# 4.1.11 - Surds

## 4.1 - Algebra - Expressions

Leaving Certificate Mathematics

Higher Level ONLY


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We can remove a square root in an equation by squaring both sides.

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$$\begin{aligned}\sqrt{2x+3}+2 &= 3x+4\\ \sqrt{2x+3} &= 3x+2\\ \left(\sqrt{2x+3}\right)^2 &= (3x+2)^2\end{aligned}$$

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\sqrt{2x+3}+2 &= 3x+4 \\
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$\therefore\ x = -\frac{1}{9}$ is the only valid solution.