4.4.4 - De Moivre's Theorem I

4.4 - Algebra - Complex Numbers

Leaving Certificate Mathematics

Higher Level ONLY

De Moivre's Theorem

De Moivre's Theorem

Theorem: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$, for all n.

De Moivre's Theorem

Theorem: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta,$ for all n.

Example 1. Evaluate $(-1 + i\sqrt{3})^5$, and express in the form $a + ib$.

De Moivre's Theorem

Theorem: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta,$ for all n.

Example 1. Evaluate $(-1 + i\sqrt{3})^5$, and express in the form $a + ib$.

Answer:

De Moivre's Theorem

Theorem: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta, \quad$ for all n.

Example 1. Evaluate $(-1+i\sqrt{3})^5$, and express in the form $a+ib$.

Answer:

$$(-1+i\sqrt{3})^5 \quad =$$

De Moivre's Theorem

Theorem: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$, for all n.

Example 1. Evaluate $(-1+i\sqrt{3})^5$, and express in the form $a+ib$.

Answer:

$$(-1+i\sqrt{3})^5 = \left[2(\cos\tfrac{2\pi}{3} + i\sin\tfrac{2\pi}{3})\right]^5$$

De Moivre's Theorem

Theorem: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta, \quad \text{for all } n.$

Example 1. Evaluate $(-1+i\sqrt{3})^5$, and express in the form $a+ib$.

Answer:

$$\begin{aligned}(-1+i\sqrt{3})^5 &= \left[2(\cos\tfrac{2\pi}{3} + i\sin\tfrac{2\pi}{3})\right]^5 \\ &= (2)^5\end{aligned}$$

De Moivre's Theorem

Theorem: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta, \quad$ for all n.

Example 1. Evaluate $(-1+i\sqrt{3})^5$, and express in the form $a+ib$.

Answer:

$$\begin{aligned}(-1+i\sqrt{3})^5 &= \left[2(\cos\tfrac{2\pi}{3} + i\sin\tfrac{2\pi}{3})\right]^5 \\ &= (2)^5(\cos\tfrac{2\pi}{3} + i\sin\tfrac{2\pi}{3})^5\end{aligned}$$

De Moivre's Theorem

Theorem: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta, \quad \text{for all } n.$

Example 1. Evaluate $(-1+i\sqrt{3})^5$, and express in the form $a+ib$.

Answer:

$$\begin{aligned}(-1+i\sqrt{3})^5 &= \left[2(\cos\tfrac{2\pi}{3} + i\sin\tfrac{2\pi}{3})\right]^5 \\ &= (2)^5(\cos\tfrac{2\pi}{3} + i\sin\tfrac{2\pi}{3})^5 \\ &= 32\end{aligned}$$

De Moivre's Theorem

Theorem: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$, for all n.

Example 1. Evaluate $(-1+i\sqrt{3})^5$, and express in the form $a+ib$.

Answer:

$$\begin{aligned}(-1+i\sqrt{3})^5 &= \left[2(\cos\tfrac{2\pi}{3} + i\sin\tfrac{2\pi}{3})\right]^5 \\ &= (2)^5(\cos\tfrac{2\pi}{3} + i\sin\tfrac{2\pi}{3})^5 \\ &= 32(\cos\left[5(\tfrac{2\pi}{3})\right] + i\sin\left[5(\tfrac{2\pi}{3})\right])\end{aligned}$$

De Moivre's Theorem

Theorem: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta,$ for all n.

Example 1. Evaluate $(-1+i\sqrt{3})^5$, and express in the form $a+ib$.

Answer:

$$\begin{aligned}(-1+i\sqrt{3})^5 &= \left[2(\cos\tfrac{2\pi}{3} + i\sin\tfrac{2\pi}{3})\right]^5 \\ &= (2)^5(\cos\tfrac{2\pi}{3} + i\sin\tfrac{2\pi}{3})^5 \\ &= 32(\cos\left[5(\tfrac{2\pi}{3})\right] + i\sin\left[5(\tfrac{2\pi}{3})\right]) \\ &= 32(\cos\tfrac{10\pi}{3} + i\sin\tfrac{10\pi}{3})\end{aligned}$$

Theorem: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$, for all n.

Example 1. Evaluate $(-1+i\sqrt{3})^5$, and express in the form $a+ib$.

Answer:

$$(-1+i\sqrt{3})^5 = 32(\cos\tfrac{10\pi}{3} + i\sin\tfrac{10\pi}{3})$$

De Moivre's Theorem

Theorem: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta, \quad \text{for all } n.$

Example 1. Evaluate $(-1 + i\sqrt{3})^5$, and express in the form $a + ib$.

Answer:

$$\begin{aligned}(-1 + i\sqrt{3})^5 &= 32(\cos\tfrac{10\pi}{3} + i\sin\tfrac{10\pi}{3}) \\ &= 32(-\tfrac{1}{2} + i(-\tfrac{\sqrt{3}}{2}))\end{aligned}$$

De Moivre's Theorem

Theorem: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$, for all n.

Example 1. Evaluate $(-1+i\sqrt{3})^5$, and express in the form $a+ib$.

Answer:

$$\begin{aligned}(-1+i\sqrt{3})^5 &= 32(\cos\tfrac{10\pi}{3} + i\sin\tfrac{10\pi}{3}) \\ &= 32(-\tfrac{1}{2} + i(-\tfrac{\sqrt{3}}{2})) \\ &= -16 + i(-16\sqrt{3})\end{aligned}$$

De Moivre's Theorem

Theorem: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta,\quad$ for all n.

Example 1. Evaluate $(-1+i\sqrt{3})^5$, and express in the form $a+ib$.

Answer:

$$\begin{aligned}(-1+i\sqrt{3})^5 &= 32(\cos\tfrac{10\pi}{3} + i\sin\tfrac{10\pi}{3})\\ &= 32(-\tfrac{1}{2} + i(-\tfrac{\sqrt{3}}{2}))\\ &= -16 + i(-16\sqrt{3})\\ &= -16 - i(16\sqrt{3})\end{aligned}$$