

Trigonometry

3rd Year

Higher

Level

Revision

A. Gilleran



Trigonometry

Trigonometry mainly appears on paper 2.
It appears on Paper 2.

1) Pythagoras

On page 16 of The Formulae and Tables Booklet you will see the following:



Right-angled triangle

$$c^2 = a^2 + b^2$$

Pythagoras' theorem

Note: Be careful with the letters a , b and c because the examiner often labels the triangles in your Junior Cert with these letters, but they may have nothing to do with these letters in Pythagoras' Theorem.

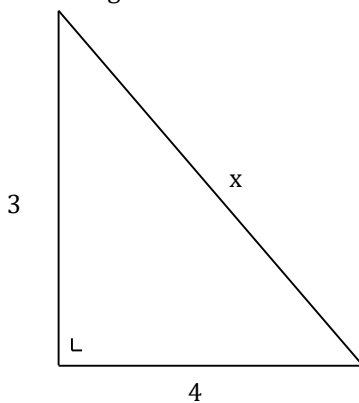
The way I like to remember this is:

$$(\text{longest side})^2 = (\text{other side})^2 + (\text{other side})^2$$

Note: The longest side is always the one across from the right angle. If you are looking for a shorter side--> square the other two sides, subtract them and get the square root of your answer.

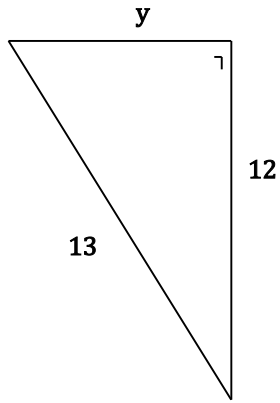
Question 1

Find the length of x .



Question 2

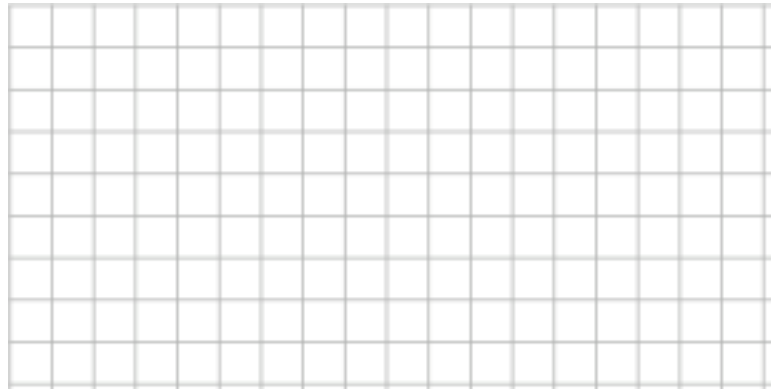
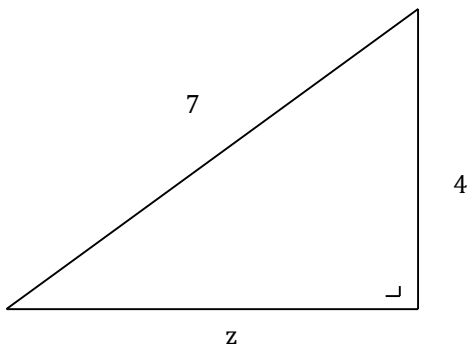
Find the length of y .



Note: Sometimes we are asked to write our answer in 'surd form'. This means to leave your answer with the square root sign.

Question 3

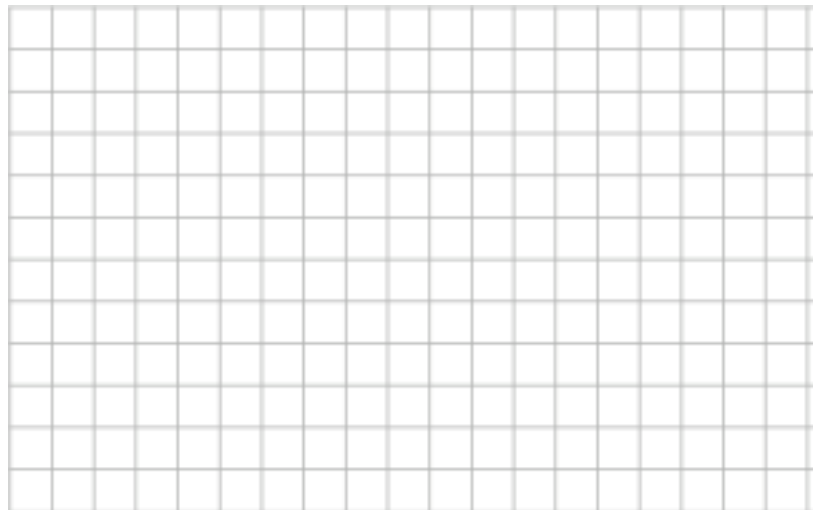
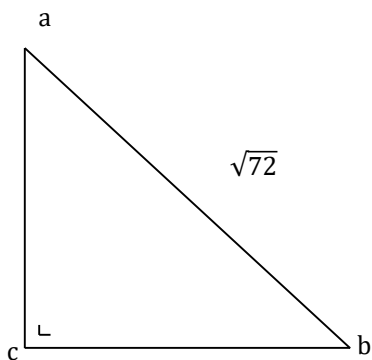
Find z , in surd form.



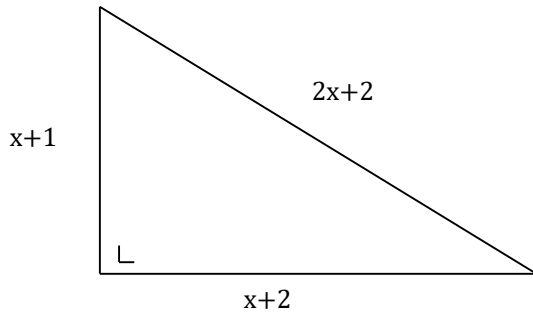
Question 4

abc is an isosceles triangle with:

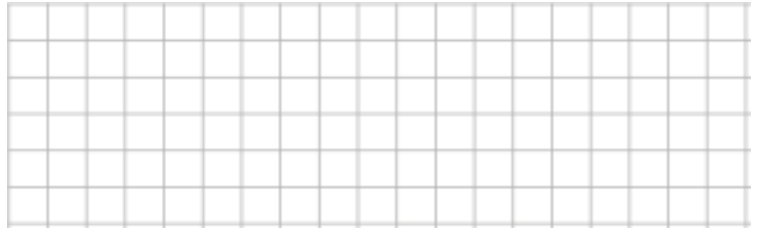
$|ac| = |bc|$, $|ab| = \sqrt{72}$ and $\angle acb = 90^\circ$. Find $|bc|$.



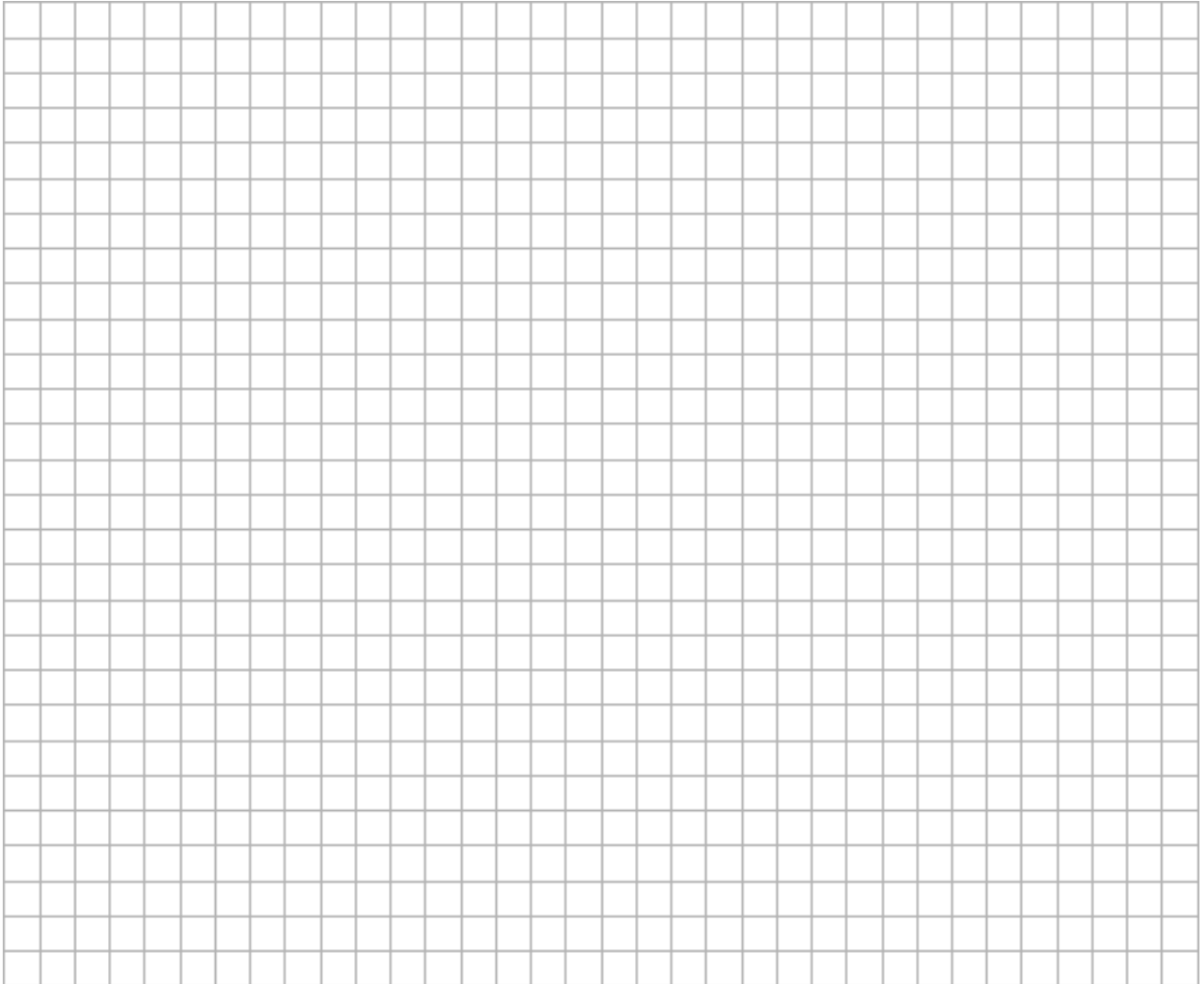
The lengths of the sides of a right-angled triangle are as shown in the diagram.



i) Using the theorem of Pythagoras, write an equation in x .



ii) Solve this equation to find x correct to 2 decimal places.



Note: Sometimes the examiner gives us a real life situation which requires the use of Pythagoras. You can also be asked to list the assumptions you have made. Don't let this confuse you. Think about the next example:

Question 6

A kite is held by a string $13m$ long. When blown in the wind it is $9m$ above the ground as shown in the diagram:



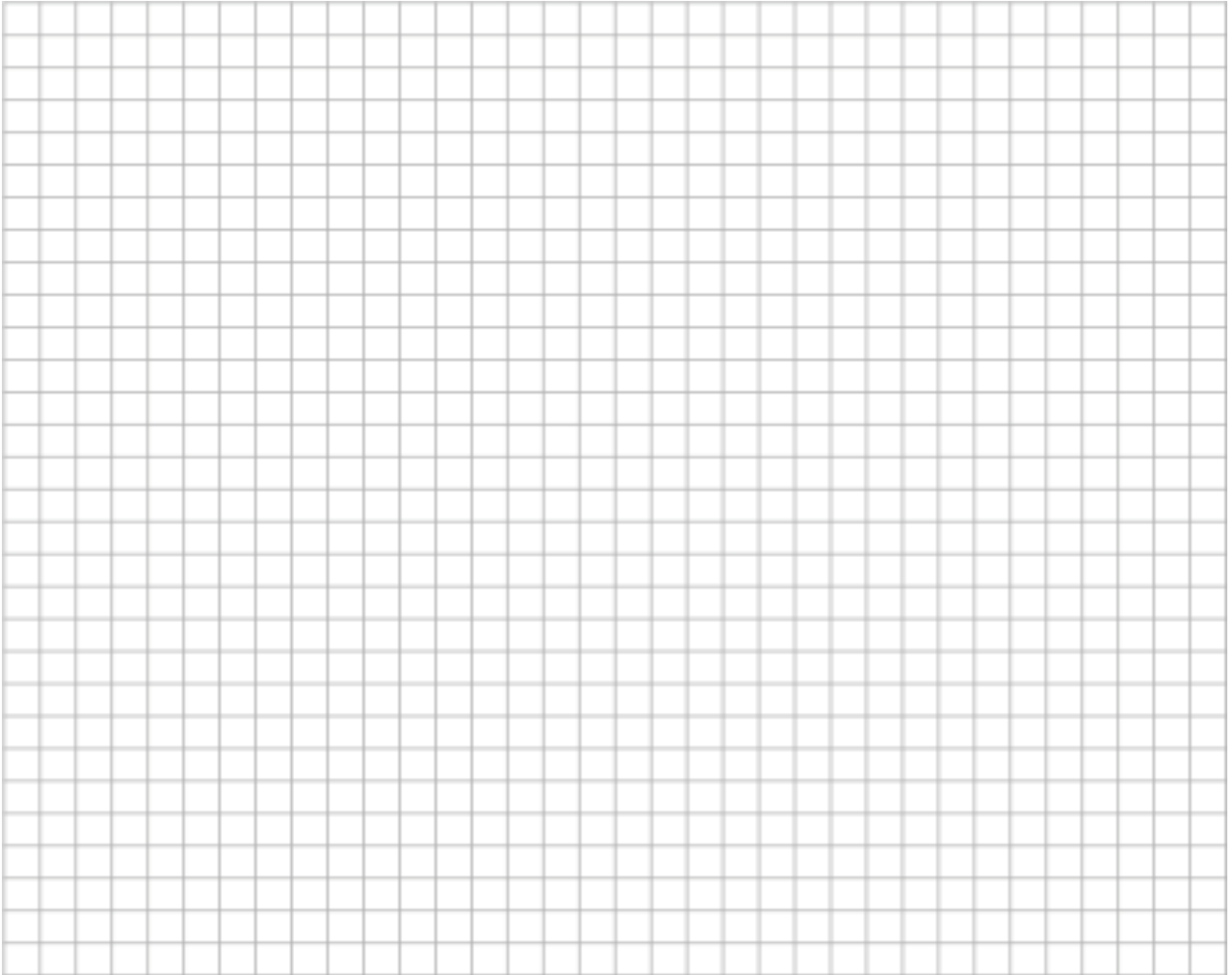
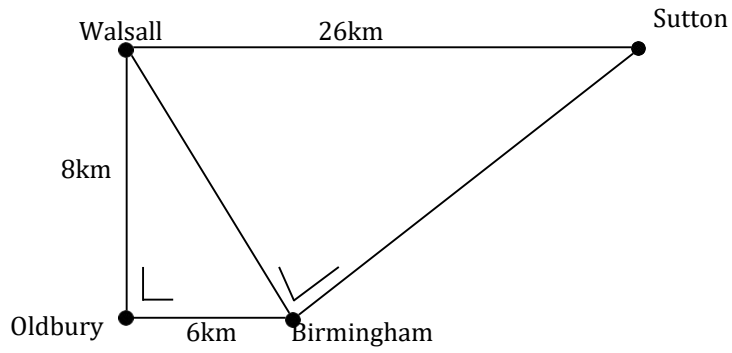
How far is the point A from B, correct to the nearest metre?

List any assumptions you have made.

A large grid of 20 columns and 20 rows, intended for the student to write their answer and list any assumptions made.

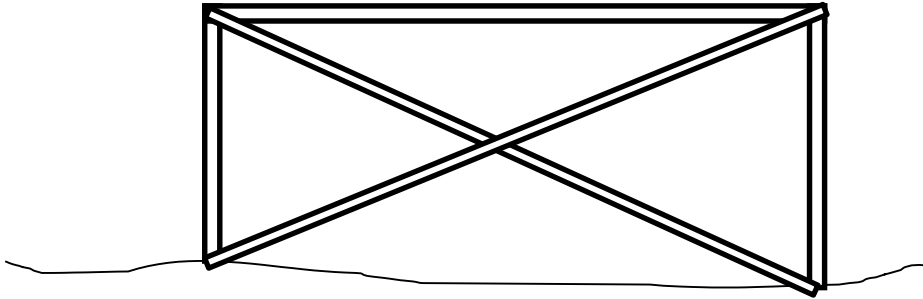
Question 7

Show that Sutton is 14km further from Birmingham than Walsall is from Birmingham.

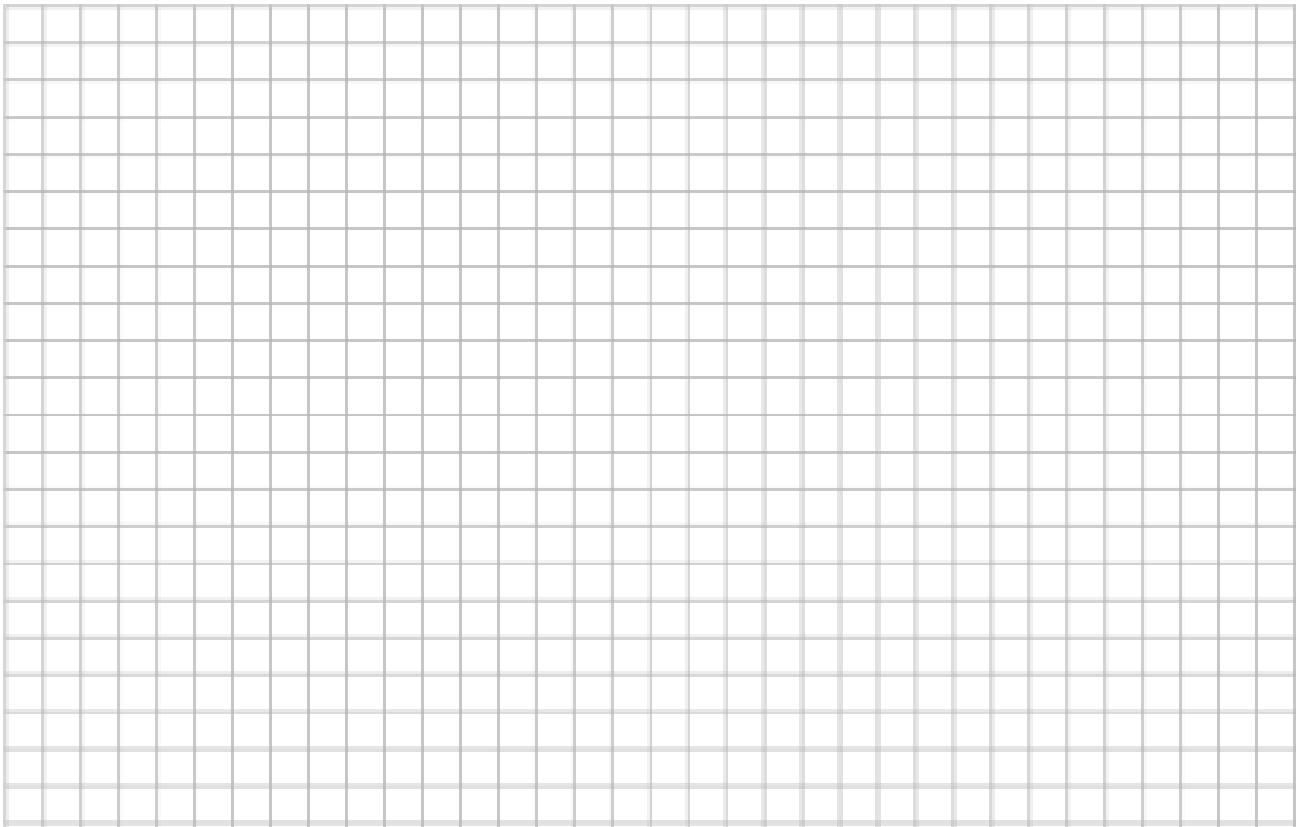


Question 8

The height of a wooden goal post is 2.5m and the width is 7m James wants to play a prank by nailing X-bars on the goal as shown:

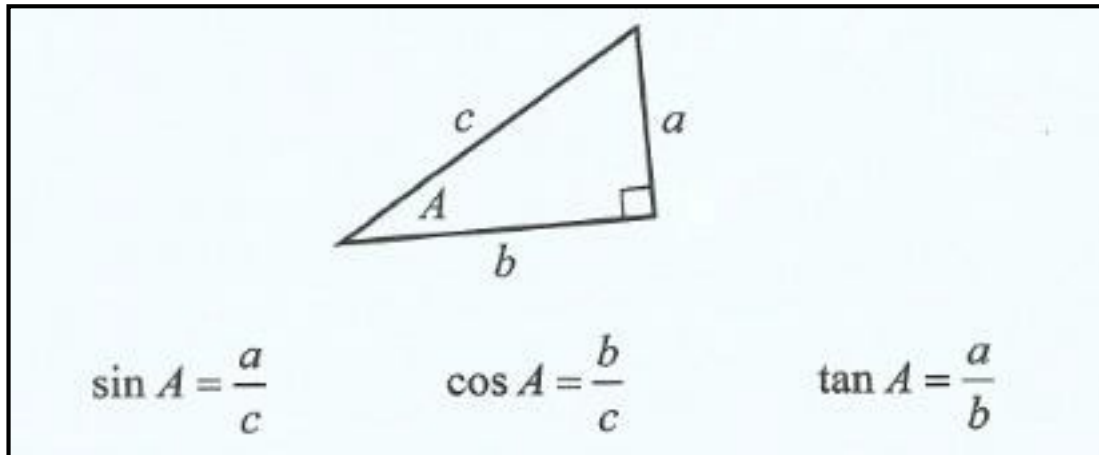


What is the minimum total length of wood that James needs to make the prank a success, correct to the nearest metre?



2) Sin, Cos, Tan

On page 16 of The Formulae and Tables Booklet you will see the following:



However, as with Pythagoras, the letters a, b and c can be very confusing. Therefore, you must learn the rules a different way, as follows:

$$\text{Sin} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{Cos} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{Tan} = \frac{\text{opposite}}{\text{adjacent}}$$

The way to remember these formulae is:

SOH CAH TOA

or

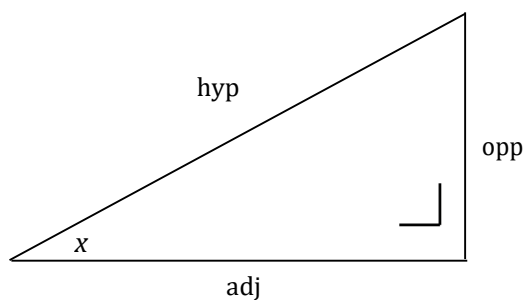
Silly Old Harry, Caught A Huge , Troun Ot Angling

To use these formulae, you clearly have to know how to label the sides of a triangle:

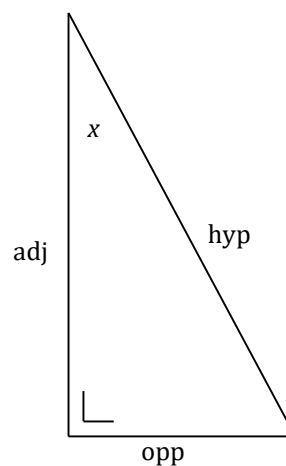
The hypotenuse is the side across from the right angle.

The opposite is the side across from the angle we are using.

The adjacent is the other side.



or



Note: You should always label your triangle sides. When labelling the sides we can use the names *hyp, opp, adj*.

Note: In these questions you will often end up with something like $\cos x = \frac{\sqrt{3}}{2}$, so to find x you write $x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$. This is called cos inverse. The way you get this on your calculator is by pressing:

Casio: *Shift, cos*

Sharp: *2nd Function, cos*

If you're using a Sharp, make sure to include the brackets after the \cos^{-1} .

If you're using a Casio, the brackets are automatically included.

You should get 30 as your answer .

Note: You need to make sure your calculator is in 'degrees' mode.

To check if your calculator is in degrees mode:

Casio: There should be a little 'D' at the top of the screen.

Sharp: There should be a little 'DEG' at the top of the screen.

If your calculator isn't in degrees mode you can change it as follows:

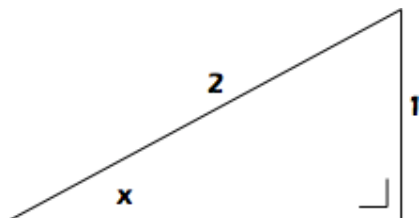
Casio: *Shift, Mode, 3*

Sharp: *Setup, 0, 0*

Note: if you have any problems with your calculator make sure to ask me. Your calculator is a sound chap, you just have to know how to deal with him.

Question 1

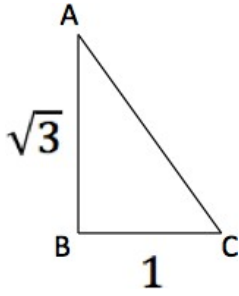
Find the measure of the angle x .



Question 2

In the triangle ABC , $|BC| = 1$ and $|AB| = \sqrt{3}$.

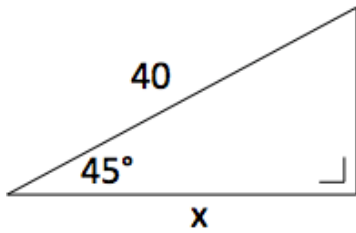
Find $|\angle BAC|$.



Note: As well as using Sin, Cos & Tan to find angles, we can use them to find sides:

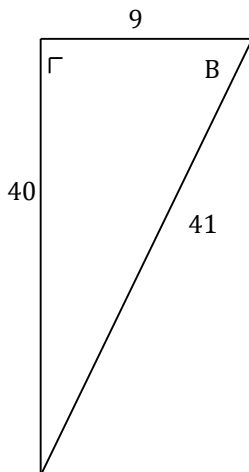
Question 3

Find x , correct to three decimal places.



Question 4

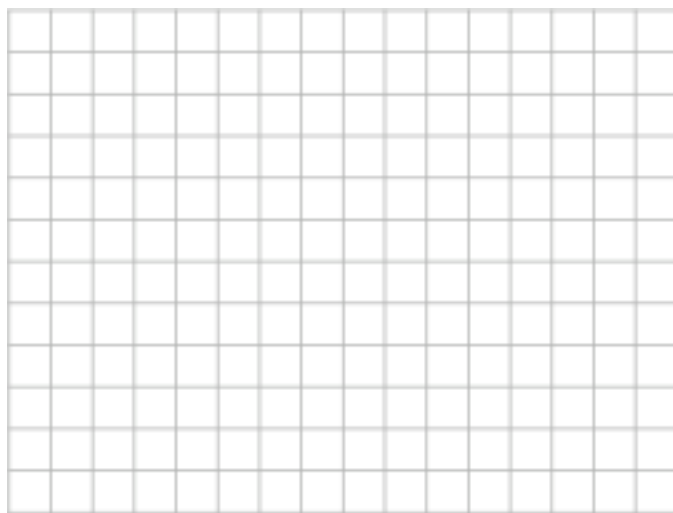
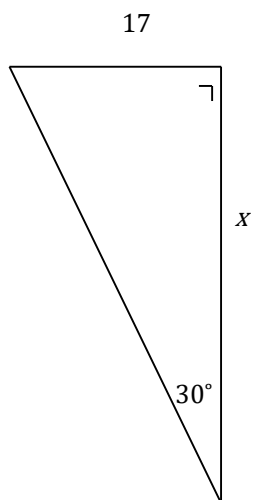
Calculate the measure of the angle B , correct to the nearest degree



Question 5

Find x , correct to one decimal place.

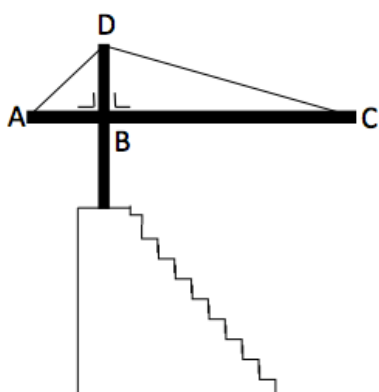
Note: Be careful with your algebra at the end of this question!!!



Question 6

An architect's view of Leinster Rugby's proposed new North Stand at The RDS is given below:

The architect has informed the engineer of the following specifications:



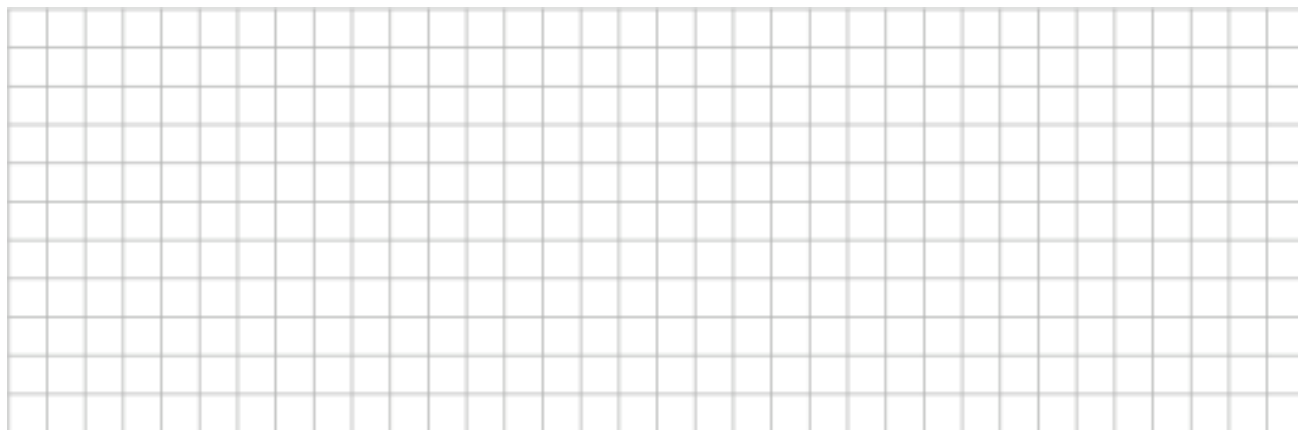
$$|AB|=5\text{m}$$

$$|AC|=15\text{m}$$

$$|\angle DAB|=40^\circ$$

Find (i) $|AD|$ correct to two decimal places

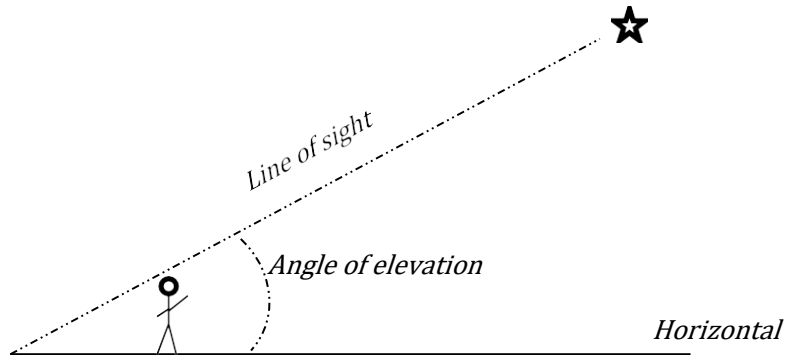
(ii) $|DC|$ correct to two decimal places



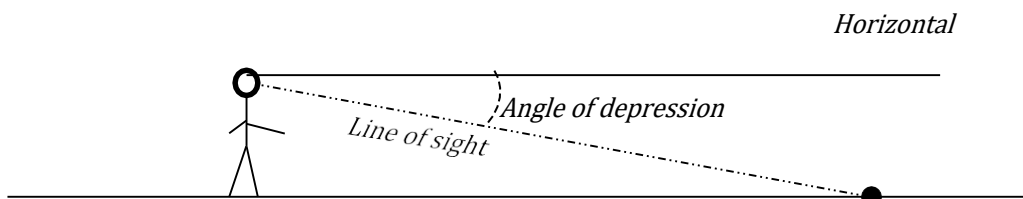
4) Angles of elevation/depression

There are two other points we must know to help us with 'real world' scenarios:

- i) **Angle of elevation:**
This is the angle your line of vision makes with the horizontal when looking up:



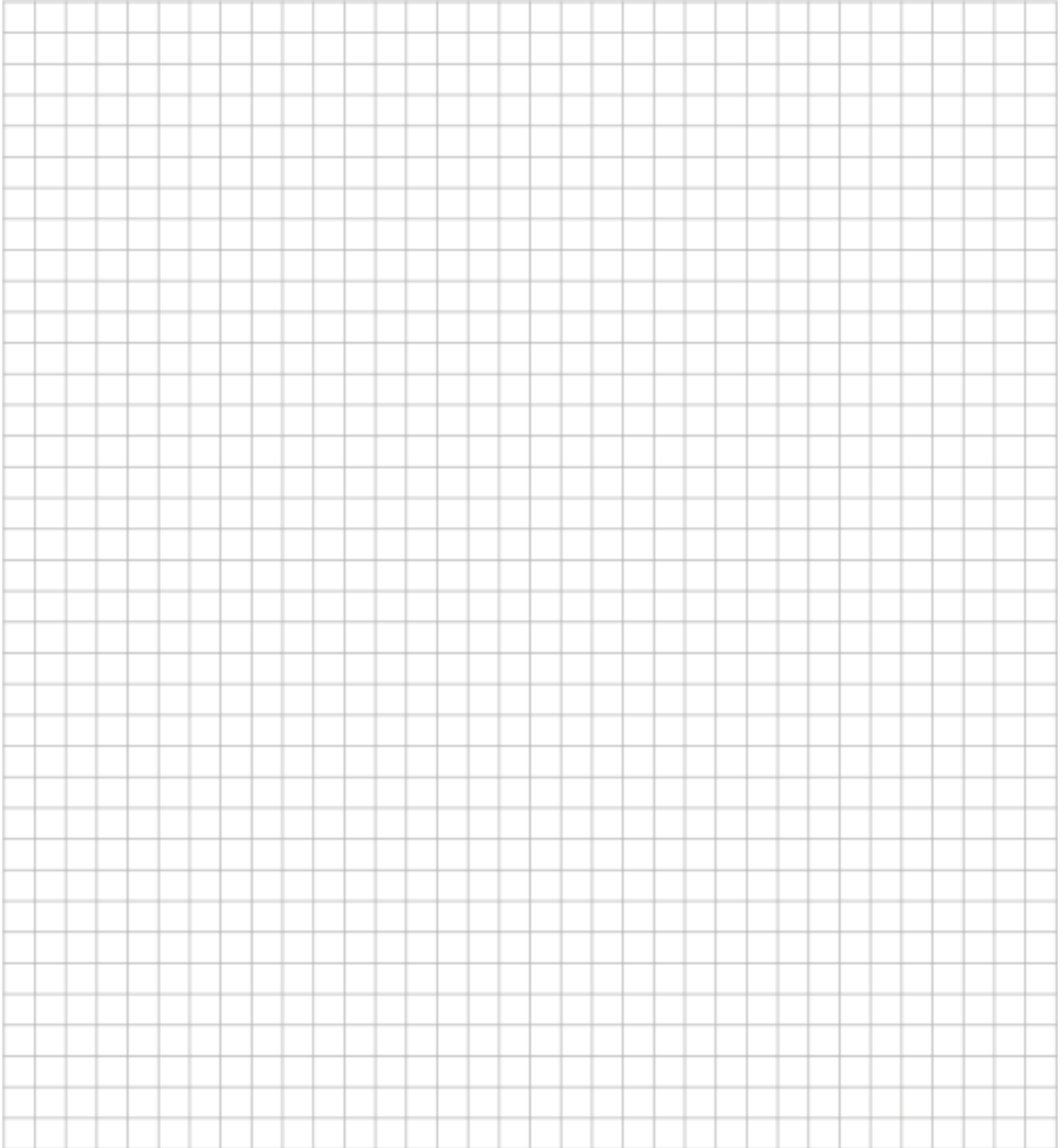
- ii) **Angle of depression:**
This is the angle your line of vision makes with the horizontal when looking down:



Question 1

A boat is anchored off the coast near $100m$ high cliffs in Co. Kerry. The angle of elevation from the boat to the cliff top is 15° . The next day the boat moves further towards land and anchors again. This time the angle of elevation is 38° . How far did the boat move?

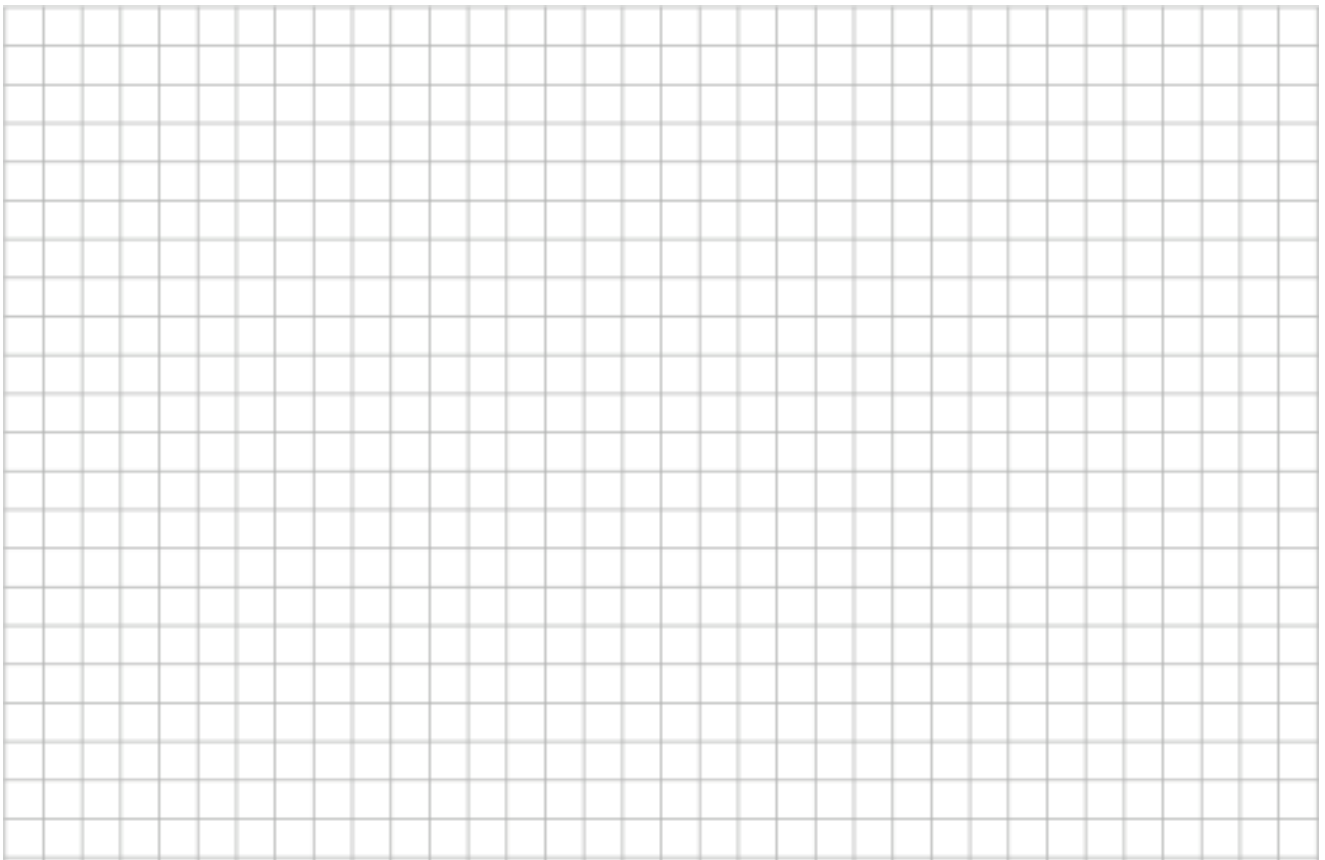
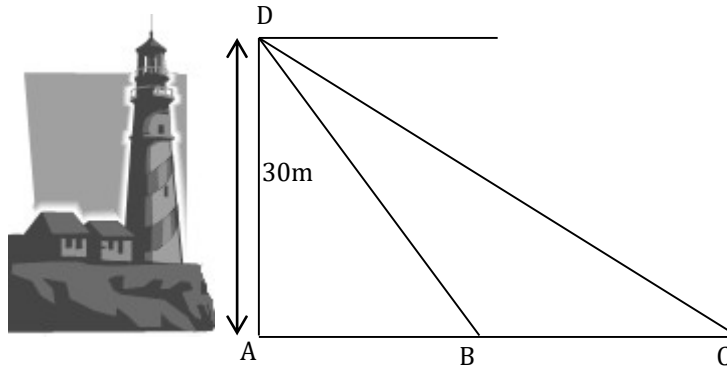
Give your answer correct to the nearest metre and make assumptions as necessary, stating them clearly.



Question 2

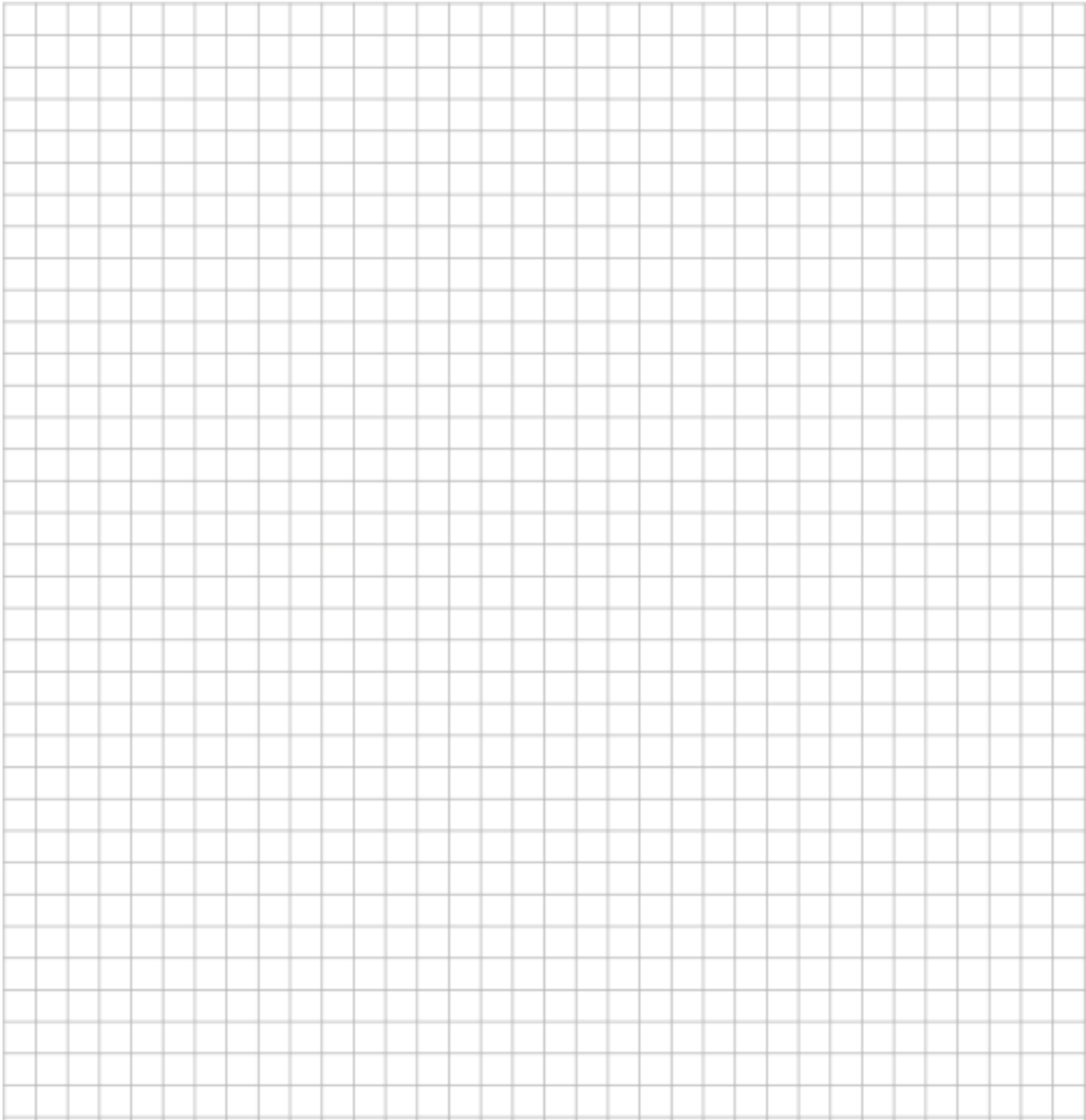
A boat sails due east from the base, A , of a 30m high lighthouse, $[AD]$. At the point B the angle of depression of the boat from the top of the lighthouse is 68° . Ten seconds later the boat is at the point C and the angle of depression is now 33° .

- i) Find $|BC|$, the distance the boat has travelled in this time.
- ii) Calculate the average speed at which the boat is sailing between B and C . Give your answer in metres per second, correct to one decimal place.



Question 3

The beach at Brittas Bay slopes downwards at a constant angle of 12° to the horizontal. How far out into the sea can a man walk before the water covers his head? Make assumptions as necessary.



Note: If you're asked to "explain your answer", simply explain it with maths.

For example, in the question below the answer is Yes or No. When they ask you to explain your choice, Explain using methods like Pythagoras, sin, cos, tan etc.

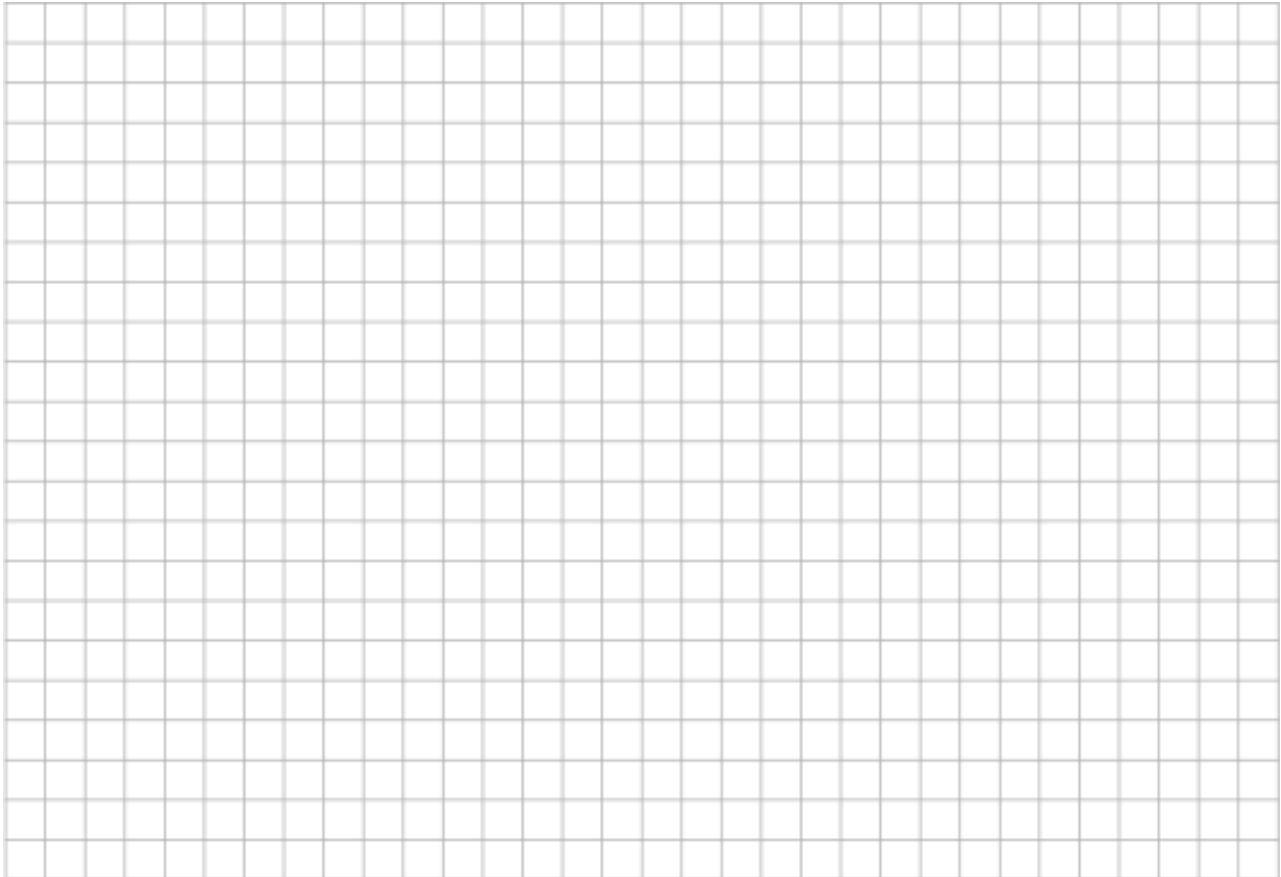
Question 4

The city council decides to construct a ramp for the local skate park. Before they start construction, they ask some skaters for their advice:

“We want a ramp that is 3 meters high at the top, but we don't want the angle of elevation to be more than 20° .”

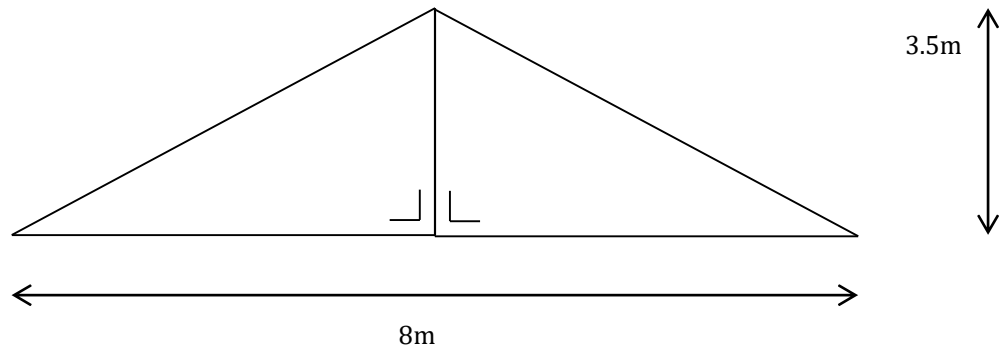
The city council comes up with a design. The design has the ratio of max height to ramp length of 3:8.

Does this designs fit in with the skaters requests? Explain your answer.



Question 5

A builder called Mark wants to build a canopy roof. Since he is worried about the chance of collapse due to the weight of snow, he asks the advice of a friend in Canada who is in the building trade. His Canadian friend says that it is best to build a roof with a pitch greater than 40° so that the snow will not accumulate. Mark's design for the roof is illustrated below:



Has Mark followed the advice of his friend?

Explain your answer.

A large grid of 20 columns and 20 rows is provided for the student to write their answer.

5) From one to another

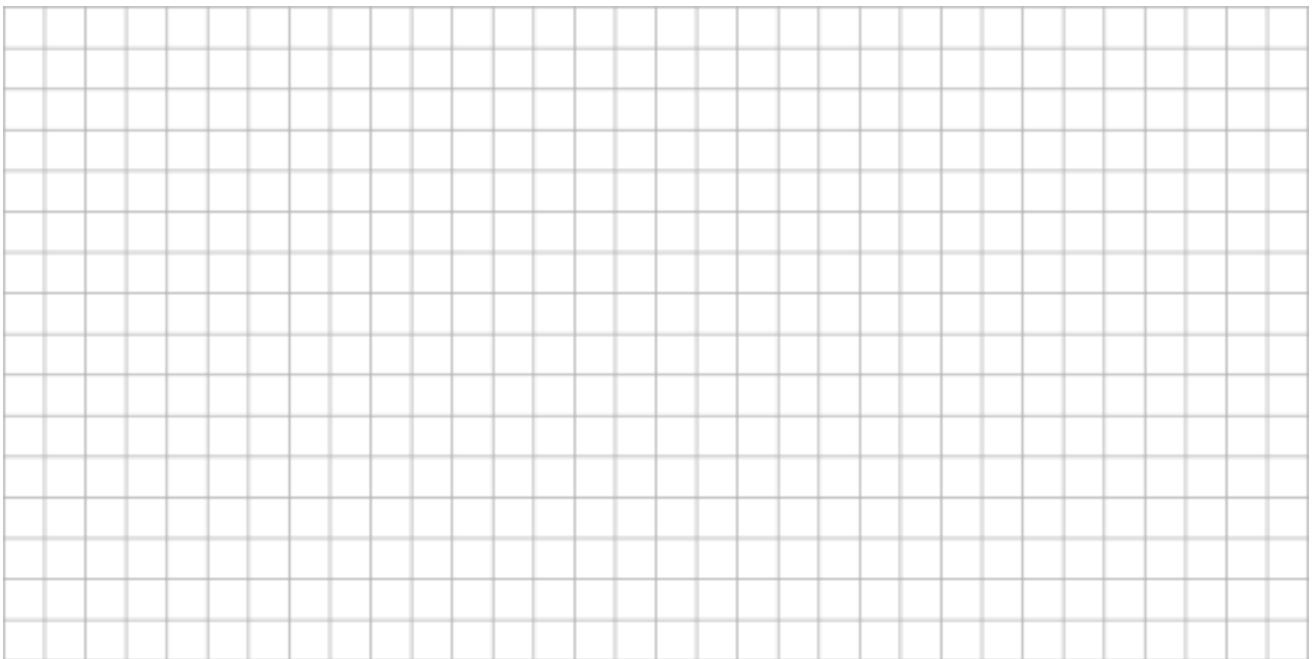
We can be given what **Sin**, **Cos** or **Tan** is equal to and then from this be asked to find another one.

There are 5 steps:

- 1) Sketch a right angled triangle.
- 2) Label the angles given.
- 3) Label the two sides given.
- 4) Find the missing side using Pythagoras.
- 5) Fill out the answer.

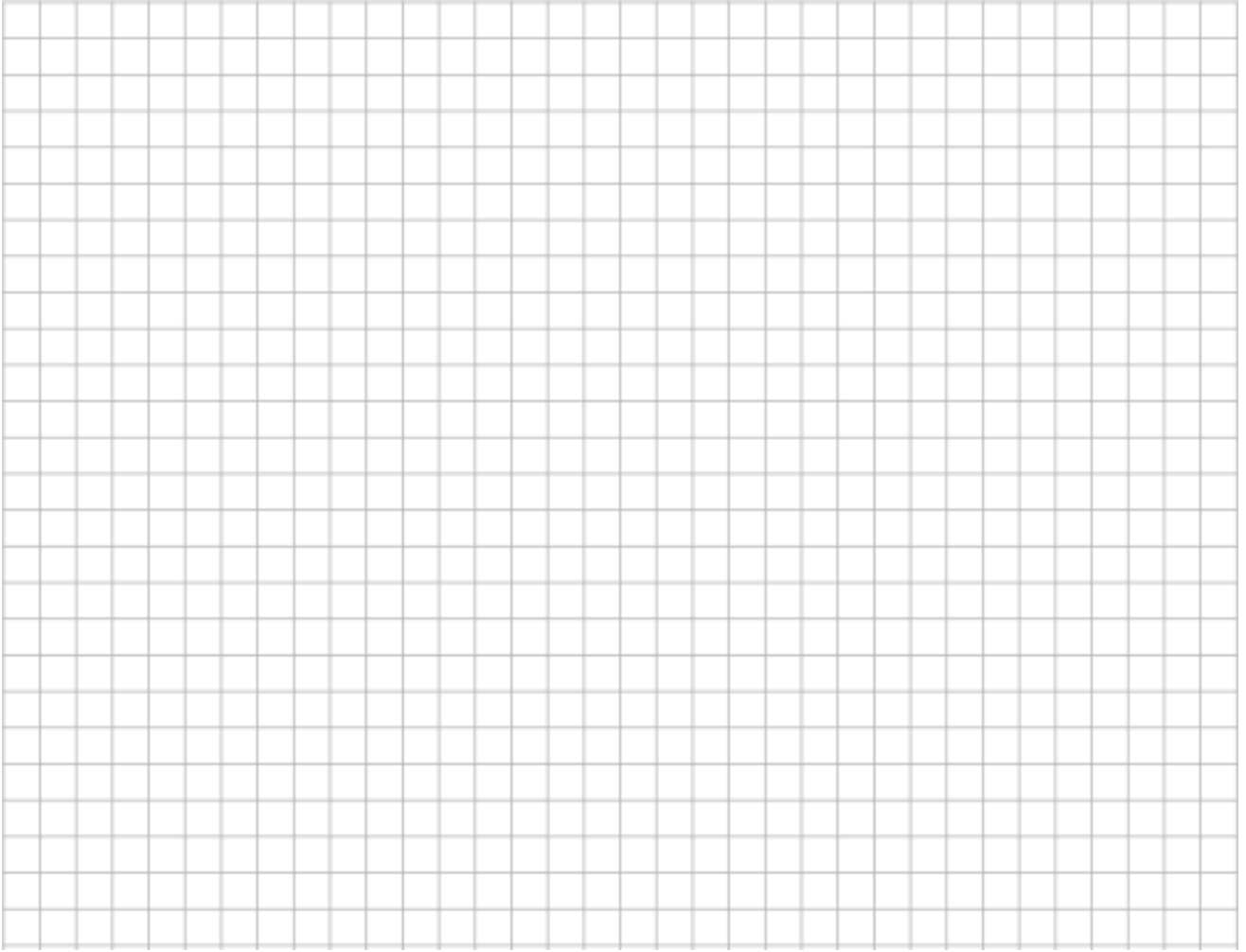
Question 1

If $\sin x = \frac{\sqrt{7}}{4}$, find the value of $\cos x$.



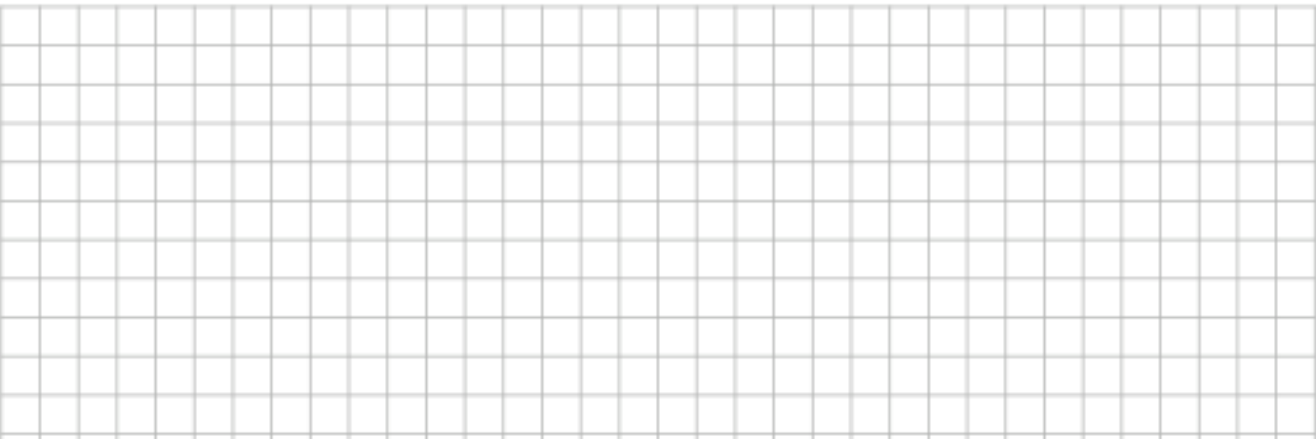
Question 2

Given that $\tan A = 4$, write $\cos A$ in the form $\frac{1}{\sqrt{x}}$, $x \in \mathbb{N}$



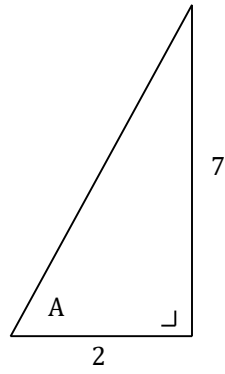
Question 3

Given that $\cos X = \frac{1}{7}$, write $\sin X$ in the form $\frac{\sqrt{a}}{b}$, $a, b \in \mathbb{N}$



Question 4

Use the information given in the diagram to find $\sin A$ and $\cos A$. Give your answer in surd form.



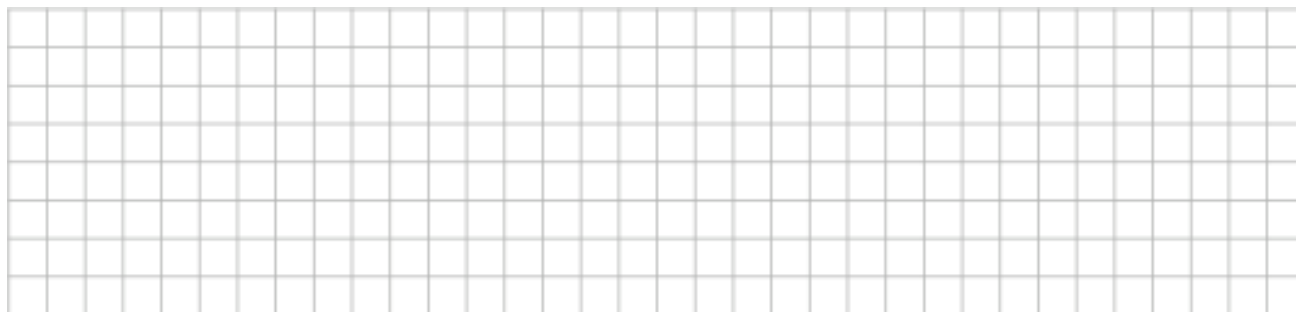
6) Evaluating trigonometric functions

This section sounds complicated but these types of questions are actually embarrassingly easy. You simply use page 13 of the Formulae and Tables booklet, or your calculator.

Note: If given $\mathbf{Sin}^2\theta$, we rewrite this as $(\mathbf{Sin}\theta)^2$. The same rule applies for \mathbf{Cos} and \mathbf{Tan} . If using your calculator, you must put in the brackets!

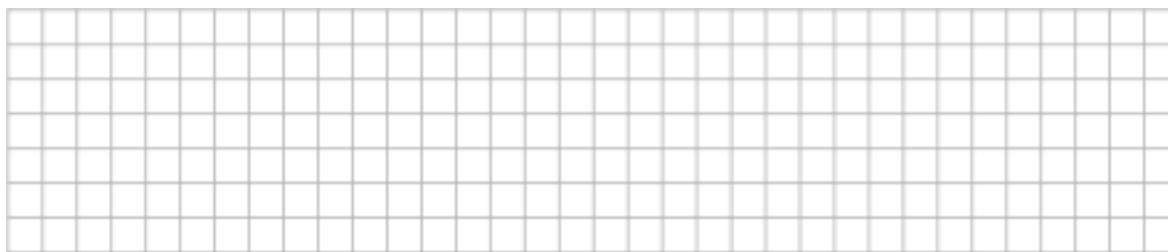
Question 1

Evaluate: $\mathbf{Sin}^2 30^\circ + \mathbf{Tan}^2 45^\circ$.



Question 2

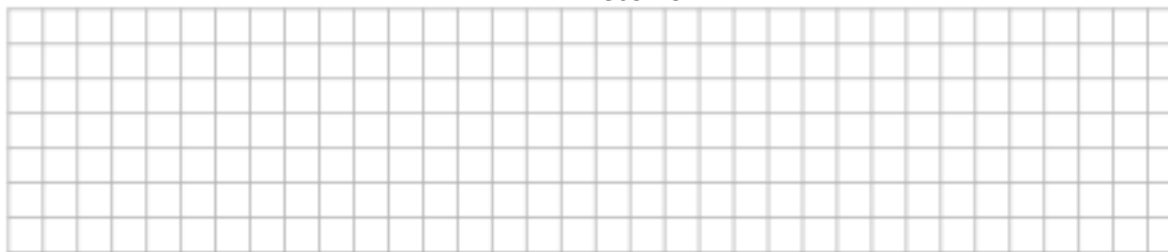
Evaluate: $1 - 2\mathbf{Tan}^2 45^\circ$.



Question 3

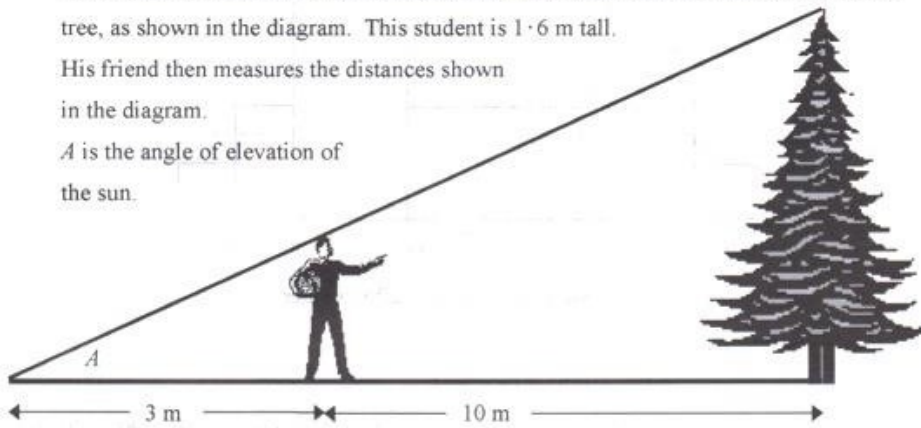
Verify that:

$$\frac{\mathbf{Tan}60^\circ\mathbf{Tan}30^\circ + 1}{\mathbf{Cos}^2 45^\circ} = 4$$



Question

- Some students wish to estimate the height of a tree standing on level ground. One of them stands so that the end of his shadow coincides with the end of the shadow of the tree, as shown in the diagram. This student is 1.6 m tall. His friend then measures the distances shown in the diagram. A is the angle of elevation of the sun.

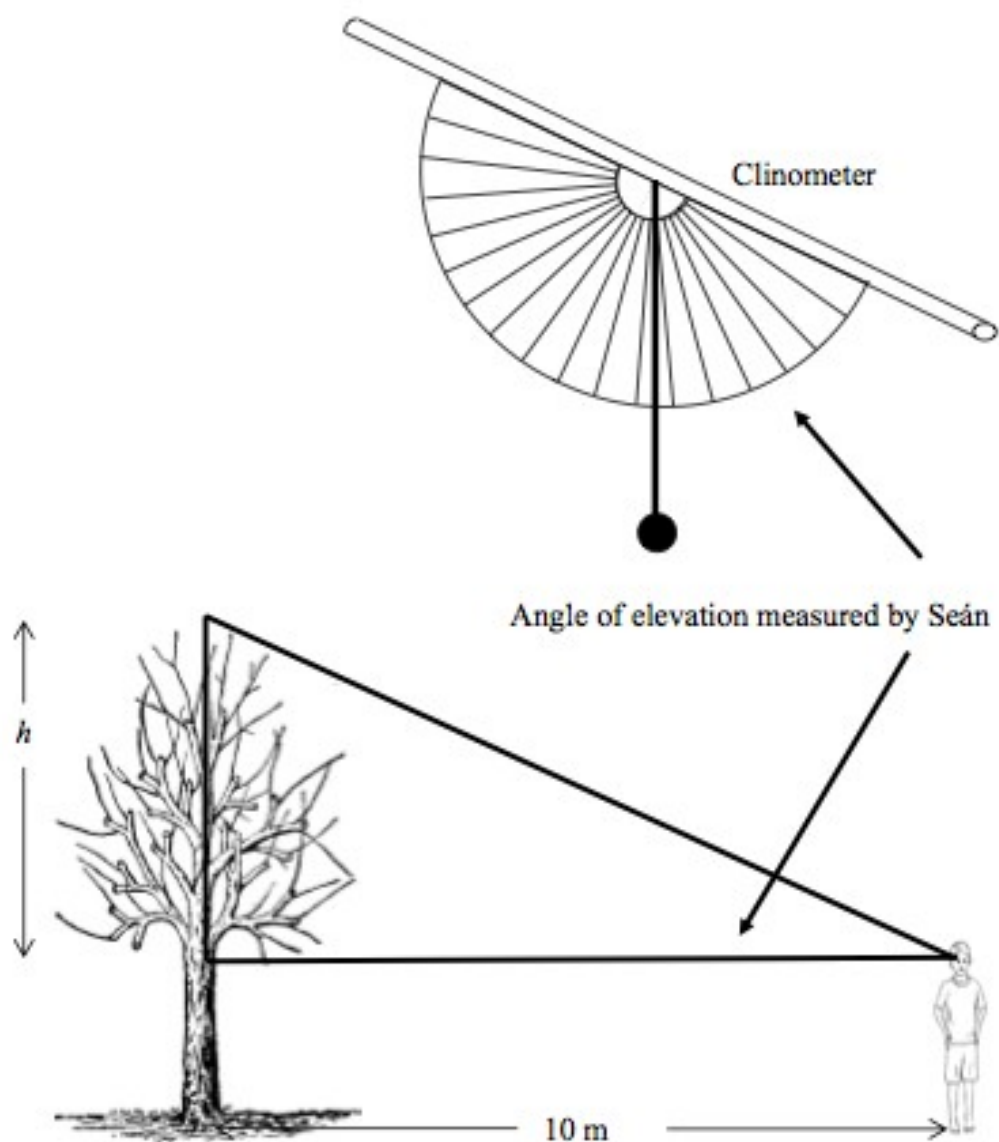


- (i) Find A , correct to the nearest degree.
- (ii) Find the height of the tree correct to one decimal place.



Question

Seán makes a clinometer using a protractor, a straw, a piece of thread and a piece of plasticine (used as a weight). He stands 10 m from a tree and uses his clinometer to measure the angle of elevation to the top of the tree as shown. Seán is 1.75 m in height.

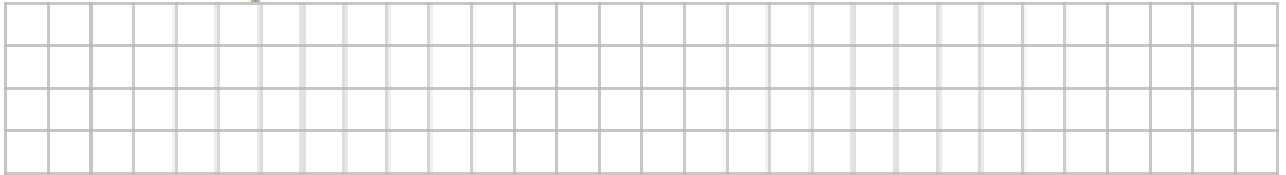


(a) Find the angle of elevation by reading the clinometer above. _____.

- (b) Calculate the height h as shown in the diagram. Give your answer correct to two decimal places.



- (c) Find the total height of the tree.

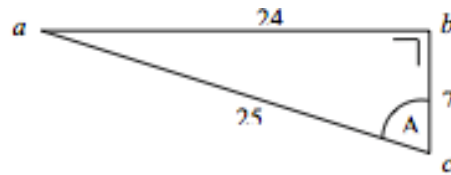


- (d) Another student uses the same method as Seán and finds the height of the tree to be 23.1 m. Seán did not get this answer. Give one possible reason why the answers might be different.



Question 8

- (a) The right-angled triangle abc has measurements as shown.

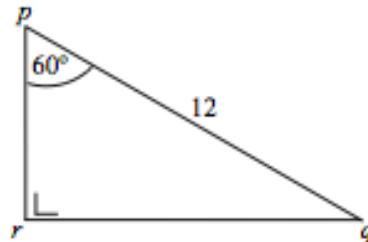


- (i) Write down the length of the side opposite the angle A .

Length of the side opposite the angle A =

- (ii) Write down the value of $\tan A$, as a fraction.

- 6(b) In the right-angled triangle pqr ,
 $|pq| = 12$ and $|\angle qpr| = 60^\circ$.

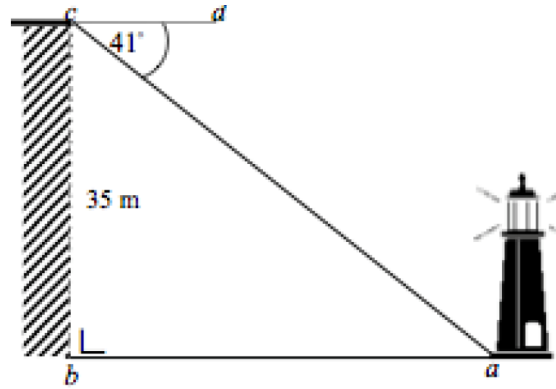


- (i) Write down the value of $\cos 60^\circ$.

- (ii) Calculate $|pr|$.




- (e) Claire is at the point c on the top of a cliff.
 The point b is at the base of the cliff.
 The height of the cliff is 35 m,
 as shown in diagram.
 She wishes to find $|ba|$, the distance from the
 base of the cliff to the base of the lighthouse.
 She measured $\angle dca$ and found it to be 41° .
 cd is parallel to ba .



(i) Find $|ba|$.

(ii) Find $\angle bac$.

-  (ii) Find, to the nearest metre, $|ba|$, the distance from the base of the cliff to the base of the lighthouse.

Question

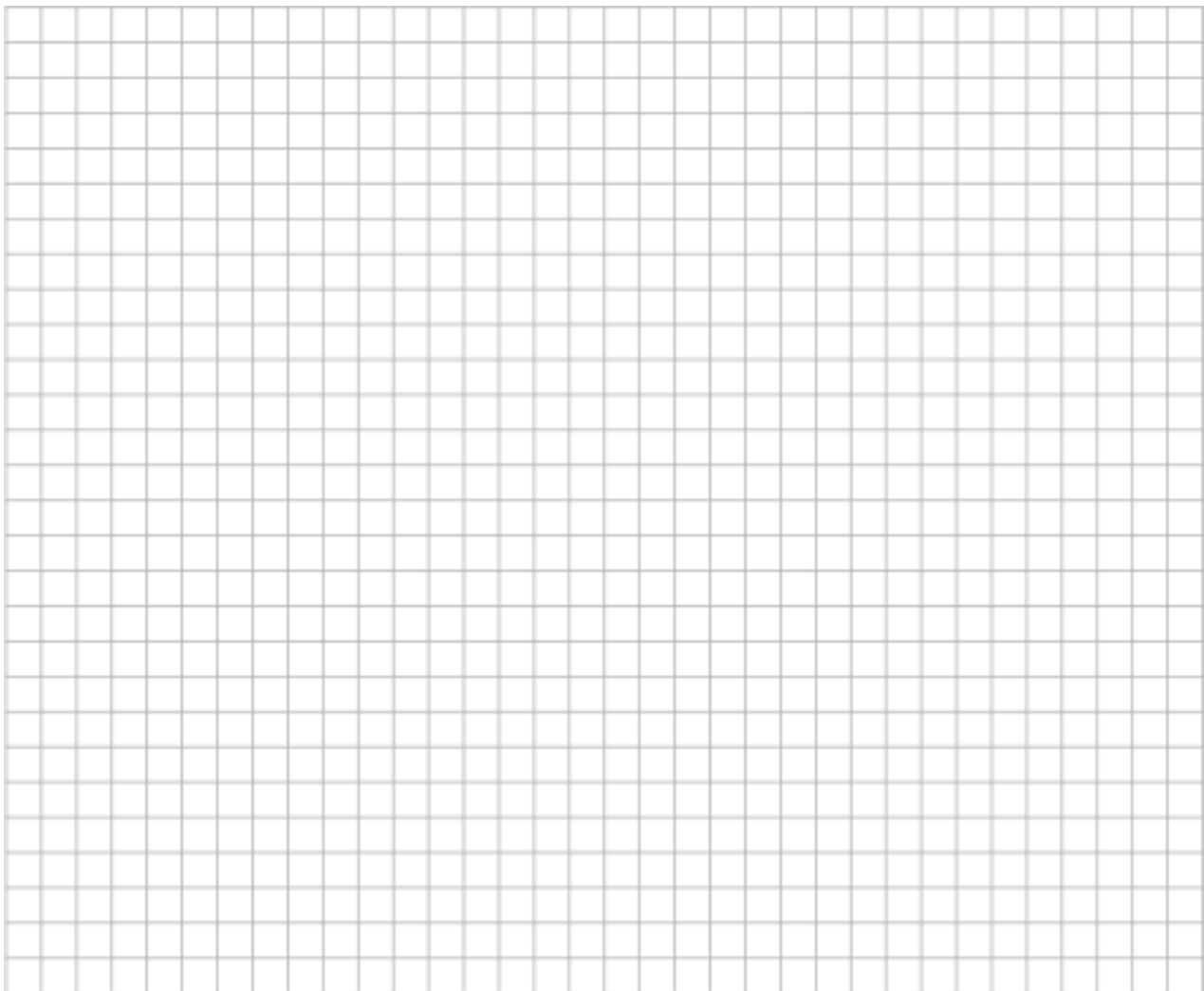
(a) $\sin A = 0.5045$. Find A .

(b) In the triangle pqr , $\angle prq = 90^\circ$,
 $\angle pqr = 50^\circ 48'$ and $|pq| = 20$ m.
Calculate $|qr|$, correct to one decimal place.

(c) A vertical pole is 6 m high.

It casts a shadow 5 m long
on level ground.

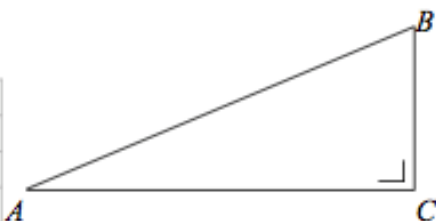
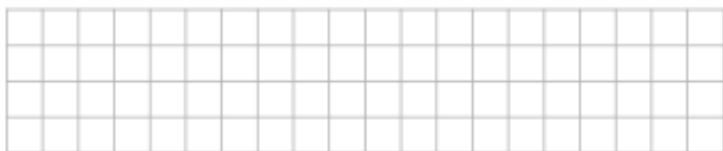
Calculate the angle of elevation of the sun,
correct to the nearest degree.



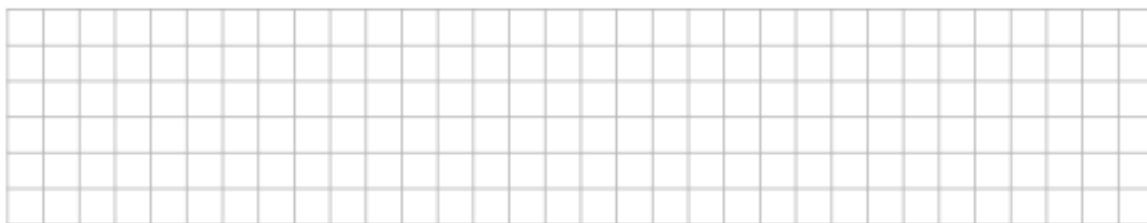
Question 35

In the triangle ABC , $|AB| = 2$ and $|BC| = 1$.

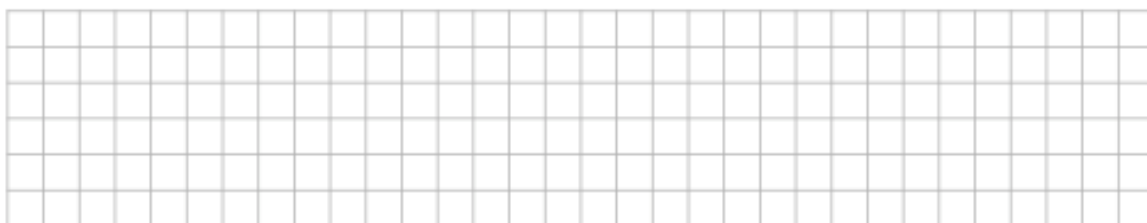
- (a) Find $|AC|$, giving your answer in surd form.



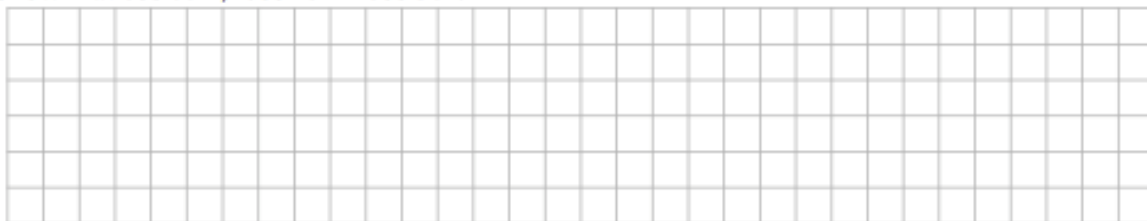
- (b) Write $\cos \angle BAC$ and hence find $|\angle BAC|$.



- (c) Sketch a right angled isosceles triangle in which the equal sides are 1 unit each and use it to write $\cos 45^\circ$ in surd form.



- (d) Show that $\cos 75^\circ \neq \cos 45^\circ + \cos 30^\circ$.



Question 41

- (a) The triangle PQR has sides of length 8, 11, and y .
Write down **one** value of y for which $\triangle PQR$ is an **isosceles** triangle.

$$y = \boxed{}$$

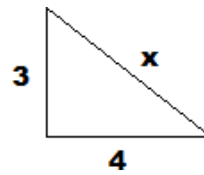
- (b) The triangle STU has sides of length 4, 7, and x .
Find the **two** values of x for which $\triangle STU$ is a **right-angled** triangle.
Give each answer in surd form.



8) Solutions to Trigonometry

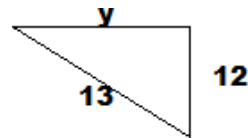
Question 1.1

$$\begin{aligned}x^2 &= 4^2 + 3^2 \\x^2 &= 16 + 9 \\x^2 &= 25 \\x &= 5\end{aligned}$$



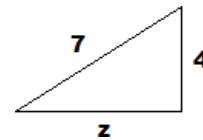
Question 1.2

$$\begin{aligned}13^2 &= 12^2 + y^2 \\169 &= 144 + y^2 \\169 - 144 &= y^2 \\25 &= y^2 \\ \Rightarrow y &= 5\end{aligned}$$



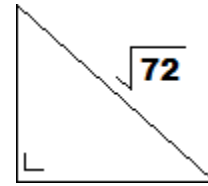
Question 1.3

$$\begin{aligned}7^2 &= 4^2 + z^2 \\49 &= 16 + z^2 \\49 - 16 &= z^2 \\33 &= z^2 \\ \Rightarrow z &= \sqrt{33}\end{aligned}$$



Question 1.4

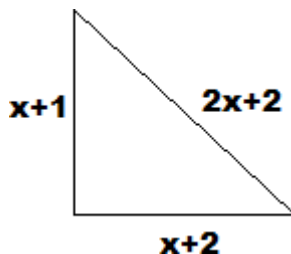
$$\begin{aligned}(\sqrt{72})^2 &= |ac|^2 + |bc|^2 \\72 &= 2x^2 \text{ (Remember: } |ac| = |bc| \text{)} \\ \frac{72}{2} &= x^2 \\36 &= x^2 \\ \sqrt{36} &= x = 6\end{aligned}$$



Question 1.5

$$\begin{aligned}(2x+2)^2 &= (x+2)^2 + (x+2)^2 \\1) (2x+2)(2x+2) &= 4x^2 + 4x + 4x + 4 = 4x^2 + 8x + 4 \\2) (x+1)(x+1) &= x^2 + 2x + 1 \\3) (x+2)(x+2) &= x^2 + 4x + 4\end{aligned}$$

$$\begin{aligned}\therefore 4x^2 + 8x + 4 &= x^2 + 2x + 1 + x^2 + 4x + 4 \\4x^2 + 8x + 4 &= 2x^2 + 6x + 5 \\2x^2 + 2x - 1 & \\a = 2, b = 2, c = -1 &\end{aligned}$$



$$\begin{aligned}\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} &= \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)} \\ &= \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)} \\ &= \frac{-2 \pm \sqrt{4+8}}{4} = \frac{-2 \pm \sqrt{12}}{4} \\ x &= \frac{-2 + \sqrt{12}}{4} \text{ or } x = \frac{-2 - \sqrt{12}}{4} \\ x &= 0.37 \text{ or } x = -1.37\end{aligned}$$

Question 46

Assume that the kite (length of string) is at its longest which is 13 m.

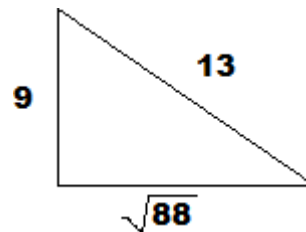
$$13^2 = 9^2 + |AB|^2$$

$$169 = 81 + |AB|^2$$

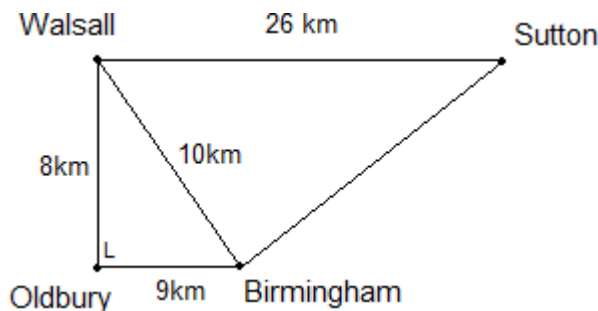
$$88 = |AB|^2$$

$$|AB| = \sqrt{88}$$

$$|AB| = 9m$$



Question 1.7



Find distance from Walsall to Birmingham
 $hyp^2 = 8^2 + 6^2 \Rightarrow hyp^2 = 64 + 36 \Rightarrow hyp^2 = 100 \Rightarrow hyp = 10$

Now find distance from Sutton to Birmingham

$$26^2 = 10^2 + x^2$$

$$676 = 100 + x^2$$

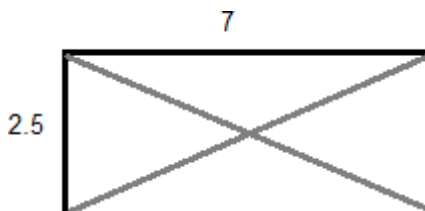
$$676 - 100 = x^2$$

$$x^2 = 576$$

$$x = \sqrt{576} = 24$$

Sutton is 14km further from Birmingham than Walsall is from Birmingham.

Question 1.8



$$hyp^2 = 7^2 + 2.5^2$$

$$hyp^2 = 49 + 6.25$$

$$hyp^2 = 55.25$$

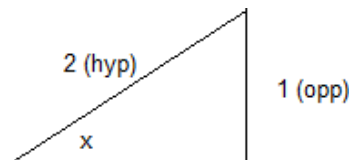
$$hyp = \sqrt{55.25}$$

$$hyp = 7.433$$

We need two planks so we need to multiply 7.433 by 2 = 14.866.
 The minimum length, correct to the nearest metre, is 15m.

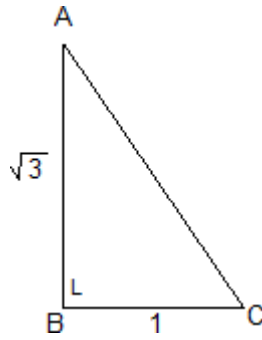
Question 2.1

$$x = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$



Question 47

$$|\angle BAC| = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$$

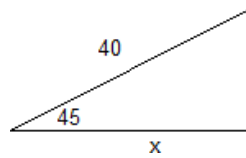


Question 2.3

$$\cos 45^\circ = \frac{x}{40}$$

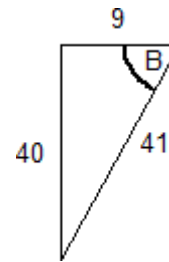
$$40 \cos 45^\circ = x$$

$$x = 28.284$$



Question 2.4

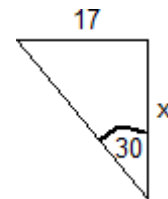
$$B = \tan^{-1}\left(\frac{40}{9}\right) = 77^\circ$$



Question 2.5

$$\tan 30^\circ = \frac{17}{x}$$

$$x = \frac{17}{\tan 30} = 29.4$$



Question 2.6

(i) Find |AD|

$$\cos 40^\circ = \frac{5}{x}$$

$$x = \frac{5}{\cos 40} = 6.53\text{m}$$

(ii) Find |DC|

First get |DB|:

$$6.53^2 = 5^2 + x^2$$

$$42.6409 = 25 + x^2$$

$$17.6409 = x^2$$

$$x = 4.2\text{m}$$

$$|DC|^2 = 10^2 + 4.2^2$$

$$|DC|^2 = 117.6409$$

$$|DC| = \sqrt{117.6409}$$

$$|DC| = 10.85$$

Question 4.8

50.27° to the nearest minute
= 50° 16'

Question 3.2

50.38° to the nearest minute
= 50° 23'

Question 3.3

Sin 35° 7' = 0.5752

Question 4.1

$\tan 15^\circ = \frac{100}{x}$ $\tan 35^\circ = \frac{100}{y}$
 $x = \frac{100}{\tan 15^\circ}$ $y = \frac{100}{\tan 35^\circ}$
 $x = 373.21\text{m}$ $y = 128\text{m}$
Distance moved = $x - y$
 $= 373.21 - 128$
 $= 245\text{m}$

Question 4.2

(i)

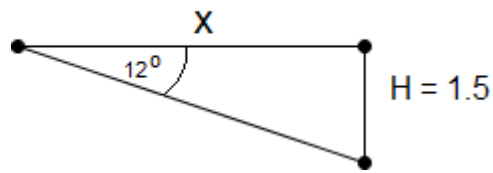
$\tan 68^\circ = \frac{30}{|AB|}$
 $\Rightarrow |AB| = \frac{30}{\tan 68^\circ}$
 $= 12.121\text{m}$
 $\tan 33^\circ = \frac{30}{|AC|}$
 $\Rightarrow |AC| = \frac{30}{\tan 33^\circ}$
 $= 46.196\text{m}$
 $\therefore BC = AC - AB = 34.075\text{ m}$

(ii)

Average Speed = $\frac{34.075\text{m}}{10\text{ sec}} = 3.41\text{ m/s}$

Question 49

Distance out to sea

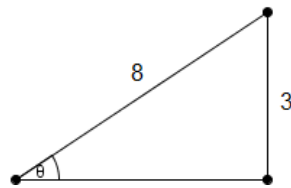


If the man is Carl, assume his height to be 1.5 m.
x is the distance Carl needs to walk out.

$$\begin{aligned} \tan 12^\circ &= \frac{1.5}{x} \\ \Rightarrow x &= \frac{1.5}{\tan 12^\circ} = 7.1\text{m} \end{aligned}$$

Question 4.4

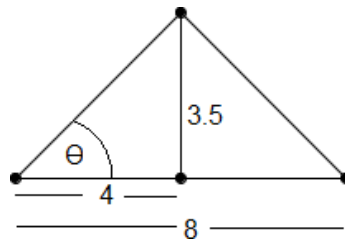
Ramp height = 3m
Ramp length = 8m



$$\theta = \sin^{-1}\left(\frac{3}{8}\right) = 22^\circ$$

No, as $22^\circ > 20^\circ$

Question 4.5

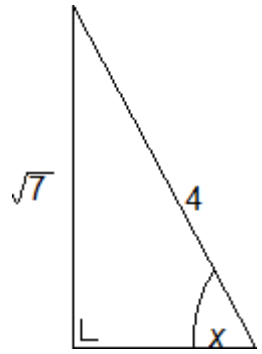


$$\theta = \tan^{-1}\left(\frac{3.5}{4}\right) = 41.2^\circ$$

He has followed the advice, as $41.2^\circ > 40^\circ$

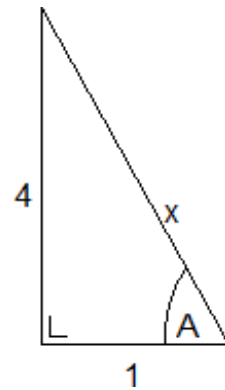
Question 5.0

$$\begin{aligned}\sin x &= \frac{\sqrt{7}}{4} \\ 4^2 &= (\sqrt{7})^2 + \text{adj}^2 \\ 16 &= 7 + \text{adj}^2 \\ 9 &= \text{adj}^2 \\ \text{adj} &= 3 \\ \Rightarrow \cos x &= \frac{\text{adj}}{\text{hyp}} = \frac{3}{4}\end{aligned}$$



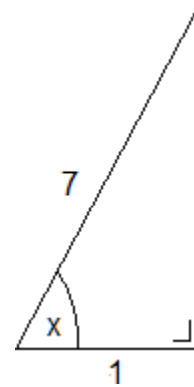
Question 5.2

$$\begin{aligned}x^2 &= 4^2 + 1^2 \\ x^2 &= 17 \\ x &= \sqrt{17} \\ \Rightarrow \cos A &= \frac{1}{\sqrt{17}}\end{aligned}$$



Question 5.3

$$\begin{aligned}7^2 &= 1^2 + \text{opp}^2 \\ 49 &= 1 + \text{opp}^2 \\ \text{opp}^2 &= 48 \\ \text{opp} &= \sqrt{48} = 4\sqrt{3} \\ \sin x &= \frac{4\sqrt{3}}{7}\end{aligned}$$



Question 5.4

$$x^2 = 7^2 + 2^2$$

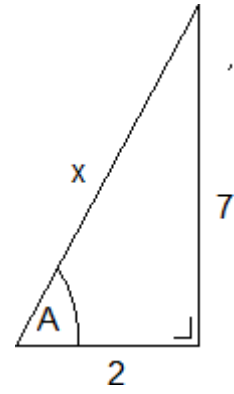
$$x^2 = 49 + 4$$

$$x^2 = 53$$

$$x = \sqrt{53}$$

$$\sin A = \frac{7}{\sqrt{53}}$$

$$\cos A = \frac{2}{\sqrt{53}}$$

**Question 6.1**

$$\sin^2 30^\circ + \tan^2 45^\circ = \frac{1}{4} + 1$$

$$= 1\frac{1}{4}$$

Question 6.2

$$1 - 2 \tan^2 45^\circ = 1 - 2$$

$$= -1$$

Question 6.3

Verify that $\frac{\tan 60^\circ \tan 30^\circ + 1}{\cos^2 45^\circ} = 4$

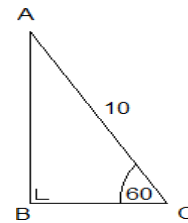
$$\frac{\sqrt{3} \left(\frac{1}{\sqrt{3}} \right) + 1}{\frac{1}{2}} = \frac{2}{\frac{1}{2}} = 4$$

Solutions to past and probable exam questions

Question 1

(a)

$$\begin{aligned}\sin 60^\circ &= \frac{|AB|}{10} \\ 10(\sin 60^\circ) &= |AB| \\ |AB| &= 8.66 \text{ cm}\end{aligned}$$



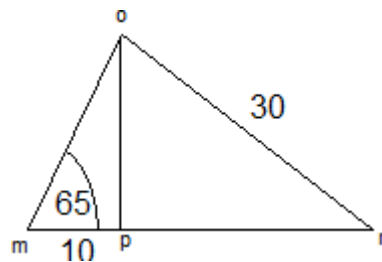
(b)

(i)

$$\begin{aligned}\tan 65^\circ &= \frac{|op|}{10} \\ 10 \tan 65^\circ &= |op| \\ |op| &= 21.4 \text{ cm}\end{aligned}$$

(ii)

$$\begin{aligned}|\angle mon| &= |\angle mop| + |\angle pon| \\ |\angle mop| &= 25^\circ \\ |\angle pon| &= \cos^{-1}\left(\frac{21.4}{30}\right) = 44.49^\circ \\ 44.49^\circ + 25^\circ &= 69.5^\circ\end{aligned}$$



Question 2

(i)

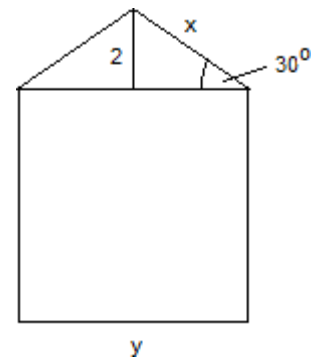
$$\begin{aligned}\sin 30^\circ &= \frac{2}{x} \\ \Rightarrow x &= \frac{2}{\sin 30^\circ} \\ x &= 4\end{aligned}$$

(ii)

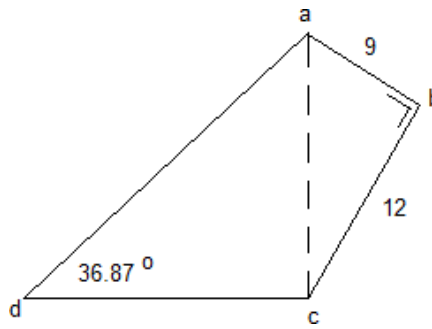
Use Pythagoras:

$$\begin{aligned}4^2 &= 2^2 + \text{adj}^2 \\ 16 &= 4 + \text{adj}^2 \\ 12 &= \text{adj}^2 \\ \text{adj} &= \sqrt{12} = 2\sqrt{3} = 3.464 \text{ m}\end{aligned}$$

$$\begin{aligned}3.464 \times 2 &= y \\ y &= 6.93 \text{ m}\end{aligned}$$



Question 3



(i)

$$\begin{aligned} |ac|^2 &= 9^2 + 12^2 \\ |ac|^2 &= 81 + 144 \\ |ac| &= \sqrt{225} = 15 \end{aligned}$$

(ii)

$$\begin{aligned} \tan 36.87^\circ &= \frac{15}{|cd|} \\ \Rightarrow |cd| &= \frac{15}{\tan 36.87^\circ} = 20 \text{ km} \end{aligned}$$

(iii)

$$\begin{aligned} &\text{Find } |ad| \\ |ad|^2 &= 15^2 + 20^2 \\ |ad|^2 &= 225 + 400 \\ |ad|^2 &= \sqrt{625} \\ |ad| &= 25 \text{ km} \end{aligned}$$

$$\text{Total length} = 25 + 20 + 12 + 9 = 66 \text{ km}$$

Question 4

(i)

$$\tan^{-1}\left(\frac{16}{3}\right) = 28^\circ$$

(ii)

$$\begin{aligned} \tan 28^\circ &= \left(\frac{x}{13}\right) \\ 13(\tan 28^\circ) &= x \\ x &= 6.9 \text{ m} \end{aligned}$$

Question 5

(a)

$$\begin{aligned} \sin 25^\circ &= 0.423, \cos 25^\circ = 0.906, \tan 25^\circ = 0.466 \\ \sin 50^\circ &= 0.766, \cos 50^\circ = 0.643, \tan 50^\circ = 1.192 \end{aligned}$$

(b)

(i)

No. While Sin and Cos are always less than 1, tan is only less than 1 for angles less than 45° . (At 45° , opp = adj $\Rightarrow \tan 45^\circ = 1$, $\tan 50^\circ = 1.192$)

(ii)

No. Trigonometric functions are not linear.

$$\text{E.g. } \frac{\sin 40^\circ}{\sin 20^\circ} = 1.8 \neq 2$$

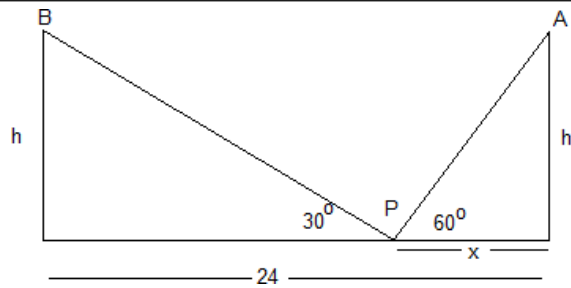
(iii)

No. Cos reduces with increasing angle (for $\theta < 180^\circ$)

$$\cos 40^\circ = 0.766, \cos 20^\circ = 0.940$$

$\therefore \cos 40^\circ < \cos 20^\circ$

Question 6



(a)

$$\tan 60^\circ = \frac{h}{x} \Rightarrow \sqrt{3} x = h$$

(b)

$$\tan 30^\circ = \frac{\sqrt{3}x}{24-x}$$

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}x}{24-x}$$

$$24 - x = 3x$$

$$24 = 4x$$

$$x = 6$$

$$h = \sqrt{3}(6) = 10.39\text{m}$$

Question 7

(a)

$$\text{Angle of elevation} = 65^\circ$$

(b)

$$\tan 65^\circ = \frac{h}{10}$$

$$h = 10(\tan 65^\circ) = 21.45\text{m}$$

(c)

$$\text{Total height of tree} = 21.45\text{m} + 1.75\text{m} = 23.2\text{m}$$

(d)

He may not be as tall as Sean

Question 8

(a) (i)

$$24$$

(ii)

$$\tan A = \frac{24}{7}$$

(b) (i)

$$\cos 60^\circ = \frac{1}{2}$$

(ii)

$$|pr| : \cos 60^\circ = \frac{|pr|}{12}$$

$$\Rightarrow 12\left(\frac{1}{2}\right) = |pr| = 6$$

(c) (i)

$$|\angle bac| = 41^\circ - \text{alternate angles}$$

(ii)

$$\tan 41^\circ = \frac{35}{|ba|}$$

$$\Rightarrow |ba| = \frac{35}{\tan 41^\circ} = 40.26$$

$$\text{Ans.} = 40\text{m}$$

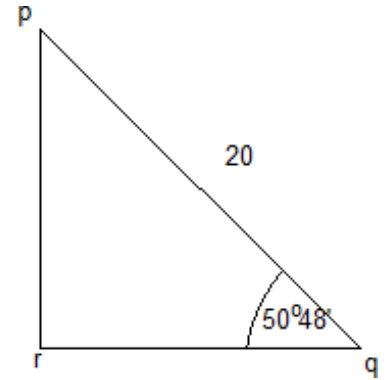
Question 9

(a)

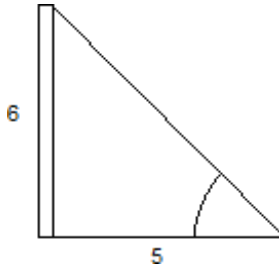
$$\sin^{-1}(0.5045) = 30.2982^\circ$$

(b)

$$\begin{aligned} \cos 50^\circ 48' &= \frac{|rq|}{20} \\ 20(\cos 50^\circ 48') &= |rq| \\ |rq| &= 12.6\text{m} \end{aligned}$$



(c)

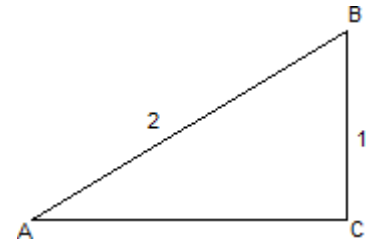


$$\tan^{-1} \frac{6}{5} = 50^\circ$$

Question 10

(a)

$$\begin{aligned} |AB|^2 &= |BC|^2 + |AC|^2 \\ 2^2 &= 1^2 + |AC|^2 \\ 4 &= 1 + |AC|^2 \\ |AC|^2 &= 3 \\ \Rightarrow |AC| &= \sqrt{3} \end{aligned}$$

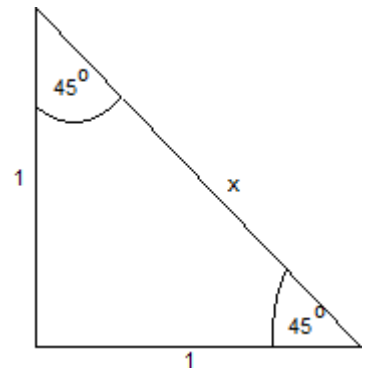


(b)

$$\cos \angle BAC = \frac{\sqrt{3}}{2} = 30^\circ$$

(c)

$$\begin{aligned} x^2 &= 1^2 + 1^2 \\ x^2 &= 2 \\ x &= \sqrt{2} \\ \cos 45^\circ &= \frac{1}{\sqrt{2}} \end{aligned}$$



(d)

$$\cos 75^\circ = \frac{\sqrt{6} - \sqrt{2}}{4} = 0.259$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = 0.707$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} = 0.866$$

$$0.259 \neq 0.707 + 0.866$$

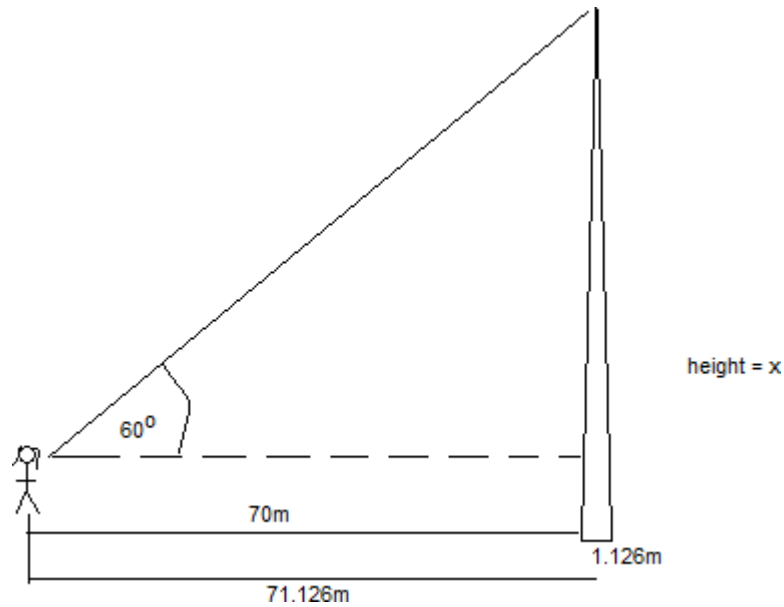
Question 56

(a)

The circumference will be used to calculate the radius of the Spire, which will help to give the full distance that Maria is from the Spire.

$$2\pi r = 7.07\text{m}$$

$$r = \frac{7.07}{2\pi} = 1.126$$



$$\tan 60^\circ = \frac{x}{71.126} = 123.194 \text{ m}$$

Add Maria's height,

$$123.194 + 1.72$$

$$\text{Ans} = 125\text{m}$$

Question 57

(a)

$$\begin{aligned} |\angle B| &= 50^\circ \\ \sin B &= 0.766 \end{aligned}$$

(b)

$$\begin{aligned} \sin 50^\circ &= \frac{h}{8} \\ \Rightarrow 8(\sin 50^\circ) &= h \\ H &= 6\text{m} \end{aligned}$$

(c)

$$\begin{aligned} \text{Area of ABC} &= \frac{1}{2} \text{Base} \times \text{Height} \\ &= \frac{1}{2} (7.5 \times 6) = 22.5 \text{ cm}^2 \end{aligned}$$

Question 13

(a)

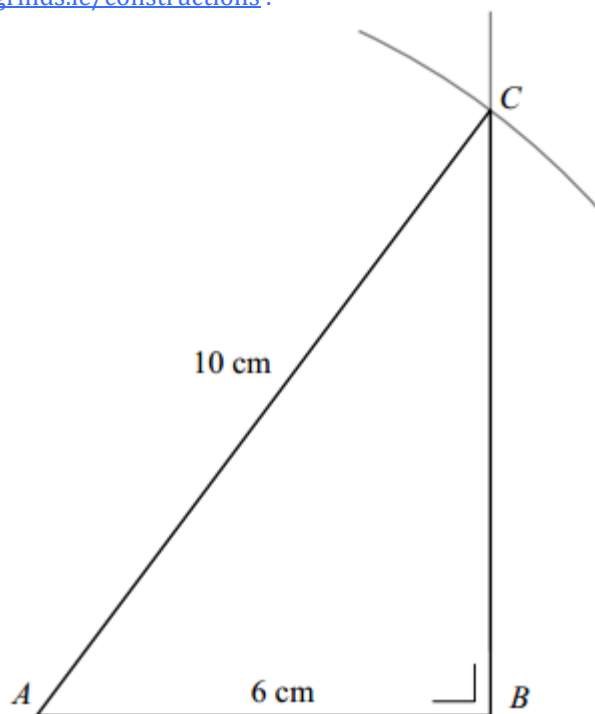
$$\begin{aligned} \tan 30^\circ &= \frac{x}{18} \\ \Rightarrow x &= 18 (\tan 30^\circ) = 10.4\text{m} \end{aligned}$$

(b)

We need to know what Jasmine's height is and add it to the result we got in part (a).

Question 14

- (i) Use construction 13, Right-angled triangle, given the length of the hypotenuse and one other side. A demonstration is available on The Dublin School of Grinds website:
www.dublinschoolofgrinds.ie/constructions.



(ii) 53°

(iii)

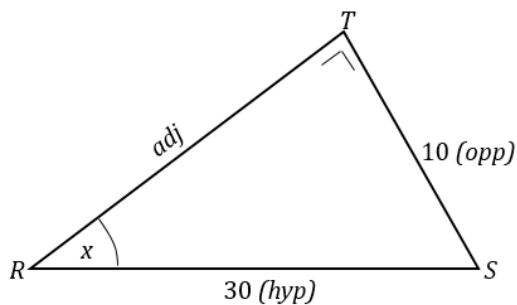
$$\begin{aligned}\cos 53^\circ &= 0.60181502 \\ &= 0.602 \text{ (correct to 3 decimal places)}\end{aligned}$$

- (iv) Because the answer in part (ii) is rounded off to a whole number it gives a different answer.

Question 15

- (a) An angle opposite a diameter in a circle is a right angle i.e. 90° or the angle at the centre of a circle is twice the angle at the circumference standing on the same arc.

(b)



$$\begin{aligned}\sin x &= \frac{10}{30} \\ \sin^{-1}\left(\frac{10}{30}\right) &= 19.471 \\ &= 19.5^\circ\end{aligned}$$

Question 16

(a)

$$y = 8 \text{ or } 11$$

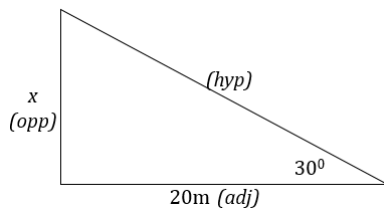
(b)

$$\begin{aligned} \textcircled{1} \quad x^2 &= 4^2 + 7^2 \\ x^2 &= 16 + 49 \\ x^2 &= 65 \\ \Rightarrow x &= \sqrt{65} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad 7^2 &= 4^2 + x^2 \\ 49 &= 16 + x^2 \\ x^2 &= 33 \\ \Rightarrow x &= \sqrt{33} \end{aligned}$$

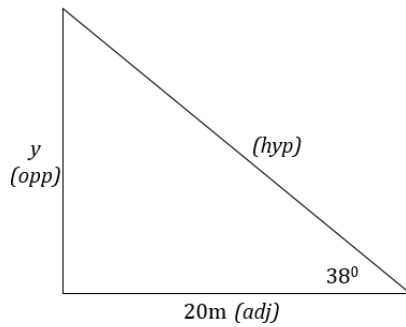
Question 17

(a)



$$\begin{aligned} \tan 30^\circ &= \frac{x}{20} \\ \Rightarrow x &= 20(\tan 30^\circ) \\ x &= 11.55\text{m} \end{aligned}$$

(b)

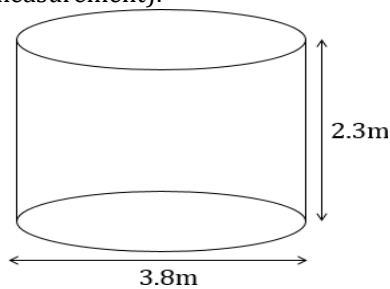


$$\begin{aligned} \tan 38^\circ &= \frac{y}{20} \\ \Rightarrow y &= 20(\tan 38^\circ) \\ x &= 15.6257\text{m} \end{aligned}$$

$$\begin{aligned} h &= y - x \\ h &= 15.6257 - 11.55 \\ &= 4.08 \\ &= 4.1\text{m} \end{aligned}$$

(c) Shape of the water tank: Cylinder

Measurements from photograph
(label each measurement):



From photo radius = 1.9

$$1.9 : 2.3 = x : 4.5$$

$$\frac{1.9}{2.3} = \frac{r}{4.5}$$

$$\Rightarrow r = 3.7$$

Calculations:

$$\begin{aligned} \text{Vol. of cylinder} &= \pi r^2 h \\ &= \pi (3.7)^2 (4.5) \\ &= 193.5378\text{m}^3 \\ &= 194\text{m}^3 \end{aligned}$$

Volume of water, in m^3 : 194m^3