# 4.4.6 - De Moivre's Theorem III

4.4 - Algebra - Complex Numbers

Leaving Certificate Mathematics

Higher Level ONLY

# Example 1

**Q.** Use De Moivre's theorem to express:

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(i) $\cos 3\theta$ in terms of $\cos \theta$
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Answer:

$$(\cos \theta + i \sin \theta)^3 =$$

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(i) $\cos 3\theta$ in terms of $\cos\theta$
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Answer:

$$(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta \qquad \ldots \text{by De Moivre's theorem}$$

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Answer:

$$\begin{aligned}(\cos\theta + i\sin\theta)^3 &= \cos 3\theta + i\sin 3\theta && \ldots\text{by De Moivre's theorem}\\ \cos 3\theta + i\sin 3\theta &= (\cos\theta + i\sin\theta)^3\end{aligned}$$

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$$\begin{aligned} (\cos\theta + i\sin\theta)^3 &= \cos 3\theta + i\sin 3\theta \qquad \text{...by De Moivre's theorem} \\ \cos 3\theta + i\sin 3\theta &= (\cos\theta + i\sin\theta)^3 \\ &= \cos^3\theta + 3i\cos^2\theta\sin\theta - 3\cos\theta\sin^2\theta - i\sin^3\theta \end{aligned}$$

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(\cos\theta + i\sin\theta)^3 &= \cos 3\theta + i\sin 3\theta \qquad \text{...by De Moivre's theorem} \\
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&= \cos^3\theta + 3i\cos^2\theta\sin\theta - 3\cos\theta\sin^2\theta - i\sin^3\theta \\
&= (\cos^3\theta - 3\cos\theta\sin^2\theta) + i(3\cos^2\theta\sin\theta - \sin^3\theta)
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