4.4.2 - The Conjugate Root Theorem

4.4 - Algebra - Complex Numbers

Leaving Certificate Mathematics

Higher Level ONLY

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$\therefore \quad \bar{z} = 2 - i$ is also a root.

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