## Estimating and Graphing Polynomials LCHL

Using Geogebra, graph the following polynomials and fill in the table below after graphing each one

NOTE: to put indices into an equation use SHIFT and $\overline{6}$ on keyboard to get ${ }^{\wedge}$ symbol, then type degree required [e.g for $(x-5)^{2}$ you'd type in $\wedge 2$ ]

| Polynomial | Leading coefficient (term with biggest degree/power) positive or negative? | Equation of degree? | Number of roots? | List of roots | Where it crosses $x=$ axis | Where it touches (but doesn't cross) $x$-axis | End behaviour = Direction of ends/arms (up or down) Both same/different/which up/which down |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)=x(x-1)(x-2)(x-3)$ | +1 | 4 | 4 | 0,1,2,3 | 0,1,2,3 | n/a | Both up |
| $f(x)=x(x-2)(x+5)$ | +1 | 3 | 3 | -5,0,2 | -5,0,2 | n/a | Left down, right up |
| $y=x(x+4)(x-7)(x-2)(x-5)^{2}$ | +1 | 6 | 6 | -4,0,2,5,7 | -4,0,2,7 | 5 | Both up |
| $y=(-x)(x+4)(x-2)(x-5)^{2}$ | -1 | 5 | 5 | -4,0,2,5 | -4, 0,2 | 5 | Left up, right down |
| $y=(x)^{2}(x+2)^{3}$ | +1 | 5 | 5 | -2,0 | -2 | 0 | Left down, right up |
| $y=x^{5}-12 x^{4}-40 x^{3}+120 x^{2}$ | +1 | 5 | 5 | $\begin{aligned} & \hline-4.22, \\ & 0,2,14.22 \end{aligned}$ | -4.22,2,14.22 | 0 | Left down, right up |
| $y=-x(x+3)(x+4)^{3}$ | -1 | 5 | 5 | -4,-3,0 | -4,-3,0 | n/a | Left up, right down |

(Reflection: What examples above have the same characteristics? Is there a pattern to how each graph looks compared to the equation?)

What conclusions can you draw from your graphs and table about the following:

| A polynomial of even degree? Similar characteristics to quadratics. Both arms up if leading coefficient positive. (or down if leading coefficient negative) |
| :--- |
| A polynomial of odd degree? Similar to cubics, if leading coefficient positive then graph goes up from left to right (left arm down, right up). If leading <br> coefficient negative, then graph goes down from left to right (left arm up, right arm down) |
| The leading coefficient is positive? Both arms up, if even degree. Graph goes up if odd degree |
| The leading coefficient is negative? Both arms down, if even degree. Graph goes down if odd degree |
| Polynomial which has a factor or root which occurs multiple times (known as Multiplicity)? Stationary point at this location |
| Polynomial which has a factor or root which occurs 2 times? Turning point is here |
| Polynomial which has a factor or root which occurs 3 times? Horizontal stationary point is here (both $f^{\prime}(x)$ and $f^{\prime \prime}(x)=0$ at this point) |

