4.4.3 - Polar Form

4.4 - Algebra - Complex Numbers

Leaving Certificate Mathematics

Higher Level ONLY

Polar Form

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θ is the **argument** of z.

The angle between the line to the point and the **positive x axis**.

Example 1

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Q. Write $1 + i$ in polar form

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$$r \;=\; |1 + i|$$

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r &= |1 + i| \\
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\therefore \ 1 + i &= \sqrt{2}\left(\cos\tfrac{\pi}{4} + i\sin\tfrac{\pi}{4}\right)
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Example 2

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