

4.1.12 - The Binomial Theorem I

4.1 - Algebra - Expressions

Leaving Certificate Mathematics

Higher Level ONLY



Notation

$$n! = n \times (n - 1) \times (n - 2) \times \dots \times 3 \times 2 \times 1 \quad \dots \text{ "n factorial"}$$

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Examples:

$$\begin{aligned} 3! &= 3 \times 2 \times 1 \\ &= 6 \end{aligned}$$

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$$\begin{aligned} 3! &= 3 \times 2 \times 1 & 5! \\ &= 6 \end{aligned}$$

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Examples:

$$\begin{aligned} 3! &= 3 \times 2 \times 1 & 5! &= 5 \times 4 \times 3 \times 2 \times 1 \\ &= 6 & & \end{aligned}$$

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Examples:

$$\begin{aligned} 3! &= 3 \times 2 \times 1 \\ &= 6 \end{aligned} \quad \begin{aligned} 5! &= 5 \times 4 \times 3 \times 2 \times 1 \\ &= 120 \end{aligned}$$

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In general,
$$\binom{n}{r} = \frac{n \times (n-1) \times \dots \times (n-r+2) \times (n-r+1)}{r \times (r-1) \times \dots \times 2 \times 1}$$

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Example:

$$\binom{10}{3} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1}$$

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