how to move from a starting point. For example, the pupil can provide directions on how to get from the school to a certain location, for example, the shop: *Stand your teddy at the front gate of the school, now walk down the street toward the park, stop when you get to the zebra crossing, cross over the road to the other side of the road, walk away from the park now, turn right when you get to the next street.*

6. Spotting Differences

Pupils need two pictures or sets of images that have slightly different details. They must describe their pictures to each other in an attempt to identify the differences, for example, *I have four big triangles in my picture, do you? I have one shape that has 6 sides, do you? I have no squares, do you have any squares?*

LEVEL A.2: SORT, DESCRIBE AND NAME 3-D SHAPES INCLUDING CUBE, CUBOID, SPHERE AND CYLINDER

TEACHING NOTES

Undirected Play

In the early stages of learning about 3-D shape, the emphasis for pupils should be practical, hands-on and concrete in nature. Before beginning more formal work on 3-D shape ample opportunity should be provided for pupils to handle many real, every day 3-D objects in a variety of informal settings. This can include sand and water play, use of construction materials, playing with a variety of toys, etc. This helps to lay a solid foundation for subsequent work.

Starting with 3-D Objects

'Solid shapes are prominent in the environment and a study of these usually precedes the work with 2D shapes'(Deboys & Pitt, 1980, p.151).

Irregular 3-D Objects

3-D objects should be represented in irregular (non-prototypical) forms and positions so that pupils do not develop a limited concept of them. Some examples are provided below.

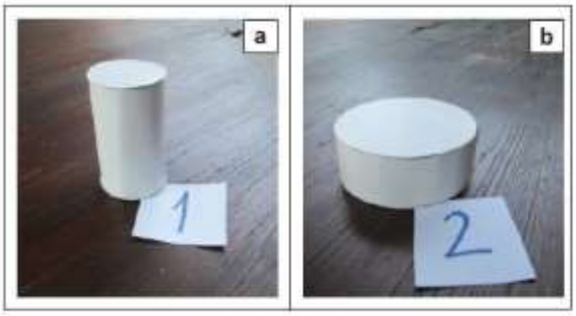


FIGURE 2: A prototype cylinder (a) and a non-prototype cylinder (b).

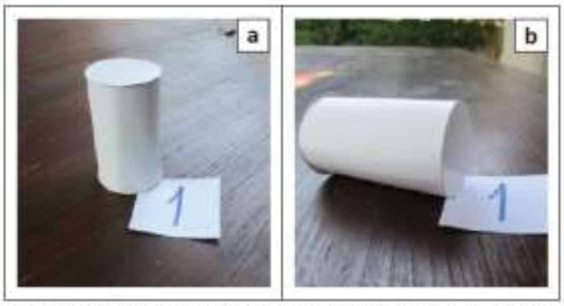


FIGURE 3: An example of a cylinder in a typical position (a) and an atypical position (b).

Shapes versus Objects and Figures

Often, concepts such as 'triangle' or 'cone' are referred to as 'shapes', as are the physical models of triangles and cones ('put the shapes into the box' or 'cut out some shapes'). Using the word 'shape' to describe both the property of the thing and the thing itself can cause confusion. Asking pupils to 'describe the shape of this shape' highlights one problem. Another problem is that pupils must be able to think of 'all rectangles' as being 'the same shape', while mathematically speaking all rectangles are not the same shape. For this reason, it is suggested that teachers model for pupils the more helpful convention of calling 2-D things ‘figures’ and 3-D things ‘objects’. Using this informal convention, a circle is a figure but a sphere is an object.[21]

> Using the convention of calling 2D things ‘figures’ and 3D things ‘objects’ should be helpful for pupils. Using this informal convention, a circle is a figure but a sphere is an object.

For consistency 3-D shapes and 2-D shapes are used in this manual because these are the terms used in the curriculum; however, teachers should certainly consider using ‘figure’ and ‘object’ in an attempt to increase clarity for pupils.

Using Real Objects

As well as using commercial products, everyday objects found in the classroom and the home should provide much of the stimulus for exploring 3-D shape. Initially, everyday objects should be used before introducing the commercial resources. Asking pupils to bring in examples of a cube, cuboid, sphere and cylinder from home will reinforce the fact that 3-D shapes are all around us. Pupils will enjoy seeing examples of 3-D shapes brought in by others. These objects can then be used for sorting activities and can also be placed on a 3-D shape display table.

Cube

Cuboid

Sphere

Cylinder

Sorting

The importance of a carefully planned programme of sorting activities for young pupils cannot be over-emphasised. In general, it provides a highly valuable context for the promotion of pupils’ skills in language, observation, and reasoning. Pupils who find mathematics difficult benefit considerably from early sorting activities. At the heart of sorting is the notion of a ‘set’: a set is a well-defined collection of objects that belong together because of a common property.[22]

[21] First Steps Space
[22] Pitt, E.

The real value of these early sorting experiences depends on the discussion that results between teacher and child (Deboys & Pitt, 1979).

Sorting[23]

Pupils should explore how shapes are alike and how they are different and then use these ideas to begin to create classes or 'sets' of shapes. Some of these sets may be based on geometric properties such as: *they are all cubes; they are all shapes with straight sides; they are all shapes that roll*. Other shapes may be described in looser terms, such as, *these look like rockets; these shapes are all curvy*. As pupils notice more properties, it is important for teachers to attach the appropriate names to them.

1. Present a collection of 3-D objects to the pupils including tins, boxes, balls, etc. However, before meaningful sorting activities occur it is a good idea to let young pupils play freely with the objects; this will allow them to become familiar with, and to explore, the properties of the objects. Also pupils will be less likely to 'play' with them when it comes to sorting work.

In any sorting activity, the pupils, not the teacher, should decide how to sort. This will allow them to own and understand the activity. By listening to the kinds of properties they use the teacher can assess which properties pupils know and also how the pupils think about shapes.

2. Pupils carefully examine a group of objects. They can choose one object to describe, then another object. The teacher prompts their thinking through questioning:

How are they the same?
Why do they belong together?
How are they different?
Jim please revoice that.
Why do they not belong together?
You chose a shape that has flat faces; can you find another one that has flat faces?

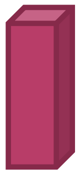
3. Pupils sort for properties including:
    - shapes/objects that roll/do not roll,
    - shapes/objects that slide/do not slide
    - shapes/objects that build/do not build

[23] Deboys, M. & Pitt, E. (1979) Lines of Development in Primary Mathematics

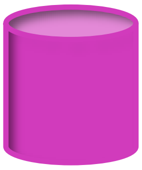
- shapes/objects that are solid/ hollow, etc.

Some more examples include:

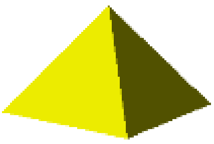
These all have flat faces

These will all roll



These all have triangles on some of their faces




Here pupils are sorting everyday 3-D objects into sets. This is important because it helps them to understand that although a cardboard tube is tall and thin and a drink can is much shorter, they are both cylinders. Pupils must be guided to recognise that it is the properties of shapes that are significant when sorting and not their size, colour, orientation, etc.

It is important when sorting that pupils are guided to focus on differences between shapes and categories of shapes, rather than just what is similar.

Naming 3-D Objects/Shapes

1. Teacher Language

When pupils describe the various characteristics of the shapes they will often use informal mathematical language. At this stage, the teacher can model the use of the correct or formal language.

You are correct - both of those shapes have bendy faces.
The faces are curved.
These faces are curved and these are straight.

2. Hunt

Many examples of 3-D shapes are required for this activity – the ones that pupils brought to school may be particularly useful. Show pupils a 3-D shape, for example a box (cuboid), and then in pairs, they hunt for something that is a similar shape in the classroom. Their experience of sorting will provide good preparation for this activity.

3. Name the shape and identify its properties

The teacher then asks if anyone knows the name of the shape, or indeed if anyone can think of a suitable name. The teacher accepts all suggestions and if the name is not forthcoming then the teacher introduces the name. It is important that in naming the shape, the teacher or pupils make direct references to its properties, for example, *a cuboid has faces, edges and corners*. If pupils do not mention all of these properties they should be prompted to do so. Note that looking at faces may lead to the naming of 2-D shapes.

This shape is called a cuboid. Who can tell me something about the cuboid?
You are correct: it has flat faces.
Yes, this face is a rectangle shape. What else?
Yes, it has no curves, what does it have instead?
It has edges - straight edges.
What do you think these bits might be called? Yes, they're corners. We call them vertices - a vertex is a corner.
So, a cuboid has flat faces, straight edges and vertices. Does the shape that you found have flat faces, straight edges and corners?
Share your findings with another group.

4. Labelling

These objects can now be labelled and added to a 3-D shape display.

The properties of each 3-D shape outlined in the infant curriculum are:
CUBOID: 6 faces, 12 edges, 8 vertices
SPHERE: 1 face
CYLINDER: 3 faces, 2 edges
CUBE: 6 square equal faces, 12 equal edges, 8 vertices

CONSOLIDATION ACTIVITIES

Sensory Bag

Pupils try to name the 3-D shape in the bag by touch alone.

1. Gather pupils together in a circle on the floor. Show them approximately five 3-D shapes laid out on the floor. Have another set of identical objects ready and place one in a feely bag. To keep the attention of the pupils several bags could be used at once.

One of these shapes is in the bag. Tell me which one just by feeling it.
Tell me something you notice about the shape in the bag.
Can you describe what you can feel?
Anne, please revoice what Michael just said.
Which shape do you think it is? Why?

2. Pass the bag or bags around the circle so that each pupil can feel the shape. Encourage them to think about the shape. Pupils shouldn't point or call out which shape they think it is so that everyone gets a chance.
3. As each pupil feels the shape in the bag, get them to say one thing they notice about it.
4. Once each pupil has had a turn to feel the shape, ask them to say which shape they thought it was.
5. Reveal the hidden shape.

Shapes Hunt

The class go on a 3-D shape hunt around the school. If possible, photos are taken of the shapes the pupils find; otherwise, pupils can take notes/sketches of what they find. On returning to class all of the 3-D shapes that were discovered on the hunt should be recorded. A display entitled 'Real Life 3-D Shapes' can be created from the photos/sketches. For example, the section on spheres might include the following:

marbles are spheres

the sun is a sphere

sliotars are spheres

the earth is a sphere

Chocolate Feely Bag

This activity is a fun way to explore the properties of 3-D shape. Use a collection of sweets and chocolates that are different 3-D shapes, for example, toblerone, walnut whip, smarties, dairy milk, rolos, mars, etc. Place one into the feely bag and have a pupil describe its properties. Other pupils can guess the 3-D shape based on the description given.

Cylinder

Cone

Sphere

Cuboid

LEVEL A.3: SORT, DESCRIBE AND NAME 2-D SHAPES INCLUDING SQUARE, CIRCLE, TRIANGLE AND RECTANGLE

BACKGROUND KNOWLEDGE FOR TEACHERS

It is worth noting that many materials that are commonly used in classrooms as 2-D shapes, for example, attribute blacks, are actually 3-D shapes as they have sides. True 2-D shapes are completely flat and some would argue that the only true 2-D shapes are faces on a 3-D shape or drawn shapes. Once you can pick up a shape in your hand it is technically a 3-D shape with sides even though the sides might be minute. Of course, the use of such materials is necessary in the teaching and learning of 2-D shape, however, a discussion about the true distinction between shapes might be useful with pupils.

TEACHING NOTES

Textbooks and educational resources often present only regular shapes leading to pupil misconceptions that, for example, a triangle that is not equilateral and does not have its' vertex shown at the top is not a 'proper' triangle or that a long and narrow rectangle is not a 'real' rectangle. To overcome this misconception, irregular shapes should be used frequently in addition to regular shapes.

Now that pupils have engaged in meaningful exploration of 3-D shape in Level A.2, they will more clearly be able to see the link between 3-D shape and 2-D - 2-D shapes are the faces of 3-D shapes. The teaching sequence outlined below begins with activities that lead pupils to make this connection and is followed by exploration of the properties of 2-D shapes.

SAMPLE LEARNING EXPERIENCES

Moving from 3-D to 2-D

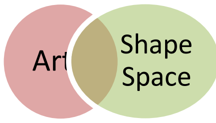
Unfolding 3-D Shapes

Even though pupils are only beginning to explore shape at an early stage, they will be fascinated to observe everyday 3-D shapes being unfolded into nets. The pupil's curiosity will be sparked by the notion that 3-D shapes can be broken down into 'flat' or 2-D shapes.

1. Take a 3-D shape such as an empty box and tell the class you are going to unfold it to make it lie out flat.
2. Ask the pupils to guess what shape it might be when it is flat.

3. Unfold the box and compare the net to pupils' predictions.
4. Repeat this for other shapes.

This introductory activity clearly demonstrates the relationship between 2-D and 3-D shape.



Using 3-D Shapes to Make 2-D Shapes

As a follow-on activity, pupils can use a variety of 3-D shapes to make prints using paint and paper. This allows pupils to see clearly that faces of 3-D shapes are 2-D shapes. Before beginning the activity, a range of shapes should be shown to the class so that pupils can predict the shape they expect to see printed on the paper. After printing, the actual prints can be compared to the predictions.

Which 3-D shapes on our shape table could we use today in art for printing?
Yes, we could use the cylinder - what shape will the face make on the paper?
I wonder what shape the curved face will make.
Yes, we could also use the cube. I want you to predict what shape this side will make on paper.
Will the cubes other faces make any different shapes?
Pat revoice what Kim said

Footprints in the Sand [24]

The teacher uses blocks of various shapes to make impressions in the sand. Pupils then identify the shapes and match the blocks to the impressions. The following poem can be read as part of the activity:

A monster made these footprints
While we were all asleep.
What funny shapes these prints are,
And they're not very deep.
I think the monster fooled us
And used our blocks instead;
And you can figure out which blocks
If you just use your head.

Naming Faces

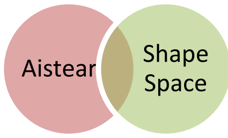
At this stage pupils will be familiar with some 3-D shapes. By inspecting the faces of cubes and cuboids, the rectangle and square can be formally introduced. By inspecting the ends of a cylinder,

[24] Miriam Leiva, ed., Geometry and Spatial Sense (Reston, Va.: National Council of Teachers of Mathematics, 1993), p. 6.

circles can be introduced. Following this, the triangle can be introduced as a face on a pyramid. Discussion about the properties of these shapes is important.

Does anyone know what we might call the side of this cube?
It is different to the rectangle. How are they different?
What makes this one a square and this one a rectangle?
Let us count the sides, let us count the corners.
They are both the same, so what is different?
Conor, revoice Lucy's reasons.

Exploring 2-D shapes



Undirected Play

As with 3-D shape, pupils should begin their exploration of 2-D shape with play. A variety of materials should be used for this, for example, coloured plastic shapes (attribute bocks), in addition to wooden and cardboard shapes. Each shape should feature in different colours and sizes. Pupils can use these shapes to create patterns, pictures and simple representations. During these activities the teacher can engage pupils in discussion about what they are doing. Remember to use example of irregular shapes - these may not be as freely available in commercial products so may need to be made from cardboard.

Tell me about the pattern you are making.
That is a lovely house, how did you make it?
Describe the steps to your partner.

Sorting

Sorting activities provide an ideal context for pupils' early mathematical reasoning. Many opportunities should be provided for pupils to orally express reasons to others. This can happen through pair work, group work and whole class discussion.

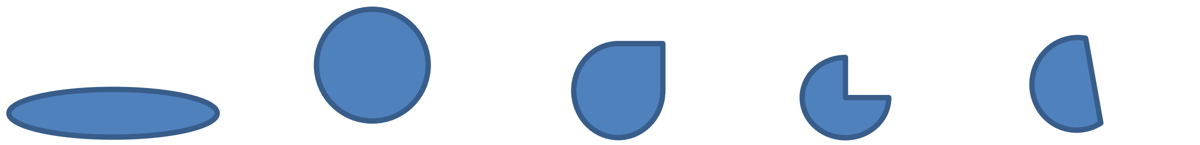
By sorting shapes pupils must focus on the properties of each shape to create groups or sets of shapes. It is important in any sorting activity that the pupil decides which property to sort for. This ensures that pupils understand and have ownership over the activity. It also provides an accurate assessment for the teacher in terms of a pupil's level of understanding regarding the properties of shapes.

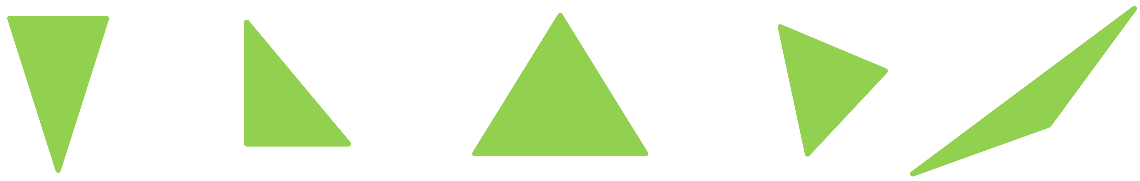
1. Provide pupils with a selection of plastic or cardboard shapes.
2. Each pupil selects a shape and shares one or two things that they find interesting about the shape.
3. When everyone is comfortable with this, one pupil is asked to select two shapes and to share something that is alike and something that is different about the shapes.

4. The group chooses one shape at random - this is the target shape. The group must now sort all of the shapes that possess the target property, for example, *this shape has all straight sides just like the target shape*.
5. A different property for the target shape can be selected and a second sort can then be conducted.

Tell me about this shape. How is it the same as this one?
Why do they belong together?
How are they different?
Why do they not belong together?
Can you find another shape that also has curved sides?

Below are some examples of 2-D shape properties that pupils might sort for.

Shapes with curved sides

Shapes with a ‘square corner’

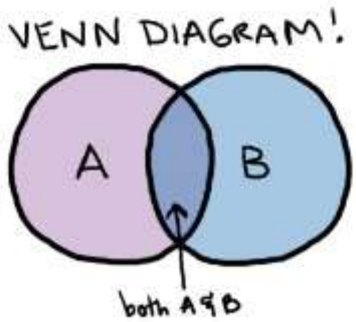
Shapes with three sides


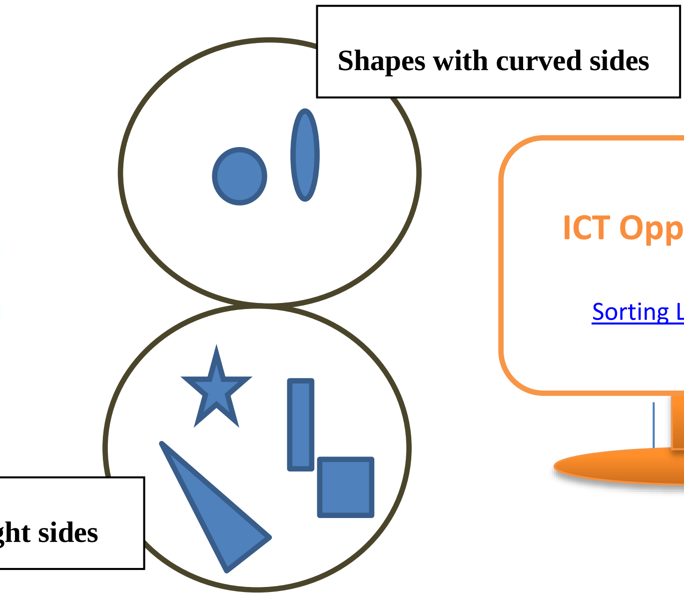
Sorting using Structured Diagrams

It is important that pupils represent and record their mathematical thinking in as many ways as possible. Introducing sorting diagrams helps to clarify pupils’ thinking and reasoning and provides a different perspective on their sorting activities. It is also a way of recording their work and can prove to be a valuable assessment tool.

Venn Diagram[25]

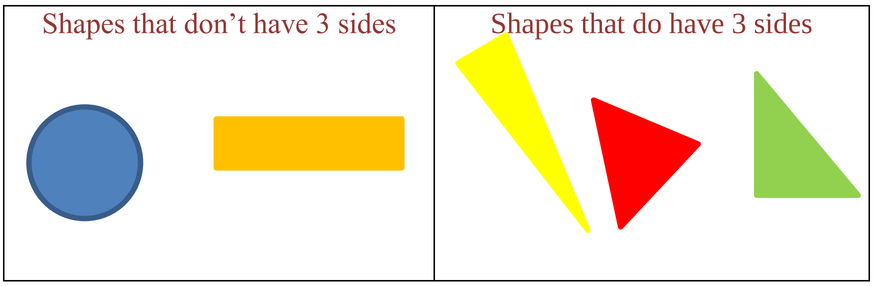
1. For young pupils, a simple Venn diagram, consisting of one set only can be used. Pupils sort for things that belong in the set and things that don't. For example, pupils might place shapes that roll into the Venn diagram and shapes that do not roll outside of the diagram.
2. As an extension, a Venn diagram with two overlapping sets can be used. For example, the pupil has sorted shapes into two sets; shapes with straight sides and shapes with curved sides; the category in the middle would have a shape with both properties- curved and straight sides.





Carroll Diagram[26]

A Carroll diagram is a diagram used for grouping things in a yes/no fashion and is named after the author Lewis Carroll. Numbers or objects are categorised as either having an attribute or not.

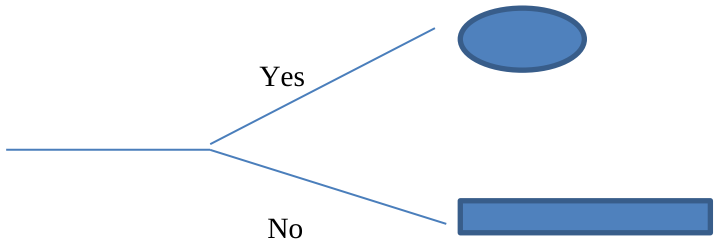


[25] Link to Sorting Logic Blocks: accessed at http://nrich.maths.org/content/id/7192/JulySh.swf
[26] Link to Sorting Rectangles in a Carroll Diagram: accessed at http://www.coppschool.lancsngfl.ac.uk/Classwork/Classwork/flashaids/carroll_diagram.swf

Tree Diagram

A tree diagram is another variation of a simple sorting diagram for younger pupils. Like the Venn and the Carroll diagram, pupils must determine whether a particular shape has or does not have a particular property.

does it have curved sides



Giant Shape Sort[27]

Use masking tape to make giant 2-D shapes on the classroom floor. Ask a group of pupils to find as many objects as they can in the classroom that can be placed in each shape category.

Shape Hunt

Pupils search the classroom to find specific 2-D shapes. These might be found as faces on a 3-D shape or as 2-D shapes. The teacher leads a discussion about the differences and similarities between the discovered shapes.

Attribute Descriptions

Pupils examine and describe cardboard shapes, noting shapes (square, rectangle, triangle, hexagon, circle, etc.). The teacher places a shape where pupils cannot see it and one by one, the pupils ask "yes" or "no" questions in a circle. If the answer to a student's question is "yes," then he or she can continue to ask questions; if the answer is "no," then it is the next pupil's turn to ask a question. For example, *does the shape have any curved sides, does the shape have any points, does the shape have four sides that are all the same length?* Continue until a pupil can deduce what shape it is.

Secret Shape folders[28]

A double set of 2-D shapes are required.

1. Cut out one set and glue each shape into a folded A4 piece of card to create secret shape folders.

[27] http://pinterest.com/pin/164240717632333678/

[28] Van de Walle (2007)

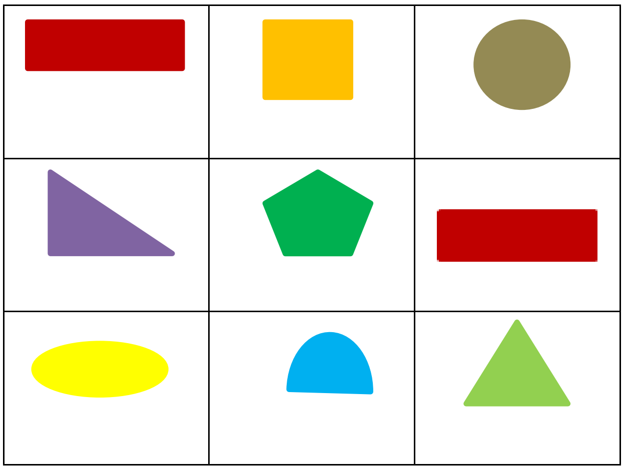
2. Choose one pupil to be the leader - they choose one shape folder and lay the other full set of shapes out on the table in front of the group.
3. The group takes turns asking ‘yes’ or ‘no’ questions to determine the secret shape in the folder. As they eliminate shapes, they should be removed from the group of shapes as they narrow down the possibilities. Pupils are not allowed to ask ‘is it this one’ but must only ask questions about the properties and characteristics of the shapes[29].

ICT Opportunities

[Printable assorted shapes]

2-D Shape Bingo

Do you have a green shape with 3 sides?
Do you have a shape that has four sides of all the same length?
Do you have a shape with curved sides that is not a circle?



Bingo can be played a) as a whole-class where the teacher calls out attributes and descriptions of shapes or b) in small groups where cards with attributes and shape descriptions are laid face down on the table. In the latter set-up, pupils take turns to pick a card. Only the pupil who picks a card can cover a corresponding shape on their bingo mat. The first pupil to fill a line calls ‘bingo’.

Memory Drawings

Pupils see a shape for a few seconds and then try to draw the shape from memory. Having many shapes prepared and available to show on an interactive whiteboard may be helpful. Alternatively, a box of attribute blocks can be used. Pupils then share their drawings with each other and discuss the characteristics of the shape in their drawings. The original shape can be shown again for comparison purposes.

Describe the shape you drew. How many sides does it have?
What else can you tell me about the sides?
What is that shape called? Why did you draw a rectangle?
Compare your shape with the person sitting beside you. Do they look the same?
Did anybody draw a different number of sides?
Now let us check the shape again - was it the same as yours? What was different?
Kate please repeat that in your own words.

[29] P

Visualising Shape and Space[30]

It is important for pupils to develop the capacity to visualise shape and space in their heads. Activities like these will support that ability.

Imagine a big triangle painted on the floor. How many corners does it have?
How many sides does it have?
Imagine a small square floating in the sky, how many sides does it have?
Does it have corners?
Imagine holding a tin of beans, and turning it round and round in your hands.
How many circles can you see?
Close your eyes. Listen while I describe a shape to you ... Now open your eyes.
Pick up the shape I was describing. Now describe a shape for someone else to guess.
Look at this collection of objects or shapes. Close your eyes while I pick one up and hide it. Open your eyes. Tell me which object or shape I hid.

Human String Shapes

Show the class a long piece of string and ask the class to describe its properties as it lies in a bundle, for example, wiggly, messy, long, bunched, etc.

1. Hold a section of the string taught and ask pupils to describe what has happened to the string (the string has made a straight line).
2. Each pupil holds part of the string and moves backwards to form a circle.
3. Encourage pupils to describe the shape in their own language.
4. Using shorter lengths of string, pupils explore the shapes that can be made.

What was the string like when we held it tight?
What was it like on the floor?
How else could we hold the string?
How could we change its shape?

5. Pupils work in small groups to see how many different shapes they can make.

[30] The National Numeracy Strategy: Shape and space activities 1999

6. Pupils record the work they have just done in a variety of ways. For example, they might:
    - stick shorter lengths of string on to paper to record the shapes made by their group
    - use geoboards
    - take photos
    - draw a picture
    - using learning logs
    - use masking tape on paper, etc.

What shape we could make using 3 children?
I wonder what other shape we could make.
How many children would we need?
Do all the sides have to be the same length?
Jim revoice what Amy just said please.

Lollipop Stick Shapes

Pupils can work in pairs for this activity. Provide them with 10 lollipop sticks and ask them to make *as many* 2-D shapes as possible. Pupils might be given 12, 15, 20 or any number of sticks as an extension to this activity. Before giving them extra sticks, encourage pupils to estimate how many extra shapes they will be able to make with more sticks.

How many shapes did you make Jim? What shapes are they?
Did you have any sticks left over? Why?
Could you do it a different way and have no sticks left over?
Did anyone else make a different number of shapes?
Why do you have more/less shapes than Jim?
So why did making triangles give you more shapes than Paddy who made rectangles?
Mai please explain that to me in your own words.

Geoboards[31]

Geoboards are useful for creating 2-D shapes.

1. Initially the teacher can use a visualiser, tablet, or any projecting device to project shapes on the geoboard for pupils to copy. Begin with simple shapes made from one band before moving to shapes made from more bands.
2. Once comfortable with this, distribute 2-D shape cards so pupils copy the shapes on the cards.
3. Finally, the teacher orally describes the properties of a shape that pupils then make.

Make a shape that has four straight sides and four corners.
What shape did you make Megan?
Why do you call it a rectangle?
Did anyone make a different shape?
Why do you call it a square Sean? How is it different from Megan's shape?
Can you make your shapes again but this time make them smaller?
Why is it a 2-D shape and not a 3-D shape?
Kim please explain that to the whole class in your own words.

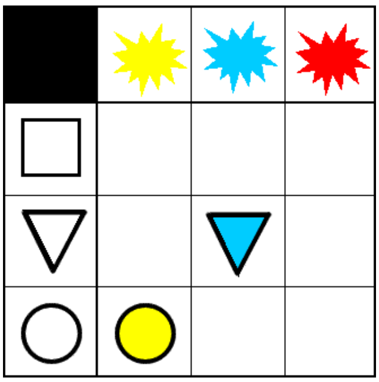
ICT Opportunities

[Online Geoboards](#)
[Online Geoboards](#)

CONSOLIDATION ACTIVITIES

What Shape What Colour?[32]

Can you find the correct shape in the correct colour to fill these boxes?



ICT Opportunities

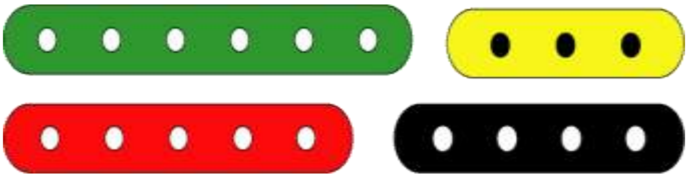
[Online Shape Game](#)

[31] Online Geoboards can be accessed at http://mste.illinois.edu/users/pavel/java/geoboard/ and also at http://nlvm.usu.edu/en/nav/frames_asid_172_g_2_t_3.html
[32] Online shape game: accessed at http://nrich.maths.org/2185

Triangle Investigation[33]

Pupils make triangles using a class construction set. Strips of different lengths are needed. There are:

- yellow strips with 3 holes
- black strips with 4 holes,
- red strips with 5 holes and
- green strips with 6 holes.



There are plenty of strips of each colour.

Some useful questions might be: How many different triangles can you make with these lengths? Can you find three strips that cannot be made into a triangle when you use them together?

Secret Sort[34]

Whoever is 'on' chooses a property (the teacher or a pupil). Pupils take turns choosing a shape and asking 'does this shape have the property?' If the shape does have the chosen property, for example, it has no curved sides then it goes into the 'set' and the person who is 'on' responds, 'Yes, this shape has the chosen property'. This continues until a pupil thinks they can attempt to identify the property.

What Shape?[35]

The set of shape cards that can be accessed on this ICT link is required. Cards are spread out on the table so that all shapes are visible. The teacher or a pupil looks at the cards and secretly chooses one. The other pupils must ask questions to identify the chosen card.

ICT Opportunities

What Shape Interactive Game

2-D Shape Jigsaw[36]

Printable jigsaw pieces can be accessed at the link opposite.

ICT Opportunities

2-D Shape Jigsaw

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[33] http://nrich.maths.org/93

[34] Pitt, E. Ready, Set, Go - Maths

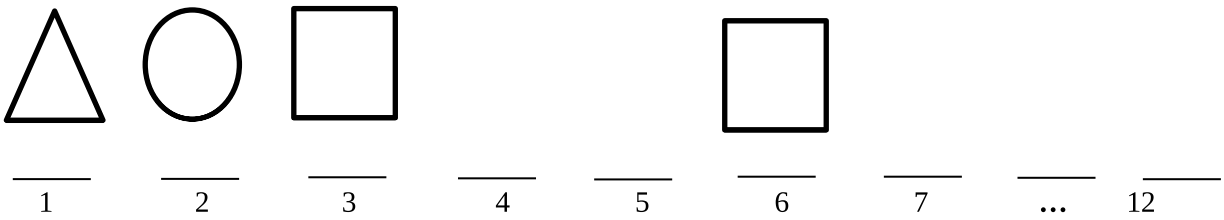
[35] Link to What Shape Interactive Game: accessed at http://nrich.maths.org/9925

[36] Link to 2-D Shape Jigsaw: accessed at http://nrich.maths.org/6886

What is the Twelfth Shape?

Experience of copying and extending patterns are pre-requisites for this activity. Similarly, an ability to count to 12 is necessary; if not, alter the numbers. Pupils describe and name each of the three shapes below using the known properties of each. As an extension to this activity, pupils can devise a similar problem for a partner.

Kitty was playing with some 2-D shapes. She began making a pattern and laid down the first three shapes and the sixth shape.
What shape will be the twelfth in the pattern?
Talk to your partner about how you might figure this out and what materials you could use to help.
Do you think the twelfth shape might be a triangle, why?
How did you figure it out?
Jim please revoice that.
Did any pair do it another way?
Now record your pattern pictorially.



Use of Level B.8 'Maths Trails' in this level will help to consolidate learning.

LEVEL A.4: COMBINE AND DIVIDE 3-D AND 2-D SHAPES TO MAKE LARGER OR SMALLER SHAPES

TEACHING NOTES

Pupils will need many hands-on opportunities to explore how 2-D and 3-D shapes fit together to form larger shapes (compose) and how larger shapes can be made up of smaller shapes (decompose). Experiences using a wide range of materials and resources will enhance learning. The following resources may be useful when composing and decomposing 2 and 3-D shapes:

- tangrams
- pattern blocks
- geoboards
- paper for folding
- dot paper
- building materials (for example, sticks and marshmallows)
- commercially available materials such as Polydron

SAMPLE LEARNING EXPERIENCES

2-D Shapes

Tangrams[37]

ICT Opportunities

Printable Tangram Pictures
Virtual Tangram Puzzles

The tangram is a Chinese puzzle consisting of seven flat shapes called tans, which are put together to form shapes. The objective of the puzzle is to form a specific shape (given only an outline or silhouette) using all seven pieces. Initially, younger pupils will enjoy engaging in undirected play with the tans, making up their own pictures and exploring the shapes. Alternatively, the tangram puzzle can be completed online.[38]

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37 Link to Printable Tangram Pictures: accessed at http://www.makinglearningfun.com/themepages/MathTangrams.htm

38 Link to Virtual Tangram Puzzle: accessed at http://nlvm.usu.edu/en/nav/frames_asid_268_g_1_t_3.html?open=activities&from=category_g_1_t_3.html

It may be more beneficial for two or three pupils to have a number of geoboards at a station rather than for more pupils to have only one each. This allows for a variety of shapes to be created and compared amongst the group.

Geoboards:[39] Creating 2-D Shapes

ICT Opportunities

Online Geoboards
Online Geoboards

Make a shape with three sides, what is it called?
Now make a different shape with three sides. Is it still a triangle? Why?
Make a shape with four sides. Is it a square or a rectangle?
How do you know?
Kate revoice that explanation please.
How could you change it into a square/rectangle?
Make a house. What shapes did you use?

Geoboards: Composing and Decomposing 2-D Shapes

Make a big triangle. How do you know that it is a triangle?
Try to turn your big triangle into two smaller ones.
How did you do it? Let us show that to the class using the visualiser.
Did anyone do it a different way?
Can you make a small square?
Could you add a shape to it to create a rectangle?
How do you know that it is a rectangle?

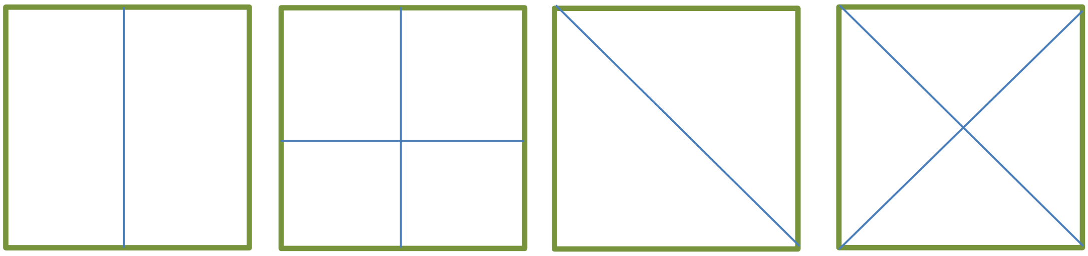
Paper Folding

1. Take a large piece of square paper and fold it in half.
2. Encourage pupils to predict what shapes have been created by this fold.

[39] Link to Online Geoboards: accessed at http://nlvm.usu.edu/en/nav/frames_asid_172_g_2_t_3.html and http://mste.illinois.edu/users/pavel/java/geoboard/

What shape is my paper? How do you know?
I am going to fold it in half, what will you see when I open the paper?
Why do you think that?
Are both shapes the same size? How do you know?

3. Provide pupils with a square of paper. Display the following diagrams one at a time (these can be drawn up onto the board or displayed on an interactive whiteboard).
4. Pupils create these shapes by folding their paper. It might prove useful to give them a second square of paper before they attempt folding number three and four.
5. Pupils can colour in the shapes when they are finished or can record them pictorially.



1 2 3 4

6. Elicit responses from pupils.

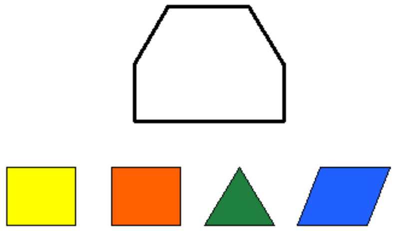
How did you fold your paper to make that shape?
Which shape was the hardest to make? Why?
If your paper was not a square could you have made that shape?
What smaller shapes did you make from your square paper?
Please draw those shapes now.

Marshmallow 2-D Shapes

2-D shapes can be made using straws (or toothpicks, matchsticks, etc.) and marshmallows (or any small jelly-like sweet). Pupils enjoy this activity for obvious reasons; however, the lesson should focus on discussion with pupils throughout the activity, for example: before - in the planning phase; during - in the making phase; and after - in the reflection phase.

What shape will you make?
How many marshmallows will you need? How do you know?
How many sticks will you need? Why?
If you wanted to turn your triangle into a square, would you need more marshmallows? Why?
Can you make your triangle bigger? How would you do it?
Make a small, medium, and large triangle. Record in a table how many sticks and marshmallows you used for each triangle.

Make a hexagon[40]



1. Provide the four smaller shapes to pupils.
2. Encourage pupils to join them together to make bigger shapes.
3. Ask pupils to talk about the shapes they made.
4. Display a large version of the hexagon (pictured) and pose the problem:

   Can you use the four smaller shapes to make this large shape?

5. Pupils work in pairs to attempt this.
6. Pupils discuss the process that they used and share solutions.

3-D Shapes

Composing and decomposing 3-D shapes can prove more difficult than 2-D.

Building to Combine

Ample opportunities should be provided for young pupils to experiment, play and build. Building materials such as blocks, Lego, straws, empty boxes, etc. may be useful for this.

Sticking 3-D Shapes Together

Commercial and non-commercial objects can be stuck together using blue-tac or masking tape to create newer bigger shapes.

Marshmallow 3-D Shapes

Marshmallows are useful for building 3-D shapes as well as 2-D shapes. These can be used with matchsticks, straws or toothpicks.

Making 3-D Objects

1. Pupils construct 3D objects using materials such as Polydron™.
2. Pupils display their structure and explain which 2-D shapes they used and how they arranged them to create the object.
3. Once pupils have made two 3-D shapes, the activity can be extended so that they have to combine the two into one new shape.

What 2-D shape did you need to make your object?
How did you put them together to make your object?
How is it that you counted five squares on your box but I can only see four?
Ann please revoice that for me.

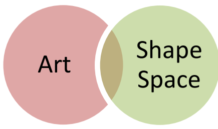
[40] This problem was sourced from www.nzmaths.co.nz

LEVEL A.5: USE SUITABLE 3-D AND 2-D STRUCTURED MATERIALS TO CREATE PICTURES

TEACHING NOTES

The activities outlined in this level all integrate Shape and Space objectives with objectives from the Primary School Visual Arts Curriculum. Suggestions are made for each strand: Drawing, Paint and Colour, Print, Clay, Construction, and Fabric and Fibre. The strand unit Looking and Responding plays a key role throughout all of these activities because discussion deepens both pupil's artistic and mathematical understanding.

SAMPLE LEARNING EXPERIENCES



Drawing

Drawing 2-D shapes accurately can be difficult for younger pupils. Instead of drawing them by hand, pupils can trace around 3-D shapes to produce 2-D shapes on paper. This activity will highlight the relationship between 3-D and 2-D shape, that is, that a face of a 3-D shape is a 2-D shape. Rich mathematical discussion should ensue from this activity.

What 3-D shape did you use to trace?
What shape did it make on the paper?
Can you only make one 2-D shape or are there other faces that will create a different shape?

Pupils can create patterns or figurative pictures from these shapes. These shapes can then be coloured or painted.

Paint and Colour

Geometric artwork

Sean Scully, an Irish-born abstract artist is well known for his trademark stripes and cubes. Exploring images of his work or any artist whose work has a strong geometric element is beneficial. Discussion can focus on the shapes and colours; the meaning; and the style. This should engage pupils and motivate them to create their own abstract works.

Sean Scully Figure in Grey 2004[41]

[41] http://emuseum.pointblank.ie/online_catalogue/work-detail.php?objectid=2196

The following list of websites for Irish galleries may be useful when sourcing work for discussion or in preparation for a gallery visit. Each website has an online collection or virtual tour:

- Hugh Lane Gallery http://www.hughlane.ie/online-collection/about-online-catalogue
- National Gallery of Ireland http://www.nationalgallery.ie/en/Collection/Search%20online%20collection.aspx
- Irish Museum of Modern Art (IMMA) http://www.imma.ie/en/subnav_115.htm
- The Douglas Hyde Gallery http://www.douglashydegallery.com/home

In addition to Irish art, the work of numerous international artists provides a strong stimulus for pupils' own shape painting.

Piet Mondrian, Composition C (No.III) with Red, Yellow and Blue 1935

Wasilly Kandinsky Squares with Concentric Circles, 1913

Print

Using 3-D Shapes to Make 2-D Shapes

Pupils can use a variety of 3-D shapes to make prints using paint and paper. This allows them to very clearly see that faces of 3-D shapes are 2-D shapes.

1. Show a range of shapes to the class and ask the pupils what shape they expect to see printed on the paper.
2. After printing, compare the actual prints to their earlier predictions.

What 3-D shapes on our shape table could we use today in art for printing?
Yes, we could use the cylinder - I wonder what shape the face at the end will make on the paper.
I wonder what shape the curved face will make.
Yes, we could also use the cube.
Predict what shape this side will make on paper.
Will the cubes other faces make any other shape?

Vegetable Printing

1. Discuss the shape and texture of various vegetables.
2. Encourage pupils to guess which shape will be created when the vegetable is cut. *What shape will be created if we cut a carrot lengthways? What shape will be created if we cut across the carrot?*
3. Cut up the vegetables so that pupils can make patterns and pictures with them by dipping them in paint.

Clay

Play Dough Imprints

Pupils press 3-D shapes into play dough or clay to create imprints of their 2-D faces.

Play Dough Shapes

Small balls of play dough or clay can be used to create 2-D shapes by using matchsticks, straws or lolly pop sticks.

How many balls of clay will you need to make a triangle? How do you know?
How many sticks will you need? How many would you need to make two triangles?
How could you make that triangle bigger?
Can anyone think of a different way?
Kim please revoice that for me.
How many balls of clay will you need to turn your triangle into a square?

Construction

Boxbots

This involves making robots out of recycled materials, boxes and containers. Pupils bring materials in to school that could be used to construct their 'boxbots'. Some materials from school can also be collected such as empty drink cartons.

Re-cycled 3-D Shapes

Use a story, poem or nursery rhyme as a stimulus, then construct the setting out of everyday 3-D materials such as cardboard boxes (cuboids and cubes), cardboard rolls from kitchen towel, wrapping paper, etc., (cylinders).

Fabric and Fibre

Fabric and Fibre Carousel

1. Collect a range of materials which can be stuck onto card to create 2-D shapes.
2. Give each table a different material and allow them a set time in which to create some shapes.
3. Move the materials around so that each table has a new material - do this until all of the tables have explored each material. These materials might include:
   - pipe cleaners
   - lolly sticks
   - drinking straws
   - string or wool
   - match sticks
   - pieces of material cut into strips

4. Discuss the advantages and disadvantages of each material for creating shapes

What material was best for making triangles? Why?
Does anyone disagree? Explain why.
What was the best material for making shapes? Why?
Which materials were best for making circles? Why?
Which material was most useful for making all types of shapes?
Explain why.

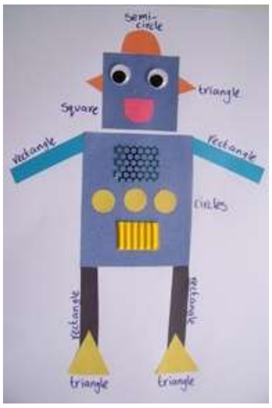
Felt Shapes

This activity has two advantages: a) it is re-usable and b) the felt shapes are easy for young pupils to manipulate.

1. Cut out several of each shape ensuring that non-prototypical or irregular examples of shapes as well as prototypical examples are included.
2. Glue a piece of felt on to a sturdy piece of cardboard (to make a durable backboard).
3. Encourage pupils to discuss the shapes they have chosen to use while they are creating patterns and figurative pictures.

Why have you chosen that shape for the roof?
Can roofs be any other shape? I wonder why builders use triangle shapes for roofs.
If you wanted to turn the house into a double story house what shape could you add?
Are there any shapes you would like to use that we have not cut out?
How would you use it?

Shape People

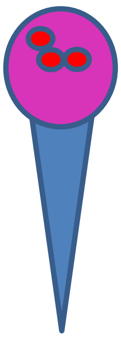


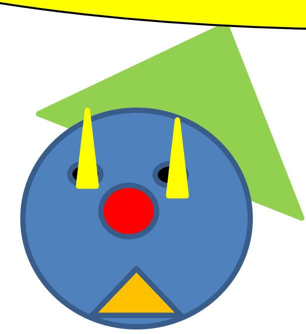
Pupils can use different polygons and circles to create a shape person. These shapes can be cut out of coloured paper or card. When pupils have completed their figure, they can explain their shape person to their partner, for example, its name, where it lives, its favourite pastimes, or a short story about the character.

Circle and Triangle Objects

1. Distribute a number of identical circles and triangles to pupils (either pattern blocks or made from card).
2. Brainstorm the possible pictures that can be made using only these two figures (for example, ice cream cone, a clown's head, a bow tie, a cat's face with ears, etc.).
3. Pupils make these pictures using their circles and triangles.
4. Encourage analysis of the pictures through questioning:

Why do all the pictures look different when we have all used the same shapes?
Why did you choose those triangles for the ice cream cones, why didn't you choose another triangle?
How did you make the mouth?





LEVEL B TEACHING AND LEARNING EXPERIENCES

LEVEL B.1: EXPLORE, DISCUSS, DEVELOP AND USE THE VOCABULARY OF SPATIAL RELATIONS (POSITIONAL AND DIRECTIONAL)

BACKGROUND KNOWLEDGE FOR TEACHERS

Many different types of maps and plans exist. The characteristics of these maps and plans are illustrated in the following table.[42]

Map Type	Representations	Distances between Objects (proximity)	Relative Position of Objects
Mud maps	Key features relevant to a particular journey drawn informally	Not drawn to scale but symbols or verbal instructions may be used	Landmarks usually drawn in order; informal indication of direction may be used
Network diagrams	Dots (nodes) represent the feature of interest; lines indicate connections between items	Not shown	Order preserved by lines that indicate connections between objects; direction not shown
Scale maps	Key features within certain boundaries are represented (mainly with symbols)	Drawn to scale	Bird's eye view showing relative position
Plans	Key features within certain boundaries are drawn to scale	Drawn to scale	Bird's eye view showing relative position and orientation

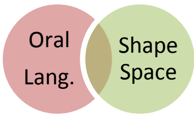
TEACHING NOTES

The learning experiences for level B.1 are the same as those for level A.1. The sample learning experiences below extend the learning in A.1 and focus on use of mapping, grids and turning in quarter and half turns.

> Young pupils initially think of direction in relation to their own bodies and therefore find directions like 'turn left' and 'move to the right' easier to learn than those associated with fixed external reference points like 'turn east' or 'turn towards Galway'. Therefore, such terms and concepts need to be used incidentally throughout the school day to ensure they are embedded.

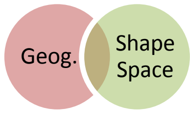
[42] First Steps in Mathematics: Shape (p.54)

Mystery Object



Extend Level A.1 learning activity 'Mystery Object' - pupils play in pairs and use directional language (for example, left, right, north, east, south, west, forwards, backwards, half turn, quarter turn, etc.) to lead partners to the object. Pupils can only direct their partner by saying how many steps to take and in what direction.

Mapping



Mud Maps[43]

Mud Maps are informal sketches that are often drawn on the spot to show how to get to a particular place and generally have a specific, immediate use (for example, to show how to get to the church, or a town). Thus, they tend to show only a few key landmarks. While they are not drawn to scale, there is often some reference made to distance and direction, through the use of symbols (e.g. arrows) or spoken words (e.g. 'Go along this road for 100 metres or so and when you get to this tree turn left.')

Many opportunities should be provided for pupils to practice drawing and following mud maps. As well as improving their spatial ability, an ability to create and follow maps is a valuable life skill that eludes many adults. Mud maps can be used in the activities below:

Virtual Tour

1. Pupils plan a series of locations to visit, such as the school library or the Principal's office.
2. Pupils walk from one stop to the next, taking digital photos of each destination as they go.
3. On returning to the classroom, pupils create a simple record in the form of a mud map.
4. Pupils' oral descriptions of their route can be recorded using free software such as audacity or Photostory3. The teacher can encourage editing by asking these questions:

Could you use some other words to help make your description clearer?

5. Drawing software and the photos can be used to create a 'virtual tour' or map of the school.
6. This map could be uploaded to the school website for visitors to use.

Local Trip

[43] First Steps: Space

Give pairs of pupils a map to decide on the best route to take to get to a venue, perhaps the local park, post office, or local attraction. Encourage pupils to suggest a route to take by giving oral instructions for their partner to draw the route onto the map. Invite pairs to compare with others in the class and discuss why they think their chosen route is the best one to take.

Playground Mapping

1. Pupils help make a physical map of the school playground on a large sheet of paper.
2. 3-D representations of the playground equipment can be used for this (boxes, toys, etc.).
3. Pupils use their memory and language (for example, near, next to, between, not far from) to establish positions.

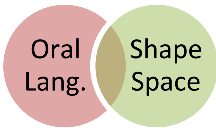
Chloe, where do you think the slide should go on our map?
Why do you think it should go there?
Does everyone agree with Chloe that the slide is beside the gate?
If you disagree, explain why.

4. When satisfied with the placement, draw and label each piece of equipment on the map.
5. Discuss and mark where other key features will be on the map, for example:

Where will the gate be on our map?
Where will our classroom be?
Where will the big tree near the oval be?

6. Take pupils to the playground with their map to match it to the actual equipment.
7. On returning to the classroom, make corrections on the map if necessary.

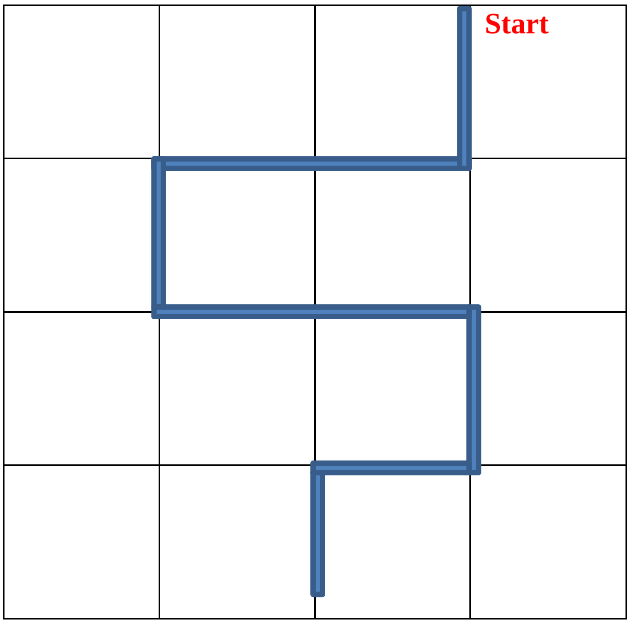
Grids



Moving on a Grid

This involves pupils follow instructions to move around a grid system.

1. Make a 4 x 4 grid on the floor using 16 carpet squares and make cards with paths drawn through the grid.
2. Invite one pupil to take a card and call out directions for another pupil to follow the path through the grid (for example, one forward, two to the right, one backwards).
3. Encourage pupils to check each other's instructions and movement through the grid.
4. Extend the activity by asking pupils to label the rows and columns to make giving instructions easier (for example, start at row 1, column 4).
5. Once explored concretely, provide grid paper so that pupils can draw their own path then work with a partner, giving directions, so that their partner replicates their path on paper.



Grid of Roads[44]

This activity can be used to introduce grid coordinates.

1. Make a grid of roads using masking tape on the floor.
2. Label each of the roads using known street names.
3. Several pupils stand on the intersections of two streets and teacher asks:

   If you wanted to meet Connor and Jamal, where would you find them?
4. When a pupil names one street (e.g. Priorswood Road) the teacher asks:

   How far along Priorswood Road? Could we tell where they are by naming two streets?
5. Invite three or four pupils to walk down the streets. When they meet someone at a corner the teacher asks:

   What are the two streets that make the corner?

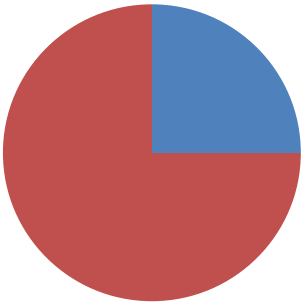
Quarter and Half Turns

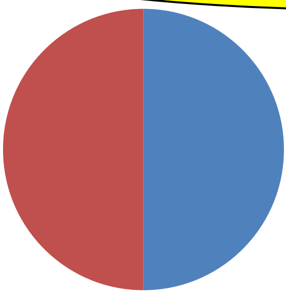
[44] First Steps: Space

Turning Discs

Two round discs, made from different coloured card can be used to explore turning in quarter and half turns. Cut a slit into each disc from the edge to the centre in a straight line. The two slots overlap and turning links the two together. Once the teacher has done this as a whole class activity or with smaller groups, pupils can work in pairs to give each other instructions.

Turn your blue disc one quarter to the right.
Now turn it one more quarter to the right.
How far have you turned the disc all together? Yes, a half turn.
Now turn it by another half turn. Where are you?
Yes, back to the beginning - you have moved the whole way around!





Turning Man[45]

This activity allows pupils to practice making quarter turns. The game can be played interactively using the link provided or can be played manually by following the link, then printing and cutting out the figure of the man.

ICT Opportunities

Turning in Quarter Turns Game

Battleships

There are many commercial versions of Battleships available but making a simple version using a 6 x 6 grid works well.

1. Pupils number the columns and rows 1 to 6 (grid references) and label the rows blue and the columns red.
2. Pupils secretly place two battleships onto their grids by choosing three adjacent squares for each ship.

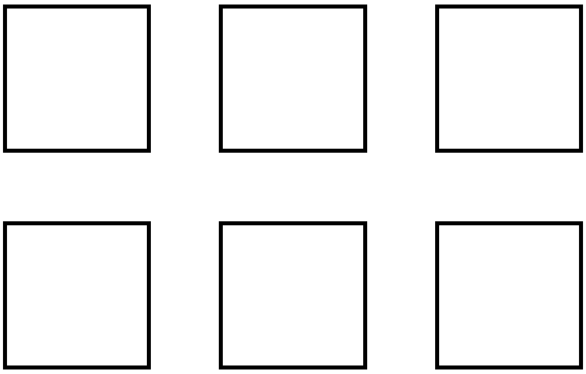
[45] Link to Turning Man: accessed at http://nrich.maths.org/5560

3. Players have three turns to 'shoot' their partner's ships by calling out grid references (for example, red 3, blue 2). Two 'hits' are required before a ship is sunk.
4. Repeat the game, this time labelling the lines not the spaces (grid coordinates). Pupils again choose three adjacent squares for each ship.

Now that we are using grid coordinates, how will this change the game?
What counts as a hit?
Would any of the intersections be a hit?
Should we call a hit one that lands inside the ship?
What numbers would we need to use for that to happen?

Coloured Squares[46]

Use these clues to colour each shape:



ICT Opportunities

[Interactive Game](http://nrich.maths.org/234)

- Blue is between green and red
- Orange is below green
- Yellow is to the left of both purple and orange

[46] Link to Interactive Game: accessed at http://nrich.maths.org/234

LEVEL B.2: DESCRIBE, COMPARE AND NAME 3-D SHAPES INCLUDING CONE

BACKGROUND KNOWLEDGE FOR TEACHERS

3-D Property Lists

Cone: 1 circular face, 1 curved edge, 1 curved surface, 1 apex,

Cube: 6 square faces, 12 equal straight edges, 8 vertices

Cuboid: 6 faces (4 or 6 rectangular faces), 12 straight edges, 8 vertices

Sphere: 1 curved surface

Cylinder: 2 circular faces, 1 curved surface, 2 curved edges

Note: a surface is curved (as in a sphere or cylinder) whilst a face is flat (as in the base of a cylinder)

TEACHING NOTES

The learning experiences for level B.2 are the same as those for level A.2. The activities in B.3 can also be used in this level. Pupils should have a strong grasp of the names and properties of 3-D shapes after completing the sorting activities in Level A.2. These activities should be repeated; however, this time including the cone. Additionally, the activities below will consolidate pupils' understanding that what makes shapes alike and different are determined by their geometric properties. Pupils should now have a sound understanding of basic 3-D shapes and their properties. This should extend beyond simply describing the properties of any 3-D object to the ability to compare the properties between 3-D shapes. See the blue box above for complete property lists.

It is important that pupils consistently use the correct terminology when describing properties of 3-D objects: **faces, surfaces, vertices (singular: vertex) and edges**. It is imperative that the teacher models the correct and consistent application of these terms.

SAMPLE LEARNING EXPERIENCES

Property Lists

1. Pupils work in pairs or small groups to compile a list of all the different properties that they can come up with for 3-D shapes.
2. Ask each group for one property, continue around the groups until no group has any new properties to add to the class list.
3. Scribe a master list for the class.

If groups are struggling to think of the general properties of 3-D shapes, this might signal that they have not engaged fully enough in the sorting activities outlined in Level A.

4. Use the class property list that the pupils have come up with to create a template like the one shown below.
5. Give each pair or group an A4 or A3 sized copy of the table and ask them to fill in as much of it as possible.
6. Ensure that pupils have a selection of 3-D shapes to hand while completing their tables.
7. Each group should contribute to a whole class property list for 3-D shapes. This table should form a classroom display. Ensure that pupils provide as much information as possible about each property, for example:

Yes, a cube has edges, but how many edges? Count them.
So 12 edges - does everyone agree?
What else can we say about those edges? Yes, they are straight.
What else? Yes, they are all equal in length.
Why is that important? If they weren't equal in length, what shape would it be?

	Faces?	**Surfaces?**	**Edges?**	**Vertices?**	**Apex?**
Cone 					
Cube 					
Cuboid 					
Sphere 					
Cylinder 					

LEVEL B.3: SORT, DESCRIBE, COMPARE AND NAME 2-D SHAPES INCLUDING SQUARE, RECTANGLE, TRIANGLE, CIRCLE, SEMICIRCLE, OVAL

TEACHING NOTES

The learning experiences for Level B.3 are the same as for A.3. Pupils should have a strong grasp of the names and properties of 2-D shapes after completing the sorting activities in Level A.3. These activities should be repeated; however, this time including the semi-circle and oval. The additional activities below will consolidate pupils' understanding that what makes shapes alike and different is determined by their geometric properties.

It is important that pupils consistently use the correct terminology when describing properties of 2-D figures - **straight side, curved side, corners.** It is imperative that the teacher models the correct and consistent application of these terms.

SAMPLE LEARNING EXPERIENCES

Face Off

As with many of the activities in level A.3, this activity highlights the relationship between 3-D and 2-D shapes.

1. Each pupil selects a 3-D object, either an everyday item (such as a toothpaste box) or a commercial 3-D shape.
2. Each pupil also needs a small piece of play dough or blue tac.
3. One pupil is the caller and so has a cloth bag of small cards with 2-D figures drawn on them (for example, rectangles, circles, triangles, etc.).
4. The caller takes out a card and calls out the shape that is on it.
5. The rest of the pupils respond by looking for that 2-D face on their 3-D object. If they have that 2-D face, they attach a dab of play dough to it. When all of the faces on their object are marked, they call out 'Bingo!'

Get Set

This activity focuses pupil's attention on the properties of 2-D figures. Pupils may already be familiar with semi-circles and ovals however, if not ensure they get particular attention in these activities.

1. The teacher needs a collection of regular and irregular 2-D shapes (either cut from card or made from plastic).
2. These are placed into a hula hoop (the set).

3. Pupils need a set of figures identical to the master set.
4. Slowly add one shape at a time to the hula hoop set. Pupils should describe the figures properties as each is added. Pupils might describe each these properties to a partner:

I have just put a 2-D figure into my set.
Person A tell your partner what it is called. Yes, it is an oval.
Person B tell their partner all the things that make this figure an oval. Remember, we call these the properties.
Yes, it has no corners. What other properties does it have?

5. Add more figures, after each, ask pairs to name and describe the properties as above.
6. Once you have two or three figures in your set, ask each pair to choose a figure from their own set of shape cards that they think could also belong to the set you have created.
7. Ask pupils to hold this figure up so that you can see it. This serves as a quick assessment tool.

So you think that a circle belongs in my set? Why?
Mary please revoice that.
Does anyone disagree? Explain why.

8. Ensure that pupils justify their choice based only on the properties of the given figures. Pupils should understand that it is the properties of shapes and not their colour or size that determines their category.
9. When pupils are familiar with the game they can play it in small groups.

Guess my Secret

Like the activity above, pupils need to be comfortable with sorting 2-D shapes and need to have a good understanding of properties.

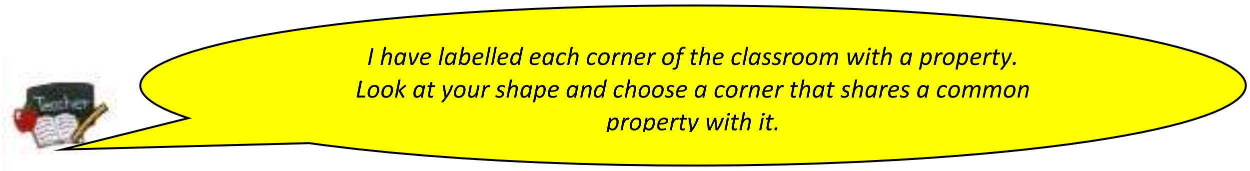
1. The teacher begins with an empty set (for example, a hula hoop) and chooses one property for the set but does not tell it to the class (for example, shapes that have four corners).
2. Pupils take turns choosing a shape from the teachers master set and ask ‘Does this shape belong in your set?’
3. Before replying ‘yes’ or ‘no’ the teacher should ask the pupil to describe the properties of the shape they have chosen.

Tell me about your shape Jack.
Are there any properties that Jack has forgotten to include?

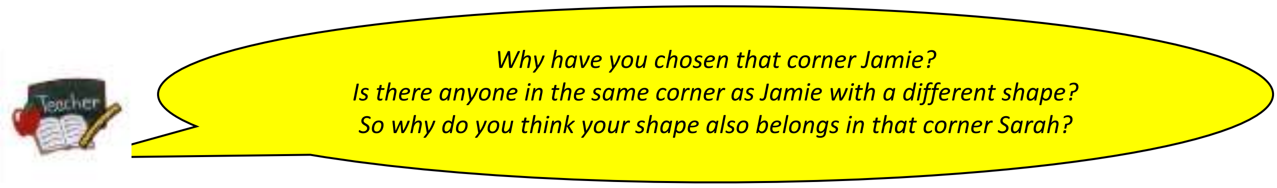
4. If the chosen shape has the property that the teacher has pre-selected then it can be added to the set, if not the teacher explains ‘Sorry Jack that shape does not belong in my set’.
5. Pupils take turns choosing shapes and describing their properties until they are able to deduce what the property might be.
6. When comfortable with the game, pupils can play it in small groups or pairs.

Four Corners

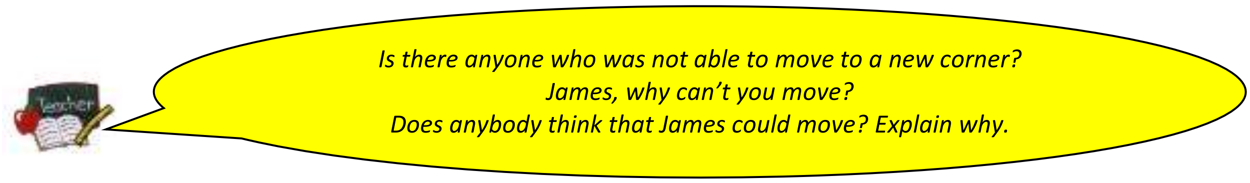
1. Allocate each pupil a cardboard or plastic 2-D shape.
2. Label 4 corners of the classroom with a property, for example, straight sides, curved sides, equal sides, less than four corners, etc.



3. Pupils go to a particular corner based on the properties of their shape.
4. Pupils from each corner must justify their choices to the rest of the class:



5. Pupils choose a different corner that shares another common property with their shape.



6. Repeat this process until all pupils agree that they have been to all possible corners for their shape.
7. Pupils record their shape along with a property list for each shape (they can use the corners they were at as a reminder).
8. This activity can be repeated by pupils swapping shapes.

Extension

After recording the above, challenge pupils to list as many properties as they can that could be used when playing the ‘four corners’ game the next time. Ask each table or group to compile their ideas into a master list. Ask each group to share their list with the whole class thus compiling a comprehensive class property list. The teacher should record and display this list. The next time the game is played the pupils' suggestions can be used to label the four corners.

Famous Shapes

1. Draw some 2-D figures onto card.
2. Choose three pupils to stand in front of the class.
3. Without them looking, stick either a figure or a written word onto their foreheads so that the rest of the class can see them clearly.
4. The three pupils take turns asking questions to determine what shape they are.
5. Pupils can only ask questions to which the answer is yes or no, for example:

   'Do I have any curved edges?; Do I have three vertices?'

6. Once comfortable with the game, pupils can play this game in small groups or pairs.

Extension

To extend this activity, write the names of the 2-D figures on card rather than drawing them. This requires pupils to visualise the figures in their heads in order to determine their properties.

Picture Gallery / Barrier Game

1. Each pair needs a card with a simple picture or pattern made from some 2-D shapes. They also need at least two of each shape.
2. One pupil takes a card, hiding it from his or her partner. Without touching any of the pieces, this pupil gives instructions to enable the other pupil to assemble the picture.
3. Once complete, the arrangement is compared with the picture on the card.

Extension

Pupils can create a simple picture and write a set of instructions for a partner to assemble without seeing the picture[47]. Some of the mathematical language used may include:

names of shapes, position, top, bottom, side, middle, corner, over, under, underneath, above, beside, next to, below, right, left, in front, behind, opposite, between, quarter turn, half turn, whole turn.

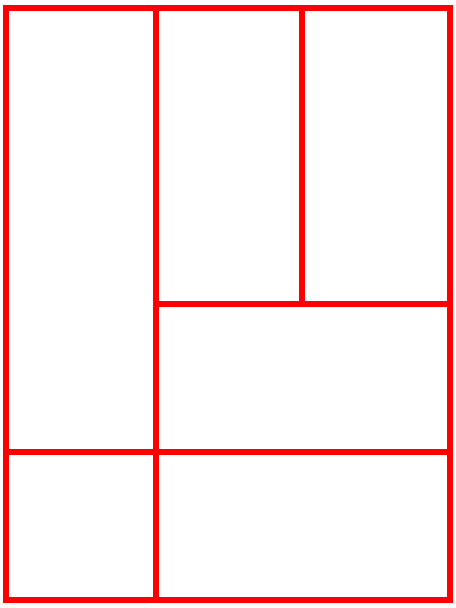
Where should that piece go in relation to the ...?
How can we move that piece into the correct position?
Should it be turned a quarter turn?
Which instructions are most helpful?

47 Th... Employment

Extension Problem

Is a square a rectangle?[48]

How many rectangles can you and your partner find in this shape?



ICT Opportunities

[Is a Square a Rectangle?](http://nrich.maths.org/191)

How many of the rectangles are differently shaped?
Find some that are 'similar', for example, the same shape but a different size. How many did you find?
Did anyone else find a different number?
What strategy did you use to keep track of how many rectangles you were counting?
Mary please tell me that in your own words.
Did anyone use a different strategy?
Shelby's strategy worked well so let us all try it.

[48] Link to Is a Square a Rectangle?: accessed at http://nrich.maths.org/191

LEVEL B.4: CONSTRUCT, DRAW, COMBINE AND PARTITION 2-D SHAPES

TEACHING NOTES

Pupils, regardless of age, should begin to explore shape through play and free exploration. Rich and stimulating instruction can be provided with free play and guided activity. Mosaics such as pattern blocks or tiles, tangram puzzles and are useful. Geoboards and geo-strips can also be used meaningfully by pupils at this level.

Please see the following sections of this manual for further activities suitable for this level:

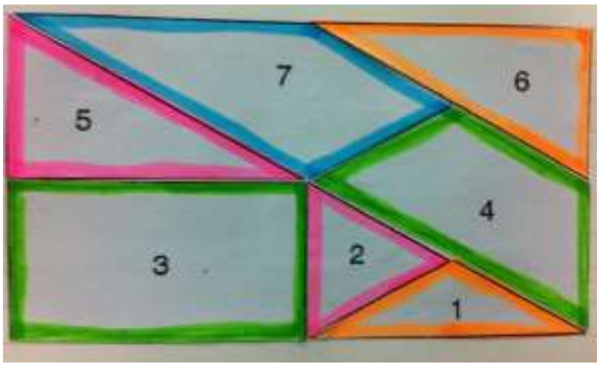
- Level A.4: Combine and divide 3-D and 2-D shapes to make larger or smaller shapes
- Level C.3: Construct and draw 2-D shapes; construct 3-D shapes
- Level C.5: Combine, tessellate and make patterns with 2-D shapes

SAMPLE LEARNING EXPERIENCES

The 7-piece Mosaic Puzzle[49]

Pupils need a durable copy of the mosaic puzzle. The pieces are numbered on their topsides to aid discussion of the activities. The 7-piece rectangular puzzle comprises:

1. An isosceles triangle (piece 1)
2. An equilateral triangle (piece 2)
3. Two right angles (pieces 5 and 6)
4. A rectangle (piece 3)
5. A trapezoid (piece 7)
6. An isosceles trapezoid (piece 4)



Play

1. Allow pupils to use their imaginations in using the pieces to create whatever they wish. This 'play' will allow teachers to assess how pupils use, think and talk about the shapes.

What can we do with these pieces?
Jenny can please share your work with us. You made a person.
Did anyone else make a 'real world' picture? Yes, Tom made a person.
Did anyone make something that is not a 'real world' picture?
Look. Karim made a pattern - tell us about it.

Combining Pieces

[49] Van Hiele, (1999)

Some pupils may have used a) two of the pieces to create a new shape or b) joined two pieces to make another piece, for example, combining pieces 5 and 6 to make shape 3. These combinations can be shared with the whole class. Combining pieces should be explored by the whole class.

Elsie has put piece 5 and 6 together and made shape 3.
Try to find other pieces that can be made by combining two other pieces.

Only piece 1 and 2 cannot be made by combining two other shapes.

Extension

Pupils extend the concept of combining two shapes to create another shape by combining 3 shapes. Pupils can either place their pieces directly on top of the shape they want to make or form it next to the piece.

There is one piece that can be made by combining 3 others.
Try to figure out which shape it is.
Work with a partner if this helps.

Recording

Pupils should record their solutions:

Who has an idea about how we can record our work to save our ideas?
Does anyone have a different method?

Pupils regularly record their learning and mathematical ideas. It is important that they become used to recording their work using a wide range of strategies. For this activity recording might include:

- recording learning in a learning log using words and /or pictures;
- recording the procedural steps and end product using a digital camera;
- recording verbally using audio recording a) software such as 'Audacity'; b)a Dictaphone; or c) using a video recording device;
- using dotted paper or grid paper to record the shapes made; and
- tracing around the larger piece and then the smaller pieces to show the shapes that were combined.

Further Investigation

Pupils will now know that combining two or more pieces can create a new shape that is not like any of the 7 pieces. They can now investigate how many shapes are possible by combining the same two shapes, for example:

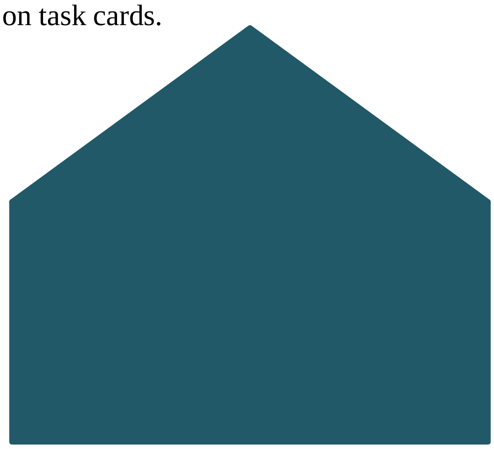
- with pieces 5 and 6, six shapes are possible;
- try the same activity with pieces 1 and 2;
- for the following shape the teacher could pose the challenges including:

Find two pieces that will make this shape. Are the pieces number-side up?
Has anyone done it a different way?
How can it be completed with the number-side down?
How can it be made with pieces 1 and 7?
Did you make it with the number-side up or down?
How did you do it Lizzie? What other two pieces will make this shape?
Record your learning in your learning log.

Further Mosaic Puzzle Tasks

These puzzles present a greater challenge. They can be presented orally or on task cards.

Task 1: House Puzzle



a) Make a house like this one using 2 pieces
b) Trace around the house to form the shape on paper
c) Make the shape with two other pieces
d) Make the shape with three other pieces.

Can you do it two different ways?

Can it be made with four pieces?

Task 2: Tall House Puzzle

a) On a piece of paper make a tall house with piece 2 as the roof and one other piece
b) Trace around the tall house
c) Make the shape with pieces 5 and 7

Can you make it with three pieces?

Task 3: Make a Puzzle

a) Use any two, three or four pieces
b) Make a shape with these
c) Trace around it on paper and colour it in

d) Can you make this shape using different pieces?

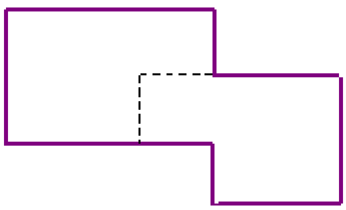
e) Write your name on your paper and give your puzzle to another pupil to try

Some pupils use strategies to solve these puzzles, for example, in part d) of both House Puzzles, pupils who know that rectangular piece 3 can be made up of pieces 5 and 6 may use this relationship to arrive at their solution. Similarly, they might know that pieces 1 and 2 form a triangle which can be used as the roof.

For the three puzzles above it is important for pupils to be given the opportunity to share their puzzles and solutions with each other. Appropriate teacher questioning will ensure that pupils gain deeper insight into the tasks and learn from others. To deepen mathematical thinking, enough time should be set aside for discussion throughout the lesson and at the end of the lesson.

Extension Activity: Same Shapes [50]

In this problem solving activity irregular shapes are provided and pupils must attempt to cut them in half in order to make two matching shapes.



ICT Opportunities

[Make Same Shapes]

[50] Link to Make Same Shapes: can be accessed at http://nrich.maths.org/990

LEVEL B.5: IDENTIFY HALVES AND QUARTERS OF 2-D SHAPES

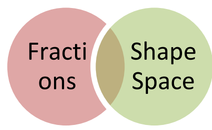
TEACHING NOTES

The use of a variety of representational models is crucial to the successful teaching of fractions. [51] Three main models are recommended: the area, linear and set models. Area models represent fractions as part of an area and are therefore useful when exploring fractional parts of 2-D shapes. Considering Level B.5 is concerned with identifying halves and quarters of 2-D shapes, only the area model that is explored here. However, to deepen pupils' understanding of fractions, it is recommended to also use the linear and set models. The PDST have produced a comprehensive resource that is available for free at http://pdst.ie/node/2509 . Where applicable for some activities below, you are referred to the Fractions manual.

Repeating the same activity with pupils using a variety of resources and models reinforces concepts about fractions. Some commonly available resources that are suitable when exploring fractional parts of 2-D shapes include:

- pattern blocks/attribute blocks
- geoboards - shapes made with rubber bands
- paper shapes of regular polygons (squares, rectangles, hexagons, triangles)
- grid paper with drawn shapes
- dot paper with drawn shapes

SAMPLE LEARNING EXPERIENCES



Exploring Halves and Quarters of 2-D Shapes with an Area Model

See PDST Fractions Manual Level A.1 for suitable teaching and learning experiences.

Fraction Game (for 2 players)[52]

This game may be helpful to consolidate pupils' understanding of halves and quarters in the context of the area model. See PDST Fractions Manual Level A.1 for a detailed description of this game.

[51] Van de Walle (2007)

[52] Deboys and Pitt (1980)

Correct Shares[53]

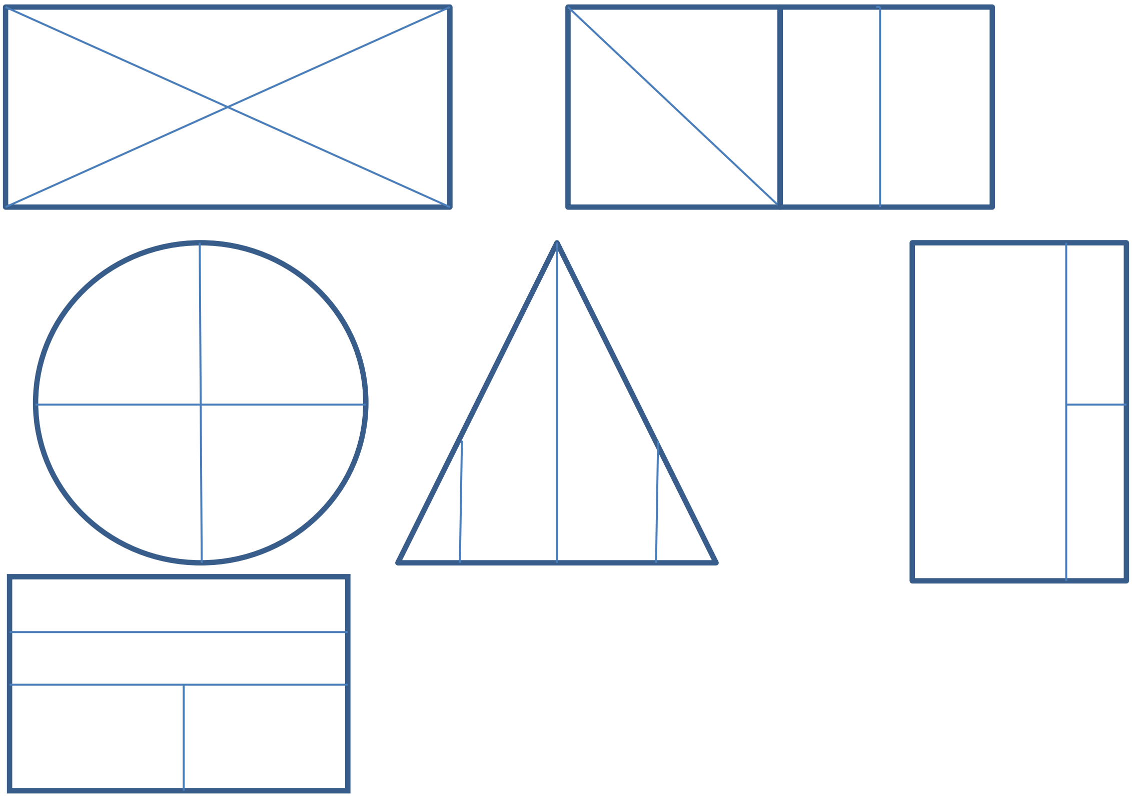
1. Draw or display shapes like the ones shown below.
2. Pupils identify those that have been correctly divided into equal fractional parts, either halves or quarters, and those that have not.
3. Pupils must explain their reasoning.

The shapes provided should cover the following categories:

- same shape, same size (equivalent)
- different shape, same size (equivalent)
- different shape, different size (not equivalent)
- same shape, different size (not equivalent)

Pupil Misconception

Many pupils incorrectly believe that for equal shares, the shape needs to be the same. This task assesses pupils' understanding - it is the size of the part that matters and not the shape.



[53] Van de Walle (2007)

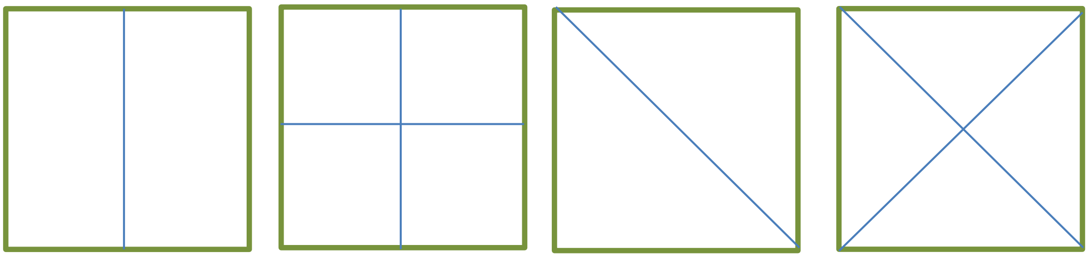
Paper Folding

1. Folding a piece of paper in two unequal parts and discuss the equality of the parts with pupils.
2. Pupils will have to consider the size of the unit as opposed to its shape.
3. Once the idea of equality has been established, pupils fold their shape into equal halves and colour in one half.

Fold your paper in half. How do you know that it is half?
Is there another way of folding it in half?
Please revoice that for me Ann.
Did anybody do it a different way?

4. Fold the shape into quarters.

Now fold your paper to make a quarter. How many quarters are there?
Which is bigger, one half or one quarter? How do you know?
How could you use quarters to make a half?
How much bigger is three quarters than one-half?
Record your work in your learning log- use pictures and words.



5. This activity can be repeated using different shaped paper.

Turning Discs

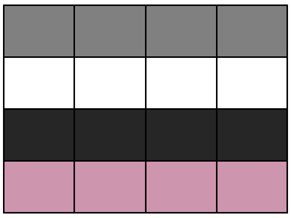
See Level B.1 of this manual for a detailed explanation of this activity.

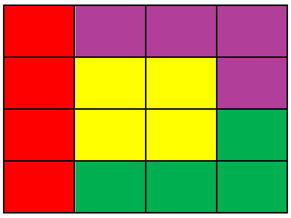
Show me one quarter/Show me one fourth.
Show me another quarter.
How much of the circle is blue now?
If half of your circle is blue how much of the circle is now red?
How many quarters would fit inside the blue half?
How do you know? How could you test that?
Record what you have discovered in your learning log using words and pictures.

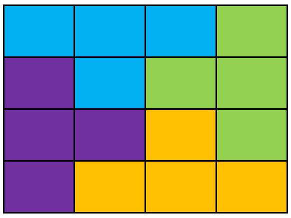
CONSOLIDATION ACTIVITY

Finding Quarters

The following open-ended activity challenges the pupil misconception that for equal shares, shape as well as the size, need to be the same. Pupils divide 4 × 4 grids into quarters in as many ways as possible.







Ensure that pupils share their solutions with others and that a class discussion ensues.

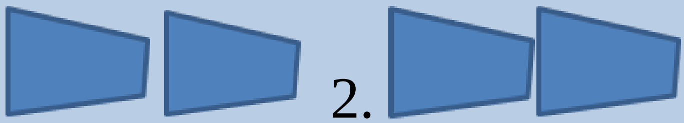
LEVEL B.6: IDENTIFY LINE SYMMETRY IN SHAPE AND IN THE ENVIRONMENT

BACKGROUND KNOWLEDGE FOR TEACHERS

Line Symmetry

An object has line symmetry if, when you draw a line down the middle, the left side is a mirror image of the right side. Often it is incorrectly suggested that a picture is symmetrical if 'it is the same on both sides'. This is not a helpful explanation of line or mirror symmetry, since the two sides are usually different as they are reflections of each other.

1. 2.



"the same" "symmetrical"

In Figure 1, the two parts are the same, but the total picture does not have mirror symmetry; in Figure 2, the two parts are different (being reflections of each other) but the whole picture is symmetrical.

TEACHING NOTES

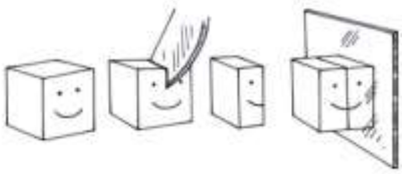
The concept of symmetry comes naturally to many children and is often evident in much of their play including construction and artwork. The classroom, home and school environment should provide much of the material required to introduce line symmetry, for example:

- children's clothes
- books
- toys
- windows, doors
- flowers, leaves and plants, etc.

SAMPLE LEARNING EXPERIENCES

Mirror Symmetry

Using a mirror is an effective way to show pupils how half of an object can be reflected symmetrically. It is important to point out that the reflected image is not 'the same' but is a reflected or mirror image that creates a whole.



Sorting

Using two hula hoops, a range of symmetrical and non-symmetrical objects and images can be sorted into two sets: objects that are symmetrical and those that are not.

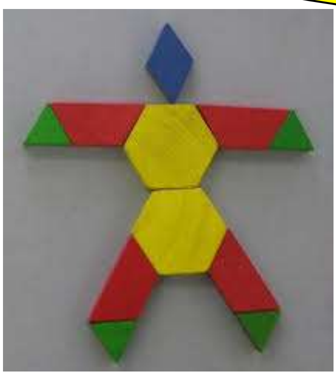
Which set does this leaf belong in? Explain why.
If I cut it in half, would both halves be a mirror image?
What if I cut it in another direction? Why? Why not?

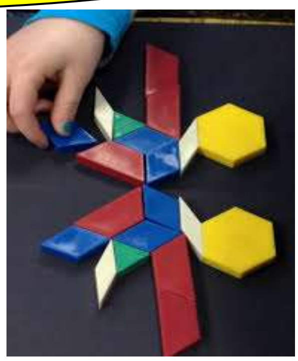
In pairs, pupils can search the classroom for something symmetrical and non-symmetrical to add to each set.

Pattern Block Symmetry

1. Show pupils real objects or images that have line symmetry and discuss each.
2. Using a visualiser, model creating pattern block designs that have line symmetry and also some that do not. Discuss with pupils why some designs are symmetrical and others are not.
3. Using pattern blocks, pupils create designs that have line symmetry.
4. Pupils share their designs with the class using a) a visualiser or b) a fish-bowl technique.
5. Pupils take photographs of their designs to create a class display or on-line slide show that could be shared with another class or parents.

Does Jodie's design have line symmetry? Explain.
It has line symmetry when the line is drawn vertically, would it still have line symmetry if we drew the line horizontally? Why?
Is there anywhere else we could draw the line?





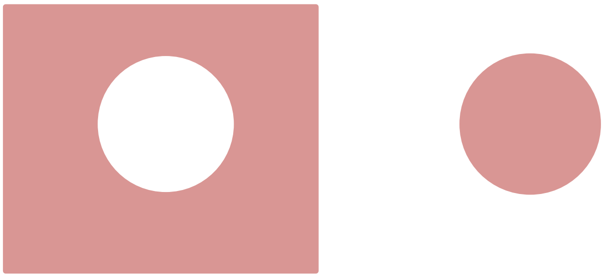
Art
Shape Space

Ink and Paint Blots

1. Pupils fold a piece of paper in half and then apply ink or paint to one half
2. Pupils fold the painted paper over to create a reflected image
3. These could be used to create a 'line symmetry' class display.

Paper Folding and Cutting

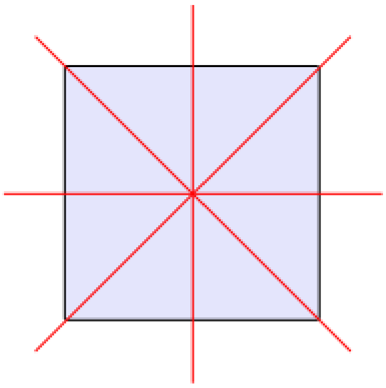
By folding a piece of paper in two then tearing or cutting out a shape, pupils discover that both the hole left in the paper and the paper that has been torn out are symmetrical.



A more common variation of this activity is making paper snowflakes or butterflies.

Shape Folding

1. Pupils are given a 2D shape on paper or card.
2. Pupils explore how many different ways they can find to create two symmetrical halves for the shape.
3. They can either a) cut out the shape and fold it or b) use a pencil to draw lines onto the shape.



This activity can be completed in pairs to promote discussion and reasoning. The example of a square is shown, there are four possible ways to create equal halves or 'line symmetry'. Pupils should also try it with different 2D shapes.

How many ways have you found to split the square into two symmetrical parts?
How do you know that this is symmetrical?
Jim please tell me that in your own words.
Has anyone discovered another way?
Record what you have done using pictures in your learning log.

Environmental Symmetry

Learning activities that support 'symmetry in the environment' can be found in other sections of this manual:

- Level B.8
- Level C.6
- Level C.7

LEVEL B.7: EXPLORE AND RECOGNISE ANGLES IN THE ENVIRONMENT

TEACHING NOTES

It is recommended that Maths Trails outlined in Level B.8 should be used in this level. Activities from level C.10 and C.11 may also be useful for this level.

SAMPLE LEARNING EXPERIENCES

Turn Investigations

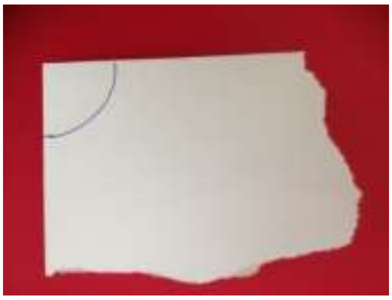
Beginning with concrete and environmental examples of ‘things that turn’ highlights the everyday application of angles for pupils. Pupils can locate things that turn in the classroom, school or at home such as door handles, wheels, clocks, etc.

Investigating Angles as Corners

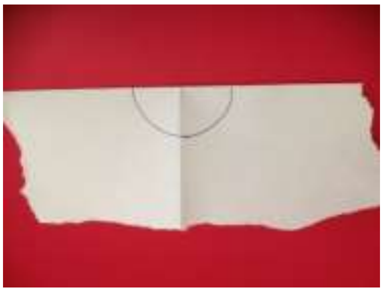
Through sorting activities, pupils will become aware that squares and rectangles have ‘square corners’. Demonstrating how to make a right angle by folding paper may be a useful tool for pupils. If a torn piece of paper is used, it allows pupils to focus on the angle that is produced when the paper is folded rather than a regular shape (for example, as would be produced if a square was folded in four).

1. Fold the paper to create four right angles

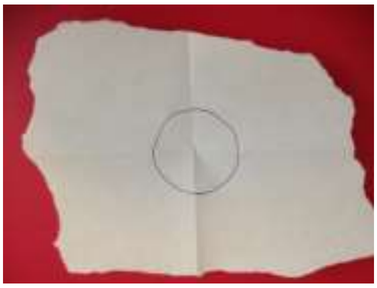
2. A right angle can now be made



3. Two of the folded shapes together show that two right angles make a straight line



4. Four fitted together create a whole circle

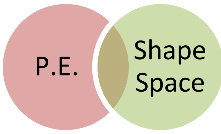


The right angle in the second picture can be used to ‘measure’ the corners on various shapes found in the environment.

Rotating Angles

An angle should not only be viewed as a static measure, but also as one that rotates. One way of doing this is to place a pin through the centre of the paper and observe the X mark moving from its starting position to:

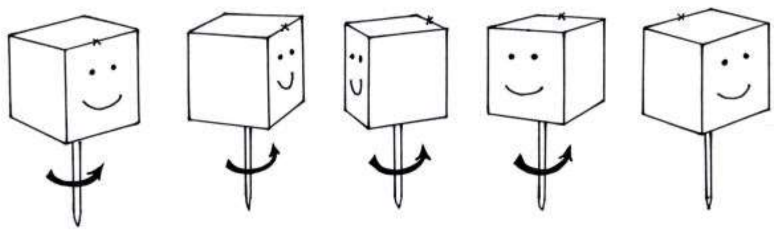
- right around back to its starting position - or a whole turn
- a right angle is a quarter turn
- a straight line is a half turn



Making Full, Half and Quarter Turns

When pupils learn about the concept of quarter, half and full turns, it is important that they have opportunities to experience these kinaesthetically. Therefore, plenty of opportunities should be provided for them to practice these turns during PE time. Putting pupils into pairs affords them the opportunity to both give and follow instructions in relation to turns. Once pupils have experienced turning their own bodies in quarter and half turns, they can make simple figures like the one shown below. Drawing a face on one side of the cube will allow pupils to give and follow instructions to each other regarding how many turns to make.

Make sure your face is facing the front of the room.
Turn your figure one-quarter turn. Where is it facing?
Turn it another quarter turn. Where is it facing now?
What other way could I ask you to make two quarter turns?



LEVEL B.8: IDENTIFY AND DISCUSS THE USE OF 2-D AND 3-D SHAPES IN THE ENVIRONMENT

TEACHING NOTES

Maths Trails

There are eight types of problems outlined in the Primary School Curriculum (2000) – maths trails being one of these. Maths trails provide a valuable opportunity for pupils to see maths as something that exists all around them. Carefully designed trails consolidate learning that has taken place in the classroom. Trails can focus on a particular topic or cover several topics. They are particularly relevant to Shape and Space as they can encompass both the positional and directional components of the curriculum that pupils have covered as well as those relating to 3-D and 2-D shape. It is important to be critical when constructing a trail or using one that already exists.

> Maths trails should challenge pupils to think mathematically, not merely focus on observational or procedural skills. Open-ended questions that require reasoning and opportunities for justifying should be included. This helps to ensure mathematical thinking is to the forefront in a trail.

The questions in any trail should provide opportunities for pupils to solve problems using a range of strategies. In line with the instructional framework, trails should be used as a stimulus for mathematical talk; this should be done before, during and after the trail. In infant classes small groups of pupils might be accompanied by a teacher or parent when completing their maths trail, an adult can help them to navigate their way around the trail, keep them on task, and read the questions to them if required.

Getting Started

Maths trails can be designed either by teachers or by pupils. However, it is recommended to work with pupils on recognising and talking about maths in the environment before embarking on trails. This introduction can take place in the classroom, school hall, playground, etc. Pupils can devise questions with a partner based on a certain area in the classroom. The teacher can also model doing this so that pupils experience rich, open-ended questions.

Looking around the computer area of the classroom, describe what you see.
Yes, bricks. Could you estimate how many bricks there are on that wall?
What strategy did you use?
What kind of shapes are those? How many sides do they have? I wonder what that shape is called?
I wonder how tall the sunflower has grown. Estimate the height. How could we measure it?

Maths trails should contain a mixture of open and closed questions. They should challenge pupil's mathematical thinking and provide opportunities for estimation, problem solving, reasoning, and communicating mathematical ideas.

Using Photographs

Mathematics in the environment can also be explored using photographs taken by pupils in the school grounds or local area. These photographs can provide a stimulus for mathematics discussion.

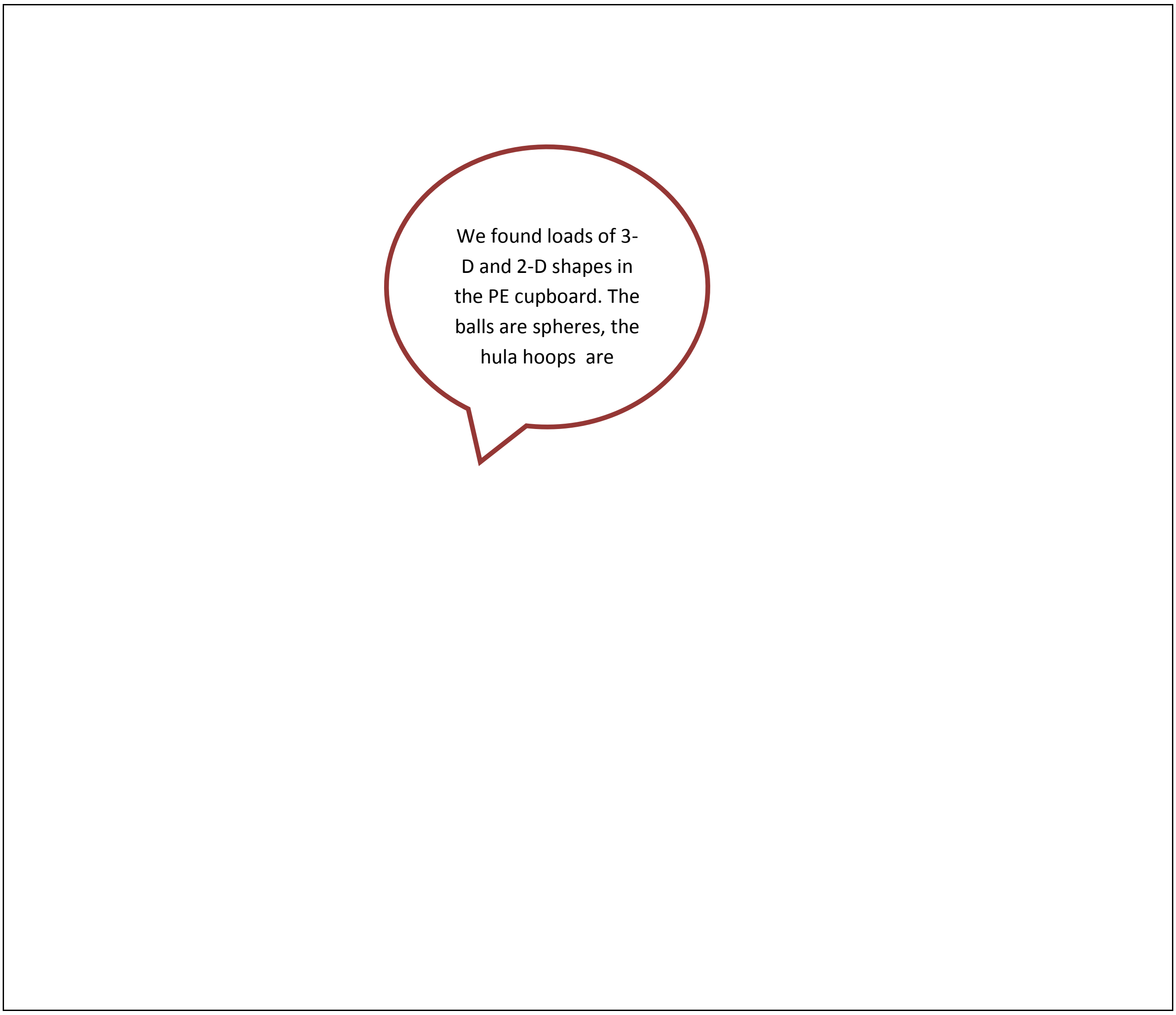
Take the digital camera out into the school corridor. Try to find two 3-D shapes and two 2-D shapes and take pictures of them.
We will compare them to the pictures taken by John's group.

The bricks on the outside of the school are all rectangles, the red ones are bigger than the grey ones, no actually they're longer

The ice cream is a sphere and the cone is a cone!

The post box is like a cuboid, it has all straight edges and

The parachute is a giant circle, but only when children are pulling the edges otherwise it's all floppy



Shape and Space Trail

A maths trail should be specific to the local area of the school; these will be more useful than generic trails. Including pupils in the design of trails should increase motivation as well as presenting greater mathematical challenge. A Shape and Space trail might contain questions such as:

- **Station 1:** Can you see two 2-D shapes? Draw and label them on your group's task sheet. Now list the properties of both shapes. Explain to your group leader why these are 2-D shapes.
- **Station 2:** Draw a picture of the item sitting under the biggest window. What 3-D shape is it most like? Why? How does the shape of this item suit its purpose?
- **Station 3:** Look at the windows beside the front door of the school. Count how many windows there are. What shapes are they? Record the total number of sides and corners on your task sheet using words and pictures.

Face the front door, take 2 quarter turns. What can you see? Draw it.

- **Station 4:** What shape is the roof of the school? If you had to change it to another shape, which would it be? Discuss this with your group. You will need to explain your answer to the class later. Take a blank piece of paper each and draw the school with the 'new roof'.
- **Station 5:** You are now in the senior yard. With the digital camera take pictures of the following:
    - A cuboid
    - A rectangle
    - A circle
    - One other 3-D shape
    - Something with line symmetry
- **Station 6:** Begin with your backs facing the wall. Walk seventeen steps straight ahead, turn right, walk ten steps. Make a quarter turn to your left. What is directly in front of you?
- **Station 7:** On the ground is a pattern of squares within squares. How many small squares make up a larger square pattern? Draw one more pattern you can see on the ground.
- **Station 8:** You will see a press in front of the office. What 3-D shape is the press? Estimate how many of these is needed to stack on top of each other to reach the roof in the hall. How many would you need if it were turned on its side?

More information on creating and using maths trails can be accessed at the archived PCSP website http://www.ppds.ie/pcsparchive/ma_trail.php

LEVEL C TEACHING AND LEARNING EXPERIENCES

LEVEL C.1: IDENTIFY, DESCRIBE AND CLASSIFY 3-DSHAPES BY COMPARING THEIR PROPERTIES INCLUDING TRIANGULAR PRISM AND PYRAMID

BACKGROUND KNOWLEDGE FOR TEACHERS

Categories of 3-D shapes[54]

The table below outlines some of the important categories of 3-D shapes. This information may be useful for teachers before engaging with the activities to support the pupils in *discovering these properties*.

Shape	**Description**
Sorted by Edges and Vertices	
Spheres and 'egglike' shapes	Shapes with no edges and no vertices (corners) Shapes with edges but no vertices (e.g. a flying saucer) Shapes with vertices but no edges (e.g. a football)
Sorted by Faces and Surfaces	
Polyhedron	A solid three dimensional shape with flat faces[55].
Cylinders	A solid shape with one curved surface and two congruent circular or elliptical bases. The base is the same as the top, and also in-between. Because it has a curved surface it is not a polyhedron[56].
Right cylinder	A cylinder with elements perpendicular to the bases. A cylinder that is not a right cylinder is an oblique cylinder.
Prism	A solid 3-D shape with two identical parallel bases, all other faces are rectangles. The name of the base determines the name of the prism[57].
Rectangular prism	A Prism with rectangles for congruent end faces
Cube	A prism with six congruent square faces
Cone	A solid shape with an elliptical or circular base and a curved surface that tapers to a point (vetex)[58]. The vertex need not be directly over the base.
Circular cone	Cone with a circular base.
Pyramid	A solid shape with a polygon as a base and triangular faces that taper to a point (apex)[59]. Pyramids are named by the shape of the base, triangular pyramid, square pyramid, octagonal pyramid and so on.

54 Van de Walle, Karp & Bay-Williams (2013, p.413) and, Bana, Marshall and Swan (2006)
55 http://www.mathsisfun.com/geometry/polyhedron.html
56 http://www.mathsisfun.com/geometry/cylinder.html
57 http://www.amathsdictionaryforkids.com/dictionary.html
58 http://www.amathsdictionaryforkids.com/dictionary.html
59 http://www.amathsdictionaryforkids.com/dictionary.html

The following structured diagrams are useful for sorting activities:

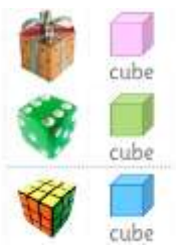
Carroll Diagram: A grid link structure for categorising results and is named after its inventor Louis Carroll.

Venn Diagram: A diagram that represents sets and their relationships and is named after John Venn who developed the method in the 1890's.

Tree Diagram: A diagram using a branching process to solve a problem.

TEACHING NOTES

Activities in this section are reflective of previous learning experiences in Level B.2. Solid shapes are prominent in the environment and a study of these usually precedes the work with 2-D shapes. Sets of solids should be available for sorting and should include commercially produced shapes as well as material collected by the teacher. Food containers can provide examples of cubes, cylinders or cuboids. Roofs, gables and Toblerone boxes are examples of triangular prisms. Marbles, beads, balls, etc. are examples of spheres. Ice cream cones, fir cones, etc. illustrate conical shapes.





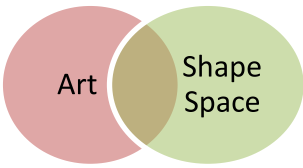
The line of development in the next section begins with:

1. Detailed investigations of the properties of shape, for example, faces, edges, vertices, etc.
2. Sorting for prisms and pyramids.
3. Consolidation of learning through open-ended investigation.

SAMPLE LEARNING EXPERIENCES

Finding Faces[60]

Investigation of 'faces' of shapes leads naturally into the activity of opening out cardboard shapes



ICT Opportunities

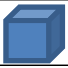
[Label Parts of Shape](#)

[60] Link to Label Parts of Shape: http://www.ngfl-cymru.org.uk/vtc/castle_shapes/eng/Introduction/MainSessionPart2.htm

to find out how they were made. Pupils open cereal boxes, chocolate cartons (prisms), 'Smarties' boxes (cylinders) etc. and put them together again.

The record of the result might appear as shown below:

Number of each 2D shape used to make the following 3D shapes

				
	6	-	-	-
	-	6	-	-
	-	1	2	-
	-	3	0	2

Find the shapes with all faces square. Name them.
Name other shapes with six rectangular faces.
Some of the cartons have square faces, some have rectangular faces. Sort them.
Describe the faces of a soup tin.
Kim revoice what Ann just said.
Name all the shapes with curved surfaces.

Edges[61]

Pupils count the edge of each shape in turn and record their result. Pupils should have a concrete example of the 3-D shape while investigating these properties.

ICT Opportunities

Exploring Shapes

How many edges has a cube? Mark each as you count.
Find another shape with the same number of edges and name it.
How many edges has the toblerone carton/triangular prism?
Count the edges of the pyramid.

[61] Link for Exploring Shapes: http://www.learnalberta.ca/content/me3us/flash/index.html?launch=true

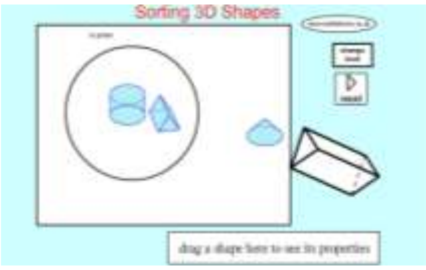
Sorting Shapes: Sorting Activity for Prisms and Pyramids[62]

Sorting in a variety of ways to highlight a particular property is a useful activity (a selection of prisms[63] and pyramids[64] are needed for this activity).

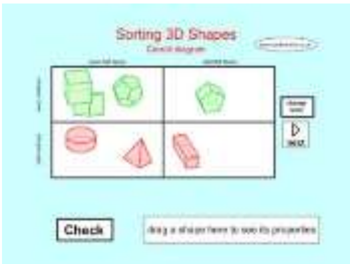
1. It is recommended that pupils decide on the common property that they are sorting for and **discover** similar properties among a variety of 3-D shapes. Once pupils have sorted for one property, the teacher can encourage them to sort for a different property using the same collection of shapes.
2. Properties pupils may discover include:
   - shapes with square faces, rectangular faces, triangular faces.
   - shapes with the same number of faces, same number of edges.
   - congruent/matching faces, parallel lines.
3. Pupils should then record their results. Venn, Tree and Carroll diagrams are useful for this as shown below:

ICT Opportunities

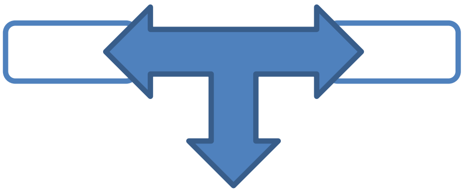
Sorting 3-D Shapes



Venn Diagram



Carroll Diagram



Tree Diagram

ICT Opportunities

Examples of Prisms and Cylinders

ICT Opportunities

Examples of Pyramids

What shapes are the faces of this object?
How many faces/edges/vertices can you count?
Show these to your partner. Are any of the faces the same?
Think of any examples of these shapes that you might see in the supermarket.
Why have you grouped these shapes together?
Can this object join them? Why, why not?

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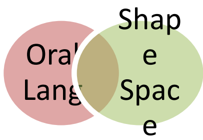
[62] Link for 3-D Shapes: http://www.mathsframe.co.uk/en/resources/category/21/sorting_and_classifying
[63] Link for Prisms and Pyramids: http://www.mathsisfun.com/geometry/prisms.html
[64] Link for Pyramids: http://www.mathsisfun.com/geometry/pyramids.html

CONSOLIDATION ACTIVITY

True or False



Prepare a set of true/false statements of the following forms. This can be used as an open-ended investigation in which pupils generate property statements for others in their class to test and justify.

"If it is a _______, then it is also a______."
"All ________ are__________".
"Some __________are_________.
For example:
"All prisms have a square face"
"If it is a pyramid then it is a prism"

BACKGROUND KNOWLEDGE FOR TEACHERS

Below is a table outlining some of the important categories of 2-D shapes for the teacher's background knowledge.

Categories of Two-Dimensional shapes[65]

Shape	Description
Concave, convex	An intuitive definition of concave might be 'having a dent in it'. If a simple closed curve is not concave, it is convex. A more precise definition of concave might be interesting to explore with pupils.
Symmetrical, non-symmetrical	Shapes may have one or more lines of symmetry and may or may not have rotational symmetry. These concepts require more detailed investigation
Polygons Concave,Convex Symmetrical, non-symmetrical Regular	A plane shape with three of more straight sides[66] All sides and all angles are congruent
Triangles	Polygons with exactly three sides
Triangles - Classified by sides Equilateral Isosceles Scalene	 All sides are congruent At least two sides are congruent No two sides are congruent
Triangles – Classified by angles Right Acute Obtuse	 Has a right angle All angles are smaller than a right angle One angle is larger than a right angle
Convex Quadrilaterals	Convex polygons with exactly four sides
Kite	Two opposing pairs of congruent adjacent sides
Trapezoid Isosceles	At least one pair of parallel sides A pair of opposite sides is congruent
Parallelogram	Two pairs of parallel sides
Rectangle	Parallelogram with a right angle
Rhombus	Parallelogram with all sides congruent
Square	Parallelogram with a right angle and all sides congruent

[65] Van de Walle, Karp & Bay-Williams 2013, p.411
[66] http://www.amathsdictionaryforkids.com/dictionary.html

TEACHING NOTES

Classifying and sorting activities require a deeper focus on the properties that make the shape what it is, not just that it looks like the others in its group. The aim of this level is for pupils to *discover* some of the properties of 2-D shapes that make them unique. Eventually this leads to pupils creating their own Minimal Defining Lists for each shape (Level C.9).

> Pupils should be encouraged to 'discover' the properties of shapes and not 'learn off the definitions'. This leads to a greater conceptual understanding of shapes.

The line of development in the next section begins with:

- Pupils discovering, identifying and recording similar properties among different 2-D shapes.
- Detailed investigations of these individual properties, for example, angles, sides, symmetry, etc.

SAMPLE LEARNING EXPERIENCES

Sorting Shapes[67]

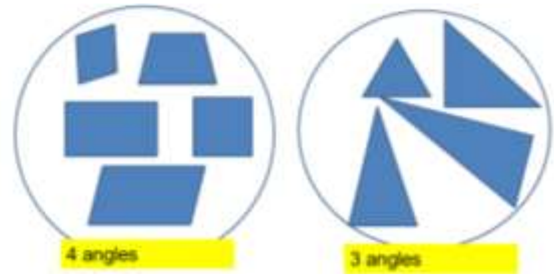
As for the 3-D sorting activity in Level C.1; however, the properties the pupils may discover in this 2-D sorting activity include:

- shapes with same type of angles, parallel sides, perpendicular sides;
- regular, irregular shapes; and
- shapes with the same number of sides, same number of angles.

ICT Opportunities

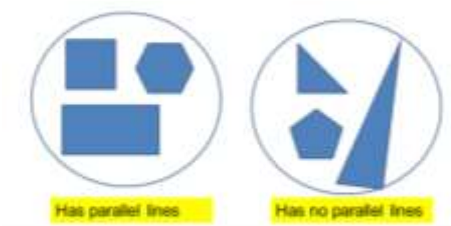
Printable Sorting Shapes

Number of angles:

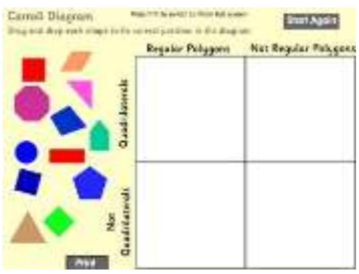


Type of Angle:

Type of Sides:



Regular / Irregular Shapes:



[67] Printing shapes http://wps.prenhall.com/ca_ab_vandewalle_mathematic_1/25/6506/1665690.cw/index.html

Examples of Regular and Irregular Shapes[68]:

ICT Opportunities

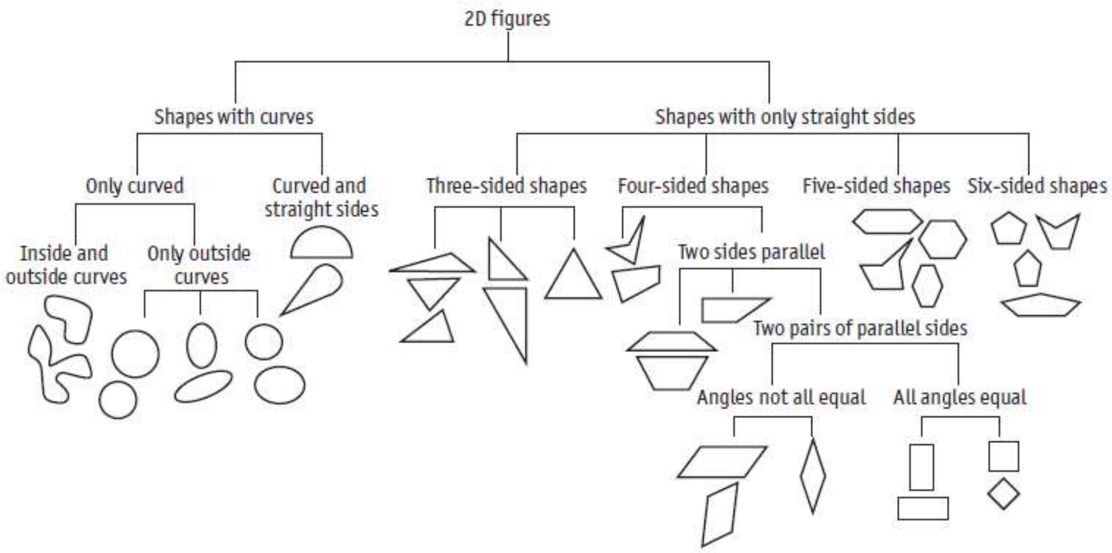
[Regular and Irregular Shapes]

Sorting Figures[69]

ICT Opportunities

[Printable Sorting Cards]

1. Extend 'Sorting Shapes' by giving pupils a collection of 2D figures (e.g. squares, rectangles, quadrilaterals, triangles, hexagons, ovals, circles).[70]
2. Ask them to construct a tree diagram to show how figures can belong to different groups.
   Invite pupils write a classification key for the figures in each group describing what they have in common (See example below).
3. Each group shares and explains their tree diagram. Through consultation and discussion all groups come together to devise a comprehensive class tree diagram. It may look like the diagram below.
4. Encourage pupils to refer to a mathematics dictionary to find the correct name for each of their groups (e.g. polygon, quadrilateral, rectangle, square).



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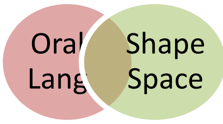
[68] Link to Regular and Irregular Shapes: http://www.iboard.co.uk/iwb/Sorter-RegularIrregular-Open-175

[69] First Steps in Mathematics: Space, Western Australia, p. 16

[70] Link to Printable Sorting Cards: http://illuminations.nctm.org/lessons/6-8/Sorting/SortingPolygons-AS-Cards.pdf



‘I’m thinking of a shape’

I’m thinking of a shape. Can you guess what it is?

The teacher lists things about the shape. Pupils make suggestions about which shape it is / draw that shape on a mini whiteboard. The student who guesses the shape then has a turn at describing a new shape.

The shape has 4 sides.
The shape has 4 corners all the same size.
The shape has two long sides and two shorter sides.

‘What am I?’ books[71].

Use A4 paper folded in half. On the inside of the ‘book’ draw a shape or paste in a cut-out shape. On the outside cover of the book get the pupils to suggest clues and record these. Pupils work in pairs to make a ‘What am I?’ book for a chosen shape. Some pupils may make more than one. The pupils share their books with others in the class to see if they can guess the shape from the given clues.

CONSOLIDATION ACTIVITY

What is My Shape?

A double set of 2-D shapes on card is required. In a group, one pupil has the secret shape folder/bag. Other pupils have to identify which shape is in the bag by asking only yes or no questions in relation to the properties of the shapes. They can eliminate shapes from their set as the activity continues and in the end compare their shape to the secret shape.

Does the shape have three sides?
Does the shape have 3 corners all the same size?
Has your shape got two long sides and two shorter sides?

The difficulty of this activity is largely dependent on the shape in the folder/bag. The more shapes in the collection that share properties with the secret shape, the more difficult the task.

[71] From: http://www.nzmaths.co.nz/resource/shape-explorers

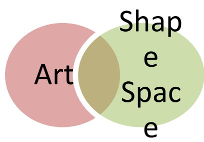
LEVEL C.3: CONSTRUCT AND DRAW 2-D SHAPES; CONSTRUCT 3-D SHAPES

TEACHING NOTES

Building three-dimensional shapes is more difficult than building two-dimensional shapes. A variety of commercial products permits construction of geometric solids. The following are approaches to making homemade models:

- Plastic coffee stirrers with twist ties or modelling clay – plastic stirrers can be cut to various lengths. Use twist ties inserted into the ends or small balls of clay to connect corners.
- Plastic drinking straws – slits can be made to the ends of straws so they can be inserted into the ends of other straws.
- Straws and pipe cleaners – are useful for construction of cubes and cuboids. Pipe cleaners are used to link together the framework of the shapes.
- Rolled newspaper rods – large skeletons can be built using newspaper and masking tape.
- Marshmallows and toothpick sticks – marshmallows can be used to highlight the vertices (corners) and toothpicks for the edges.

SAMPLE LEARNING EXPERIENCES



Polyhedrons[72]

1. Give pupils a range of polyhedrons (for example, cube, square, pyramid, tetrahedron) to examine to make sets of polyhedrons.
2. Pupils choose from a selection of materials (for example, play dough, straws, cardboard, etc.).
3. Pupils share and solve construction problems related to attaching the component parts.
4. Pupils explore different aspects of polyhedrons, for example, ask pupils to choose one polyhedron and make three models of it:

A skeleton model (dowelling or straws)

Straws: If pupils are making a cube they need twelve pieces of straw all the same length. For a cuboid they also need twelve pieces, either four each of three different lengths, or eight of one length and four of another (cuboid with square ends).

[72] First Steps in Mathematics: Space (2005, p.72)

A solid model (clay or play dough)

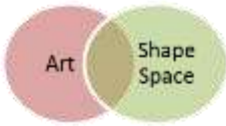

Hollow model (cardboard)




5. Pupils could write out the steps to construct each model for other pupils integrating procedure writing with the activity

What part of the shape do you need to focus on when you are using straws (play dough, cardboard)?
Which features of the objects are highlighted by the different materials?
Why are the corners not fitting quite right?
Pat please re-tell me that in your own words.
Is it the lengths of the sides or angles that need to be checked?
What did you need to focus on to make each type of model?
What is different?

Opening 3-D Objects[73]



1. Provide a collection of different cardboard boxes.
2. Pupils choose one of the boxes, trace around each of the faces and cut them out.
3. Pupils lay out the shapes so that they would remake the box if folded.
4. Pupils compare the net of the box with the way they laid out the faces.

ICT Opportunities

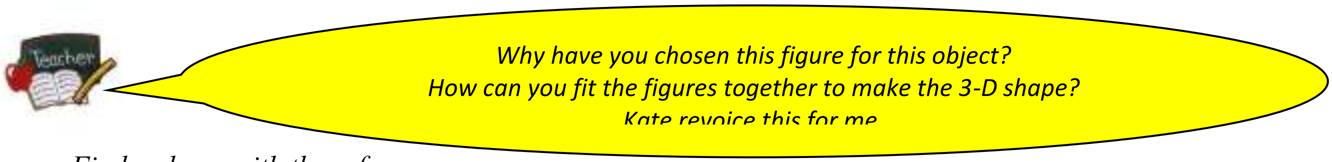
Match Nets to Shapes

Can you open up your object in a different way and still be able to put it back together to make the same object?
Is there another way still? Is it the same? If not, how is it different?
Does it matter that it is different?
Is there a number of different ways of putting the faces together so that they could be re-folded to make the box?

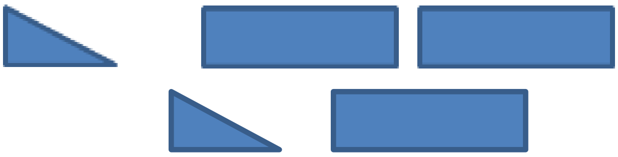
[73] Link to Nets of Shapes: http://www.sadlier-oxford.com/math/enrichment/gr4/EN0411b/EN0411b.htm

Matching 2-D with 3-D

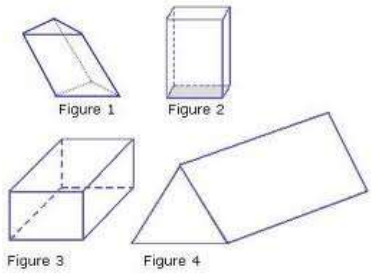
Provide pupils with a choice of 3D objects (for example, cubes, rectangular prisms, triangular prisms) and a sheet of paper showing a selection of different 2-D figures or alternatively a selection of 2-D figures on cards. Pupils select the 2-D figures needed to cover the 3-D object.

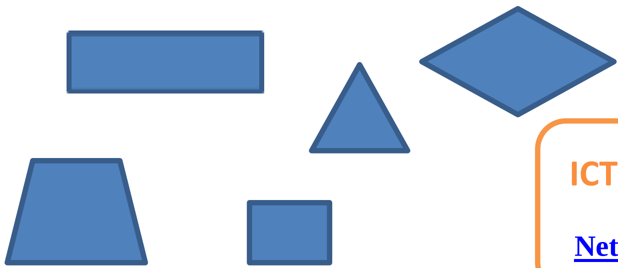


Find a shape with these faces…



Which cards do you need for each shape?

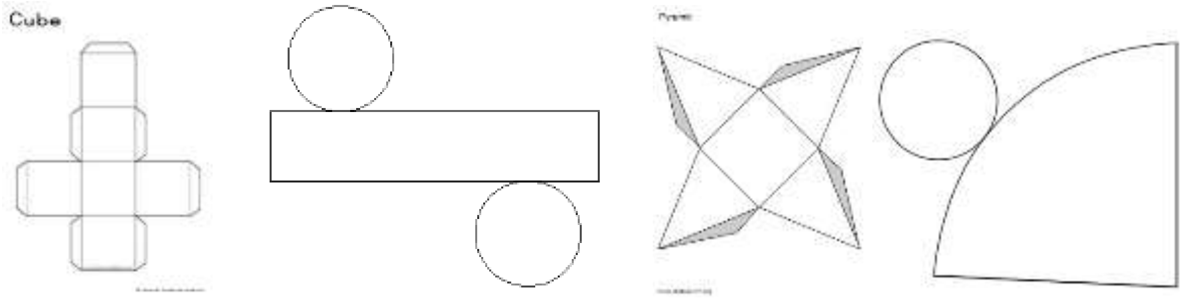


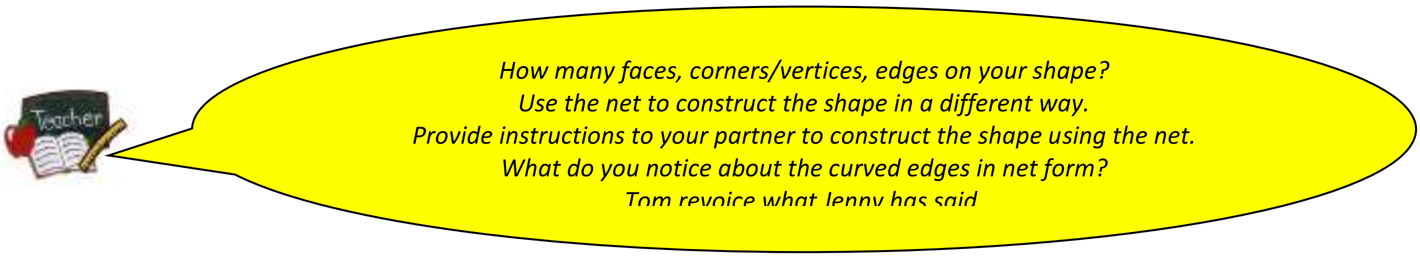


Nets[74]

Provide pupils with various nets of shapes to construct.

Print off various nets of shapes available here[75]





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[74] Link to Nets to Shapes Demo: http://illuminations.nctm.org/ActivityDetail.aspx?ID=70
[75] Link to Various Nets of Shapes: http://lrt.ednet.ns.ca/PD/BLM/table_of_contents.htm

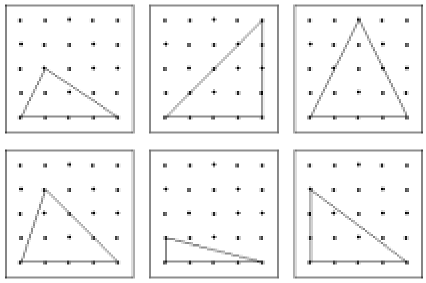
Extension Dice

1. Provide pupils with a dice and a net of a cube.
2. Pupils place the dots on the cube net so that when it is folded the dots will be in exactly the same place as on the dice.
3. Focus pupils on the position of the dots in relation to each other on the dice.
4. Ask the question:

How can you be sure the dots are in the correct face on your net?

Different Dice

The same as the activity above; however, provide dice in more complex shapes (for example, pyramids, prisms, etc.) and their nets.

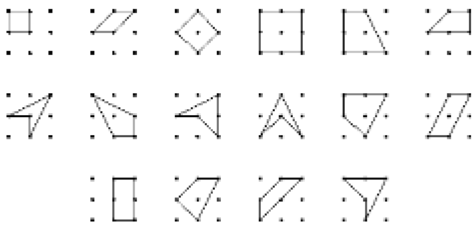


2-D Shapes

Constructing Triangles using a Geoboard

1. Pupils contruct any triangle on a geoboard.
2. Pupils make another triangle that is different in some way to the first.
3. Pupils create another triangle that is different to the first two and so on.
4. Pupils record their results on dotted paper.

How many different triangles can you make?
How are they different? What did you change?



Four-sided Shapes

1. Pupils make a four-sided figure on the Geoboard.
2. Pupils make as many different four-sided figures as possible. These are called quadrilaterals.
3. Finding similar quadrilaterals in shape and size might follow.

Are any of these shapes similar in any way? (Sides-parallel, congruent, angles-right angles, equal etc.)
Can you group similar shapes together?
What makes them similar?
Record these shapes on dotted paper

CONSOLIDATION ACTIVITY

Templates/Solids Game[76]

1. Pupils construct a set of 2-D cardboard figures they can use as templates to produce nets for a number of different polyhedrons.
2. Pupils should be in small groups to play this game.
3. Pupils take turns to roll a dice and trace around a figure from their 2-D shape template set that has that number of sides. (For example, if they roll a 4, they might choose their rectangle, rhombus or square)
4. As they build up their collection of figures, pupils write a list of polyhedrons that can be constructed from their figures. (For example, if they have two triangles and three rectangles, they could have a triangular prism)
5. The winner is the person with the most polyhedrons listed.
6. In order to elicit a mathematical discussion the teacher might ask the following questions:

Was there a figure that enabled you to construct more polyhedrons than others?

If so, what was that figure?

Why do you think you were able to use that one more?

[76] First Steps in Mathematics: Space (2005, p.85)

LEVEL C.4: IDENTIFY, DESCRIBE AND CLASSIFY 2-D SHAPES INCLUDING EQUILATERAL, ISOSCELES AND SCALENE TRIANGLE; PARALLELOGRAM; RHOMBUS; PENTAGON; OCTAGON

BACKGROUND KNOWLEDGE FOR TEACHERS

A four-sided figure with four right angles is often called a ‘rectangle’. However, rectangles are four-sided and therefore are a subset of a bigger class called quadrilaterals. Similarly, a four-sided figure with four right angles and four equal sides is often called a square when in fact it is a subset of a bigger class called quadrilaterals.

Rectangles are special kinds of quadrilaterals (the equiangular quadrilaterals), so, too, squares are the special rectangles that have all their sides the same length (the equilateral rectangles). Therefore, squares belong in the bigger set called rectangles. In a similar way, rhombuses are special kinds of quadrilaterals (the equilateral quadrilaterals) and squares are the special rhombuses that have all their sides the same length (the equiangular rhombuses).

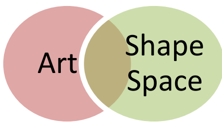
TEACHING NOTES

The distinctions between types of quadrilaterals are very important and need to be explored in significant detail with pupils.

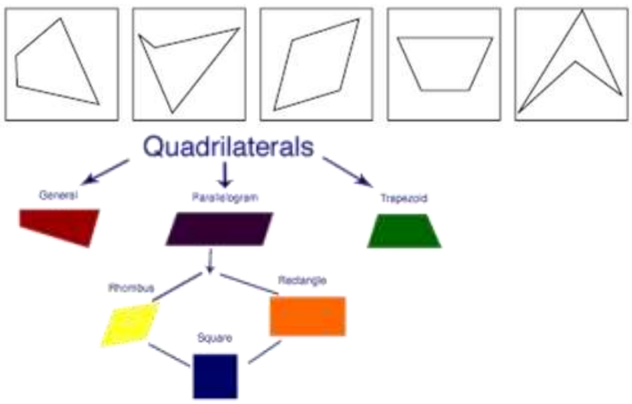
It may be appropriate here to revise some of the activities in level C.2 and for teachers to re-familiarise themselves with the categories of 2-D shapes. The aim of this level is to build on pupil’s prior knowledge and to *discover* some of the properties of 2-D shapes that make them unique and eventually lead to pupils creating their own Minimal Defining Lists for each shape.

SAMPLE LEARNING EXPERIENCES

Quadrilaterals: Open-ended Investigations



1. Pupils draw three different four-sided figures or quadrilaterals on separate cards and describe how each is different from the others.
2. Pupils work in groups to sort their figures for different properties.
3. Pupils share the different ways they sorted.



How are they different? What is different about the length of the edges?
What is different about the angles? Are these all quadrilaterals?
Have you made any familiar shapes that you can name?
Why have you put these figures together?
Try to think of a name for this category of quadrilateral.

4. Pupils label each group with an appropriate name and justify their choices.

ICT Opportunities
[Templates for Quadrilaterals]

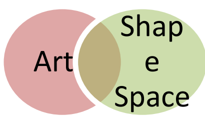
Property Lists for Quadrilaterals[77]

1. Prepare hand-outs[78] for parallelograms, rhombi, rectangles and squares.
2. Assign groups to work with each type of quadrilateral.
3. Ask pupils to list as many properties as they can that apply to all of the shapes on their sheet.
4. Pupils will need tools such as index cards to check for right angles, compare side lengths and draw straight lines and symmetry.
5. Pupils should prepare their property lists under the headings:

	Parallelograms	Rhombi	Rectangles	Squares
Sides				
Angles				
Diagonals				
Symmetries				

6. Encourage pupils to use words such as 'at least' when describing how many of something, for example, 'rectangles have at least two lines of symmetry because squares have four'.

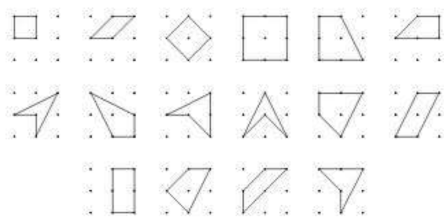
> In the classification of quadrilaterals and parallelograms, some subsets overlap. For example, a square is a rectangle and a rhombus. All parallelograms are trapezoids, but not all trapezoids are parallelograms. Burger (1985) highlights that pupils correctly use such classification schemes in other contexts. For example, individual pupils in class can belong to more than one club. A square is an example of a quadrilateral that belongs to two other clubs.



Geoboard-Changing Quadrilaterals

Pupils work on a Geoboard to change:

1. An irregular quadrilateral to a trapezoid
2. A trapezoid to a parallelogram



[77] Van de Walle, Karp & Bay-Williams (2013, p.405), Image: http://www.math-salamanders.com/images/shapes-clip-art-quadrilaterals-col.gif
[78] Link to Templates for Quadrilaterals: **http://lrt.ednet.ns.ca/PD/BLM/table_of_contents.htm**

3. A parallelogram to a rectangle

4. A rectangle to a square

5. In their maths journal or learning log, pupils describe how to change an irregular quadrilateral to a square step-by-step.

Every Square a Rectangle[79]

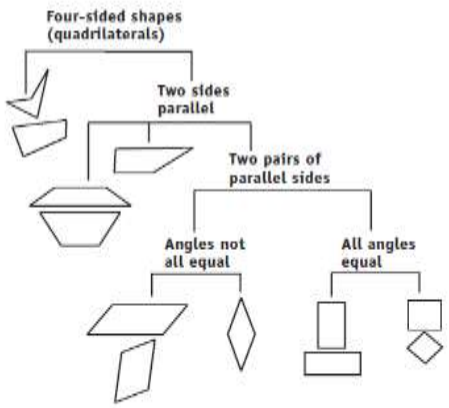
My sister said that every square is a rectangle but every rectangle is not a square.

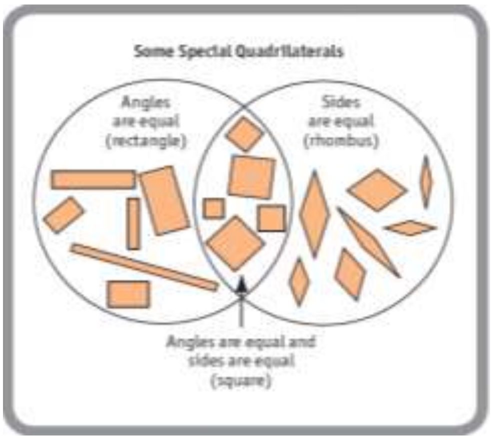
How can that be so?

Invite pupils to list all the properties of each and tick those that are the same for both shapes. The teacher asks:

What is the only property that a square has that a rectangle doesn't?

Are there any properties of a rectangle that a square doesn't have?





Minimal Defining Lists[80]

1. Once property lists for parallelograms, rhombus, rectangle and square have been generated the lists should be displayed.
2. Pupils work in groups to find 'minimal defining lists' or MDLs, for each shape. Defining here means that any shape that has all the properties on the MDL must be that shape. Minimal means that if any single property is removed from the list, it is no longer defining. For example, one MDL for a square is a quadrilateral with four congruent sides and four right angles.
3. Pupils should try to find at least two or three MDLs for their shape. A proposed list can be challenged as either not minimal or not defining. A list is not defining if a counter example (a shape other than one being described) can be produced using only the properties on the list.

[79] First Steps in Mathematics: Space, Western Australia (2010, p. 205, 209)
[80] Van de Walle, Karp & Bay-Williams (2013, p.406)

If the sides of a four-sided shape are all congruent, will you always have a square?
Can you find a counter-example?
Is that true for all triangles or just equilateral ones? Explain why.
What properties of diagonals do you think will guarantee that you have a square?

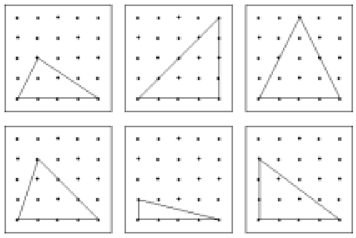
Loop Card Game using MDL's.

Pupils can create a MDL loop card game, for example, each card can contain a 2-D shape and the MDL for another 2-D shape. Pupils call out their MDL and the person with that shape has to then call out their MDL and so on, for example:

I am looking for a four-sided shape with sides that are all congruent.

The pupil with the square picture on their card then goes next and calls out the MDL on their card.

Grouping Triangles (Extended form Level C.3)



1. Pupils can use Geoboards[81] to construct any triangle.
2. Next pupils make another triangle that is different in some way to the first.
3. Pupils create another triangle that is different to the first two and so on.
4. Pupils can record their results on dotted paper.

How many different triangles can you make?
How are they different?
Pat please tell me that in your own words

5. Pupils place the triangles into their groups, compare their groups with others and redefine their groups if necessary to make the sorting easier.
6. Pupils write a description of what the triangles in each group have in common.

ICT Opportunities

Online Geoboard

What do all of these triangles in this group have in common?
Do you have any triangles that have three different-sized angles?
Do you have any triangles that have two angles the same size?
Could you use this to group your triangles?

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[81] Link to Online Geoboard: http://nlvm.usu.edu/en/nav/frames_asid_125_g_1_t_4.html?open=activities&from=search.html?qt=geoboard

PDST Professional Development Service for Teachers

Sorting Triangles

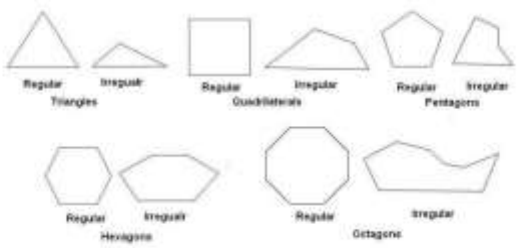
1. Pupils sort a collection of triangles of different shapes and sizes into groups according to their properties.
2. Pupils may begin to discover that there are some commonalities between various triangles, for example, two angles equal, have right angle, all sides equal, all sides unequal, opposite angles equals.
3. Pupils should sort the triangles in as many ways as possible. They may find that all triangles fit somewhere into three/four groups.

Why have you grouped these triangles together?
What do they have in common?
How are they different to this one?
Why did you not put this triangle in that group?
What do you notice about the angles in this triangle?
Is there a common relationship between the angles of the triangles in this group? If so, explain it.
Jack please revoice that.
Can anv trianale belona to two sets vou have created?

4. At this point the teacher can ask pupils if they know the names of any of these groupings. Then the formal name for each group can be introduced:
   a. Equilaterial
   b. Isosceles
   c. Scalene
   d. Right-angled

Pupils may discover that right-angled triangles belong to either set of isosceles triangles or to the set of scalene triangles

Sorting: Pentagons, Octagons and other Polygons



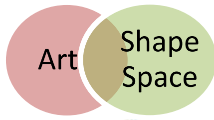
1. Pupils sort various shapes such as regular pentagons, irregular pentagons, regular octagons, irregular octagons, etc. into categories.
2. Pupils choose the category label and justify why each shapes fits into their category, for example, number of sides, angles, symmetrical, etc.
3. Pupils may classify the shapes by the number of sides, the teacher can then invite the pupils to see if they know the formal terms for this type of shape.
4. Pupils can then apply the formal classification label to each shape.

Each shape has 5 sides? What do we call this?
Why do some five-sided shapes look so different?
Is this still a pentagon? Can we further sub divide this group of shapes?
What can we call these groups?

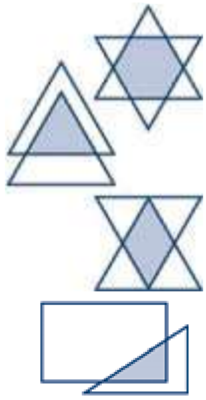
It is important that pupils experience sorting regular and irregular representations of octagons and pentagons.

CONSOLIDATION ACTIVITY

Overlaps[82]



The teacher needs shapes to demonstrate with - preferably a) made from transparent film to use with a visualiser or b) shapes on the IWB. Alternatively, sketch the shapes on the board. Each pair of pupils needs two card squares, tracing paper, scissors, plain paper and pencil.



1. Take a rectangle and a triangle. Using the visualiser/IWB show one shape placed over the other. Tell the pupils that when two shapes overlap each other, a third shape is made.

2. Discuss the 'overlap' shape you have made including its properties.
3. Ask the pupils to imagine two equilateral triangles overlapping to make a triangle. *Are there different ways of doing it? What about making a rhombus? A hexagon?*
4. Invite different pupils to demonstrate their ideas with the visualiser/IWB.[83]
5. Ask pupils to investigate what overlap shapes they can make with two squares. *What quadrilaterals can be made? Are there any special quadrilaterals that cannot be made?*

ICT Opportunities

Interactive Overlaps

How could two equilateral triangles overlap to make another equilateral triangle?
What are the other ways of doing it? What other types of triangle could you make?
Can you make a triangle with an obtuse angle? With three acute angles?
What if you rotate, reflect or slide the top triangle?
How can you make an overlap shape with four sides?
A different shape with four sides? A shape with five sides? Six sides?
What if the two triangles were right-angled?
What if you used a rectangle and a triangle?

[82] The National Numeracy Strategy, Shape and Space Activities (1999)
[83] Link to Interactive Overlaps: http://nrich.maths.org/content/id/5820/OverLap2.swf

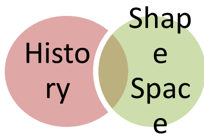
LEVEL C.5: COMBINE, TESSELLATE AND MAKE PATTERNS WITH 2-D SHAPES

BACKGROUND KNOWLEDGE FOR TEACHERS

What are Tessellations?

The word 'tessera' in latin means a small stone cube. They were used to make up 'tessellata' - the mosaic pictures forming floors and tilings in Roman buildings.

Notice, in the example to the right, the artist used many small square tiles to create one big picture of a bull. A tessellation is a tiling of the plane using one or more shapes in a repeated pattern with no gaps or overlaps.



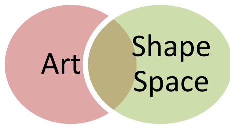
A regular tessellation is made of a single tile that is a regular polygon (all sides and angles congruent). Each vertex of a regular tessellation has the same number of tiles meeting at that point. A draughts board is a simple example of a regular tessellation.

A semi-regular tessellation is made of two or more tiles, each of which is a regular polygon. At each vertex of a semi-regular tessellation, the same collection of regular polygons comes together in the same order.

TEACHING NOTES

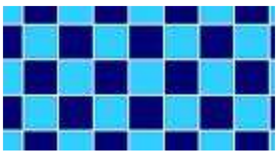
Pupils need to freely explore how shapes fit together to form larger shapes (compose) and how larger shapes can be made of smaller shapes (decompose).

SAMPLE LEARNING EXPERIENCES

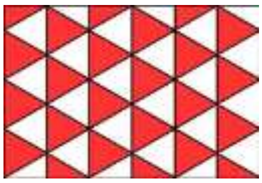


Tessellations of Regular Shapes

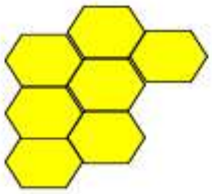
Pupils should be given the opportunity to explore which shapes tessellate using manipulatives. They should make tile and brick patterns to reinforce this. It is also a good link with covering surfaces – area. Regular shapes have equal angles and sides so start with investigating tessellating properties of these.



Square



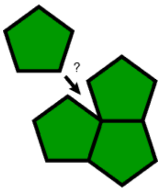
Equilateral Triangle



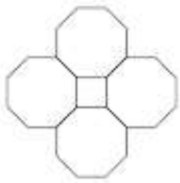
Regular Hexagon

What is a regular shape? Give examples. Regular shapes have equal angles and sides. How can you fit these together?
How many ways can you do this?
Will all regular shapes tessellate? Why? Why not?
Show me an example of a regular shape that will not tessellate.

Pupils discover that the square, the equilateral triangle, and hexagon fit together without any spaces between. The investigation may also include the regular shapes that do not tessellate.



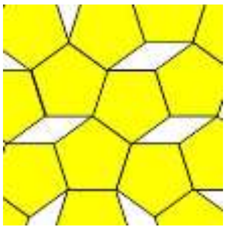
Pentagon



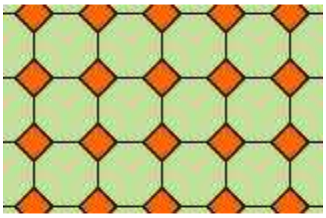
Octagon

Semi-Regular Tessellation[84]

Two or more sets of regular polygons may be fitted together to form a tessellation.



Regular Pentagon & Rhombuses



Regular Octagon & Squares

ICT Opportunities

[Tessellation of Polygons]

Tessellation[85] of Non-regular Shapes.

Non-regular shapes have sides and angles are not equal.

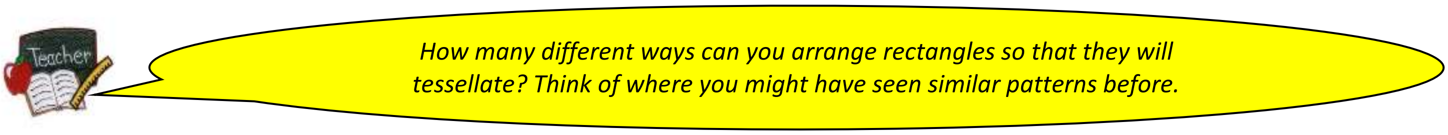
Rectangles: Many patterns of rectangular tessellations are possible.



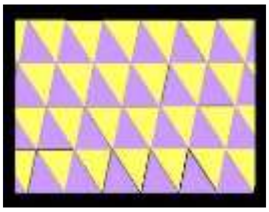
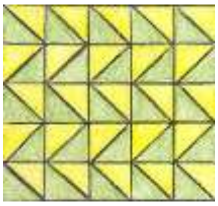
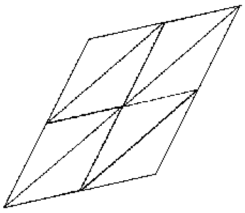

ICT Opportunities

[Tessellation Creator]

[84] Link to Tessellation of Polygons: http://nrich.maths.org/content/id/4832/polygons.swf
[85] Link to Tessellation Creator: http://illuminations.nctm.org/ActivityDetail.aspx?ID=202

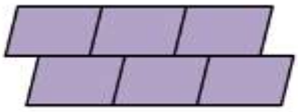
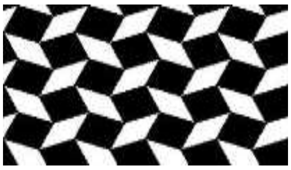


Triangles: Pupils may use different shaped triangles, for example, isosceles, right angles, scalene to form tessellations (equilaterial triangles are regular shapes).

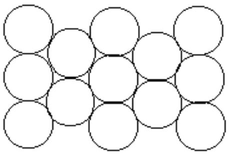
Isosceles triangles Right-angled triangles Scalene triangles

Quadrilaterals

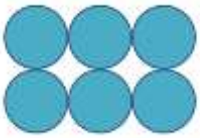
Parallelograms Rhombuses

As pupils sort, tessellate and commit the work to dotted paper, they become aware of the properties of the shapes studied. Parallel lines are illustrated in many of these tesselations and pupils may be introduced to the concept.



Non-Tessellating Shapes

Pupils should have experience of tessellations with shapes of various properties including those that do not tessellate. Pupils should be given opportunities to investigate the tessellating properties of circles, to discover that no matter how they arrange them, they do not fit together without spaces.



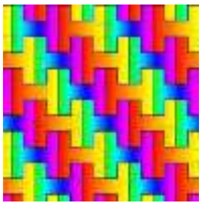
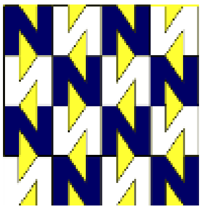
Tessellation Project

Pupils can create tessellating projects similar to those found on www.haveyougotmathseyes.com

Pupils can identify tesselations in the environment using a digital camera. (See Level D.3)

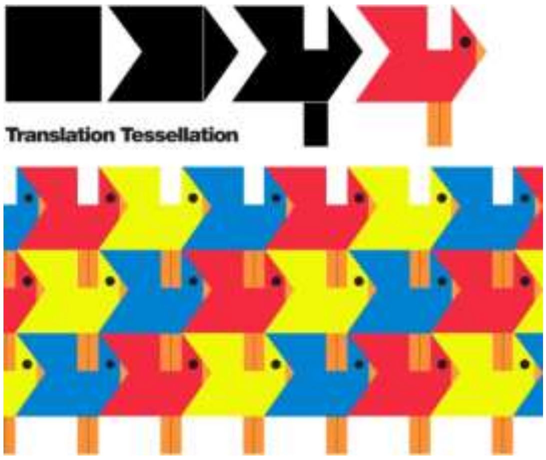
Alphabet Tessellation

Pupils can explore which letters of the alphabet have regular and semi-regular tessellation properties.

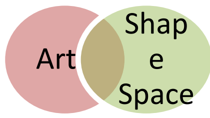
Tessellations: Addition and Subtraction

Pupils can start with a familiar shape and systematically add and remove from it to create tessellation patterns

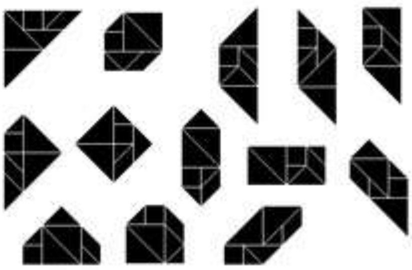


Start with a familiar shape like a square.
Remove or add sections to the shape to create new tessellating shapes.

Tangram Puzzles[86]



The Chinese Tangram puzzle is an interesting and effective way of demonstrating conservation of area. The seven pieces of the puzzle may be used to construct geometrical shapes, for example, triangle, trapezium, parallelogram, etc. Alternatively, representational figures, for example, a girl running, the letter E, candle, etc[87] can be made.



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[86] Link to Virtual Tangrams: http://nlvm.usu.edu/en/nav/frames_asid_268_g_1_t_3.html?open=activities
[87] http://mrrgteacher.wordpress.com/category/tangrams/

LEVEL C.6: IDENTIFY, DRAW AND RECOGNISE LINE SYMMETRY IN THE ENVIRONMENT AND IN SHAPES

BACKGROUND KNOWLEDGE FOR TEACHERS

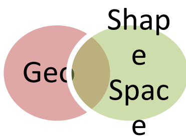
If a shape can be folded on a line so that the two halves match, then it is said to have line symmetry or mirror symmetry. The fold line is actually the line of reflection – the portion of the shape on one side of the line is reflected onto the other side, demonstrating a connection between line symmetry and transformations (flips).

TEACHING NOTES

Pupils need structured experiences that progressively build up their understanding of symmetry. For mirror symmetry these might include:

- Folding, measuring and looking for symmetry.
- Discovering and explaining that matching parts of figures are the same distance from the mirror line.
- Investigating the symmetry properties of some regular shapes such as squares and isosceles triangles and then the regular pentagon and hexagon and other regular polygons.[88]

SAMPLE LEARNING EXPERIENCES



Line Symmetry in the Environment

Pupils should be encouraged to identify line symmetry in the:

Built environment




Natural Environment


First Steps in Mathematics: Space, Western Australia, (2010 p. 141)

People

Plants

Animals

Symmetry Trail[89]

Pupils use a digital camera and go on a symmetry hunt in the classroom or on the school grounds to identify symmetry in the environment.

Describe your picture.
How do you know it is symmetrical?
Please revoice that for me Kate.
Where would the lines of symmetry appear?

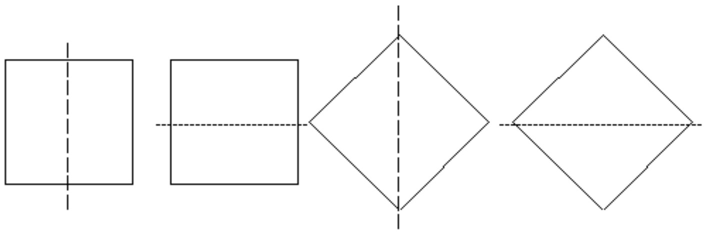
ICT Opportunities

Maths Eyes

Symmetry of Mathematical Shapes

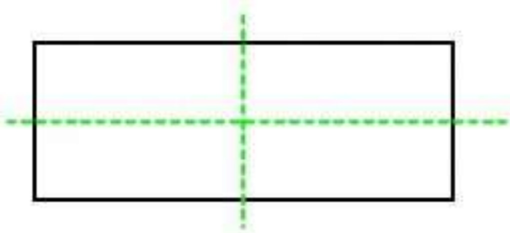
Attention may be focused on the symmetry of mathematical shapes already handled.

1. Squares, cut out of paper are folded in as many ways as possible.
2. Pupils discover that there are four ways of doing this, therefore it can be stated that



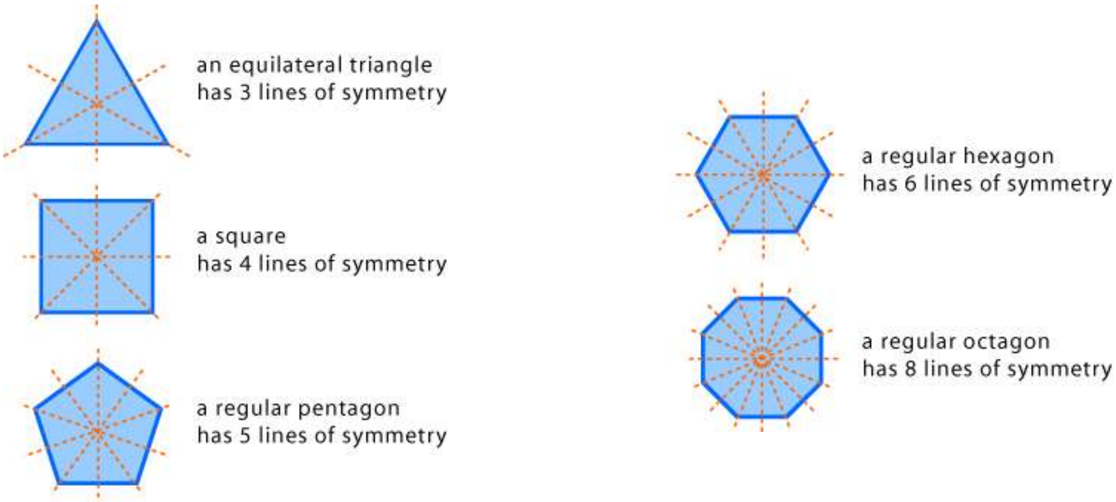
[89] Link to Maths Eyes: http://www.haveyougotmathseyes.com/

squares have four lines of symmetry.



3. Repeat with a rectangle. Pupils discover that when folded diagonally, one half does not coincide with the other. A rectangle has only two lines of symmetry[90].
4. Encourage pupils to discover how many lines of symmetry various shapes have by folding.

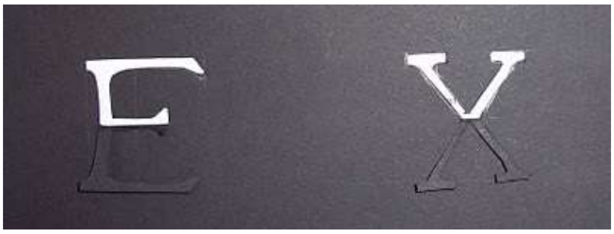
The pictures below are *for teacher reference only* and pupils should be encouraged to discover the possible lines of symmetry in each shape.



Letters[91]

Some letters of the alphabet are symmetrical and pupils may look for lines of symmetry in them. Pupils can experiment with horizontal and vertical lines of symmetry in letters and integrate these concepts into art activities.

Horizontal



Vertical

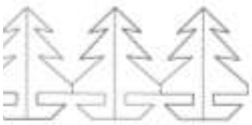
90 Deboys and Pitt (2007, p. 287)

91 Images: http://j-warburton1013-dp.blogspot.ie/2010_09_01_archive.html)

Paper Folding[92]

Pupils fold paper concertina-style, cut out a shape on one folded edge and open it out to see what frieze they have produced. Invite them to then decide and sketch what new frieze they want to produce, plan their cuts and make their frieze.

Which part of your frieze do you want to connect?
How will you cut your folded paper to make that happen?
How will you cut the paper if you want to give your 'person' some eyes?

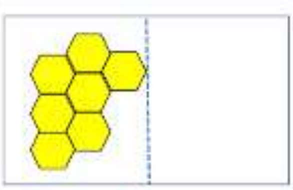
Snowflakes

Pupils fold paper diagonally, first in quarters and then in eighths, and cut a design into the folded edge. Pupils carefully draw a copy of the cut-out piece. They open out their snowflakes and pin them up. Encourage other pupils to inspect the traced outline and decide which cut-out shape goes with which snowflake. Pupils should justify their choice.

What is it about the cuts in the folded paper that match that snowflake?
What other clues did you use?
Were some easier to match than others? Explain why/why not.

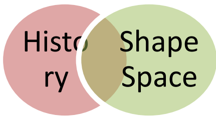
Pattern Block Symmetry / Mirror imaging

On a piece of paper, pupils use six to eight pattern blocks to make a design completely on one side of a line. When one side is finished pupils try to make a mirror image on the other side of the line.



92 First Steps in Mathematics: Space, Western Australia,(2010, p. 115)

After it is built pupils can check their work with a mirror. With the mirror on the line, they should see exactly the same image as they see when they lift the mirror. Pupils can also be challenged to make designs with more than one line of symmetry.

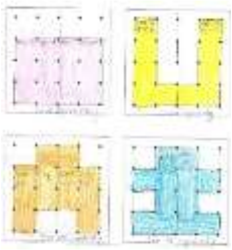


Archaeologist Activity

This activity requires pupils to complete the missing elements of a picture using their knowledge of symmetrical shape. Click on the picture/link to be brought to the print out of the activity on line.[93]

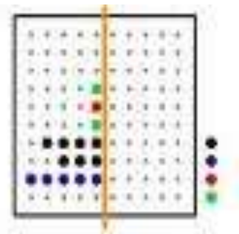
Recording Symmetry

Pupils can be encouraged to record shapes and their corresponding lines of symmetry on various mediums including using drawing, geoboards, dotted paper, etc.



Investigation of Symmetry of Reflection[94]

Pegboards and pegs or multi-grids and pegs are suitable media for exploring mirror patterns. Pupils use one half of a shape or pattern and they complete it. Diagonal, horizontal, and vertical lines or symmetry or mirror lines should be explored.



[93] Link to Archaeologist Symmetry: http://www.nzmaths.co.nz/resource/sketching-etching

[94] Deboys and Pitt (2007, p.161)

Pegboard Games[95]

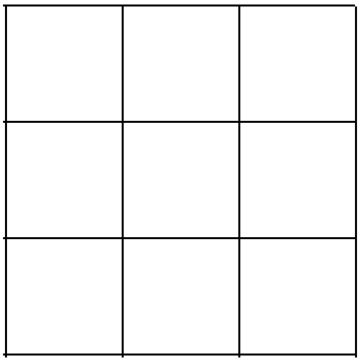
Pupils can devise games to be played on pegboards, multi -grids or squared paper. One suggested game is as follows:

1. The line of symmetry or mirror line is marked and each player takes turns to put a peg on the board.
2. The second player has to mirror the location of their opponent's peg on their side.
3. Then they place a peg on the board and the other player had to mirror its position.
4. Pupils can begin to use positional language that is further developed in Level D.

Investigation: Symmetry Challenge[96]

Systematically explore the range of symmetric designs that can be created by shading whole squares of the grid below. Pupils can use blank grids or squared paper to record their results.

How many can you find? Did you classmates find other examples / solutions?



ICT Opportunities

[Archaeologist Symmetry]

[95] Deboys and Pitt (2007, p.161)
[96] Link to Archaeologist Symmetry: http://nrich.maths.org/1886

LEVEL C.7: IDENTIFY, DESCRIBE AND CLASSIFY PARALLEL, PERPENDICULAR, VERTICAL, HORIZONTAL AND OBLIQUE LINES

BACKGROUND KNOWLEDGE FOR TEACHERS

An oblique line is a slanting line that is not vertical or horizontal.

SAMPLE LEARNING EXPERIENCES

Parallel Lines

Pupils should be provided with opportunities to explore lines in a concrete manner using geostrips.

Pupils can be instructed to pick two geostrips.

Is there any way to arrange your two geostrips so that they are an equal distance apart.

Do you know the name for these types of lines?

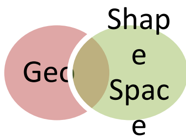
Can you relate these to any maths symbol that we use?

Can you identify lines similar to these in the classroom environment.

We call these lines parallel lines.

Pupils can draw two lines that never cross; keeping the distance between them always the same.

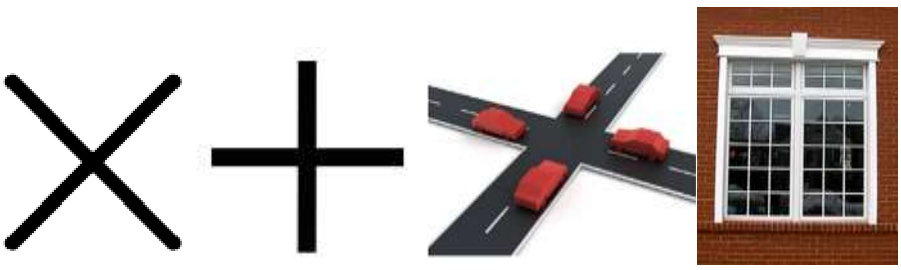
Examples of where parallel lines can be seen in the real world should be discussed.



What do all parallel lines have in common?
Try to identify any other parallel lines in the school.
Where have you located these lines?
Please sketch them and record your learning in your learning log.

Pupils can explore parallel lines by drawing them horizontally, vertically, and diagonally. They discover that they never intersect.

Perpendicular Lines



What does the word intersect mean?
Jack please revoice what Tom said.
Point to the intersections in the images.
Look at the crossroads image, what do you notice about all the angles that are formed?
If you rotate the image do the angles change?

The term perpendicular can be introduced here, which refers to lines that intersect to form a right angle. Pupils can be given the opportunity to create perpendicular lines using geostrips and discuss the angles formed.

Vertical Lines

Pupils can explore line in the environment.

What is the name of a line that goes straight up and is perpendicular to the ground?
Think of any examples of such lines in our classroom.

The term vertical can be introduced here. Pupils can discuss vertical lines in the environment. It may be useful for pupils to relate vertical lines to height.

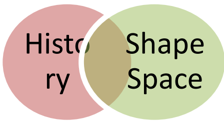
Horizontal Lines

A horizontal line is level and runs parallel to the ground – the term can be introduced here. Pupils can identify horizontal lines in the environment. One such example occurs naturally all around the world at sunrise and sunset. Images like that below can be used as a stimulus.

Show me where the sky and the sea appear to meet.
Is this an illusion or does this line actually exist?

The purpose of this instrument in construction can be discussed (a level):

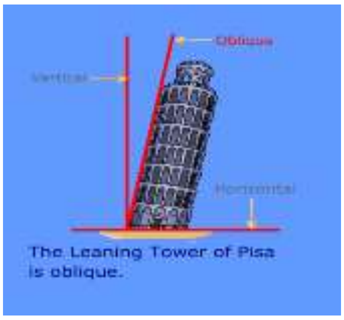
Who uses this instrument? How is it used?
Why is it so important?
What does it tell us about the surface that it is placed on?



Oblique Lines

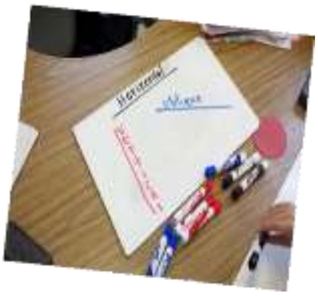
Pupils can be encouraged to discover and identify oblique lines in the environment.

What do you know about this building?
Why is it famous? Where is it located?
It took almost 200 years to build it.
Give me some reasons regarding why it may be slanted.
Identify lines on this building. Label them.



CONSOLIDATION ACTIVITY

Lines in the Environment: Revision and Consolidation[97]



Find and list 5 surfaces in the school which are horizontal and 5 which are vertical.
Try to find lines or surfaces that are parallel.
Try to find lines or surfaces that are perpendicular to each other.
Try to find perpendicular lines in the classroom where the lines are not vertical or horizontal.
What types of lines are used in the pages of your copy?
Why is it important that walls are vertical?
Record your group's learning.

[97] Adapted from: http://www.projectmaths.ie/documents/teachers/tl7-_introduction_to_angles.pdf

PDST Professional Development Service for Teachers

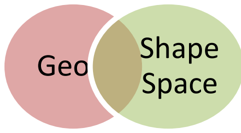
LEVEL C.8: CLASSIFY ANGLES AS GREATER THAN, LESS THAN OR EQUAL TO A RIGHT ANGLE

TEACHING NOTES

The intended line of development in this level is:

1. Identifying different angles in the environment.
2. Using a right angle static measure to assist in classifying angles
3. Using geostrips to create angles greater than, less than, and equal to a right angle
4. Introducing the language associated with angle size (acute, obtuse, right angle, reflex, straight).

SAMPLE LEARNING EXPERIENCES



Recognising Angles in the Environment

Pupils should be encouraged to identify a range of angles in the environment. Pupils can identify different angles and discuss how they look.



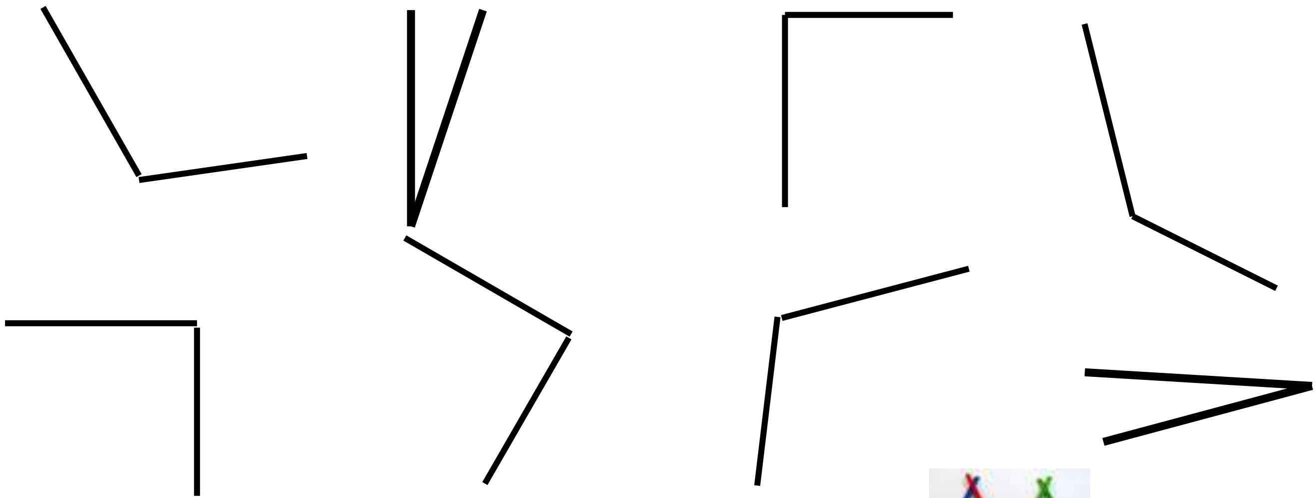

Identify any angles in the classroom.
What do these angles have in common? How are some different?
Group any of these angles.
Try to create four different categories of angles from the images above.
Which angles can you name?
Name or identify any other category of angle above.

Making a Right Angle

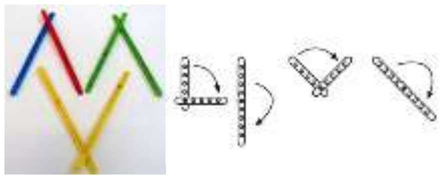
A detailed description of *making a right angle from torn paper* is illustrated in Level B.7. This activity is also highly recommended for this level. This right-angle measure can then be used for the following activity.

Using the Right Angle Static Measure

Use your right angle measure to check which of the following angles are right angles.
What other angles less than or greater than a right angle can you see?
Draw a circle around them.



Experimenting and Discovering Different Angles[98]



Pupils can use geostrips or anglegs for this activity.

1. Let the class rotate their geostrips to make a sharp angle.

Notice how sharp the point of the angle looks, acute means sharp.
Write the words acute angle in your learning log.
Have you ever met the word acute before? Acute appendicitis, acute shortage of…..

2. Where the two arms meet is called the vertex of the angle.

Show on your geostrips the vertex of your acute angle.
Write the word vertex in your learning log recording the language with a drawing of the geostrips.

3. Rotate the strip further. The teacher moves the strip to the vertical position to show a right angle.

This is called a right angle/90 degree angle.
Why do you think it is called a right angle?
Record the new vocabulary.

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[98] Teaching and Learning Plan 7: Introduction to Angles Project Maths Development Team 2009. http://www.projectmaths.ie/documents/teachers/tl7-_introduction_to_angles.pdf

4. Rotate the strip further. The teacher rotates the strip further to show an obtuse angle.

Do you know what we call this angle?
Record the word obtuse.
Notice how the point of the angle looks less sharp. The word obtuse means

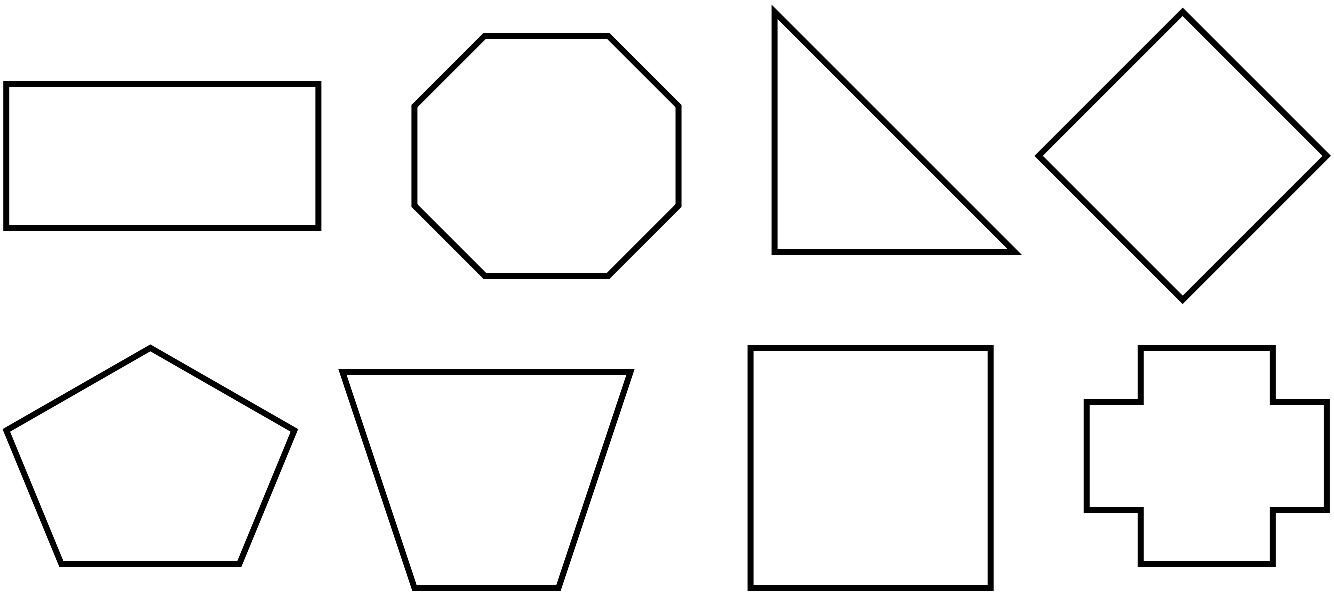
5. Rotate the strip even further. Pupils mirror what the teacher has demonstrated.

Move the strips to a position to show a reflex angle.
Record the vocabulary in your learning log.

6. Rotate the strip further until you have come back to where you started from originally. Pupils mirror with their geostrips what the teacher has demonstrated.

Angle Identification

What type of angles can you identify in these shapes?



ICT Opportunities
Interactive Right Angle Challenge

Extension Activity: Interactive Right Angle Challenge[99]

Make a right-angled triangle on a peg-board by joining up three points round the edge. Can you work systematically to prove this? Try changing the number of points round the edge. Can you do it now? Can you show by calculation that the angle is a right angle? What do you notice about the side of the triangle opposite the right angle? Try this with other numbers of points round the edge. When is it possible to make a right-angled triangle?

[99] Link to Interactive Right Angle Challenge: http://nrich.maths.org/2847

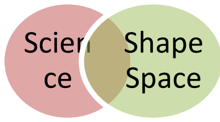
LEVEL C.9: RECOGNISE AN ANGLE IN TERMS OF A ROTATION

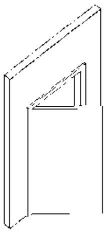
TEACHING NOTES

The intended line of development of this level progresses from recognising angles of rotation in the environment to constructing and experimenting with lines and angles of rotation.

SAMPLE LEARNING EXPERIENCES

Recognising Angles in the Environment



Pupils should be encouraged to identify objects that rotate in the environment. Pupils can investigate the relationship between rotation and how an angle is made.

What does "rotate" mean?
Give me other of words meaning the same thing.
Give examples of things that you have seen rotating.
What about when we open the door? The door rotates around the hinges.
See how the door makes an angle with the wall as I rotate it.
Can you think of any other everyday examples of rotation?

Creating Angles

Pupils can use strips of paper or commercially bought material such as anglegs or geostrips to create different sized angles.

It may be appropriate here to revise the activities in Level C.10 where pupils discover different angles by rotating two geostrips.

Pupils should be encouraged to manipulate the strips of equal length to form bigger and smaller angles and to discover that the angle is formed at the meeting point of the lines or point of rotation (vertex). During teacher-led instruction, emphasis should be placed on the language of rotations and other related terms such as, *angle, ray, vertex, common point, extend*. Throughout experiences with concrete materials such as paper strips, geostrips, anglegs, etc. pupils should be encouraged to use the language above as modelled by the teacher.

Now let us rotate the strip of paper further - we get a bigger angle.
If we rotate more, we get a bigger angle still.
The important thing is that the angle is the rotation of one ray about another ray through a common point.
The angle changes as we change the amount of rotation.
Show your partner the vertex, the arms/rays, angle formed?

Pupils investigate what happens when they change the length of the strip used to make the angle. Pupils should share what happens/doesn't happen to the angle when they use a longer strip. They should also make conjectures about the reasons for this. (rotation and angle size are unaffected by the length of the strip used)

Rotating Angles

To develop the concept of an angle not only as a static measure but as a measure of turning, pupils can rotate the folded paper by putting a pin in the centre and turning until the circle (1) returns home again. This can also be done with a paper plate and a straw. If we call this a whole turn, then a right angle is a quarter turn and a straight line is half a turn. These activities help pupils to form the concept of an angle as a measure of the amount of turning and not just as a static measure.

Clock Face

The movement of the minute hand around a clock face draws attention to an angle in terms of rotation.

What does the movement from 12 o'clock to 3 o'clock show? How much of a turn?
What does the movement from 12 o'clock to 6 o'clock show? How much of a turn?
Explain it to me in terms of angles. Jim please tell me that in your own words.
What does the movement from 12 o'clock to 9 o'clock show?
What does the movement from 12 o'clock to the starting point? How much of a turn? Yes, a full turn - we can also call this a revolution.
Who can explain revolving doors to me? Why do you think they were named so?

Extension Activity: Move the Hands[100]

How many times in twelve hours do the hands of a clock form a right angle? Use the + and - buttons to move the hands of the NRICH clock (ICT link), then click on the right angle symbol to check your answer. Record your answers and those of your classmates in your Maths Journal.

ICT Opportunities

Right Angle Clock

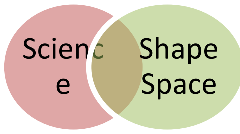
[100] Link to Right Angle Clock: http://nrich.maths.org/1159

LEVEL C.10: DRAW, DISCUSS AND DESCRIBE INTERSECTING LINES AND THEIR ANGLES

TEACHING NOTES

This section requires a range of mathematical language to be included in discussions. It is recommended that relevant mathematical language such as intersecting, obtuse, acute, perpendicular, parallel, etc. should be taught in context rather than memorised as an abstract definition.

SAMPLE LEARNING EXPERIENCES



Intersecting Lines and Angles

Pupils should be encouraged to identify lines and angles in the environment. Pupils can identify different lines and how they intersect, creating various angles. Pupils can then discuss these angles.



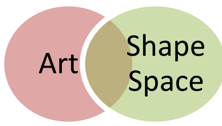


How many different types of lines can you identify in these pictures/ in the school?
How many different angles can you identify?
How could we group different angles together?
What angle can we easily recognise?
How could we group angles in relation to this angle?
Is the angle formed greater than/less than/equal to a right angle?
Does anyone know the name we use for angles less than/greater than a right angle?

Drawing

Pupils can draw different sets of intersecting lines and discuss the angles formed in relation to a right angle benchmark.

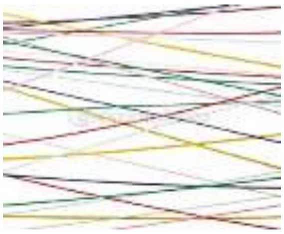
Exploring Perspective in Art Work



Pupils should be encouraged to draw a range of intersecting lines and discuss the angles. They can their partner to identify as many different lines and angles as they can. This can be linked effectively with art activities.

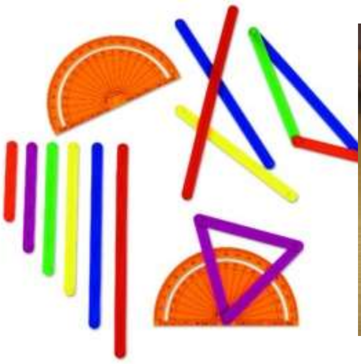
String

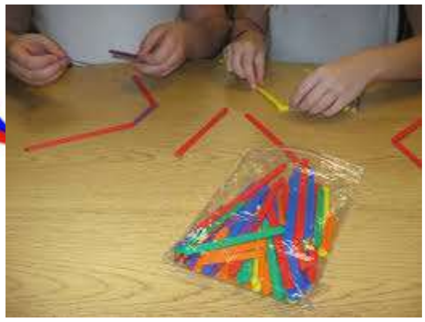
Pupils should be encouraged to form lines and angles with pieces of string and to discuss the angles formed.



Anglegs

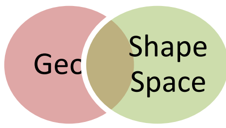
Pupils should be encouraged to form lines and angles with anglegs and to discuss the angles formed.

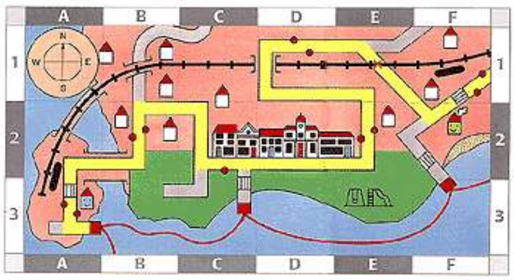




Mapping

Pupils can draw and discuss their own street maps based on their local town, route to school, etc. Discuss the different angles formed at cross sections of roads, giving directions, etc. (Google maps). This activity can be extended to co-ordinate mapping in Level D.8.





Extension Activity: Teddy Town Map

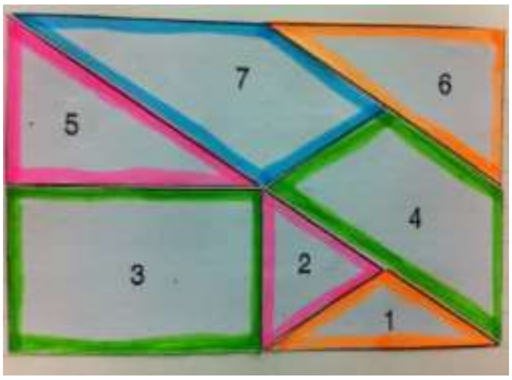
In Teddy Town the streets are very special. If you walk along a street from east to west, or west to east, all the houses are a different colour and the teddies living in the houses are a different colour too. Can you complete the online investigations in order to create street maps and grids for the whole town?

ICT Opportunities

Teddy Town Map

Mosaic Puzzle

Pierre Van Hiele describes and interesting set of tiles he calls the ‘mosaic puzzle’. The value of this puzzle is that the set contains five different angles. You can use the pieces to talk about square corners (right angles) and angles that are more or less than right angles (obtuse and acute). This activity builds on pupils’ previous learning experiences in Level B.4 where they have completed extensive investigations on this puzzle.



LEVEL D TEACHING AND LEARNING EXPERIENCES

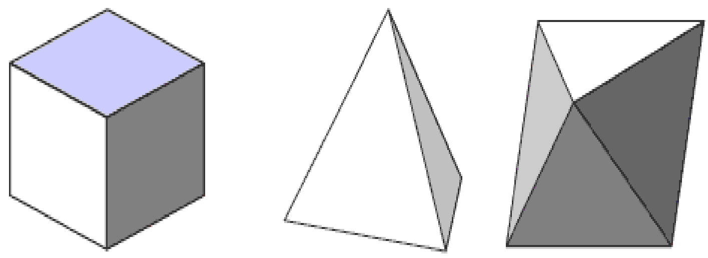
LEVEL D.1: IDENTIFY AND EXAMINE 3-D SHAPES AND EXPLORE RELATIONSHIPS INCLUDING TETRAHEDRON OCTAHEDRON

BACKGROUND KNOWLEDGE FOR TEACHERSI

Properties of Shapes[101]

The Platonic solids are the regular polyhedra and derive their name from the Greek philosopher Plato (428–347 BC). This term comes from the Greek words *poly*, meaning "many," and *hedron*, meaning "face". Therefore, quite literally, **a polyhedron is a three-dimensional object with many faces.** The platonic solids are the only polyhedra whose faces are exactly the same. Every face is identical to every other face. For instance, a cube is a Platonic solid because all six of its faces are congruent squares. Similarly, the same number of faces meet at each vertex. Every vertex has the same number of adjacent faces as every other vertex. For example, three equilateral triangles meet at each vertex of a tetrahedron. No other polyhedra satisfy both of these conditions. Consider a pentagonal prism. It satisfies the second condition because three faces meet at each vertex, but it violates the first condition because the faces are not identical — some are pentagons and some are rectangles. **The Platonic solids are the cube, the tetrahedron, the octahedron, the dodecahedron, and the icosahedron.**

Platonic Solids (included in Primary School Mathematics Curriculum)



cube (hexahedron) tetrahedron octahedron

Properties of Platonic Solids

Name of Shape	No. of faces	No. of vertices	No. of edges
Cube	6	8	12
Tetrahedron	4	4	6
Octahedron	8	6	12

[101] http://www.nzmaths.co.nz/resource/caught-nets

TEACHING NOTES

The work in level C.1 and C.3 can be repeated at this stage if necessary.

a) Construction of 3-D shapes using card to draw attention to the number of faces.
b) Construction of 3-D shapes using pipe cleaners and straws to draw attention to the number of edges and number of vertices.

SAMPLE LEARNING EXPERIENCES

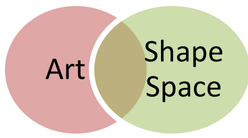
Tetrahedron and Octahedron

Pupils can be encouraged to identify examples of tetrahedrons and octahedrons in the home and school environment. See everyday examples below

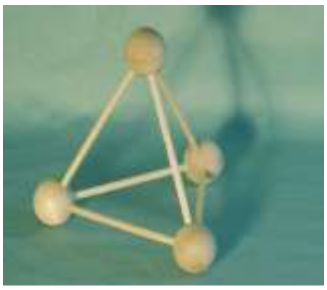
How many faces has each shape?
How many corners can you count?
Trace your finger around the vertices of each shape.
What do you notice about the shape of each face?
What 2-D shapes are combined to make this 3-D shape?
Which 2-D shape is common to both?

.

Construction Activity[102]



Having explored the faces pupils can be encouraged to experiment with simple materials to construct a tetrahedron and octahedron in the classroom. Simple models can constructed from match sticks and blue tac, cocktail sticks and jellies etc. can be used to extend and consolidate learning.



[102] Link to Interactive Platonic Solids: www.learner.org/interactives/geometry/platonic.html

Professional Development Service for Teachers

How many faces will you need to make to create a tetrahedron/octahedron?
How will you begin?
Why did you do this? Did you sketch a plan? Who did it in another way?
What 2-D shapes can you combine to create this 3-D shape?
In your learning log, write the procedure of how you constructed this shape.

Recording the Properties of Platonic Solids

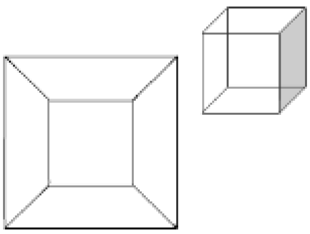
Having constructed their shapes pupils can use these to explore and record the properties of a cube, tetrahedron and octahedron. Associated work on faces, vertices and edges should be included.

Fill in the table to demonstrate your learning using the platonic solids.
Share this with your partner. Transfer this learning to your learning log.
Include your labelled sketches of cubes, tetrahedrons and octahedrons in your learning log.

Shape	No. of Faces	Type of Face	Vertices	Edges
Cube				
Tetrahedron				
Octahedron				

Extension Activity: Platonic Challenge

This problem relates to investigating whether it is possible to start at one vertex of a platonic solid and visit every other vertex once only - returning to the vertex where you started. This is quite difficult to visualize in 3-D so this problem required drawing 2-D skeletons of the five Platonic solids (known as Schlegel graphs) and using these to try to answer the challenge set below. Use the ICT link below to find the paths along the edges of the solids.[103]



ICT Opportunities

Platonic Challenge

[103] Link to Platonic Challenge: http://nrich.maths.org/897

LEVEL D.2: DRAW THE NETS OF SIMPLE 3-D SHAPES AND CONSTRUCT THE SHAPES

TEACHING NOTES

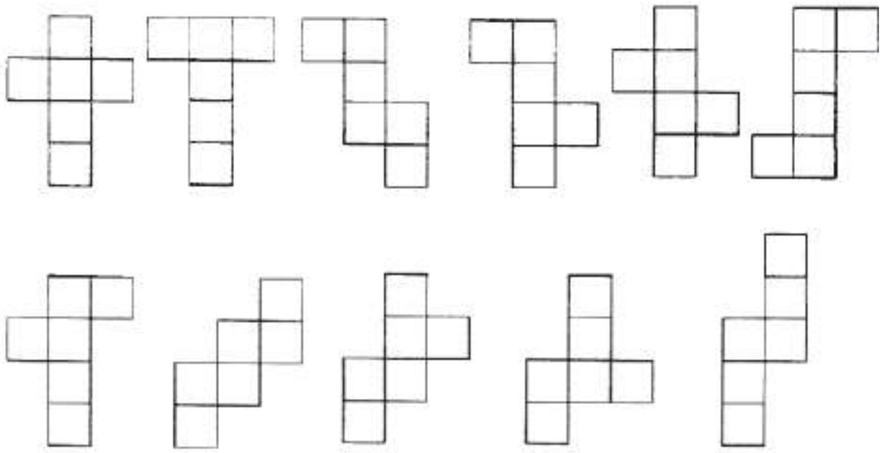
The activities in this level are based on pupils' learning experiences in Level C.3. This level engages pupils in visualising, constructing, and modelling 3-D shapes. Pupils may need to revise and consolidate their experiences of constructing simple 3-D shapes as outlined in Level C.3.

SAMPLE LEARNING EXPERIENCES

Art | Shape Space

Drawing Nets for 3-D Shapes – Cubes[104]

Pupils should experiment drawing various nets to explore whether a cube can be formed from them. Pupils will discover that various combinations of six square nets can be re-configured to construct a cube. Pupils should a) discover these nets and b) construct the cubes.



the 11 nets for a cube

ICT Opportunities

[Interactive Nets for Cubes]

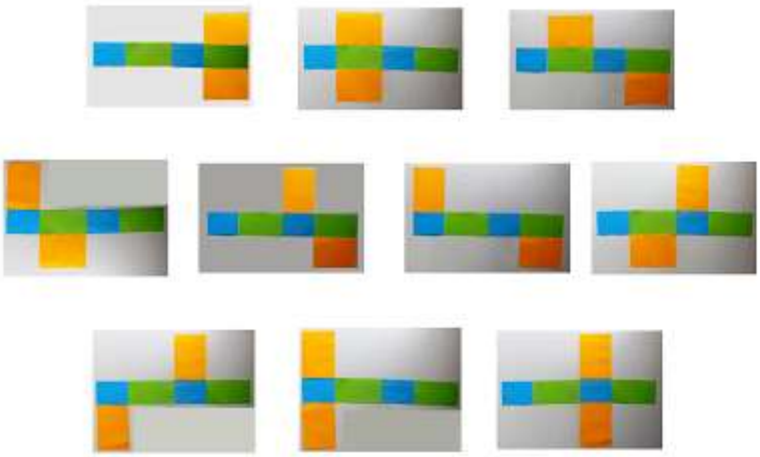
There are eleven different ways to construct a cube (illustrated above). This excludes rotations (turn it around) and reflections (turn it over).

To make a net of a cube, first look at one such as a dice in your classroom.
How could you create a net for this?
Does anyone have a different way?
Now you must work out a way to do this so that it will fold up into a cube.
How many different ways have you found?
Has anyone discovered a different method?
How can we use our nets to construct cubes?

[104] Link to Interactive Nets for Cubes: http://gwydir.demon.co.uk/jo/solid/cube.htm

Drawing Nets for 3-D Shapes – Cuboids[105]

Pupils experiment drawing various nets to discover which ones form a cuboid. Pupils will discover that various combinations can be re-configured to construct a cuboid.



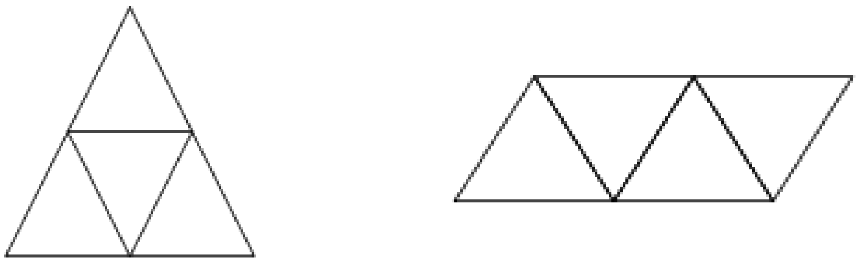
ICT Opportunities

[Visualising Cuboids]

To make a net of a cuboid, first look at one such as a cereal box.
How many face does it have? Six. What do the faces have in common?
Are they all the same shape? Do any of them match?
What do you notice about the angles?
Kate please revoice what Kim said.
Now you must work out a way to arrange six shapes so that they will fold up into a cuboid. How many different ways have you found?
Has anyone discovered another method?
Can we use our nets to construct cuboids?
Record your nets and procedure in your learning log.

Drawing Nets for 3-D Shapes – Tetrahedron

It is recommended that pupils revise and consolidate their learning in relation to platonic solids and in particular, the tetrahedron in Level D.1. Pupils should discover which combination of four equilateral triangles form a tetrahedron. Pupils can experiment with the net they have drawn to test whether they can construct the shape.



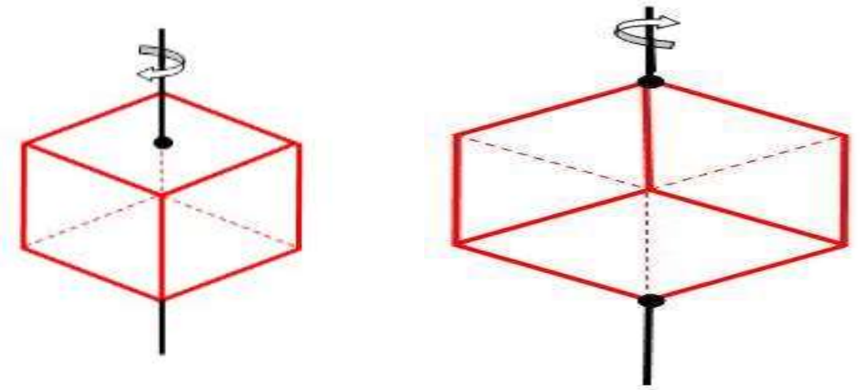
105 Link to Visualising Cuboids: http://donsteward.blogspot.ie/2013/05/nets-of-cuboid.html

Extension Activity: Interpenetrating Solids

This problem involves visualisation and rotation of cubes:

> *Imagine that you place a cube on a flat table. You rotate the cube 45 degrees about the axis joining the centre of the top face to the centre of the bottom face. A significant and important part of any visualisation problem is understanding clearly what the problem is asking of you and then thinking about the key features of the problem, so please be prepared to spend some time thinking about what the problem is about before putting pen to paper. Don't forget that sketches, diagrams and models are to be encouraged as they will help you as you work towards the solution.*

The accompanying ICT Link can be explored for further investigations.[106]



ICT Opportunities

[Visualising Cuboids](http://nrich.maths.org/5751)

[106] Link to Visualising Cuboids: http://nrich.maths.org/5751

LEVEL D.3: TESSELATE COMBINATIONS OF 2-D SHAPES

BACKGROUND KNOWLEDGE FOR TEACHERS

Polyominoes are shapes made by fitting squares together so that they make a unit, which can be cut out without falling apart.

Domino: made from two squares
Tromino: made from three squares
Tetromino: made from four squares
Pentomino: made from five squares
Hexomino: made from six squares

TEACHING NOTES

These activities extend previous knowledge and experiences from Level C.5 where pupils combined, tessellated and made patterns with 2-D shapes (regular, irregular, non-tessellating shapes, etc.) In this level, pupils extend their previous knowledge of tessellating shapes to a) combining shapes such as squares and equilateral triangles in various ways and b) experimenting with and recording their tessellating properties.

SAMPLE LEARNING EXPERIENCES

Tessellation: The Local Environment

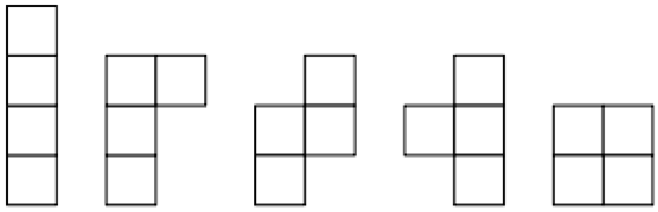



What common feature is shared by the honeycomb, fish, pineapple and snowflake?
Name some of the shapes on each object.
List other examples of tessellating shapes in the natural and built environment.

Four Square Combinations: The Five Tetrominoes

1. Using plastic squares, pupils attempt making as many different patterns as possible using four squares.

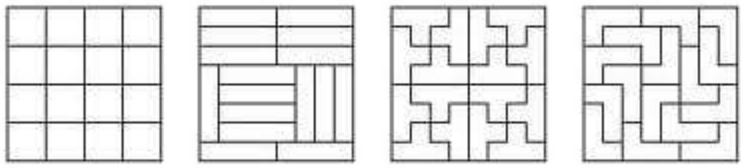
2. Results can be recorded on squared or spotted paper. All possible arrangements for four squares are illustrated below. ICT opportunities for manipulating these squares can be accessed also (see link below). [107]



ICT Opportunities

[Interactive Tessellation]

3. These shapes also tessellate so pupils can explore this and record their findings on squared paper making various patterns as shown below.



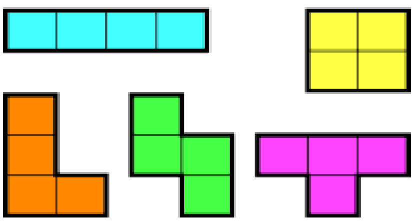
Why did you choose this combination of tetrominoes?
Describe the pattern they created.
How did you record this pattern on grid paper?
Arrange the tetrominoes in another way.
Share and compare your arrangements to others in the class.

Extension Activity: Tetrafit Problem

A tetromino comprises four squares joined edge to edge. This problem involves further investigation of tetrominoes:

Can this tetromino together with 15 copies of itself be used to cover an eight by eight chessboard? Investigate this claim and then share your findings.

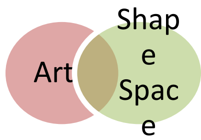
The ICT link may be useful for pupils to explore this problem. [108]



ICT Opportunities

[Tetrafit Problem]

[107] Link to Interactive Tessellation: http://illuminations.nctm.org/ActivityDetail.aspx?ID=202
[108] Link to Tetrafit Problem: http://nrich.maths.org/814



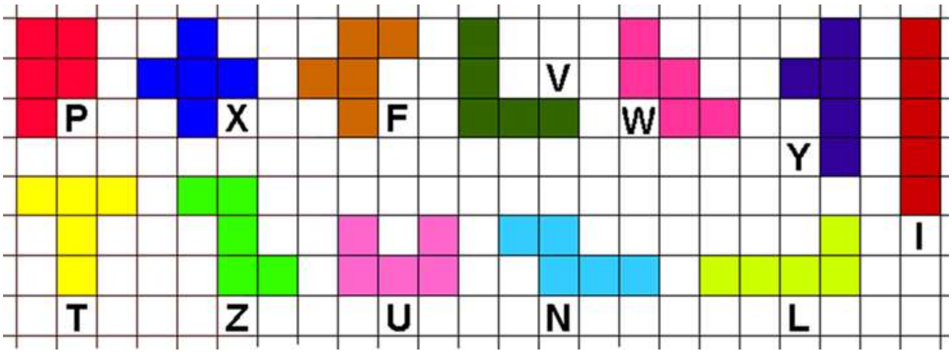
Extension Activity: Pentominoes, Triamonds and Hexiamonds

Pentominoes comprise five squares whilst hexiamonds comprise six equilateral triangles.

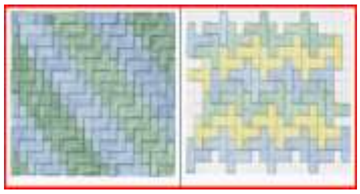
Pentomioes

1. Pupils tessellate five squares to discover all the possible combinations.
2. Pupils record these on dotted paper or graph paper.

The investigation of pentominoes should lead to the discovery of twelve different arrangements of five squares, which are illustrated below:

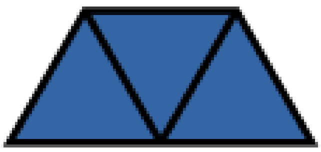


3. Pupils investigate the tessellating properties of some of the shapes they have created from their five squares (pentominoes). Pupils cut out/draw a selection of each and try fitting them together in a repeating pattern. A sample of two of the shapes are illustrated here.



Three Equilateral Triangles

A triamond is formed by joining three equilateral triangles. Only one arrangement is possible. Pupils should investigate this and justify their findings. The ICT link below can be used to investigate all the possible combinations.[109]

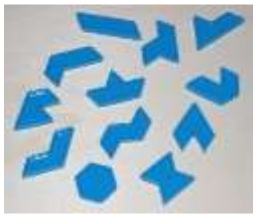


What are the properties of an equilateral triangle?
What shape can you make if you combine two equilateral triangles?
Moving on, now combine three equilateral triangles to form a new shape.
How many different shapes can you create?
Describe these shapes and include reference to sides, symmetry, and angles.

ICT Opportunities

Combining Triangles

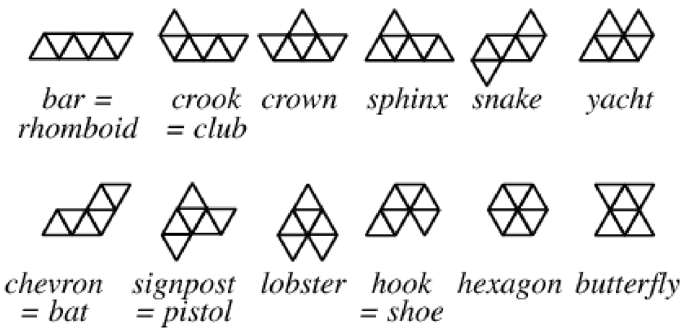
[109] Link to Combining Triangles: http://nrich.maths.org/4869



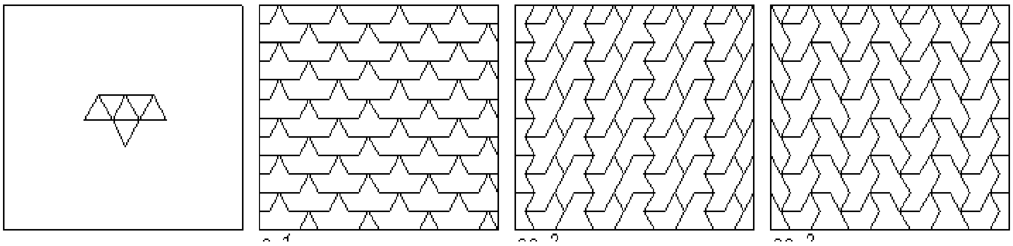
Six Equilaterial Triangles (Hexiamonds)

1. Pupils tessellate six equilateral triangles to discover all the possible combinations.
2. Pupils record these on dotted paper or graph paper.

The investigation of hexiamonds should lead to the discovery of twelve different arrangements as illustrated below:



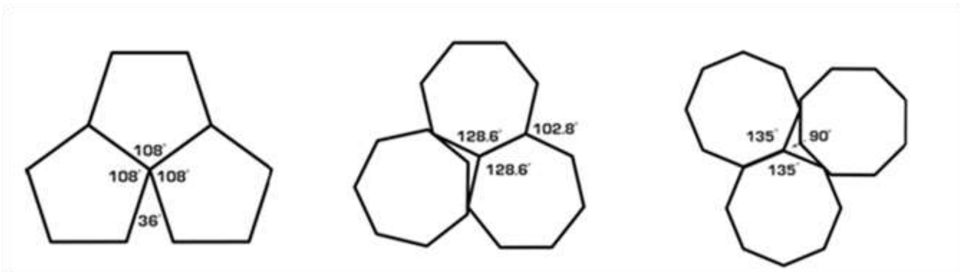
3. Pupils can create names for the shapes they have discovered.
4. Pupils investigate the tessellating properties of some of the shapes they have created from their six equilateral triangles (hexiamonds). Pupils cut out/draw a selection of each and try fitting them together in a repeating pattern. A sample of some tessellations are illustrated below:



How did you tessellate your six equilateral triangles?
Arrange your hexiamonds in more than one way.
How many possible tessellations can you record?

Extension: Why do Shapes Tessselate?

Tessellation activities are useful for studying the properties of angles. Pupils can suggest reasons why some of these shapes can be fitted together while others have spaces between. This discussion may lead to an investigation which may eventually lead to the assertion that the angles of tessellating shapes at their meeting points total 360 degrees, so that the sum of the angles is four right angles.

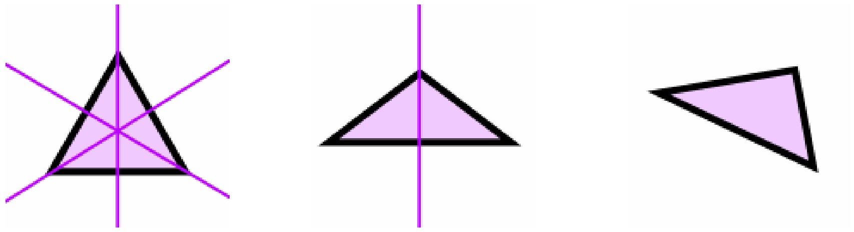


LEVEL D.4: CLASSIFY 2-D SHAPES ACCORDING TO THEIR LINES OF SYMMETRY

BACKGROUND KNOWLEDGE FOR TEACHERS

Triangles

A Triangle can have **3**, or **1** or **no** lines of symmetry:



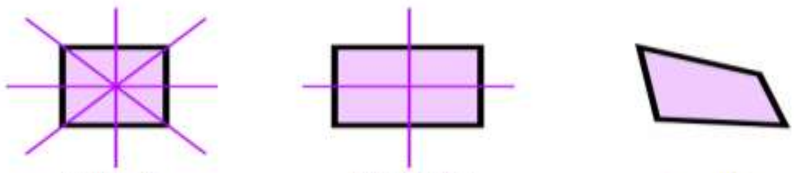
Equilateral Triangle
(all sides equal,
all angles equal)
3 Lines of Symmetry

Isosceles Triangle
(two sides equal,
two angles equal)
1 Line of Symmetry

Scalene Triangle
(no sides equal,
no angles equal)
No Lines of Symmetry

Quadrilaterals

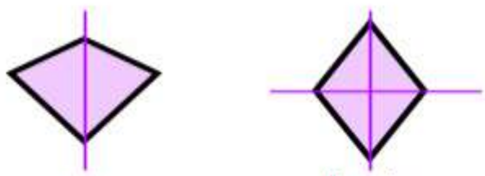
Different types of Quadrilaterals (a 4-sided plane shape):



Square
(all sides equal,
all angles 90°)
4 Lines of Symmetry

Rectangle
(opposite sides equal,
all angles 90°)
2 Lines of Symmetry

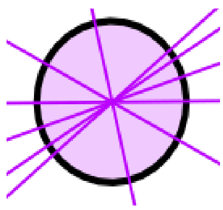
Irregular Quadrilateral
No Lines of Symmetry



Kite
1 Line of Symmetry

Rhombus
(all sides equal length)
2 Lines of Symmetry

Circle

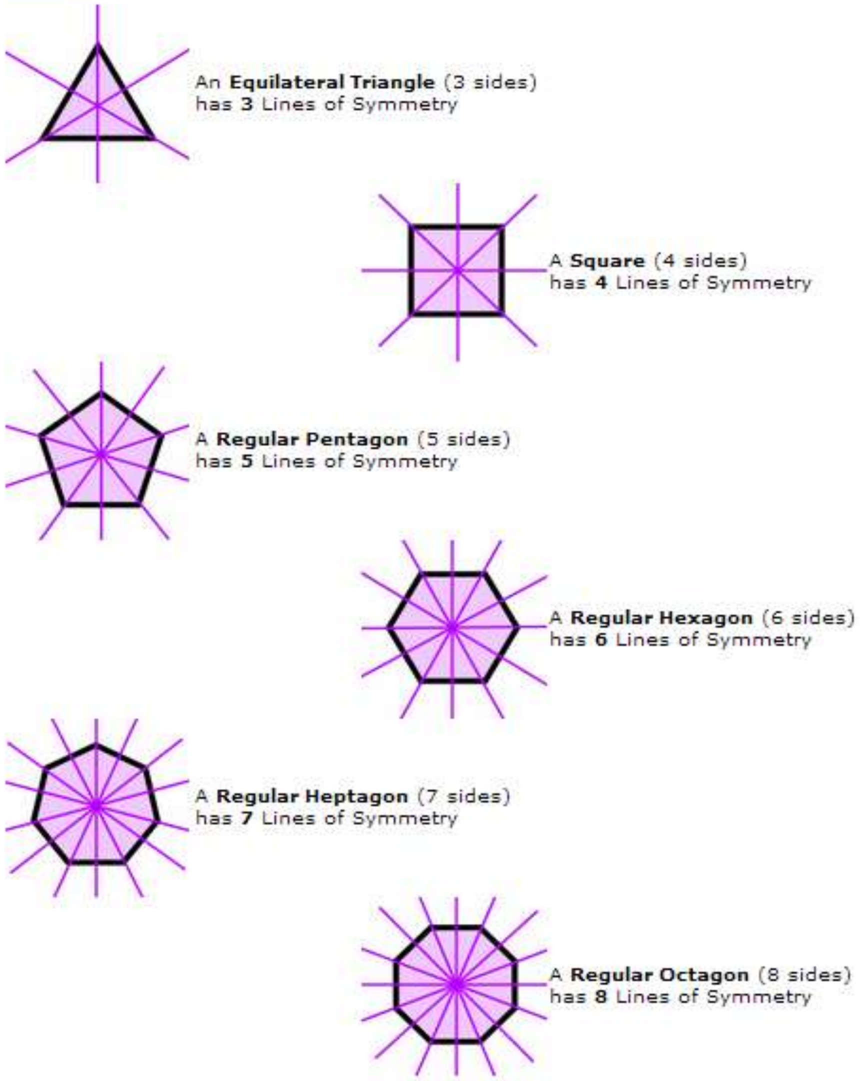


A line (drawn at any angle) that goes through its center is a Line of Symmetry.

So a Circle has **infinite** Lines of Symmetry.

Regular Polygons

A regular polygon has all sides equal, and all angles equal:



TEACHING NOTES

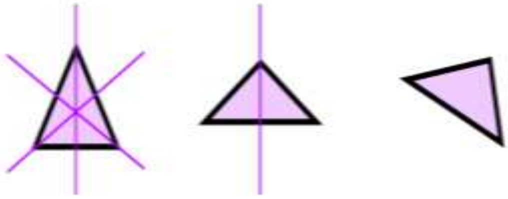
It is recommended that pupils consolidate and extend learning from previous experiences of identifying horizontal, vertical and diagonal lines of symmetry in Level C. The suggested line of development should extend from examining different types of triangles, quadrilaterals and regular polygons for lines of symmetry. Final consolidation activities involve grouping a selection of these shapes according to their symmetrical properties.

SAMPLE LEARNING EXPERIENCES

The main purpose of these activities is to remind pupils that each shape below has one or a number of line(s) of symmetry or mirror lines. The following activity challenges pupils to sort by the number of lines of symmetry in an individual shape before categorising into two separate collections.

Triangles

Pupils begin by identifying lines of symmetry in various triangles as can be seen in background knowledge section above.

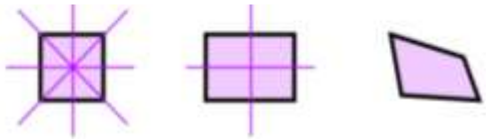


Do all triangles have the same number of (a) sides (b) lines of symmetry?
Why? Why not?
Who has a different opinion?

These questions should be explored with various triangles such as scalene, obtuse, and right-angled.

Quadrilaterals

Pupils begin by identifying lines of symmetry in various quadrilaterals as can be seen in background knowledge section above.



Do all quadrilaterals have the same number lines of symmetry?
Why? Why not?
Jim please revoice what Kate said.

These questions should be explored with various quadrilaterals such as a square, rhombus, and rectangle.

Extension Activity: Stringy Quadrilaterals

Groups of four pupils and one loop of string are required for this activity.

1. Stretch the string out so that each pupil is holding a corner to make a quadrilateral.
2. Pupils should attempt making one that has exactly one line of symmetry.
3. The teacher asks questions such as:

   Is it possible? Can you make any other quadrilaterals with just one line of symmetry?
4. Pupils should repeat this; but now attempt one that has exactly a) two lines of symmetry; b) three lines of symmetry; and c) four lines of symmetry.

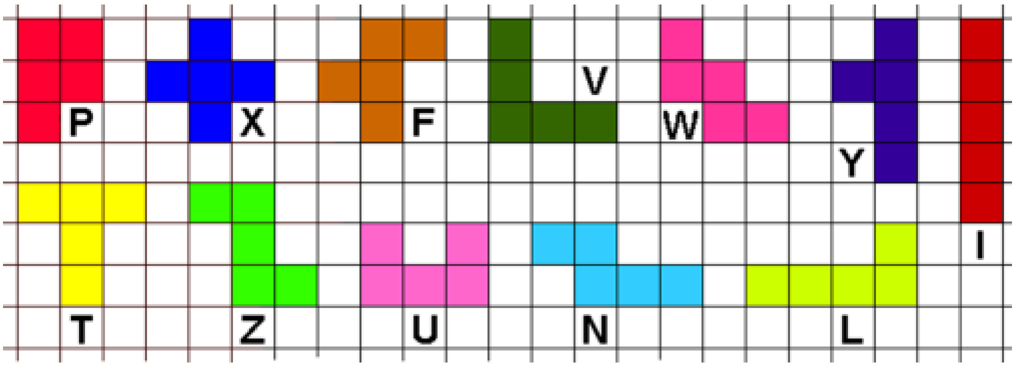
Triangles, Quadrilaterals & Polygons

This activity challenges pupils to group a collection of triangles, quadrilaterals and polygons according to their symmetrical properties, for example, one or more lines of symmetry.

Why have you grouped these shapes together?
What do they have in common?
How many lines of symmetry does this shape have?
Can this shape fit in more than one group? Why? Why not?
Who has a different opinion?
Draw groups of shapes and write a short paragraph underneath each group to explain why they are in this category.

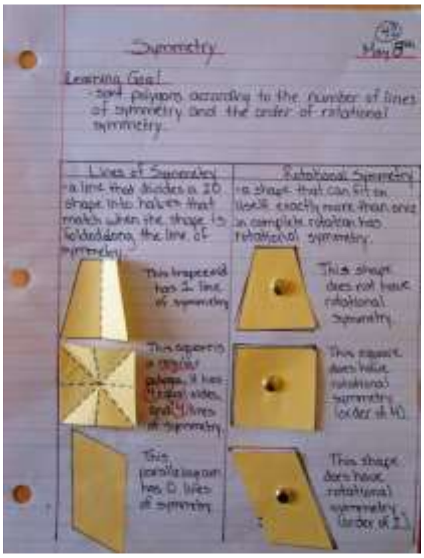
Extension Activity: Pentominoes[110]

This activity involves investigating which of the shapes below (pentominoes) have at least one line of symmetry. Pupils should be encouraged to explore horizontal, vertical and diagonal symmetry through this challenging activity. They can also be encouraged to draw or construct the pentominoes. A further extension activity could focus on exploring rotational symmetry with pupils before completing online investigations – see ICT opportunity below.



Extension Activity: Rotational Symmetry[111]

Investigate rotational symmetry related to a range of shapes, both regular and irregular. The ICT link may be sueful for this. Pupils record their learning in maths journals or learning logds – including cut- out shapes, lines of symmetry and information about their rotational symmetry.



ICT Opportunities

[Exploring Rotational Symmetry]

[110] http://nrich.maths.org/2913

[111] Link to Exploring Rotational Symmetry: http://www.bbc.co.uk/bitesize/ks3/maths/shape_space/symmetry/revision/4/

LEVEL D.5: MAKE INFORMAL DEDUCTIONS ABOUT 2-D SHAPES AND THEIR PROPERTIES

BACKGROUND KNOWLEDGE FOR TEACHERS

Informal Deduction: Van Hiele's Levels of Geometric Thinking

The Van Hiele theory puts forward a hierarchy of levels of thinking spanning the ages of about five years through to academic adults. This theory is for geometry (shape and space). Van Hiele concentrates on the three levels that cover the normal period of schooling. The main focus is on two-dimensional shape.

Level 0: Visual

This level begins with 'non-verbal thinking'. Shapes are judged by their appearance and generally viewed as 'a whole', rather than by their distinguishing features. Pupils can name and recognize shapes by their appearance, but cannot specifically identify properties of shapes. Although they may be able to recognize characteristics, they do not use them for recognition and sorting. Suggestions for Instruction using Visualization include:

- sorting, identifying, and describing shapes;
- manipulating physical models;
- seeing different sizes and orientations of the same shape so to distinguish characteristics of a shape and the features that are not relevant; and
- building, drawing, making, putting together, and deconstructing shapes

Level 1: Analysis

At this level, children can identify and describe the component parts and properties of shapes. For example, an equilateral triangle can be distinguished from other triangles because of its three equal sides, equal angles and symmetries. However, at this stage the properties are not 'logically ordered', which means that pupils do not perceive the essential relationships between the properties. Suggestions for Instruction using Analysis include:

- shifting from simple identification to properties by using concrete or virtual models to define, measure, observe, and change properties;
- using models and/or technology to focus on defining properties, making property lists, and discussing sufficient conditions to define a shape;
- solving problems including tasks in which properties of shapes are important components; and
- classifying using properties of shapes.

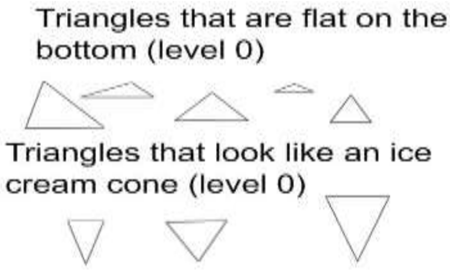
Level 2: Deduction

In this level, the properties of shapes are logically ordered. Pupils are able to recognise relationships between and among properties of shapes or classes of shapes and are able to follow logical arguments using such properties. Pupils are able to see that one property precedes or follows from another, and can therefore deduce one property from another. They are able to apply what they already know to explain the relationships between shapes, and to formulate definitions. In order to challenge and extend pupil's ability to make informal deductions about 2-D shapes they need to engage in tasks and games that deliberately develop the vocabulary associated with the ideas that have been encountered so far, while continuing to explore the properties of the shapes. Suggestions for instruction using Deduction include:

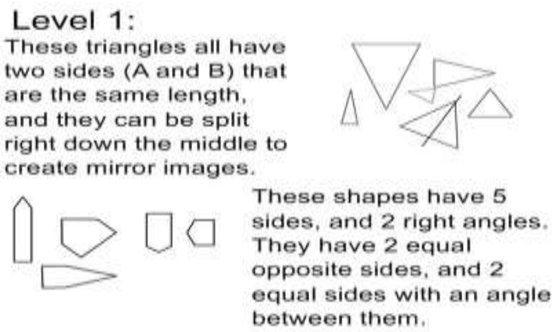
- problem solving including tasks in which properties of shapes are important components;
- using models and property lists, and discussing which group of properties constitute a necessary and sufficient condition for a specific shape;
- using informal, deductive language ("all," "some," "none," "if-then," "what if," etc.);
- investigating certain relationships among polygons to establish if the converse is also valid (for example, "If a quadrilateral is a rectangle, it must have four right angles; if a quadrilateral has four right angles, must it also be a rectangle?"); and
- using models and drawings as tools to look for generalisations and counter-examples.

The description of the groups in a shape sorting activity can reflect the level of geometric thinking that was used for the shape sorting:

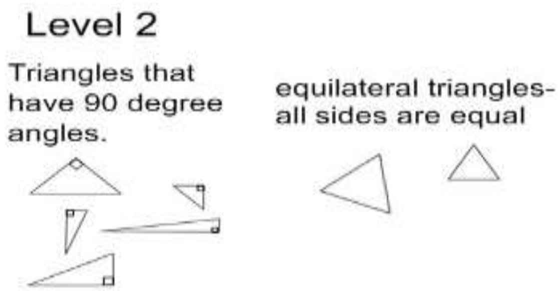
- Level 0 descriptions often have orientation or visual similarity cited as sorting rules



- Level 1 descriptions often have a lot of properties listed



- Level 2 descriptions often have more efficient lists of properties. The descriptions are usually more careful include vocabulary that is more mathematical.



and

TEACHING NOTES

Activities in this section are reflective of previous learning experiences in Level C. The emphasis is on creating relationships and connections between 2-D shapes and their common or unique features / characteristics.

SAMPLE LEARNING EXPERIENCES

‘Walking’ 2-D Shapes: Visualising & Describing

To reinforce the visual and descriptive elements of Van Hiele’s Levels (see section on Background Knowledge for Teachers), pupils can ‘walk’ a shape.

1. One pupils instructs another pupil to ‘walk’ a square by giving them explicit descriptions, for example:

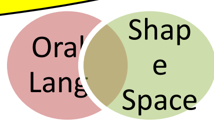
   Walk forwards 2 steps, turn 90 degrees to the right.
   Walk forwards 2 steps, turn 90 degrees to the right.
   Walk forwards 2 steps, turn 90 degrees to the right.
   Walk forwards 2 steps, turn 90 degrees to the right.

2. Pupils swap over and try this with a different sized square.
3. Repeat this using various triangles and other shapes by following explicit directions.

This activity is very valuable in its own right; however, it also provides a very sound foundation for some of the shape activities using Scratch Mathematical Programming (see below).

What shape have you created?
Record this in your learning log.
How can you apply this learning in order to create other familiar shapes?

Guess My Rule

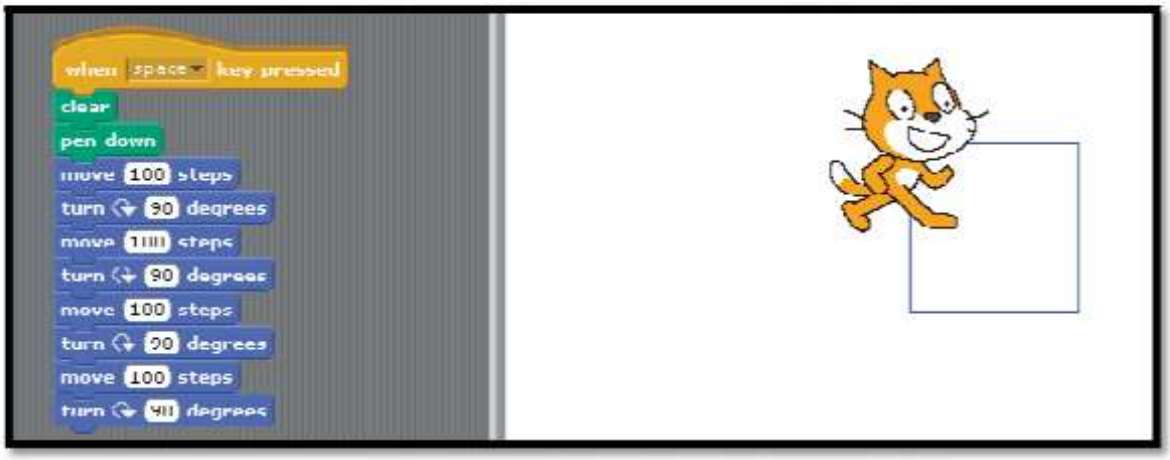


In this activity, pupils must identify the rule that produced a particular group of shapes, for example, after a sorting activity. This task strengthens pupils’ understanding regarding which rules can be used to define/sort a set of shapes. Additionally, it helps pupils move from Van Hiele’s Level 1 to Level 2 by focusing their attention on the properties needed to define a group. By looking for properties, which shapes have in common, pupils also strengthen their ability to group shapes using different rules. Furthermore, by comparing the rule they found with rules that other pupils found, or with the rule stated by the person who created the group, pupils learn that it is possible to define the same group in different ways. Finally, by providing groups of shapes that include other known types of shapes (rectangles in a group of parallelograms); the teacher can introduce the children to examples where some groups of shapes are subsets of other groups of shapes.[112] Throughout this activity, pupils can record their learning through compiling a folder of digital images detailing their sets, subsets and property rules. This is an invaluable assessment tool for teachers.

Scratch Mathematical Programming

Scratch is a programming language developed by the MIT Media Lab that makes it easy for pupils to create their own interactive stories, animations, games, music, and art and to share their creations on the web. Scratch is free to use and can be downloaded or used on-line from the Scratch website http://scratch.mit.edu. There is also an Irish scratch website with resources for teachers including manuals and lesson plans for using Scratch in the classroom. These are available at www.scratch.ie. An example of how scratch can be used for exploring shapes is illustrated below. The central purpose of using Scratch in this level is to challenge pupils to build on prior knowledge in constructing shapes, explicitly focusing on the individual properties of each shape, for example:

How do I draw a square using the Scratch Programme?

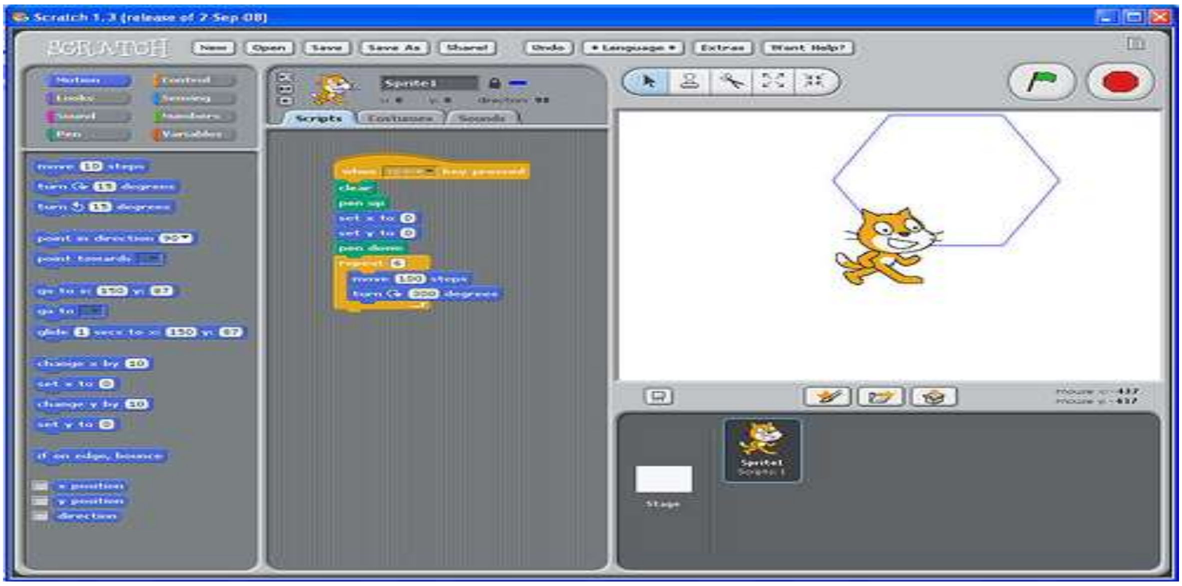


The code in the image is repetitive (because the movements and turns are all equal). In Scratch, a repeat loop can be used to create a shorter piece of code. The code is dependent on the pupils' understanding of the central properties of a specific shape; for example, an equilateral triangle is constructed by using a code containing three steps of equal length and three turns of 120° (reflex angle). This is similar to the 'walk' a shape activity above.

A sound understanding of the composition of angles, particularly the reflex angle is vitally important in constructing shapes through the Scratch Programme. One suggested method is to divide 360 by the number of sides.

[112] http://langfordmath.com/ECEMath/Geometry/VHLevPropertiesText.html

Use the same basic controls to draw a rectangle, triangle, pentagon, and other shapes.
How did you decide the measure of your angles?
How did you draw a hexagon?
Who did it in another way?
Share your solution / code with others.



Draw a circle in Scratch.
Combine 2-D shapes to create other shapes.
How do you design these shapes in the Scratch Programme?
Share with us if you have another way of doing it.

Whilst engaged in Scratch, pupils should be encouraged to employ key mathematical terminology, for example:

- congruent
- irregular
- diagonal
- tessellate
- symmetry
- polygon
- angle.

LEVEL D.6: IDENTIFY THE PROPERTIES OF THE CIRCLE AND CONSTRUCT A CIRCLE OF GIVEN RADIUS OR DIAMATER

BACKGROUND KNOWLEDGE FOR TEACHERS

Definitions of Circle Vocabulary

Radius: A straight line extending from the centre of a circle to the circumference (outer edge) or surface. The radius of a circle is half the diameter.

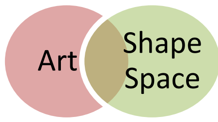
Diameter: A straight line passing through the centre of a circle and meeting the circumference or surface at each end; a line segment connecting two points on the circle and passing through the centre.

Circumference: The outer boundary of a circular area or the distance around a circle.

TEACHING NOTES

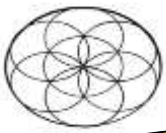
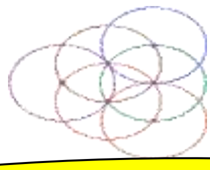
The line of development in this level begins with pupils identifying the diameter, radius, and circumference of a circle. Finally, pupils construct a range of circles of given radius or diameter.

SAMPLE LEARNING EXPERIENCES



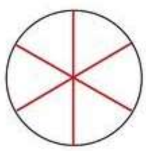
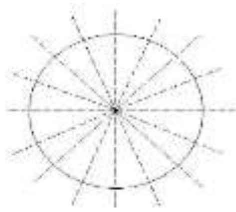
Exploring Circles

Pupils begin by drawing round counters, tin lids, coins, etc., in an attempt to produce circular patterns such as those below.


What patterns can you make using circles?
How many circles can you inscribe within a bigger circle?
What do you notice about the lines formed?
Who noticed something different? Describe the shapes.
Find a sporting flag where the circles are represented in this way.
Complete a WebQuest to investigate the significance of each of the five circles.

Pupils can also investigate the tessellating properties of circles. They discover that no matter how they arrange them they do not fit together without spaces. Pupils should fold circles in half to make semi-circles. Children will discover a variety of ways to fold them as shown below. Extend this activity to discussing and recording the symmetrical properties of circles.

Describing the Circle

Pupils describe a circle to someone who has never seen one before, for example, an alien. Pupils record their responses on index cards and share them with classmates. Conclude this activity by agreeing on a short list of the properties of a circle written by the whole class.

Exploring Circumference

To explore circumference:

1. Pupils cut a length of string that traces around the outside of a circle, for example, a bottle top, a paper place, a Frisbee, a lid, etc.
2. Ask pupils a) if they know the name for this and if not ask them to b) research the name.
3. Pupils estimate, measure and record their results as shown on an A4 sheet before transferring their learning points to their maths journal or learning log.

Exploring Radius

1. Using a piece of string, one pupil acts as the centre point of the circle.
2. Other pupils stand an equal distance from the pupil in the centre - this can be measured using the length of string.
3. The pupil in the middle holds one end of the string while different pupils on the outside hold the other end.
4. Ask pupils if they know the name of this line (the line from the centre point to the outside of the circle).

Exploring Diameter

1. As above; however, two pupils on opposite sides of the circle hold the ends of the string.
2. The teacher asks important questions such as:

   Will this always show the diameter? Why? Why not?

3. Pupils discover that the line is only called the diameter when it is straight and passing through the centre point.

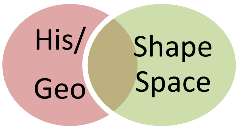
Describe the relationship between the radius and diameter.
Jim explain that to me in your own words.
What patterns did you notice when you compared the radius and diameter?
Do you think this pattern will continue for every circle? Justify you answer.

4. The relationship between circumference and diameter can be explored at the ICT Link.[113]

ICT Opportunities
[Circumference & Diameter Exploration]

[113] Link to Circumference and Diameter Exploration: http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.SHAP&ID2=AB.MATH.JR.SHAP.CIRC

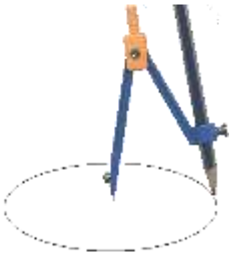
The properties of circles fascinated Greek mathematicians and they explored them for centuries.



Pupils should be encouraged to look for circular shapes in the environment and to consider the significance of circles in everyday life, for example, the importance of the discovery of the wheel, etc. Exploration of the historical importance and significance of circles in heritage sites such as Stonehenge and Newgrange is also recommended. A follow-on Web Quest activity could look at the significance of circles in history.

Drawing Circles

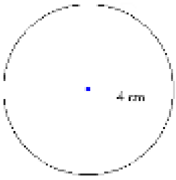
Pupils should be encouraged to experiment with the process of drawing circles with a given diameter or radius. Possible teaching points may include:



To draw a circle with a compass:

1. Make sure that the hinge at the top of the compass is tightened so that it does not slip
2. Tighten the hold for the pencil so it also does not slip
3. Align the pencil lead with the compass's needle
4. Press down the needle and turn the knob at the top of the compass to draw a circle (or arc)

Use a compass to draw a circle of radius 4 cm



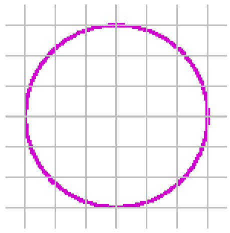
Step 1: Use a ruler to set the distance from the point of the compass to the pencil's lead at 4 cm

Step 2: Place the point of the compass at the centre of the circle

Step 3: Draw the circle by turning the compass through 360º

.

Extension Activities



Possible extension activities include:

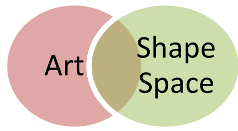
a) pupils exploring how to inscribe a square, triangle, hexagon, etc., within the circle

b) instruction on finding the area of a circle

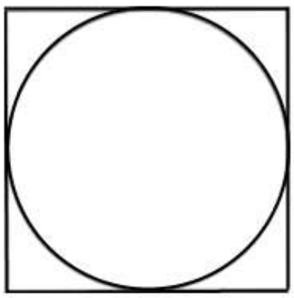
c) using the circle on grid paper (see image) to estimate or measure the area of the circle

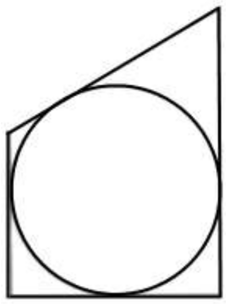
Extension Activity: Circles in Quadrilaterals

Another extension activity involves constructing circles within quadrilaterals. For each of the following types of quadrilaterals, pupils decide whether it is always, sometimes or never possible to construct a circle inside which just touches all four sides. If pupils decide *always* or *never*, they need to justify the decision with a convincing argument. If they decide *sometimes* they need to be precise about when it is possible and when it is not possible, and why. Pupils should draw and label their solutions in their learning log.



- Square
- Rectangle
- Rhombus
- Parallelogram
- Kite
- Trapezium

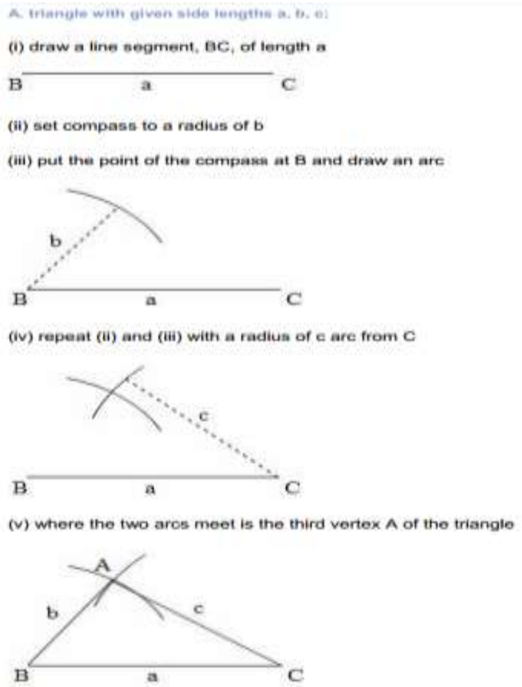




LEVEL D.7: CONSTRUCT TRIANGLES FROM GIVEN SIDES OR ANGLES

BACKGROUND KNOWLEDGE FOR TEACHERS

The following methods are intended for the teacher's reference only.[114] Pupils should be encouraged to experiment with various methods to discover how to draw triangles given sides or angles.



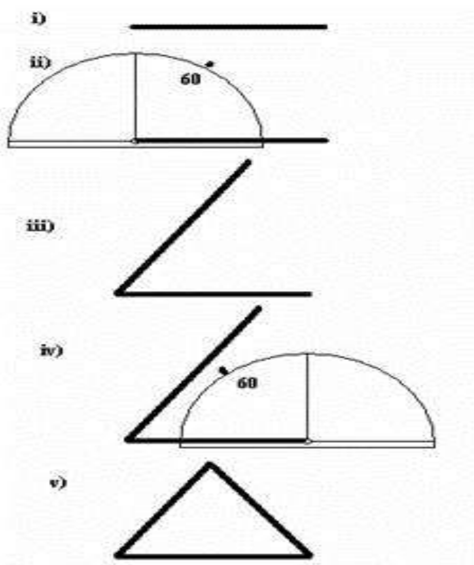
A. Using Compass and Ruler

Method A can be used to construct triangles as long as you know the lengths of all the sides.

B. Using Ruler and Protractor

Method B can be used to construct triangles as long as you know two side lengths and one angle.

1. Draw one side to the required length, using a ruler.
2. Measure an angle of 60 degrees from one end of the first side.
3. Draw a second side to the same length as the first, again using a ruler. Connect the ends, measuring to ensure the third side is the same length as the first two.

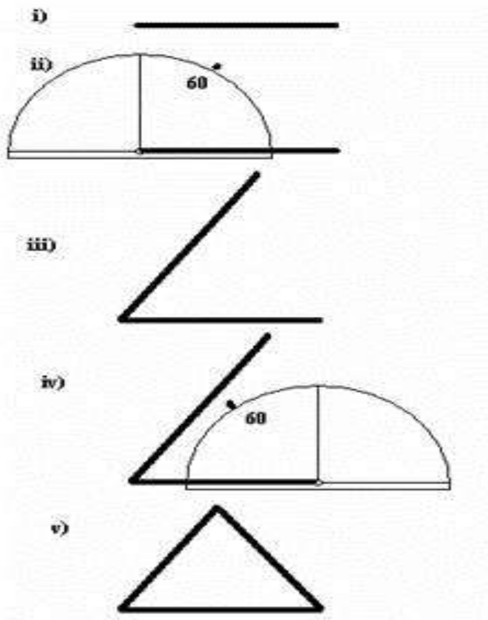


[114] http://www.nzmaths.co.nz/sites/default/files/RulerandCompassConstructions.pdf

C. Using Ruler and Protractor

Method C can be used to construct triangles as long as you know two angles and one side length.

1. Draw one side to the required length, using a ruler. Measure an angle of 60 degrees.
2. Draw a second side through your mark at 60 degrees.
3. Measure another angle of 60 degrees from the other end of your first side. Draw the third side and rub out any extra lines.



TEACHING NOTES

Beware of pupils saying that triangles have all sides the same length or all angles the same. This is not a property of triangles, but a property of regular polygons (including equilateral triangles). Ensure that this is emphasized and modelled through pupil-led investigations.

This level revises and consolidates the properties of triangles through pupils' exploration of constructing triangles. Pupils should identify the following key properties: three sides, three angles/corners, angles add to 180 degrees, different types (scalene/isosceles/equilateral/right-angled). The following activities are intended to be developmental in nature. Pupils first experiment and discover how to form triangles knowing a) all the lengths of the sides; b) having two sides and one angle; and c) having two angles and one side.

SAMPLE LEARNING EXPERIENCES

Pupils should **discover** ways to construct triangles rather than being told or shown how to do it. This can be achieved through group-work, guided discussion, trial and error, and sharing ideas.

Constructing Triangles: Knowing the Lengths of All Sides

How do you construct triangles when you know the lengths of all the sides?
Please revoice what Kate said.
What equipment do you think you will need?
Will this work every time even if the sides are the same length/different?

Extension Activity: Constructing Triangles Game

1. Take a ten-sided die (or other random number generating tools - a pack of cards with the picture cards removed, a calculator, a phone app, etc.) and generate three numbers.
2. Construct a triangle using these three numbers as the side lengths.
3. See the ICT below to continue your investigation and to see how a ruler and compass if used in constructing your triangles.[115]

ICT Opportunities

[Constructing Triangles]

Constructing Triangles: Knowing Two Side Lengths and One Angle

How can you construct a triangle as long as you know two side lengths and one angle?
What method could you use?
Is the method the same method as when you knew three sides?
What equipment do you need?

Constructing Triangles: Knowing Two Angles and One Side Length

How can you construct a triangle as long as you know two angles and one side?
What method could you use?
Is the method the same method as when you knew three sides/ two sides and one angle?
What equipment do you need?

Pupils should list advantages and disadvantages of each method and record these in their learning logs.

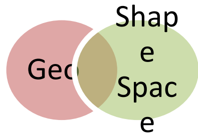
[115] Link to Constructing Triangles Game: http://nrich.maths.org/8098

LEVEL D.8: PLOT SIMPLE CO-ORDINATES AND APPLY WHERE APPROPRIATE

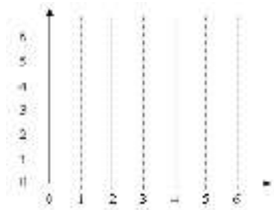
BACKGROUND KNOWLEDGE FOR TEACHERS

The horizontal axis is known as the ‘x axis’ and the vertical axis as the ‘y axis’.

SAMPLE LEARNING EXPERIENCES



Hidden Co-Ordinates Game



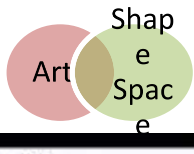
1. The teacher explains how to use two numbers to designate an intersection point on the grid. The first number tells how far to move to the right and the second number tells how far to move up.
2. Pupils should have co-ordinate grids to assist them in developing these concepts.
3. Initially words can be used along with the numbers, for example, 3 right and 0 up (it is important to include 0 in the introduction stage).
4. Then select a point on the gird and decide what two numbers name that point. If your point is (2,4) and pupils confuse this with ‘four, two’ – further teacher-led instruction and modelling is needed to reinforce this key concept before labelling ‘x axis’ and ‘y axis’.

Identify where the co-ordinate 4,1 is and mark this on your grid.
Now, identify where 1,4 is. How can you be sure which one is correct?
Justify this with your partner/group.
Co-ordinates on a grid can be used to read positions on maps such as this that shows two ships at sea.
Write 5 questions based on this map.
Now design, colour and generate questions for your own treasure grid map.

Chairs Co-ordinates

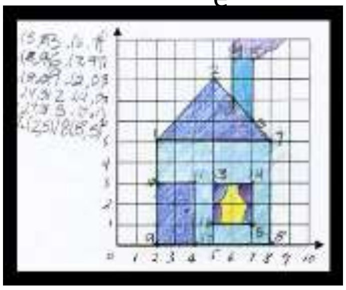
A simple way for pupils to visualise the named point could be a comparison in your classroom of pupils sitting in the second row fourth seat as opposed to fourth row second seat. This activity can also be developed to play human battleships; pupils can develop their grid and co-ordinates on tiled or playing surfaces.

Who is sitting in first row, third seat?
Name the co-ordinates of John’s seat.
Name the co-ordinates of someone with blonde hair.
Add another row and another column – how many new questions can you generate?



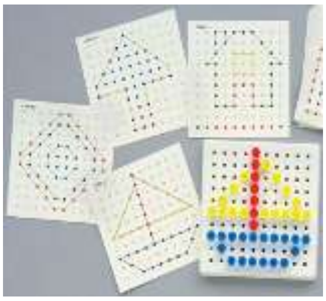
Co-ordinate Shapes

Exercises in plotting co-ordinates to give familiar shapes can add interest to prior activities. For example, pupils are asked to plot the following (1,1) (4,1) (1,3) (4,3) (2,4) (3,4) (2,5) (3,5) on a 5 x 5 grid. Pupils must describe what shape it makes. Pupils can be challenged to create and share the co-ordinates of other familiar shapes or to combine shapes into a design to share with classmates.



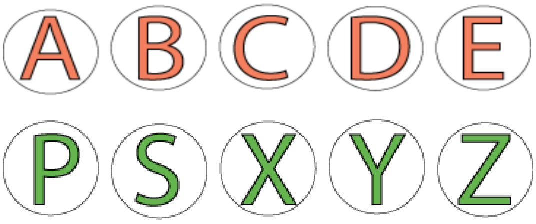
Pegboard Co-ordinates

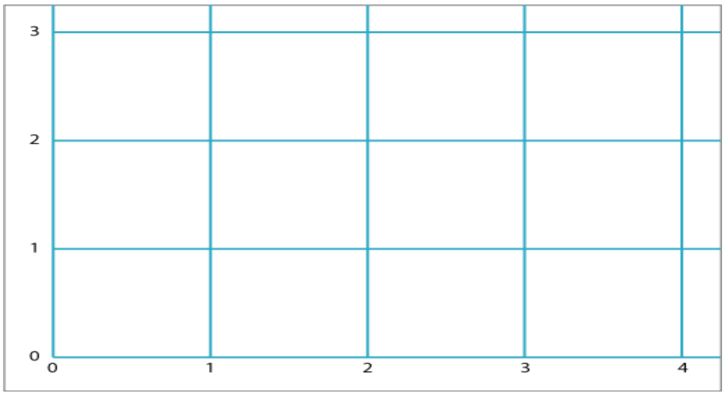
Pegboards can also be used for sample investigations such as placing the pegs at (1,3), (2,6), (3,9), etc. Pupils must share what they notice. Pupils can then create and record other interesting patterns. As a recording exercise, pupils can record these co-ordinates on grid paper in order to share with classmates.



Co-Ordinate Challenge

Pupils must position these ten letters in their correct places according to the eight clues below.





Clues:

- The letters at (1,1), (1,2) and (1,3) are all symmetrical about a vertical line.
- The letter at (4,2) is not symmetrical in any way.
- The letters at (1,1), (2,1) and (3,1) are symmetrical about a horizontal line.
- The letters at (0,2), (2,0) have rotational symmetry.
- The letter at (3,1) consists of just straight lines.
- The letters at (3,3) and (2,0) consist of just curved lines.
- The letters at (3,3), (3,2) and (3,1) are consecutive in the alphabet.
- The letters at (0,2) and (1,2) are at the two ends of the alphabet

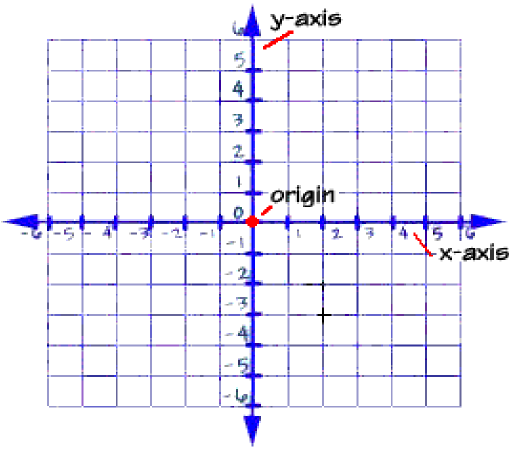
Number Line Co-ordinates

Positive and negative numbers once used to show movement to the right and left of zero can also be used to show movement up or down in terms of co-ordinate positioning on the horizontal 'x axis'. As pupils' understanding progresses these activities can be extended to the vertical axis or 'y axis'.

CONSOLIDATION ACTIVITY

Co-ordinates Board Game

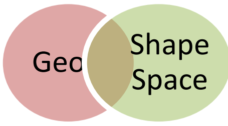
A useful consolidation activity at this stage involves the use of a co-ordinates board game. Two dice are needed - red for horizontal postion and blue for vertical position so that the co-ordinate can be marked using a counter on the grid. This can be done on a desk top, sporting or tiled surface within the school environment. One of the values of doing this activity is to determine if pupils have developed their sense of positioning on the 'x' and 'y' axis.



Professional Development Service for Teachers

LEVEL D.9: RECOGNISE, CLASSIFY AND DESCRIBE ANGLES AND RELATE ANGLES TO SHAPE AND THE ENVIRONMENT

SAMPLE LEARNING EXPERIENCES



Pupils should be encouraged to recognise angles in the environment and to classify these angles into distinct groups. Language utilised can include acute, obtuse, right, straight and reflex angles. Pupils begin by locating and describing angles in the built environment below.

Built Environment

Look at the images below; identify and label the acute, obtuse, right, straight, or reflex angles.
Why are angles important in construction and design?
Who can add to this?




Natural Environment

Look at the images; what type of angles can you locate?
Widen your investigation on angles in nature by looking at various angles formed in flowers, trees, rivers, etc.



CONSOLIDATION ACTIVITIES

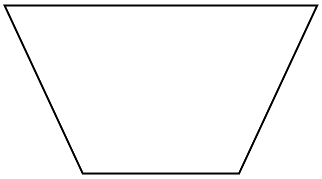
The following two activities are intended as revision and consolidation activities / investigations relating shapes and their corresponding angles. Pupils can complete the following tasks as a whole class or group activity. They should record their learning in their learning logs.

Labelling Angles

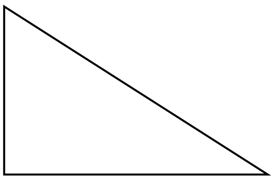
Look at the shapes; identify and label the shapes that have acute, obtuse, right, straight or reflex angles.
Share this with each other.

acute, obtuse, right, straight or reflex

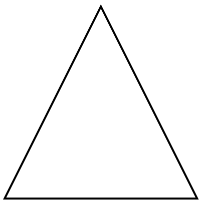
acute, obtuse, right, straight or reflex



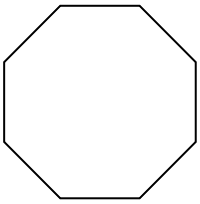
acute, obtuse, right, straight or reflex



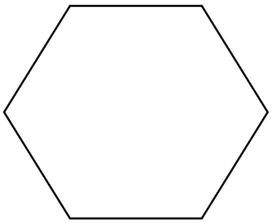
acute, obtuse, right, straight or reflex



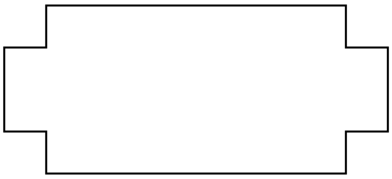
acute, obtuse, right, straight or reflex



acute, obtuse, right, straight or reflex



acute, obtuse, right, straight or reflex



acute, obtuse, right, straight or reflex

Drawing Shapes and Identifying Angles

Draw the following shapes and indicate by circling/highlighting if they have right or acute angles.

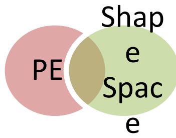
Square	Rhombus
Rectangle	Parallelogram
Trapezium	Kite

LEVEL D.10: RECOGNISE ANGLES IN TERMS OF A ROTATION

TEACHING NOTES

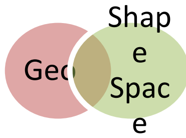
This level extends on the learning and activities of Level C.11. The emphasis is on active hands-on experience of angles of rotation in the classroom environment.

SAMPLE LEARNING EXPERIENCES



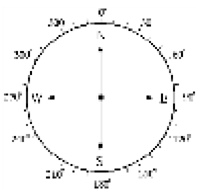
Rotations in the Environment

Rotations and the angles formed by rotations are best explored in the environment. Sporting action captured in pictures such as those below are particularly effective in demonstrating this relationship see http://nrich.maths.org/8191 for more activities and images.



Compass

The compass is a very useful tool for extending the concept of an angle in terms of a rotation. It further develops the concept that was explored using the clock face in Level C.11. Points on a compass are useful to show movement through 1,2,3 or 4 right angles. A sound starting point is for pupils to draw their own four-point compass on card.



Name the angles made by the following rotations (clockwise).
From North to East, is this a quarter or a half turn?
East to South, where is your starting point, what type of turn is this?
North to South, is this a half turn or two quarter turns?
From what points on the compass would you complete a full turn?
Justify your answer and share with your partner.
Pose questions for your partner or group.

Extension Activity: Compass

Pupils draw their own eight-point compass on card. Ask pupils to locate a range of point on the compass before discussing the size of the angles made by rotations. The sizes of the angles made by rotations could be explored, labelled and recorded by pupils in their individual learning log.

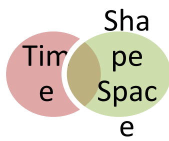
Name the turn made by the following rotations:
North to South East
North East to South East
South West to North West
Which points would you begin and end at to form a half, quarter, full turn?
Who has a different solution?

Chalk Compass Rotations

Pupils can draw a compass with chalk in the yard and explore the following rotations.

In which direction would you be facing if you turned clockwise /anticlockwise:

- *Quarter turn from N*
- *Half turn from W*
- *Full turn from E*
- *Three auarter turn form S*

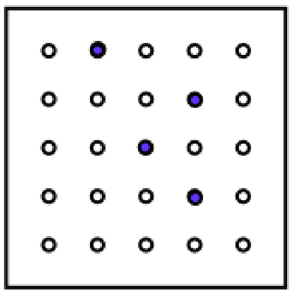


Clock Face

These activities are extensions from the clock face activities in level C.11. Find the new time if the hour hand makes a:

CLOCKWISE half turn from 3 o'clock
CLOCKWISE quarter turn from 12 o'clock
ANTI-CLOCKWISE half turn from 12 o'clock
CLOCKWISE quarter turn from 6 o'clock
CLOCKWISE half turn from 11 o'clock

Angle of Rotation Investigation

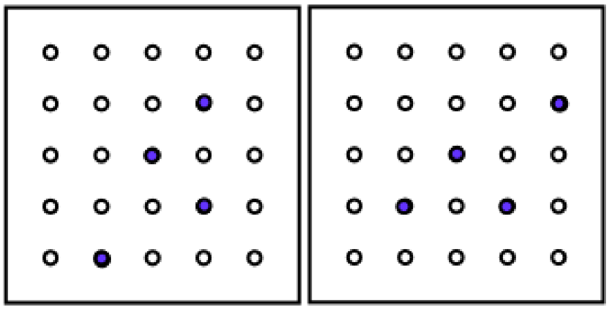


Here we have a kind of pegboard. The holes go all the way through so the pegs are visible from the top or underneath. You will see the 4 blue pegs.

Now the pegboard is either:

- Flipped over - north to south, OR east to west, OR north-east to south-west, OR north-west to south-east;
- Or it is rotated clockwise or anticlockwise

Find out which movements could produce these two views:



LEVEL D.11: ESTIMATE, MEASURE AND CONSTRUCT ANGLES IN DEGREES

TEACHING NOTES

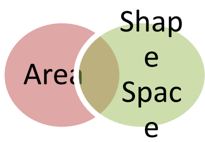
A logical progression to teaching angle measurement could be similar to that adopted in the teaching of area:

1. An angle unit (angle unit wedge non-standard size)
2. A right angle static measure
3. Non-standard protractor
4. Standard protractor
5. Measuring angles –standard units

> The protractor is one of the most poorly understood measuring instruments. Part of the difficulty arises because the units (degrees) are so small. It is almost physically impossible for a pupil to cut out a single degree to measure an angle accurately. Furthermore, the numbers on most protractors run clockwise and counter-clockwise along the edge making the scale hard to interpret without a strong conceptual foundation.

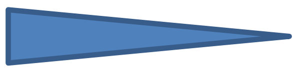
Angle measurement can be challenging for two reasons: a) the attribute of angle size is often misunderstood and b) protractors are commonly used without pupils understanding how they work. A unit for measuring an angle must be an angle. Nothing else has the same attribute of spread that we want to measure. Contrary to what many people think, degrees do not have to be used to measure angles[116]. Measuring an angle is the same as measuring length or area: unit angles are used to fill or cover the spread of an angle just as unit lengths fill or cover length.

SAMPLE LEARNING EXPERIENCES



Non-Standard Wedge

1. Each pupil draws a narrow angle on an index card (using a straight edge as a guide).
2. Pupils cut this angle out.



3. The resulting wedge can then be used as a unit of angular measure by counting the number that will fit in a given angle. This wedge is a non-standard unit of measure but can still be used to compare spread or rotation.
4. Distribute a pre-made page with assorted angles on it and have pupils use their angle unit to measure the angle. Pupils' wedges will vary in size and so results will differ. This can form a discussion in terms of unit size.
5. Once this concept is well understood pupils can progress to using measuring instruments.

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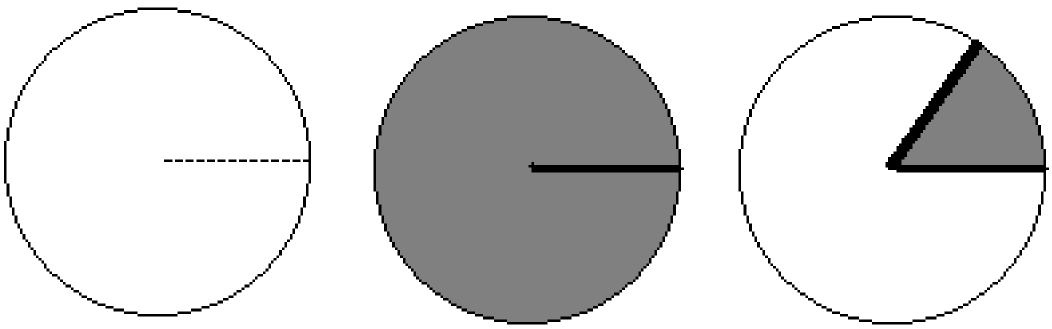
[116] Van De Walle (2013, p.395)

It is recommended revise the right angle static measure from Level C.8.

Non-Standard Protractor

Throughout this level, pupils need opportunities to estimate, measure and construct angles in degrees. In order to do so pupils can begin by constructing an angle estimator of their own. This is equates to a non-standard protractor. The design is shown below:

White paper (cut along line)	Black paper (cut along line)	Insert two circles along cut lines



Pupils should use this non-standard protractor in a variety of estimating and measuring tasks.

Estimation

As an initial activity, pupils can be invited to sit back to back. One pupil makes a certain amount of turn using the angle estimator and tells the amount of turn to their partner, for example, a quarter turn, a half turn, etc. The partner makes the turn and they then compare their results using the angle estimator. The angle estimator can be used to demonstrate two amounts of a turn for example two quarter turns or two half turns. The benchmark of a right angle should be referred to throughout these initial explorations.

Which are the easiest turns on your angle estimators for your partner to copy?
Show these to your classmates.
Which turns are more difficult for your partner to copy?
Explain why.
Can you think of a more effective and accurate way to show or measure these turns?

ICT Opportunities

[Angle Estimator Game](http://nrich.maths.org/1235)

Pupils can estimate angles using a game on the ICT link.[117]

[117] Link to Angle Estimator Game: http://nrich.maths.org/1235

Introducing Degrees

At this stage, the teacher can introduce the pupils to another unit for measuring an amount of turn called the **degree**. The teacher can begin by referring to a full turn as representing a turn of 360 units (or small amounts of turn) called degrees and denoted by "° ". Ask pupils to relate the benchmarks used on their angle estimator to a full turn of 360°. In particular, a ½ turn becomes 180°, a ¼ turn becomes 90, and a ¾ turn becomes 270°.

360 is a strange number to have for a full circle. It is believed that this number was discovered by the ancient Babylonians of 2000BC and their way of measuring the time in a day and year. Do a web quest challenge to find out more.

In pairs, one pupil draws an angle, with the direction of turn, and gives it to their partner. The partner must first estimate the angle of turn and then use the angle estimator to check and refine this estimate.

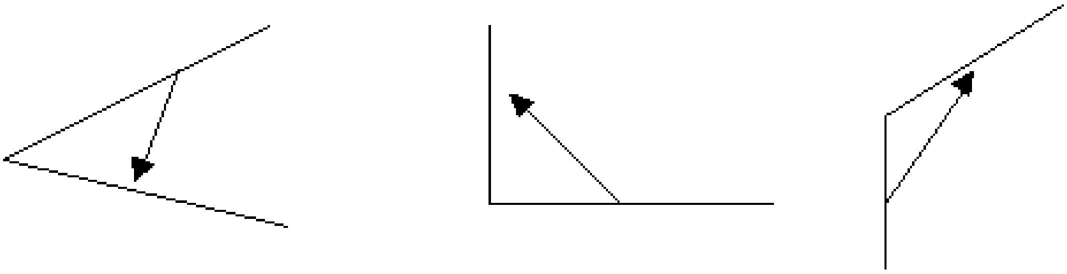
What are the most accurate angles in degrees to demonstrate and why?
Standing behind your chair demonstrate some of these turns before showing them on your angle estimator.
Using the angle estimator try to measure other angles such as 172° or 16 °.
Why is this so difficult? What do you suggest we do?
Who has another solution?

The Protractor

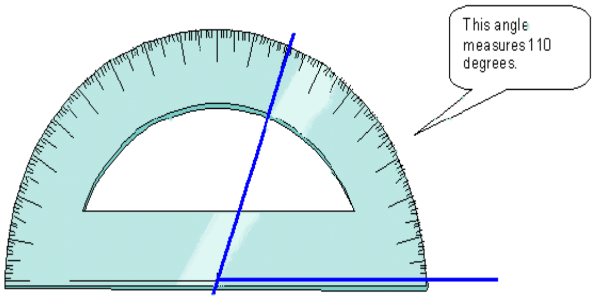
As we have established the need for a standard method of measuring angles, the next step is to introduce pupils to the protractor.

1. Hand out the protractors explaining that they are the measuring tool commonly used to measure angles.
2. In pairs, pupils explore the protractor attempting to find out how it works.
3. During the activity, it is important to ask the pupils to write down and record their estimates of the different angle measurements before formally measuring each with the protractor method.

It is important to ensure that the angles explored have a range of orientations. Some examples are shown below:



4. Pupils use protractors to measure accurately ¼ ; ½ ; ¾ turns.
5. Pupils also measure accurately 30°, 60 ° and 120°.
6. Ask the pupils how they would help someone who had measured the pictured angle incorrectly.

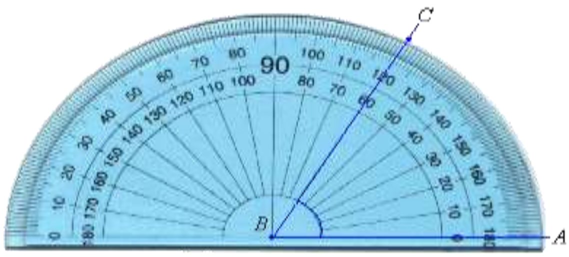


What is the measure of this angle? Is it 110° or 70°?
How can you justify your answer?
Using your learning record a short procedural piece of writing entitled 'How to measure an angle using a protractor'.

As a follow-on activity, pupils can draw an estimate of an angle, for example, 60° and then measure it with a protractor to see how close their estimate is.

1. In pairs, one pupil draws an angle and gives it to their partner.
2. The partner first gives a rough estimate of the angle, and then using the scale on the protractor, gives the accurate value.

In order to construct angles in degrees using a protractor the teacher can demonstrate the following step-by-step approach building on pupils' ideas expressed in their learning log as above.



The following steps can be adopted as a possible whole-class approach:

- Draw a straight line (an arm of the angle).
- Place a dot at one end of the arm.
- This dot represents the vertex of the angle.
- Place the centre of the protractor at the vertex dot and the baseline of the protractor along the arm of the angle.
- Find the required angle on the scale and then mark a small dot at the edge of the protractor
- Join the small dot to the vertex with a ruler to form the second arm of the angle.
- Label the angle with capital letters.

Using this procedure, draw an angle of 80° using your protractor.
Draw an angle of 115°. Show your partner the vertex and arms of your angle.
Which profession needs the ability to draw angles in everyday work?

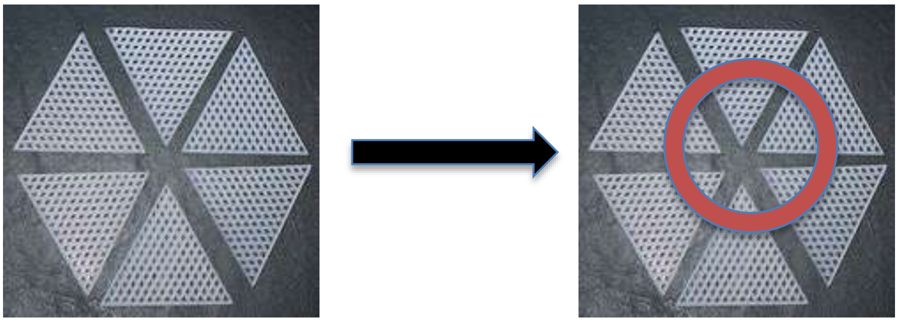
For more information on measuring and constructing angles click on this link.[118]

[118] Link to Measuring and Constructing Angles: http://www.mathsteacher.com.au/year7/ch08_angles/01_ang/ang.htm#angle

LEVEL D.12: EXPLORE THE SUM OF THE ANGLES IN A TRIANGLE AND QUADRILATERAL

SAMPLE LEARNING EXPERIENCES

Angles of an Equilateral Triangle



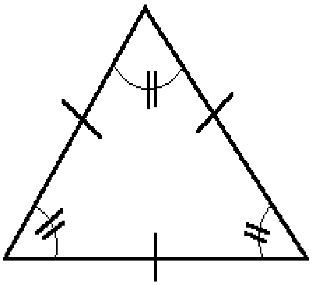
These images demonstrate the opportunity for teachers to show how six equilateral triangles tesselate. The central purpose of such an activity is for pupils to discover that the sum of six angles amounts to 360°**.** Pupils can be given six equilateral triangles to arrange as above before discussing the relationship between angles and degrees.

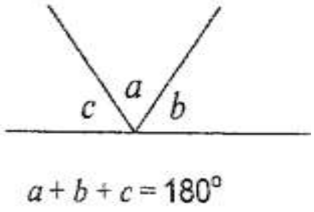
How many ways can you arrange these triangles so that six angles meet?
What do you notice about their meeting point?
If you rotate the triangles what do you notice about the meeting point?
Draw a circle around the six angles that form the meeting point.
What can you now conclude from this activity?
Sue re-tell this in your own words

Pupils should be guided to discover through the pupil-led activity above that the sum of the six angles amount to 360° before concluding that 1 angle is equal to 60°**.** After experimenting with a range of differing rotations of the triangles as outlined above pupils will conclude that the equilateral triangles will tessellate, no matter how each one is turned. Therefore, with activities based on effective teacher questioning, pupils can discover that 60°is the size of any angle in an equilateral triangle. This is a key learning point that can be recorded, analysed and justified in a variety of classroom formats.

How many ways can you arrange these three triangles so that three angles meet?
What do you notice about their meeting point?
James please revoice what Tom said.
What happens to the meeting point if you rotate the three triangles?
What shape is formed at the base where the three lines meet?

The angles of an equilateral triangle may be cut out; then fitted together as shown in order to make a straight line or two right angles, that is, 180° as seen below. As each combination is equal, we can again state that an equilateral triangle has three angles each measuring 60°.

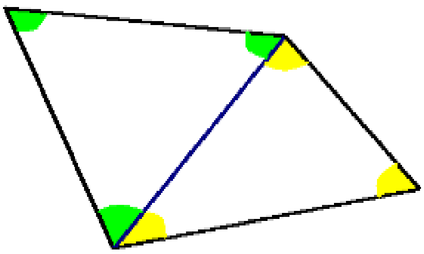


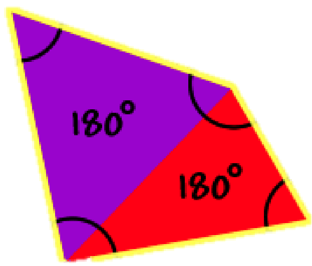


Having discovered that the three angles of an equilateral triangle are equal to 180° pupils can experiment with other types of triangles such as scalene and isosceles to determine if this rule applies to all triangles. They can then share their conclusions with other pupils.

Angles of a Quadrilateral

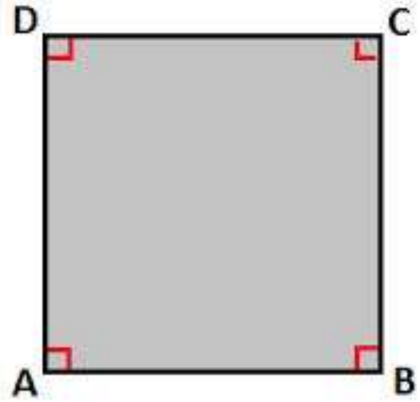
Following on from the initial activities above, pupils can investigate the combination of various triangles with at least one common side in order to form quadrilaterals such as those illustrated below.

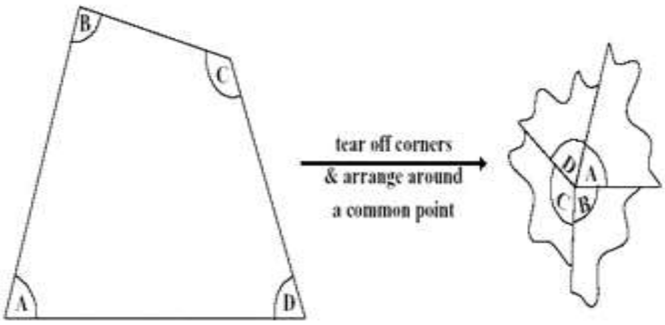




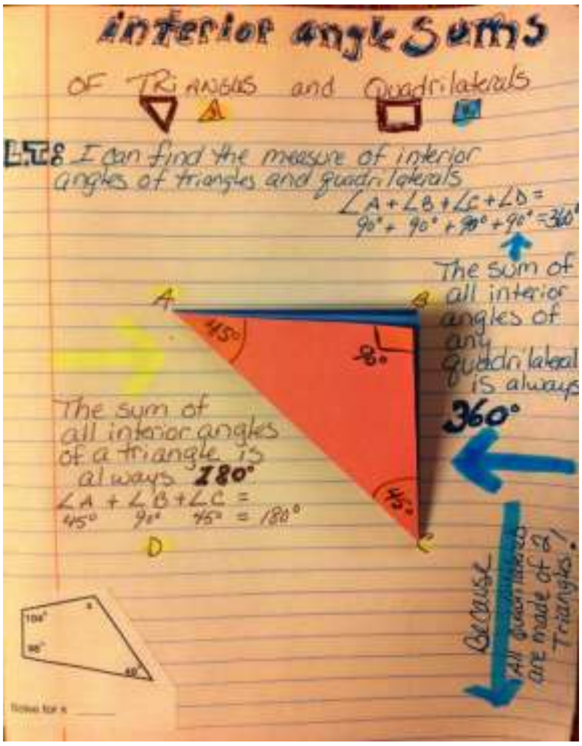
How many ways can you combine these two triangles to form a quadrilateral?
What do you notice about this combination?
What is the sum of the angles in any triangle?
Now estimate the sum of the angles in the quadrilateral that you have made.
With your partner, explain and justify your answers.

Pupils will readily agree that a square and a rectangle each have four right angles. They can investigate the idea of tessellating angles in a quadrilateral to meet a point of intersection. The angles A, B, C, D of the quadrilateral as shown below can fit completely over the four right angles. When fitted together the shapes make a complete revolution of 360° - which equals four right angles. Further pupil-led investigations with any quadrilateral will show that the sum of the angles in any quadrilateral will amount to 360°.

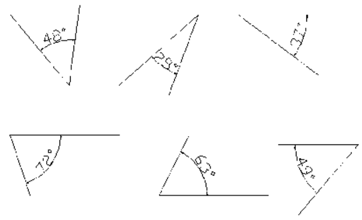
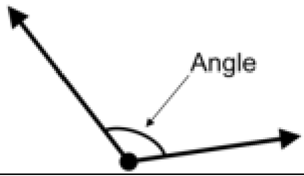
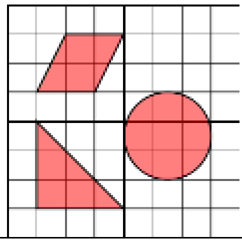
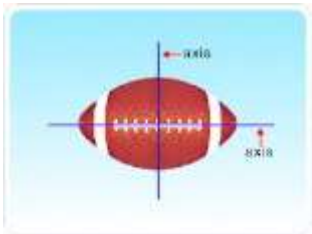


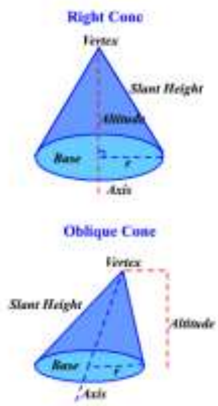
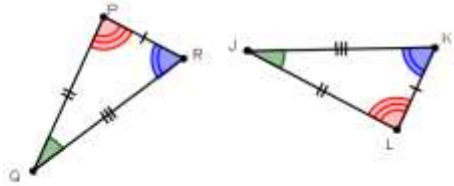
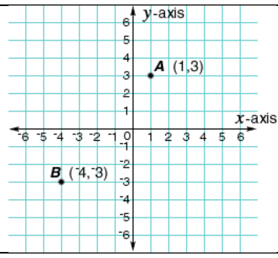
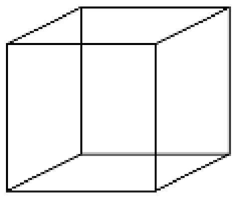
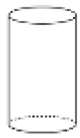
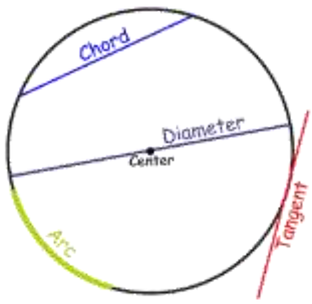


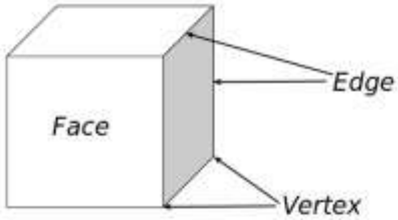
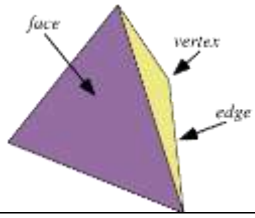
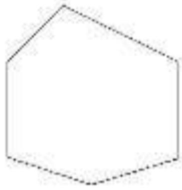
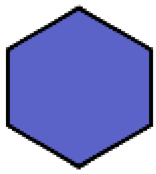
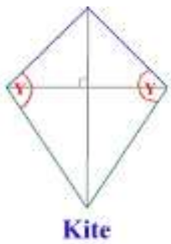
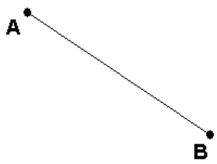
We are going to record our learning by writing in our learning log.
We begin by stating that 'I can find the measure of the interior angles of triangles and quadrilaterals by…'.
Make sure to include detailed, colourful diagrams or cut out models to show your learning.

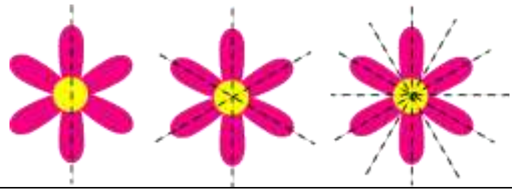
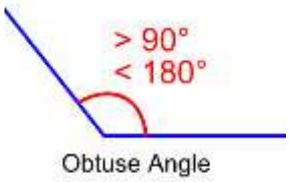
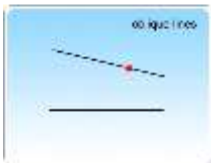
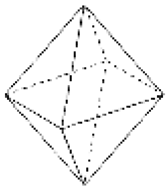
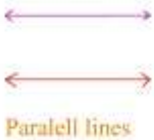
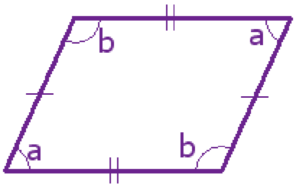
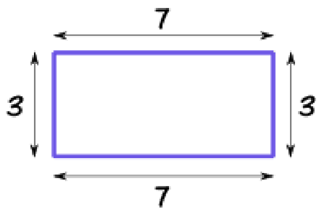


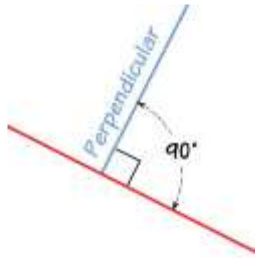
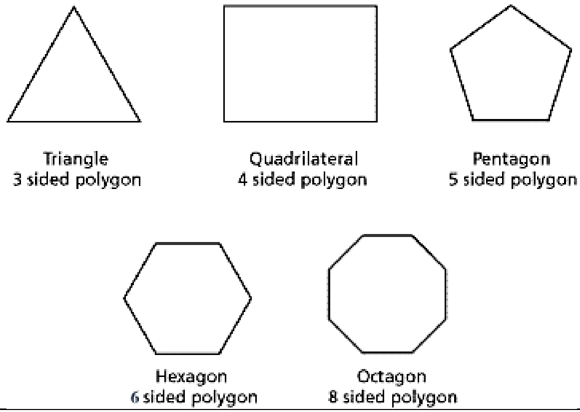
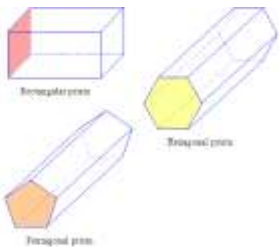
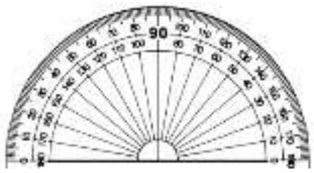
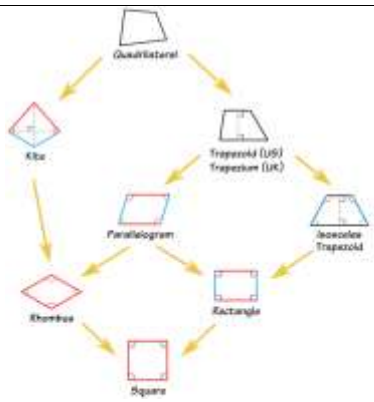
GLOSSARY OF SHAPE AND SPACE TERMS

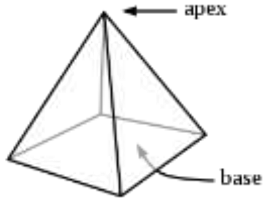
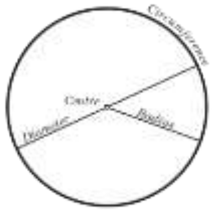
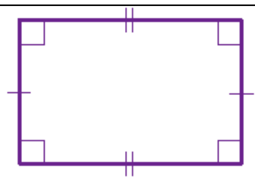
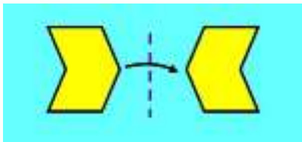
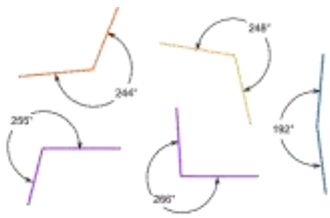
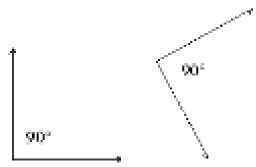
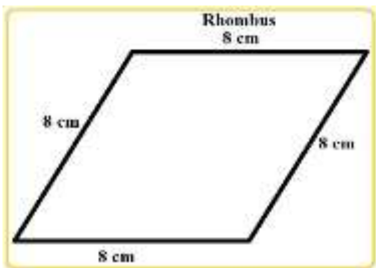
Letter	Term	Explanation
a	acute	This angle is greater than 0° but less than 90°. 
	angle	This is made when two line segments meet at a point (vertex), or when two lines intersect. It is be measured in degrees and can be acute, right, obtuse, or reflex. 
	area	This is the amount of a plane enclosed by a 2D shape measured in square units. 
	Axis of symmetry	This is a line drawn through a plane figure, so that one-half of the shape can be folded over along the line to fit exactly onto the other. A shape can have more than one axis of symmetry. 
c	Carroll diagram	This is a grid like structure for categorising results. It is named after its inventor Louis Carroll. 
	circumference	This is the length of the perimeter of a circle. 

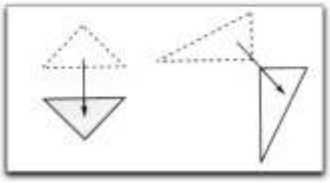
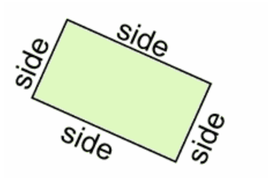
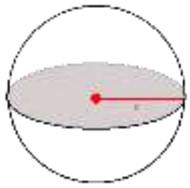
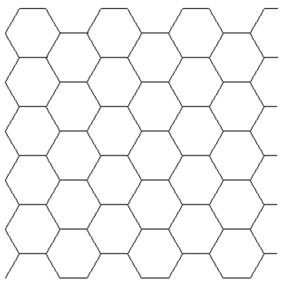
	cone	This is a solid shape with an elliptical or circular base with a curved surface that tapers to a point (vertex). The vertex does not need to be directly over the base.	
	congruent	These are 2D shapes that have identical properties and are exactly the same size, shape, and measure of angle.	
	Co-ordinates	These are the numbered pairs used to locate points on the plane. The plane is a flat surface, often referred to as the Cartesian plane.	
	cube	A prism with six congruent square faces	
	cylinder	This is a three-dimensional shape consisting of two identical circular ends joined by one continuous curved surface. It has a curved surface and so is not a polyhedron.	
d	Diameter	This is a chord through the centre of a circle. It is twice the radius in length.	

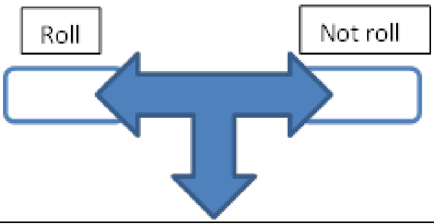
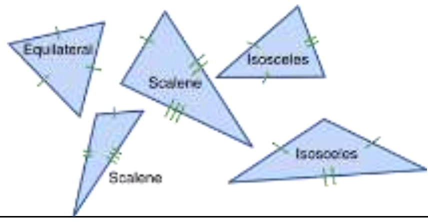
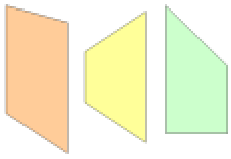
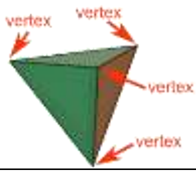
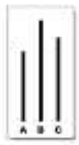
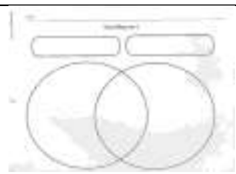
e	edge	This is the intersection of two surfaces; in particular, the straight line where two faces of a polyhedron meets. 
	face	One of the place surfaces of a polyhedron, a cube has six faces. 
g	Geostrips	These are used to construct 2-D shapes 
h	hexagon	This is a six-sided polygon. It can be regular or irregular in shape  
	horizontal	A line that is level and runs parallel to the ground 
k	kite	This is a symmetrical quadrilateral with two shorter congruent sides and two longer congruent sides. 
l	Line segment	Parts of a line, it has endpoints by which it is identified or named 

	Line symmetry	This is when one-half of the shape can be folded exactly onto the other half. 
n	net	This is the plan of a 3-D object. 
o	obtuse	This angle is greater than 90° but less than 180°. 
	oblique	This slanting line is not vertical or horizontal. They would form either an acute or an obtuse angle if they intersected. 
	octahedron	This is a 3-D shape with eight faces. 
p	Parallel	This is when a line runs at an equal distance apart from another line and they never meet. 
	parallelogram	A quadrilateral with both pairs of opposite sides parallel. Sides: Opposite sides are congruent. Angles: Opposite angles are congruent. Diagonals: Triangles formed by each diagonal are congruent and diagonals bisect each other. 
	perimeter	This is the sum of the length of the sides of a figure or shape. 

	perpendicular	This is when two lines meet at right angles (90°). 
	polygon	This is a two-dimensional (2-D) closed shape made up entirely of straight edges. It does not have to be regular. 
	polyhedron	This is a three-dimensional (3-D) shape made up entirely of flat surfaces. It does not have to be regular. 
	prism	This is a shape made up of two identical polygons at opposite ends, joined up by parallel lines. The name of the polygon determines the name of the prism. 
	protractor	This is a geometric instrument for measuring angles. 
	quadrilateral	This is a shape with four sides. 

	pyramid	This is a solid shape with a polygon as a base and triangular faces that taper to a point (apex). Pyramids are named by the shape of the base, triangular pyramid, square pyramid, octagonal pyramid and so on. 
r	radius	This is a line joining the centre of a circle to the edge of the circle. It is half the diameter in length. 
	rectangle	A quadrilateral with all angles right angles. A rectangle has all the properties of a parallelogram and its diagonals are the same length. 
	Reflection (flip)	Reflection is the mirror image of an object or figure where each point of the object is the same distance from the mirror line as its corresponding point on the image. 
	Reflex angle	This angle is greater than 180° but less than 360°. 
	Right angle	This is an angle of 90°. 
	Rhombus	This is a parallelogram with four sides congruent. A rhombus has all the properties of a parallelogram and the diagonals bisect each other at right angles and bisect the angles of the rhombus. 

	Rotation (turn)	The process by which an object or figure changes position by rotating about a fixed through a given angle 
s	Scalene triangle	This is a triangle with three sides of different length and, therefore, three different sized angles 
	side	These are the straight edges of a closed two-dimensional shape. 
	sphere	This is a 3-D shape with one curved surface, where every point on that surface is the same distance from the sphere's centre. 
	square	This is a special rectangle and a special rhombus with all sides and all angles congruent. A square has all the properties of a parallelogram/rhombus/rectangle. 
t	tangram	This is a Chinese puzzle made up of seven simple geometric shapes: 2 large triangles, 1 medium triangle, 2 small triangles, 1 square and 1 parallelogram which are capable of being recombined in many different figures. 
	tessellation	These shapes fit together exactly, form a repeating pattern, and make an angle of 360 at the points of contact. 

	Tree diagram	This diagram uses a branching process to solve a problem. 
	triangle	This is a three-sided shape. Example: An equilateral triangle had 3 sides of equal length, an isosceles triangle has 2 equal sides and a scalene triangle has no sides of equal length. 
	trapezium	This is a quadrilateral with only one pair of parallel sides. (Some texts define a trapezium as a quadrilateral with at least one pair of parallel sides) 
	vertex	This is a point or corner on a 3-D shape or where two shapes meet. 
	Vertical line	A line which goes straight up and is perpendicular to the ground 
	Venn diagram	This diagram represents sets and their relationships. It is named after John Venn who developed the method in the 1890's. 

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Website References

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All other ICT links are fully referenced in the footnotes.