

## 4.2.8 - The Factor Theorem

### 4.2 - Algebra - Solving Equations

Leaving Certificate Mathematics

Higher Level ONLY



# The Factor Theorem

# The Factor Theorem

$(x - a)$  is a factor of the polynomial  $f(x)$  if and only if  $f(a) = 0$ .

## Example 1

**Q.** Suppose  $f(x) = 5x^3 - 14x^2 + 12x - 3$ .

(i) Is  $(x - 2)$  a factor?

(ii) Is  $(x - 1)$  a factor?

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$$\therefore a = 2$$

$$\therefore f(2) = 5(2^3) - 14(2^2) + 12(2) - 3$$



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$$\begin{aligned}\therefore f(2) &= 5(2^3) - 14(2^2) + 12(2) - 3 \\ &= 40 - 56 + 24 - 3\end{aligned}$$

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$\therefore (x - 2)$  not a factor

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$$\therefore a = 1$$

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$\therefore (x - 1)$  **is** a factor

## Example 2

Q. Find  $p$  if  $(x + 3)$  is a factor of  $f(x) = 4x^3 + 21x^2 + px + 12$ .

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$$-108 + 189 - 3p + 12 = 0$$

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$$-3p + 93 = 0$$

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$$-3p + 93 = 0$$

$$-3p = -93$$

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$$-3p + 93 = 0$$

$$-3p = -93$$

$$\therefore p = 31$$