4.4.7 - nth Roots Of Unity

4.4 - Algebra - Complex Numbers

Leaving Certificate Mathematics

Higher Level ONLY

Example 1

Q. Use De Moivre's Theorem to solve $z^3 = 8$.

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$\mathbf{n=1}$ **:**

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$$\mathbf{n=2:}$$

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$$\mathbf{n=2:} \quad z = 2\left(\cos\tfrac{4\pi}{3} + i\sin\tfrac{4\pi}{3}\right)$$

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$$\begin{aligned} \mathbf{n=2:} \quad z &= 2\left(\cos\tfrac{4\pi}{3} + i\sin\tfrac{4\pi}{3}\right) \\ &= 2\left(-\tfrac{1}{2} - i\tfrac{\sqrt{3}}{2}\right) \end{aligned}$$

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\textbf{n=1}: \quad z &= 2\left(\cos\tfrac{2\pi}{3} + i\sin\tfrac{2\pi}{3}\right) \\
&= 2\left(-\tfrac{1}{2} + i\tfrac{\sqrt{3}}{2}\right) \\
&= -1 + i\sqrt{3}
\end{aligned}$$

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\textbf{n=2}: \quad z &= 2\left(\cos\tfrac{4\pi}{3} + i\sin\tfrac{4\pi}{3}\right) \\
&= 2\left(-\tfrac{1}{2} - i\tfrac{\sqrt{3}}{2}\right) \\
&= -1 - i\sqrt{3}
\end{aligned}$$

$$\therefore \quad z = 2, -1 \pm i\sqrt{3}$$