Teaching & Learning Plans

Introduction to Calculus

Leaving Certificate Syllabus

Project Maths
Tionscadal Mata
Development Team

The Teaching & Learning Plans are structured as follows:

Aims outline what the lesson, or series of lessons, hopes to achieve.

Prior Knowledge points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

Learning Outcomes outline what a student will be able to do, know and understand having completed the topic.

Relationship to Syllabus refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

Resources Required lists the resources which will be needed in the teaching and learning of a particular topic.

Introducing the topic (in some plans only) outlines an approach to introducing the topic.

Lesson Interaction is set out under four sub-headings:

i. **Student Learning Tasks – Teacher Input:** This section focuses on possible lines of inquiry and gives details of the key student tasks and teacher questions which move the lesson forward.

ii. **Student Activities – Possible Responses:** Gives details of possible student reactions and responses and possible misconceptions students may have.

iii. **Teacher's Support and Actions:** Gives details of teacher actions designed to support and scaffold student learning.

iv. **Assessing the Learning:** Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

Student Activities linked to the lesson(s) are provided at the end of each plan.

Teaching & Learning Plans: Introduction to Calculus

Aims

The aim of this series of lessons is to enable students to:

- understand what is meant by, and the difference between, average and instantaneous rates of change
- recognise the need for differential calculus in terms of real-world problems
- understand the concept of the derivative of a function
- understand that differentiation (differential calculus) is used to calculate instantaneous rates of change
- understand how to apply differentiation to calculate instantaneous rates of change

Prior Knowledge

It is envisaged that, in advance of tackling this Teaching and Learning Plan, the students will understand and be able to carry out operations in relation to:

- Functions
- Constant rates of change and calculating slopes from graphs
- Pattern analysis
- Describing graphs without formulae
- Distance, speed and time
- Indices
- Limits
- Tangents

Learning Outcomes

Having completed this Teaching and Learning Plan the students will be able to:

- describe rates of change in the real world
- use mathematical language to describe rates of change
- use the slope formula to calculate rates of change of linear functions
- use the slope formula to calculate average rates of change
- recognise that average rate of change and instantaneous rate of change are identical for linear functions
- recognise that average rate of change and instantaneous rate of change are not necessarily identical for non-linear functions

- Recognise that the slope of the secant line between two points on a curve is the average rate of change of the curve between those points
- Understand that the average rate of change over shorter intervals around a point on a curve is a better estimate of the instantaneous rate of change at that point
- Understand that the instantaneous rate of change is given by the average rate of change over the shortest possible interval and that this is calculated using the limit of the average rate of change as the interval approaches zero.
- Recognise the notation associated with differentiation (e.g. slope, rate of change, $f'(x)$, dy/dx)
- Understand $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$
- Understand that when you differentiate a function you generate a new function (the slope function) which gives the slope of the original function at any point
- Find the derivative by rule

Catering for Learner Diversity

In class, the needs of all students, whatever their level of ability level, are equally important. In daily classroom teaching, teachers can cater for different abilities by providing students with different activities and assignments graded according to levels of difficulty so that students can work on exercises that match their progress in learning. Less able students, may engage with the activities in a relatively straightforward way while the more able students should engage in more open-ended and challenging activities

In interacting with the whole class, teachers can make adjustments to meet the needs of all of the students. **For example, some students may engage with some of the more challenging questions for example question number 12 in Section A: Student Activity 1.**

Apart from whole-class teaching, teachers can utilise pair and group work to encourage peer interaction and to facilitate discussion. The use of different grouping arrangements in these lessons should help ensure that the needs of all students are met and that students are encouraged to articulate their mathematics openly and to share their learning.

Relationship to Leaving Certificate Syllabus

Sub-Topic	**Learning outcomes**		
Students learn about	**Students working at FL should be able to**	**In addition students working at OL should be able to**	**In addition students working at HL should be able to**
5.2 Calculus		Find first and second derivatives of linear, quadratic and cubic functions by rule Associate derivatives with slopes and tangent lines Apply differentiation to • rates of change • maxima and minima • curve sketching	Differentiate linear and quadratic functions from first principles Differentiate the following functions • polynomial • exponential • trigonometric • rational powers • inverse functions • logarithms Find the derivatives of sums, differences, products, quotients and compositions of functions of the above form Apply the differentiation of above functions to solve problems

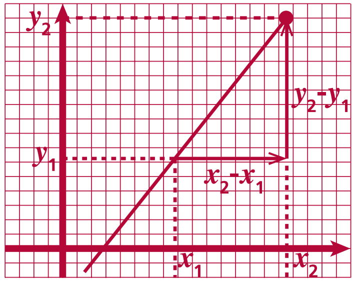
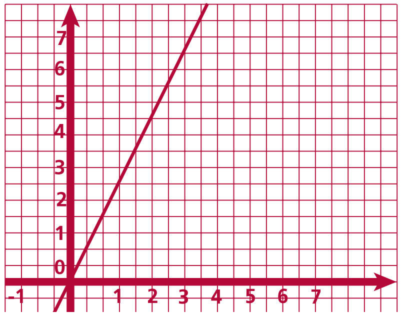
Resources Required

Whiteboards, rulers, *Geogebra*, calculator.

Teacher Reflections

Lesson Interaction

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
Section A – Rates of Change			
» We are going to look at rates of change. Where have we looked at rates of change before?	• The slope of a line • $m = \frac{y_2 - y_1}{x_2 - x_1}$ • Lines have a constant rate of change • Rate of change of a line can be found using 'rise over run' • Investigating the change from a table • Investigating the 'change of the change' from a table • If the first change is constant the pattern is linear • If the second change (change of the change) is constant then the pattern is quadratic. • If both 'change columns' develop in the same ratio, the pattern is exponential • If the rate of change is positive the pattern is increasing • If the rate of change is negative the pattern is decreasing • The money box problem	» Encourage students to recall as much as they can remember about rates of change from their Junior Certificate learning. » Write all the answers on the board. **Note:** Remind students that the money box problem is a function of Natural Numbers mapped to Natural Numbers. This will be important to remember later in the lesson as we can only analyse a continuous function using calculus.	» What prior knowledge do the students display?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Can you explain in words what the slope formula measures?	• How slanted a line is. • How steep a line is. • How the ys are changing as the xs change	» Draw a graph of a general line on the board going through points (x_1, y_1) and (x_2, y_2)  » Revise with the students that the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ measures how the ys are changing as the xs change	
» In pairs, write a sentence on your white board to explain how the ys are changing as the xs are changing when the slope of a line is 3.	• The ys increase by 3 units every time the xs increase by 1 unit.	» Draw a graph of $y = 3x$, $x \in R$ on the board as students write their sentence.  » Circulate to monitor progress. Facilitate discussion if there are difficulties. » Ask a pair of students to call out the answer.	» Can students verbalise the rate of change when the slope is 3?

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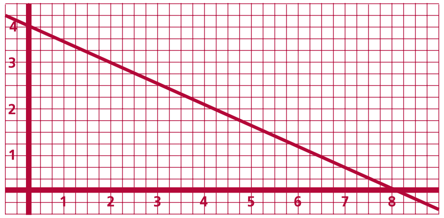
Teacher Reflections

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» In pairs, write a sentence on your white board to explain how the ys are changing as the xs are changing when the slope of a line is -0.5.	• The ys decrease by 0.5 units every time the xs increase by 1 unit.	» Draw a graph of $y = 4 - 0.5x$ on the board as students write their sentence.  » Circulate to monitor progress. Facilitate discussion if there are difficulties. » Ask a pair of students to call out the answer.	» Can students verbalise the rate of change when $m = -0.5$?
» Let's give some context to the xs and ys. Can anyone remember what the formula for speed is in terms of distance and time?	• $speed = \frac{distance}{time}$	» Write the formula on the board. **Note:** The accurate formula for speed is: $average\ speed = \frac{distance}{time}$ There is no need to mention the accurate formula here as students will discover this formula themselves on Student Activity 5.	

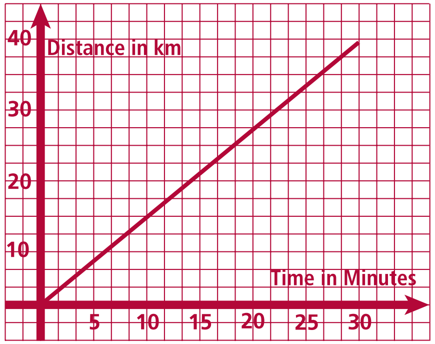
KEY: » next step • student answer/response

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Do the following question on your whiteboards. A woman drives along a 40km straight stretch of motorway in America as part of her journey along route 66. She puts on her cruise control to drive this section at a constant speed. It takes the woman 30 minutes to drive to the end of this stretch of motorway. At what speed did she travel along this road?	• $speed = \frac{40\,km}{30\,min} = 1\tfrac{1}{3}\,km/min$	» Write the question on the board. » Observe what students are writing. Assist them as required. » Ask a student to come to the board and write out the answer.	» Can students work with the speed formula? » Do they understand the units of measurement km/min?
» Let's represent this journey on a graph. Which variable is dependent? Distance or time?	• The distance depends on the time.	» Ask a student for the answer.	
» Which variable is independent?	• Time, because that is going to happen anyway.	» Ask a student for the answer.	

KEY: » next step • student answer/response

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Which variable do we traditionally put on the x-axis?	• The independent variable	» Ask a student for the answer.	
» Now draw a graph to represent the journey.	• Students draw a graph	» Ask a student to draw their graph to represent the journey on the board.  	» Do the students know how to set up their axes? » Can students draw the graph based on the information given?
» Can we relate the formula for speed, $speed = \frac{distance}{time}$ to the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$?	• They are both fractions. • Distance is on the y-axis • Time is on the x-axis		
» Can we use the slope formula to find the speed?	• Yes $m = \frac{40-0}{30-0} = 1\frac{1}{3}$	» Write on the board $m = \frac{40-0}{30-0} = 1\frac{1}{3}$ » Help students understand this on the graph drawn on the board.	» Can students see that slope and speed are the same formula in this example?

<table>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible and Expected Responses</th><th>Teacher’s Supports and Actions</th><th>Checking Understanding</th></tr>
<tr>
<td>» We can see that speed is a rate of change in this example. In the slope formula we measure how the ys change as the xs change. What rate of change are we measuring when we find speed?

» We will now look at other examples of rates of change. Working in pairs, complete Section A: Student Activity 1. Take five minutes to read it first without a pen in your hand.</td>
<td>• How the distance changes as time changes.</td>
<td>» Write this on the board: "Speed measures how the distance changes as time changes."</td>
<td>» Can students verbalise the changing quantities in speed?</td>
</tr>
<tr>
<td>» After 5 minutes ask the students to complete the table.</td>
<td>Students complete table.
<table>
<tr><th></th><th>Rates of Change</th><th>Independent Variable</th><th>Dependent Variable</th></tr>
<tr><td>1</td><td>y's change as the x's change</td><td>x</td><td>y</td></tr>
<tr><td>2</td><td>Distance travelled changes as time changes</td><td>t</td><td>d</td></tr>
<tr><td>3</td><td>Height of water changes as time changes</td><td>t</td><td>h</td></tr>
<tr><td>4</td><td>Length of metal rod changes as temperature changes</td><td>r</td><td>l</td></tr>
<tr><td>5</td><td>Number of bacteria changes with time</td><td>t</td><td>n</td></tr>
<tr><td>6</td><td>Production costs change with respect to the quality of the product manufactured</td><td>q</td><td>p</td></tr>
<tr><td>7</td><td>Height of a flower changes with respect to time</td><td>t</td><td>h</td></tr>
</table></td>
<td>» Distribute Section A: Student Activity 1.

» Observe what students are writing. Assist them as required.

» Ask students for their answers.</td>
<td>» Do students understand the concept of dependent and independent variables?

» Can students make the connection between slope, rate of change and the examples in the activity?

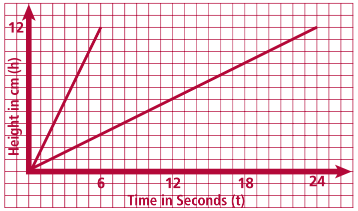
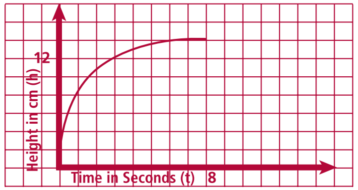
» Can students verbalise the rates of change?

» Can students think of another example of a rate of change?</td>
</tr>
</table>

Teacher Reflections

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» We have looked at rates of change from first year. Rates of change are all around us and are very important.		» Offer some more examples of rates of change like "10c per text" or "85c per minute for a phone call to a landline"	» Do students appreciate that they are surrounded by rates of change?
» Now, let's look at some of the vocabulary we use to describe rates of change. Working in pairs, complete **Section A: Student Activity 2**. Again, discuss the activity for five minutes first without pens in your hands and then fill in the Word Bank.	• Students complete the Word Bank. **Word Bank** slope rate of change increasing fast steep slower level no change constant decreasing the rate of decrease slows down	» Distribute **Section A: Activity 2** » Circulate to monitor progress. Facilitate discussion if there are difficulties. » By asking different groups create a class Word Bank on a poster for the wall. **Note:** Discuss and expand any misconceptions regarding rates of change here. **Note:** If "the rate of decrease slows down" arises as an observation, this could be expanded to informally discuss the underlying concept of the second derivative as the rate at which the slope is changing. If this arises keep the discussion very informal.	» Can students verbalise their reasoning for applying different rates of change to different parts of the graph? » Are student using the correct terms to describe rates of change?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Now we are going to look at **Section A: Student Activity 3 – Part 1.**		» Distribute **Section A: Activity 3 – Part 1.**	
» Describe the rate of change in Question 1.	• Height changes as time changes	**Note:** The volume of water in both containers increases at the same rate.	» Do students understand how to describe this rate of change?
» Working in pairs, work through this activity. Take five minutes to read it first without a pen in your hand.	• Students sketch two graphs.  	» Give students time to discuss what is happening in the containers. » Circulate to monitor progress. Facilitate discussion but allow time for students to arrive at what is happening.	» Do students understand that the rate of change of the height of water, as a cylinder fills, is a linear graph whereas the rate of change of the height, as the inverted cone fills, is curved?
» After 5 minutes ask the students to commence writing.	• It takes container B longer to fill because its radius is wider than the radius in container A. It takes the inverted cone 8 seconds to fill as the volume of a cone is one third the volume of a cylinder.	» Encourage students to see the link between the radius not changing in a cylinder and the constant rate of change in the height of the water level with time. » Ask one of the pairs of students to put their graphs on the board. » Use *GeoGebra* files to show the graphs of the containers filling.	» Do students understand that that cylinder B fills slower because container B has a bigger radius? » Can students represent these situations on a graph?
» What vocabulary from your Word Bank can you use to describe how the containers are filling?	 	» Ask a number of students to describe the graphs in words using the vocabulary from their Word Bank.	» Can students describe in words how the cone is filling?

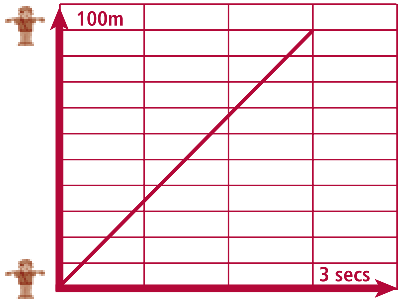
Teacher Reflections

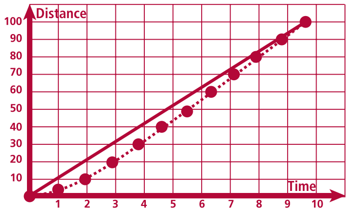
Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Now we are going to look at **Section A: Student Activity 3 – Part 2.**		» Distribute **Section A: Student Activity 3 – Part 2.**	
» Working in pairs, work through this activity.	• Students find that the slopes of [AB] and [OA] are both m = 2. Students conclude that the slope at any point on [OA] is 2 as a line has a constant rate of change. Students then conclude that the slope at the point C is 2 because it is one of the points that lie on the line [OA] that has a constant rate of change 2. • Students find that the slope of [AB] is m = 0.5. Students conclude that the height of water in the cylinder changes at a rate of 0.5 cm/sec. Students see that the height of water changes at a rate of 0.5 cm/sec at any time between 0 and 24 seconds because a line has a constant rate of change. Students then conclude that the rate of change at the point C is 0.5 cm/sec as C lies on the line whose constant rate of change is 0.5 cm/sec. • Students find that the slope of [AB] is m = 1.03 and the slope of [OB] is m = 1.50. Students conclude that the slopes of [AB] and [OB] do not help them find the slope at the point C as a curve does not have a constant rate of change.	» Observe what students are writing. Assist them as required. » Ask students to explain to the class their answers and reasoning to Q1, Q2 and Q3.	» Can students use the slope formula? » Can students relate the slope to the rate of change of the height of the water level with time? » Can students understand that the slope along a straight line is always the same? » Can students understand that the slope along the curve is changing?

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Recall the example of finding the constant speed of the car on cruise control along a long stretch of road.		» Remind students of the question if they cannot recall it.	» Can students remember this example?
» Do you recall the formula for speed in terms of distance and time?	• $speed = \frac{distance}{time}$	» Write the formula on the board. **Note:** Again, the accurate formula for speed is: $average\ speed = \frac{distance}{time}$ There is no need to mention the accurate formula here as students will discover this formula themselves on **Section A: Student Activity 5.**	» Do they know the general formula for speed?
» Can you describe the rate of change 'speed' in terms of distance and time?	• Distance changes as time changes	» Write the wording on the board.	» Can they verbalise constant speed as a rate of change in terms of distance and time? **Note:** There is no need to talk about average or instantaneous speed at this point. This will be developed in the following activities.
» Let's now look at **Section A: Student Activity 4.**		» Distribute **Section A: Student Activity 4.**	

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Working in pairs, work through this activity. Take five minutes to read it to yourselves without a pen in your hand. » After 5 minutes ask students to commence writing. » Wrap up: We see from our investigations that if we know the slope of a line, there is no difficulty in getting the slope of any other point on that line.	• Students draw a graph.  • The train passes the two students at a speed of 120 km/hr. • The train also passes the teacher at 120 km/hr because the train is travelling at a constant speed.	» Circulate to monitor progress. Facilitate discussion if there are difficulties.	» Can students represent the situation on a graph? » Do they understand that the graph is linear? » Can students use the speed formula? » Do students understand that the train will pass the teacher at the same speed because it passes at a constant rate of change?
» Let's look at another example of speed. Look at **Section A: Student Activity 5.**		» Distribute **Section A: Student Activity 5.**	

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Working in pairs answer the first 3 questions. Take five minutes to read it to yourselves first.	• Usain Bolt's speed for the race is 10.44 m/s • His speed was not 10.44 m/s during the whole race because we can see from the graph that he was slower at the beginning of the race. • As he is not running at the same speed during the whole race, 10.44 m/s represents his average speed.	» Circulate to monitor progress. Facilitate discussion if there are difficulties. » Engage students in a classroom discussion about these three questions. » Write the words 'Average Speed' on the board as a new term. » Introduce the correct formula for the speed of a journey: $average\ speed = \dfrac{distance}{time}$ **Note:** Discuss and expand on any misconceptions regarding these questions.	» Do students understand that the rate of change is not constant during the whole race? » Can they see this from the graph? » Do they therefore understand that 10.44 m/s is a representation of the average speed for the race?
» Working in pairs, answer question 4.	• Students draw in a line on the graph.  ii m = 10.44 iii The slope of the secant line is the same as the average rate of change.	» Observe what students are writing. Assist them as required. » Ask a student to draw their graph on the board. » Ask a student for the answer to part (ii) » Ask a student to give their answer to part (iii) and explain their reasoning. » Engage the students in a discussion about secants and how they represent the average rate of change between two points.	» Can students see that the slope of the secant line between the two end points of this curve is the same as Usain Bolt's average speed?

Teacher Reflections

KEY: » next step • student answer/response

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» In conclusion, the slope of the secant line between two points is the same as the average rate of change between two points.		» Write the conclusion on the board.	
» In pairs, have a discussion about question 5.	• Any discussion is good – it is not necessary for the students to come up with the correct strategy as this will be developed in the next worksheet.	» Facilitate and encourage any discussion and ideas.	

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Teacher Reflections

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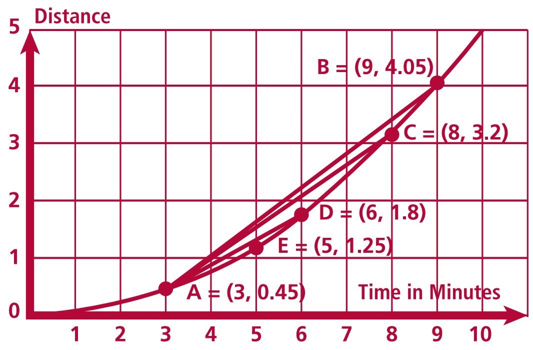
<table>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible and Expected Responses</th><th>Teacher’s Supports and Actions</th><th>Checking Understanding</th></tr>
<tr><td>» So far we have investigated a few different situations where we see rates of change. The cylindrical containers and the passing train had a constant rate of change whereas the cone container and Usian Bolt’s race did not have a constant rate of change.</td><td></td><td>» Write a comparison table on the board.
<table>
<tr><td>Filling cylindrical containers
Train passing</td><td>Filling an invertd cone container
Usain Bolt's race</td></tr>
<tr><td>• Linear Graph
• No problem finding the slope at any single point on the line</td><td>• Curved Graph
• Unable to find the exact slope at a single point on the line
• Best we can do is find an average rate of change</td></tr>
</table></td><td>» Are students able to recall what they have explored about rates of change this far?</td></tr>
<tr><td>» The situations that had a constant rate of change were represented as a line whereas the situations without a constant rate of change were represented as a curve.</td><td></td><td></td><td>» Do students recognise that there is a difficulty in finding the slope at a single point on a curve?</td></tr>
<tr><td>» On a linear graph we had no problem finding the slope of any point on the line whereas on a curved graph we were unable to find the exact slope at a single point on the curve; the best we could do was find an average rate of change.</td><td></td><td></td><td></td></tr>
<tr><td>» Now we are going to look at **Section A: Student Activity 6 – part 1** to see if it is possible to get the slope at a single point on a curve.</td><td></td><td>» Distribute **Section A: Student Activity 6 – part 1.**</td><td></td></tr>
</table>

KEY: » next step • student answer/response

<table>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible and Expected Responses</th><th>Teacher’s Supports and Actions</th><th>Checking Understanding</th></tr>
<tr>
<td>» Work through this Activity in pairs. Take five minutes to read it to yourselves first.</td>
<td>• Students write their answers
1. Victoria’s average speed is 0.5km/min.
2. Students draw in secants.


3. Students fill in the table.
<table>
<tr><td>Slope of Secant [AB]</td><td>0.60</td><td>Average speed between A and B = 0.6 km/min</td></tr>
<tr><td>Slope of Secant [AC]</td><td>0.55</td><td>Average speed between A and C = 0.55 km/min</td></tr>
<tr><td>Slope of Secant [AD]</td><td>0.45</td><td>Average speed between A and D = 0.45 km/min</td></tr>
<tr><td>Slope of Secant [AE]</td><td>0.40</td><td>Average speed between A and E = 0.4 km/min</td></tr>
</table>
4. The slope of secant AE will be the best estimate of the slope at the point A.
5. By looking at the slopes of secants nearer to the point A. The slope of the secant closest to point A will be the best estimate.</td>
<td>» Circulate to monitor progress. Facilitate discussion if there are difficulties.
» Encourage students to discuss this activity.
» Ask a student to put the graph on the board.
» Ask another student to put the table on the board.
» Ask a student to verbally give their answer to question 4 and explain their reasoning.
» Ask a number of students to verbally answer question 5 and explain their reasoning. Encourage a classroom discussion on this question.</td>
<td>» Do students understand that the slope of the secant between two points is the same as the average rate of change between two points?
» Have students recognised that the slope of the secant closest to point A will be the best estimate for the rate of change at the exact point A?</td>
</tr>
</table>

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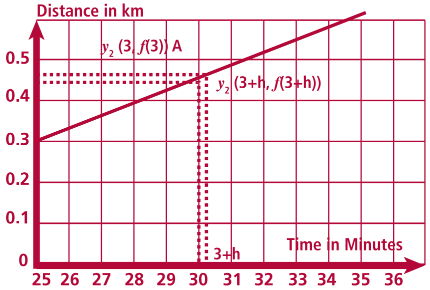
Teacher Reflections

<table>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible and Expected Responses</th><th>Teacher's Supports and Actions</th><th>Checking Understanding</th></tr>
<tr>
<td>» Now let's look at Section A: Student Activity 6 – part 2.</td>
<td>• Students fill in the table.
<table>
<tr><td>The interval of x values for secant AB=</td><td>6</td></tr>
<tr><td>The interval of x values for secant AC=</td><td>5</td></tr>
<tr><td>The interval of x values for secant AD=</td><td>3</td></tr>
<tr><td>The interval of x values for secant AE=</td><td>2</td></tr>
<tr><td>The interval of x values for secant AF=</td><td>1</td></tr>
<tr><td>The interval of x values for secant AG=</td><td>0.5</td></tr>
</table></td>
<td>» Observe what students are writing. Assist them as required.
» Ask a student to write up the table on the board.
» Encourage a class discussion on question 2.</td>
<td>» Do students understand how to find the interval of x values?</td>
</tr>
<tr>
<td>» Is there any way we can make the interval so small that it is practically zero?</td>
<td>• Students discuss the smallest possible interval they can think of.
• Let the interval be the smallest number in the world.
• Use a limit and let the interval approach 0.</td>
<td>» Remind them of their study on limits and how we can use the limit as the interval approaches 0 to make the interval as close as possible to zero without actually becoming 0.
» Introduce students to the variable 'h' or 'Δx' as how we describe this interval of x values that we would like to make very small.
» Therefore the slope of the secant closest to the point A will be found using
$$\lim_{h\to 0}\frac{y_2-y_1}{h} \quad \text{or} \quad \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$
» Draw a graph on the board to help explain this, see Appendix 1.


» Show the <i>GeoGebra</i> file of <i>Angry Birds</i> to reinforce this.</td>
<td>» Do the students recall their study of limits?</td>
</tr>
</table>

Development Team

Teacher Reflections

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» We've seen how rates of change appear in lots of different contexts. Let's look at one more such context. » In *Angry Birds* what is the aim of the game? » How do you do this? What is your strategy?	• To kill the pigs using the least number of angry birds. • Launch the angry birds at different angles and different speeds to hit the target.	» Open up the *GeoGebra* file Angry-Birds-and-Calculus.ggb. Make sure 'Show Background' is clicked. **Note:** there is a static diagram of the *Angry Birds* file in the Appendix of the T&L plan. » Demonstrate the game by flying the angry bird across the screen using the slider Fly.	
» What shape does the flight path of the angry bird make? » How might we calculate the rate of change of the angry bird at a given point during its flight?	• A quadratic. • Measure it. • Construct a secant. • Measure the slope at the point.	» Fly the angry bird again this time with 'Show Flight Path' clicked and 'Show Background' unclicked. » Demonstrate that the bird's flight path is part of a quadratic function by clicking on 'Show Full Quadratic'. » Show the point on the graph by clicking 'Show Point'. » Remind students that they faced a similar problem with Victoria Pendelton. » Show the secant on the graph by clicking on 'Show Secant'.	» Do students recognise the flight path as being quadratic in shape? » Can students apply a similar approach to that introduced with Victoria Pendelton to suggest a solution?

KEY: » next step • student answer/response

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» How might we modify our secant so that it would provide a better estimate of the rate of change of the angry bird at the point?	• Construct a secant using a point closer to the point of interest.	» Drag the purple point towards the blue point as a demonstration of the idea of a better secant. » Show the slope of the secant and average rate of change calculation by clicking on 'Show Secant Workings'.	» Do students understand that the closer the two points on the function are, the better the secant estimates the slope at the point of interest?
» How might we get the actual rate of change at the point of interest – as opposed to an estimate?	• Drag the purple point until it is directly on top of the blue point.	» Drag the purple point over the blue point such that the secant disappears and the slope and average rate of change calculations are undefined.	» Do students understand the closest the two points can be is when one sits directly on top of the other?
» Does this approach work for getting the instantaneous rate of change? Why doesn't it work?	• No. • The two points are on top of each other so we no longer have a secant and so cannot get the slope. • When we go to calculate the slope we end up dividing by zero which is undefined.	» Discuss the secant workings and highlight division by zero.	» Do students recognise that the approach breaks down due to division by zero?
» We would like to know what happens when the purple point approaches the blue point so that the two points are ever closer but never directly on top of each other. Is there a tool in mathematics which allows us to investigate such an occurrence?	• Limits.	» Remind students of various examples of limits e.g. the *introduction to* e.	» Do students understand that we could use a limit to find the slope at a point?

KEY: » next step • student answer/response

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» What limit do we want to find?	• The limit of the slope formula between the two points as the distance between the two points gets smaller and smaller.	» Write up a semi-mathematical expression for this limit e.g. $\lim (slope)$ as distance between points gets smaller.	» Do students understand that we are looking at the limit of the slope formula as the distance between the two points gets smaller?
» This expression is messy. Could we use some variables to write it in a more mathematical form?	• Yes, using the slope formula.	» Re-write this expression using the standard slope formula $\lim \frac{y_2 - y_1}{x_2 - x_1}$ as distance between points gets smaller.	
» The choice of the second point depends on the location of the first point. For this reason, could we re-write the denominator in terms of the location of the point of interest?	• Yes, x_2 - x_1.	» Suggest that we could use the horizontal distance between the two points as a measure of the distance between the two points. » Demonstrate that this distance is just x_2 - x_1 and suggest that we re-name this distance h. » Re-write the expression in the following form $\lim_{h \to 0} \frac{y_2 - y_1}{h}$ encouraging students to discuss how we might write the limit (as distance between points gets smaller) of the expression mathematically.	» Do students recognise that this expression is not written mathematically? » Do students understand that the horizontal distance is a good measure of how close the two points on the function are? » Can students write the limit as $h \to 0$?

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» This is Calculus. Calculus is the branch of mathematics that allows us to calculate instantaneous rates of change.		» Demonstrate how y_1 may be written as $f(a)$ and how y_2 may be written as $f(a+h)$. » Show this representation in the *Geogebra* file by clicking on 'Show Labels'. » Write out the full definition of the derivative $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ » Show students the definition of the instantaneous rate of change in *GeoGebra* by clicking on 'Show Calculus'. » Discuss that the closest secant offers a very good approximation for the instantaneous rate of change but that even such a small error can have consequences for real-world calculations.	» Do students understand where the new notation has come from? » Do students understand that we are simply looking at the limit of the standard slope formula re-written in terms of a single point? » Do students understand that calculus provides the instantaneous rate of change as opposed to the slope of the secant which only provides an estimate? » Do students understand that the error on the rate of change provided by the secant can be significant when carrying out rate-of-change calculations in the real world?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» We have seen that a secant provides a geometrical representation of average rate of change. There is also a geometrical representation of instantaneous rate of change. Could you suggest what that is?	• It's where the secant only touches the function at one point • It's the tangent to the curve at that point	» On the *Geogebra* file, move the purple point away from the blue point then bring them back together. Ask students if they can identify the geometrical relationship which the secant is approaching?	» Do students recognise that the secant is getting closer to being a tangent to the curve as the points get closer together?
» What do you notice about the slope of the tangent to the curve?	• It is the same as the instantaneous rate of change at that point	» Calculate the slope of the tangent by clicking on 'Show Tangent'.	» Do students understand that the slope of the tangent to the function is the instantaneous rate of change at the point of contact?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
Section B – Rules of Differentiation			
» We now see the need for being able to calculate the instantaneous rate of change. We also see that we can calculate instantaneous rate of change at a given point using $\lim\limits_{h \to 0} \frac{f(a+h)-f(a)}{h}$			
» This approach may be extended to calculate a general expression which tells us the instantaneous rate of change at any point (x) along the function.		» Write up the generalised limit $\lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$ and discuss the differences between this limit and $\lim\limits_{h \to 0} \frac{f(a+h)-f(a)}{h}$	» Do students understand the difference between the two limits presented? » Can students calculate derivative functions by first principles?
» Proceed to apply the limit $\lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$ to calculate derivative functions of linear functions and quadratic functions (Differentiation by First Principles - H.L. only).			» Do students understand what the derivative function means? » Can students correctly apply the derivative function to calculate instantaneous rates of change?

KEY: » next step • student answer/response

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» We will now investigate how to work out the instantaneous rate of change practically – such that we can perform these calculations efficiently. This will allow us to use calculus to solve real-world problems. » We will do this by looking at how the slope of various functions changes from one location to the next on a function and see if there is a pattern to this change. Identifying a pattern in the slopes would allow us to predict slope (and instantaneous rate of change) at any point. » We will start by examining linear functions.			
» For each graph A-D in **Section B: Student Activity 1**, can you write down their slopes at each point given in the table?	• The slope is zero. • The slope is the same everywhere. We are looking at a straight line. • A horizontal line has a slope of zero everywhere. • The rise is zero – therefore the slope is zero.	» Distribute **Section B : Student Activity 1** to students. » Circulate to check if students understand the task. » Circulate to check/reinforce students' understanding of slope of linear functions through questioning – e.g. Do you need to calculate the slope at each point? Why?	» Do students understand that slope means rate of change? » Do students understand that the slope of a horizontal line is zero? » Can students apply their Junior Cert. understanding of slopes of linear functions? » Do students recognise that the slope of a linear function is the same at all points?

KEY: » next step • student answer/response

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» If I came up with a new function e.g. $q(x) = 95$, could you tell me about the slope of that function just by looking at the form of the function?	• The slope is 95 (incorrect). • The slope is zero. • The slope is zero everywhere. • That's the equation of a horizontal line – therefore it has zero slope.	» Ask students what type of function is represented by $f(x) = c$? » Suggest to students that they sketch this function? » Ask students if they could have predicted the shape of the graph without sketching the function?	» Do students recognise that the function $f(x) = c$ represents a line parallel to the x-axis? » Can students read the slope from the function form?
» If you were given any horizontal line could you explain how to find the slope at any point.	• It's just zero. • Its slope is zero everywhere.	» Encourage students to generalise what they have just investigated.	» Can students generalise the pattern they have just discovered? » Can students explain in words how to predict the slope of a horizontal line?
» Can you explain why the rate of change of a function $f(x) = c$ should be zero?	• Rate of change means slope and we've seen slope is zero. • The function remains steady at the same value all the time so it doesn't change. • The y-value is constant so there is no change.	» Remind students that rate of change and slope are the same thing. » Encourage students to discuss what rate of change means. » Encourage students to explain what a function would need to do to have a zero rate of change. » Encourage students to sketch out an example of a graph where the rate of change is zero.	» Do students understand that rate of change is equivalent to slope? » Do students understand that a horizontal line means no change in the function and that this means a zero rate of change?

KEY: » next step • student answer/response

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» We have looked at a really simple example of a linear function – a horizontal line. We will now look at other types of linear function.		» Distribute **Section B: Student Activity 2** to students.	
» For each graph A-F in **Section B – Student Activity 2**, can you write down its slope at each of the points indicated in the table?	• The slope is given by $\frac{rise}{run}$ • The slope is the same at all points along each graph – we're dealing with a straight line. • We only need to calculate the slope at one point along the line.	» Circulate to check that students can calculate slope. » Circulate to check that students are calculating (or predicting) the slope at all points on each line. » Encourage students to calculate slope in different ways (formula, graph etc.). » Sketch the graph on the board or graph it using *GeoGebra*.	» Can students apply $\frac{rise}{run}$ to calculate slope? » Can students read slope directly from the graph? » Do students understand that slope is the same at all points on a straight line?
» If you were presented with a straight-line graph in the domain $-4 < x < 4$ and you were asked to determine the slope of the line when $x = 15$ how would you do it?	• Extend the graph • Calculate the slope at any point – this is the slope at every point since we have a straight line		» Do students understand that knowing the slope at one point on a straight line means that they know the slope at all points on the line?
» If you were presented with a new linear function $f(x) = 13.5x + 5$ could you describe what its slope is for all values of x?	• It's slope is 13.5 • Its slope is the same everywhere i.e.13.5 • Its slope does not change since we have a straight line. • The slope is just the co-efficient of x.	» Make links to the function form of the equation of a straight line $y = mx + c$. » Encourage students to sketch out the function – ask them how they are doing so (using their knowledge of slope and intercept).	» Can students read slope directly from a function without the need for a graph?

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Can you generalise the pattern you have just discovered? Suppose you were given a function $a(x) = 7x$ or $b(x) = -8x$ or $g(x) = nx$, how would you calculate its slope?	• Take the co-efficient of x.	» Encourage students to look back through their table together and explain how their slope was related to the original function each time.	» Can students apply their learning to find the slope of any linear function $f(x) = nx$?
» Suppose we modified the previous three functions so that they now read $a(x) = 7x + 2$ or $b(x) = -8x + 5$ or $f(x) = nx + c$, how would you calculate their slopes?	• The same way as before. • Slope is not affected by the number added on – only by the co-efficient of x. • Each graph is just a shifted version of the previous graphs so the slope is unaffected.	» Encourage students to sketch the new graph with the corresponding old graph and compare their slopes. » Make links with shifting and scaling of functions. » Discuss the functional form of a linear function ($f(x) = mx + c$) and what each term in the function represents. » Make the link between this approach and that used for functions which produce horizontal lines. Does our current approach work for those functions? Yes the previous horizontal line functions may be written as $y = 0x + c$).	» Do students understand the difference between slope and intercept? » Do students understand what each term in a linear function represents? » Do students understand that slope is unaffected by adding a constant amount to a function? » Can students apply their learning to find the slope of any linear function $f(x) = nx + c$? » Can students describe verbally how to calculate the slope of a linear function?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Many real-life situations are modelled by quadratic functions e.g. a projectile falling through the air. One way of determining how quickly a projectile falls is to calculate its rate of change as it falls.			
» Given the quadratic function $f(x) = x^2$, if you were told that the slope of the function at the point x = -3 is -6, could you say what the slopes of the function are at all other points?	• No. • The slope is not the same at all points on a curve. • The slope changes as you move along a curve. • Because of symmetry I can say that the slope at x = 3 is 6.	» Display the function $f(x) = x^2$ on the board using a sketch or *GeoGebra*. » Move around the curve and discuss slope as you go.	» Do students appreciate that the rate of change is different at all points on a curved graph?
» Given that we are dealing with a curve what do we mean by the slope at a point?	• The slope of a tangent to the curve at that point.	» Using a sketch or *GeoGebra* add a tangent to the curve at one point. Move the tangent around the curve and ask what is happening to the slope as we do so. Does the slope remain the same throughout? » Calculate the slope of the tangent at a couple of points along the curve.	» Do students understand that when we say slope at a point on a curve what we mean is the slope of a tangent to the curve at that point? » Do students understand what a tangent is? » Can students calculate the slope of different tangents from the graph?
» We have now identified a problem with rates of change for quadratic functions – they are not the same so knowing the rate of change at one point does not mean we know the rate of change at any point. Do you think it's the same for other functions that are non-linear?	• Yes. • Any function whose graph is curved will not have the same rate of change everywhere.	» Sketch different examples of non-linear functions on the board.	» Do students understand that non-linear functions do not have a constant rate of change?

Teacher Reflections

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Can we predict what the rate of change (slope) of a quadratic function will be at any point?	• Maybe there is a pattern to how the slopes change.		
» Ask all students to complete **Section B: Student Activity 3(a)** only – for the function $f(x) = x^2$.		» Distribute **Section B: Student Activity 3** to students.	
» Explain how to complete the task • Measure the slope of the tangents at each point on the graph(A-G). • Fill in these slopes in the table. • Complete the Change column in the table. • Graph the slopes as a function of x on the graph paper provided. • Investigate if there is a pattern in the slope values using the table and the graph. • Write down the pattern of the slopes.		» Support activity with *GeoGebra* file if possible. » Circulate to ensure students understand the task and can read slopes from the graph. » Encourage students to use rulers to help measure the slopes. » Remind students that each tangent is a straight line so you can measure its slope at any point on the tangent.	» Can students calculate the slope of a tangent accurately? » Can students complete the Change column in the table and recognise what the values represent in terms of the pattern of the slopes? » Can students complete the graph of $f'(x)$ and recognise the resulting shape?

KEY: » next step • student answer/response

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Are the slope values you measured predictable in any way?	• Yes – they form a linear pattern. • Yes – they all lie on the same straight line.	» Encourage students to discuss the change column in the table and the shape of the graph.	» Do students recognise that the slopes follow a linear pattern? » Do students understand what a constant change and a straight-line graph means in terms of the underlying pattern?
» For the function $f(x) = x^2$, could you tell me what the slope of that function is when $x = 8$? How about when $x = 57$? How might you do it?	• Extend my graph and measure the slope of the tangent at that point. • Use my slope pattern to predict what the slope will be at each point.	» Encourage students to discuss different ways in which this could be done? Are some approaches better than others? Explain. » Demonstrate to students that their prediction is correct by measuring the slope of a tangent at $x = 57$ using *GeoGebra*.	» Do students recognise that the pattern they have discovered allows them to predict the slope of their function $f(x) = x^2$ at any point along the curve?
» The slopes of the function $f(x) = x^2$ themselves form a pattern. The slopes form a linear pattern and can be represented by a linear function. We call this function the slope function, the differential function or simply the derivative. There are various different notations used to denote the slope function, including $f'(x)$ and $\frac{dy}{dx}$. » This slope function allows us to calculate the slope of our original function at a given point.		» Write up new language and notation on the board and encourage students to record it. » Link the new language and notation to the table and graph » Stress the fact that we have created a new function and that this allows us to calculate slopes at any point.	» Do students recognise that the slopes themselves make up a function? » Do students appreciate that the slope function tells us what the slope of our original function is at any point? » Do students understand the language and associated notation used to describe the slope function?

KEY: » next step • student answer/response

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» If you were given a different quadratic function – e.g. $f(x) = 2x^2$, $f(x) = 3x^2$ or $f(x) = 4x^2$, could you predict what their slopes would be when $x = -20$? What would you need? How might you find this out?	• We would need the slope functions for each of those functions. • We would just multiply our previous slope function by the co-efficient of x^2. • We could graph our function and measure the slope of the tangent at that point. • We could graph a piece of our function and use the slopes of the tangents to work out our slope function.	» Give different groups of students one of the three functions ($f(x) = 2x^2$, $f(x) = 3x^2$ or $f(x) = 4x^2$) to complete in the same way as the function $f(x) = x^2$ (see **Section B: Student Activity 3(b), 3(c) and 3(d)**). » Circulate to ensure students are on task. » Ask groups of students to come to the board and describe their results. » Encourage different groups who investigated the same function to agree/disagree with the results presented at the board. » Encourage groups who investigated a different function to listen and to compare and contrast what's presented at the board to their own findings. » Ask students from the groups who are not presenting to predict what the slope of the function being discussed function will be at a specific point.	» Do students understand that the new functions are different in shape to the function $f(x) = 2x^2$ and so will have different slopes? » Do students understand that each new function will have its own slope function which is needed if we want to calculate their slopes? » Do students recognise similarities and differences between their own work and that of their peers? » Can students use the different slope functions to predict what the slope at any point on a given quadratic will be?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Can we summarise the work the class as a whole has done? Is there anything common across all of the functions investigated?	• The slope functions for each quadratic are different. • The slope functions of each quadratic is a linear function.	» On one side of the board, summarise the work completed in a table consisting of the functions investigated and their slope functions. » Write up the main findings on the board and encourage students to make a note of these.	» Do students understand why each function's slope function is different? » Do students recognise that the slope functions of all our quadratic functions are linear functions?
» When we investigated linear functions we discovered that we don't actually need a graph to determine what the slope is at any point – we can read the slope directly from the function itself. Can we do the same for a quadratic?	• No, because the slope is not constant across the function – it changes from place to place.	» Bring student's attention to the summary table on the board and in **Section B: Student Activity 3(e)**. » Ask them if they can identify a pattern which relates the slope function to the original function? » Ask them to discuss this pattern and to describe how they might use the original function to find the slope function.	» Do students recognise the relationship between the original function and the slope function?

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» If you were given the function $f(x) = 5x^2$ could you write down its slope function? Could you predict what the slope of this function would be when $x = 12$? How about when $x = -4$? How did you do it?	• The slope function is $f'(x) = 10x$. • We got it by multiplying the co-efficient of x^2 by the power and reducing the power by 1. • The slope of the function when $x = 12$ is 120 and the slope when $x = -4$ is -40.	» Encourage students to discuss their approach. » Encourage students to compare results and discuss any differences. » Ask students how they might check if their predictions are correct. » Using *GeoGebra* confirm students' results by measuring the slope of the tangent at each point. » Ask students to write down their own quadratic functions and get their partner to work out their slope functions. Swap and discuss answers.	» Can students recognise the relationship between a function and its slope function and use this to write down the slope function of $f(x) = 5x^2$? » Can students use the slope function to predict the slope of the function at the points given? » Do students understand that they can determine the slope function of a quadratic simply by inspecting the original function?
» Given the general quadratic function $f(x) = ax^2$ could you write down its slope function?	» $f'(x) = 2ax$	» Write a few possible options on the board and ask students to determine which is correct » Encourage students to explain to each other why they chose their answer	» Can students apply their approach to finding a slope function to a more general example? » Are all students comfortable using calculus language and notation?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Several real-world situations, including the volume of three-dimensional objects like cuboids, spheres and cones, are modelled by cubic functions. Accordingly we would also like to be able to calculate the rate of change at any point on a cubic function.		» Sketch a cubic function (or graph it with *GeoGebra*) so students understand what it looks like.	» Do students recognise that the rate of change of a cubic function changes from one point to the next? » Do students recognise that when they are asked to find the rate of change at a point on the cubic function they are faced with the same problem as they were met with when investigating quadratic functions?
» Given a cubic function, can you determine how to calculate its slope at any point? How might you go about this?	• See if the slopes of the cubic function form a pattern? • Construct a tangent at that point and measure its slope. • Measure the slopes of different tangents along the cubic function and see if there's an obvious pattern. • Find the slope function for the cubic function.	» Distribute **Section B: Student Activity 4** to students. » Explain to students that the slopes of the tangents at each point are already calculated (they are very difficult to read from the graph). » Ask students to complete the tables for $f(x) = x^3$ and $f(x) = 2x^3$ (**Section B: Student Activity 4(a) and 4(b)**), and to see if they can identify a pattern among the slopes. » Ask students to complete the graphs for $f(x) = x^3$ and $f(x) = 2x^3$, and to see if they can identify a pattern in the graph of the slopes. » Circulate to see if students understand the task.	» Can students read the slopes at each point correctly? » Can students complete the table and use it to determine the pattern of the slopes? » Do students recognise from the shape of the graph that it represents a quadratic function? » Can students link the pattern in the change columns of the table to the shape of the graph?
» Could you calculate what the slope of the function $f(x) = x^3$ is at the point on the function when $x = 16$? How would you do it?	• Use the slope function for $f(x) = x^3$. • Extend my graph, construct a tangent at that point and measure its slope.	» Remind students that they have completed a similar task when investigating quadratics. » Ask students to discuss the advantages/disadvantages of the different approaches.	» Do students understand that every cubic function has its own slope function and that this may be used to calculate its slope at any point?

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Can we summarise what we've discovered about the slope function of a cubic function?	• Each cubic function has its own slope function • The slope function of all cubic functions is a quadratic function.	» Write each of the original functions on the board with their slope function beside them. » Ask students why each cubic function has a different slope function?	» Do students recognise that the derivative of a cubic function is a quadratic function? » Do students appreciate that each cubic function has its own slope function and why this is so?
» Could we write down the slope function of a cubic without the need for a table or graph? How would you do so?	• Multiply the coefficient by the power and decrease the power by 1.	» Highlight the summary table for cubic functions on the board. » Direct students to their own copy of the summary table in **Section B: Student Activity 4(c)**. » Include some additional functions and their slope functions in the table if required. » Ask students to discuss their ideas with their partners and to provide a written explanation of how they would find the slope function in their Activity sheet.	» Can students identify the general rule for finding the slope function of a cubic function?
» Given the function $r(x) = -2.5x^3$, could you calculate its slope when $x = 4$? How would you do so?	• $-2.5 \text{ x } (4)^3 = -160$ (incorrect) • Find the slope function and substitute $x = 4$ into it. • Plug $x = 4$ into $r'(x) = -7.5x^2$ to get $r'(4) = -120$	» Demonstrate the correct solution by constructing the function and tangent in *GeoGebra.* » Stress how our ability to find the slope function directly from the original function reduces our workload significantly. » Encourage students to make up their own cubic functions, swap over with their partner and find the corresponding slope functions.	» Do students understand that they need to find the slope function first? » Can students find the slope function by inspecting the original function. » Do students recognise the type of function they need to create? » Can students determine the slope functions correctly? » Can students identify correct and incorrect answers and explain their reasoning?

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» We have now discovered a quick rule for writing down the slope function of any cubic function $f(x) = ax^3$. Does this rule/approach look familiar in any way?	• We applied the same approach when finding the slope function of a quadratic function.	» Ask students to think back to our previous activity on quadratics. » Display the summary tables for both quadratics and cubics on the board.	» Do students recognise that they applied the same approach to reading the slope function from both a quadratic and cubic function?
» Quadratic and cubic functions are all part of the same family of functions known as polynomials. Let's look at a higher-order polynomial - $k(x) = 5x^4$. Based on your investigation of quadratics and cubics so far, could you suggest what its slope function is? How did you do it?	• Graph it and investigate the pattern of the slopes. • Do the same as we did with quadratics and cubics – multiply by the power and decrease the power by 1. • $k\,'(x) = 20x^3$.	» Encourage students to use inspection of the function to try to get to the slope function. » Confirm their answer by graphing the derivative of the function using *GeoGebra* or by choosing a point on the function and measuring its slope at that point.	» Do students understand what a polynomial is? » Can students extend what they have discovered for quadratics and cubics to higher-order polynomials?
» Linear functions are also members of the polynomial family. Can we find the slope of a linear function using the same rule we used for quadratics and cubics?	• No – it won't work (incorrect). • Yes – it will work for some linear functions if we write in the power of x as x^1 but it doesn't work for the functions which represent horizontal lines (incorrect). • Yes it works for all functions if we write $f(x) = 3$ as $f(x) = 3x^0$.	» Distribute **Section B: Student Activity 5**. » Ask students to complete the table by filling in their results from **Section B: Student Activities 1 – 4**. » Encourage students to discuss if they think the same rule applies across all the polynomial functions we investigated. » Highlight that for the rule to work the polynomial must be written in the form $f(x) = ax^n$. » Highlight the presence of this rule in the maths tables (Page 25).	» Do students recognise that x may be written as x^1 and that 1 may be written as x^0? » Do students appreciate that the same simple rule may be applied to find the slope function of any function of the form $f(x) = ax^n$?

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Given any polynomial could you now write down its slope function?	• Yes.	» Ask students to complete the table in **Section B: Student Activity 5 Q2 & Q3**. » Circulate to check that students are completing the task correctly. » Ask students how they would find the slope of a given function at a specific point e.g. What is the slope of $f(x) = 0.5x^2$ at the point on the function when $x = 7$? » Check if students are re-writing the functions into the required form before applying their rule. » Encourage students to make up additional questions, to swap these over and to try them out.	» Can students apply their rule to correctly find the slope function? » Do students recognise when a function must be re-written in the correct form before differentiating by rule? » Can students use their knowledge of indices to re-write functions in the correct form? » Can students write down the general rule using the same mathematical notation as presented in the maths tables? » Do students understand what a polynomial function is?
» We have investigated various functions of the form $f(x) = ax^n$ and have discovered a reliable and quick approach to determining their slope functions. » We now want to do the same for polynomials which are a combination of a quadratic, a linear and a horizontal line.		» Write the quadratic $f(x) = x^2 - x - 6$ on the board. Explain how it may be viewed as a combination of three separate functions added together. » Write each part out as a separate function. » $g(x) = x^2$ » $h(x) = -x$ and » $k(x) = -6$ and ask students to write down the slope function of each one. » Write each slope function beside its original function.	» Do students recognise that this function is formed by adding three separate functions? » Can students apply their knowledge of differentiation to write down the correct slope function for each part?

KEY: » next step • student answer/response

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Given a function $f(x) = x^2 - x - 6$, how could we determine how to find its slope function?	• Graph it. • Construct tangents to the function and measure their slopes.	» Distribute **Section B: Student Activity 6** to students. » Ask students to complete Part (a) [and (b) if time permits or give different quadratics to different groups] in a similar way to the activities on quadratic and cubic graphs.	» Can students complete the table and graph, and use them to write down the correct slope function?
» Having worked out the slope function by examining tangents to the function – could you suggest a more efficient approach?	• Find the slope function of each part and add them together to get the complete slope function.	» Ask students to relate their slope function to what we have already written on the board? Can they spot any similarities? » Link the various parts of the slope function to the individual parts on the board by highlighting or using arrows.	» Do students recognise that the slope function is simply the slope functions of the individual parts added together? » Do students understand how to find the slope function of any co-polynomial function?
» Can you use this knowledge to find the slope functions of different polynomials?	• Yes.	» Ask students to complete **Section B: Student Activity 6(c)**. » Circulate to check student's work. » Ask students to write down the rule for a general function formed by adding different functions together.	» Can students apply their knowledge to find the slope function of various co-polynomial functions?

KEY: » next step • student answer/response

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Can you complete the Tarsia in **Section B: Student Activity 7** by matching the correct function with its corresponding slope function?	• Yes.	» Distribute **Section B: Student Activity 7** to students. » Ask students to complete activity in groups. » Circulate to discuss any problems students are having with the activity.	» Can students apply their knowledge of differentiation by rule to match up the correct functions? » Are students comfortable with the different notation used? » Do students recognise that the function must be written in a specific form if their rule is to be applied? » Can students use their knowledge of indices to correctly re-write each function into the required form?

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Can you write down a list of the major learning outcomes of this lesson?	• Rate of change and slope are equivalent. • The slope function of a function allows us to predict the slope of the function at any point. • The slope function of a horizontal line is zero. • The slope function of a linear function is a constant value. • The slope function of a quadratic function is a linear function. • The slope function of a cubic function is a quadratic function. • Slope functions may be determined by examining the pattern of the slopes of tangents to the function. • Slope functions may be determined using a simple rule.	» Circulate to question students as to the main points of the lesson. » Why is the slope function important? » Can you describe two different ways of finding the slope function? » Can you identify strengths/ weaknesses of each approach? » If you were asked to find the rate of change of a function at a particular point on that function how would you do it? » For any quadratic function what shape is its slope function? » Encourage students to share their summaries. » Write up the main points on the board.	» Can students pick out the important learning outcomes from the lesson?

Section A: Student Activity 1

Read the following examples of rates of change.

The slope of a line $m = \frac{y_2 - y_1}{x_2 - x_1}$ measures the rate at which the ys change as the xs change.

The speed of a vehicle measures the rate at which the distance travelled (d) changes as time (t) changes.

We might be interested in the rate at which the height of water in a container (h) changes as time (t) changes.

An engineer might be interested in the rate at which the length of a metal rod (l) changes as temperature (r) changes.

A microbiologist might be interested in the rate at which the number of bacteria (n) on a piece of cheese changes with time (t).

An economist could be interested in the rate at which production costs (p) change with respect to the quantity of the product manufactured (q).

A gardener could be interested in the rate at which the height of a flower changes with respect to time.

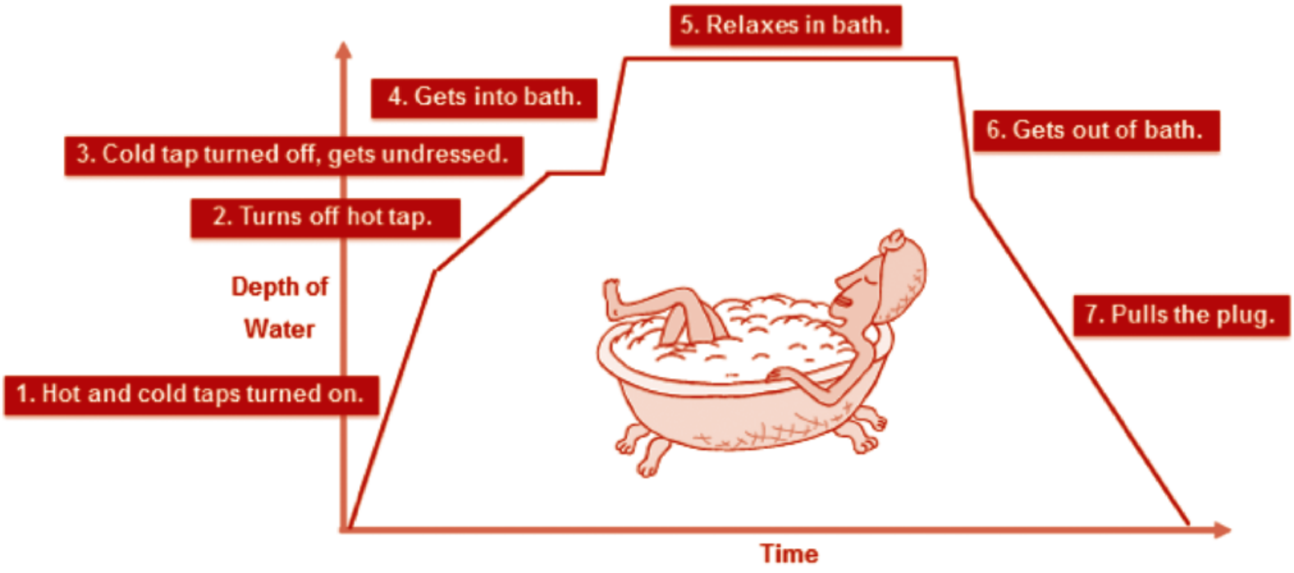
From the above examples, fill in the following table.

	Rates of Change	Independent Variable	Dependant Variable
1	ys change as the xs change	x	y
2	distance travelled changes as time changes	t	d
3			
4			
5			
6			
7			

Can you think of another rate of change that we might be interested in?

8			

Section A: Student Activity 2



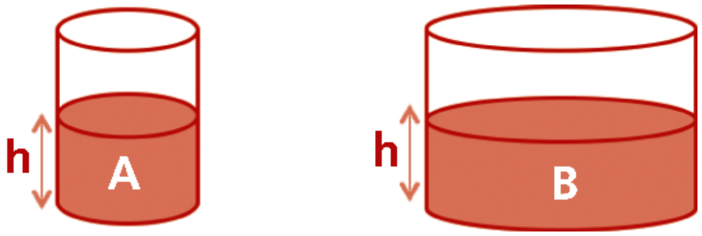
Looking at the graph above, discuss in groups how the depth of water changes with time as Isabelle takes her bath.

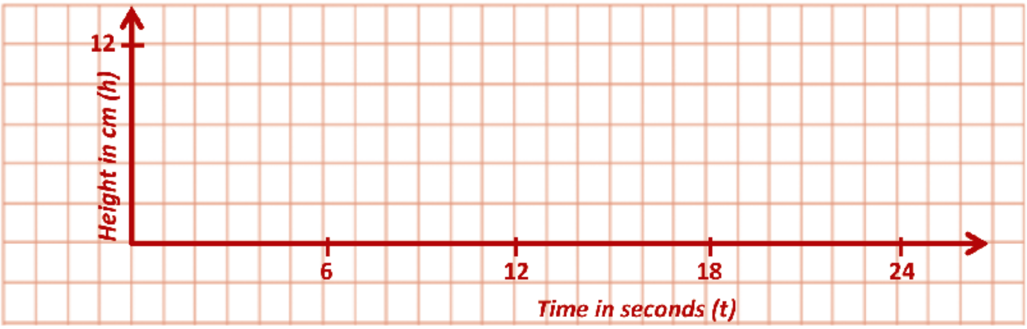
Create a word bank of terms that were used during your discussion.

Word Bank

Section A: Student Activity 3, part 1

1. (i) Two cylindrical containers, A and B, are filled with water. The volume of water increases at the same rate in both and the height of both containers is 12cm. Sketch a graph to show the rate at which the height of the water level changes with time for both containers. Put both containers on one graph. Container A is full after 6 seconds and container B is full after 24 seconds.

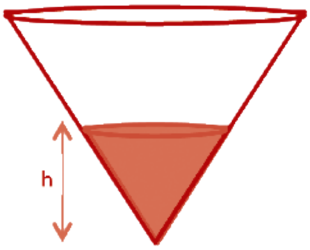


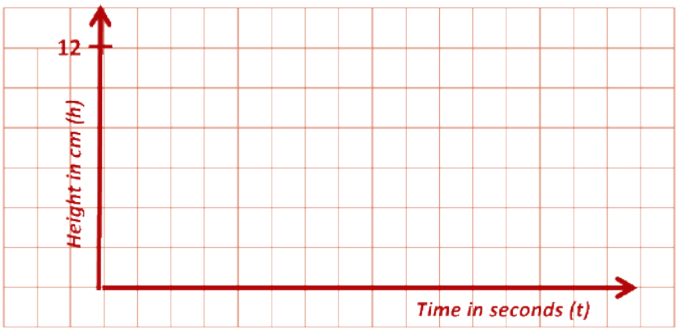


(ii) Why does it take container B longer to fill?

2. (i) Water flows into a vessel in the shape of an inverted cone as shown below. The volume of water increases at the same rate as for the two cylinders above. The vessel has the same height and radius as container B. How long will it take to fill the vessel?

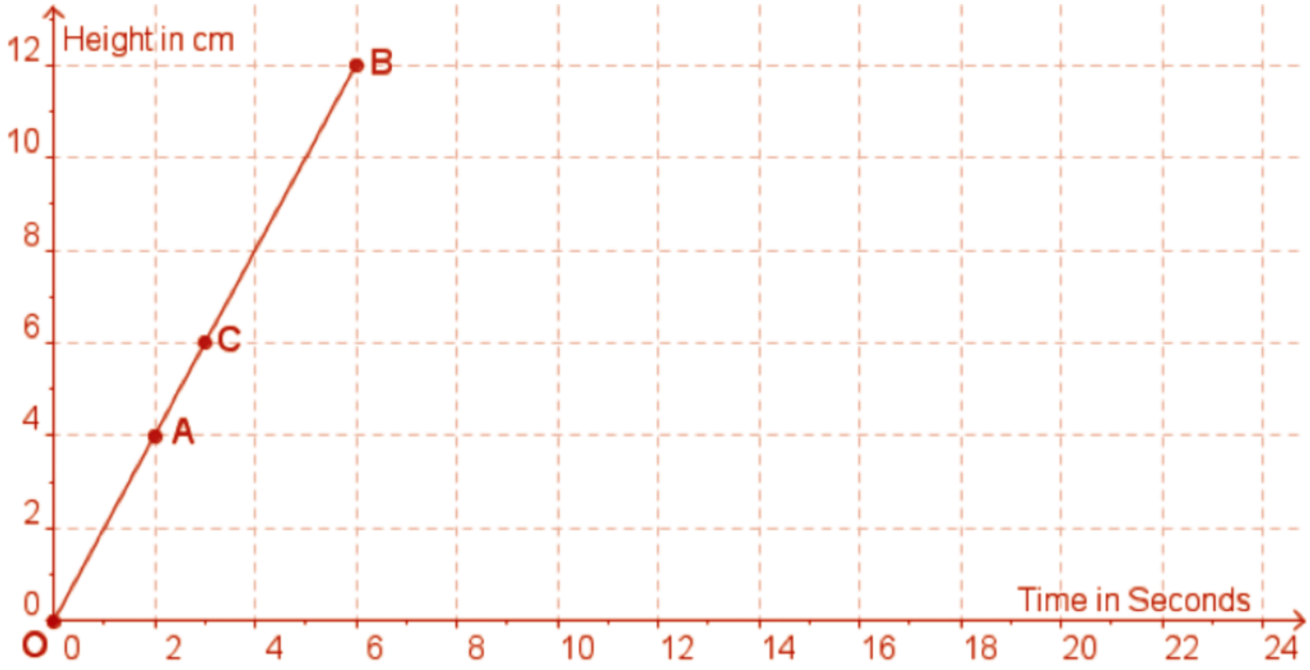
(ii) As water is poured into the vessel, sketch a rough graph to show how the height of the water level changes with time.





Section A: Student Activity 3, part 2

3. Graph of cylinder A being filled



(i) Using $m = \frac{y_2 - y_1}{x_2 - x_1}$ or rise over run, find the slope of [AB]

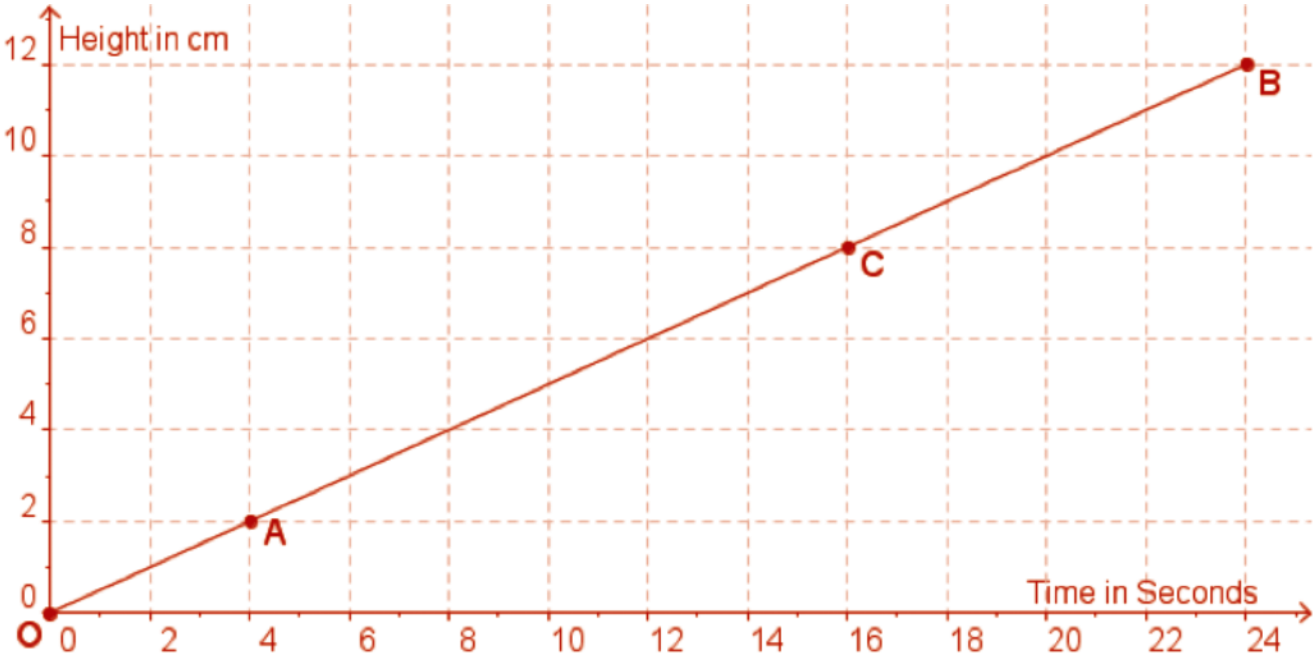
(ii) Using $m = \frac{y_2 - y_1}{x_2 - x_1}$ or rise over run, find the slope of [OB]

(iii) What is the slope of the line at any point on the line segment [OB]? Explain your reasoning.

(iv) What is the slope of the line at the point C? Explain your reasoning.

Section A: Student Activity 3, part 2

4. Graph of cylinder B being filled



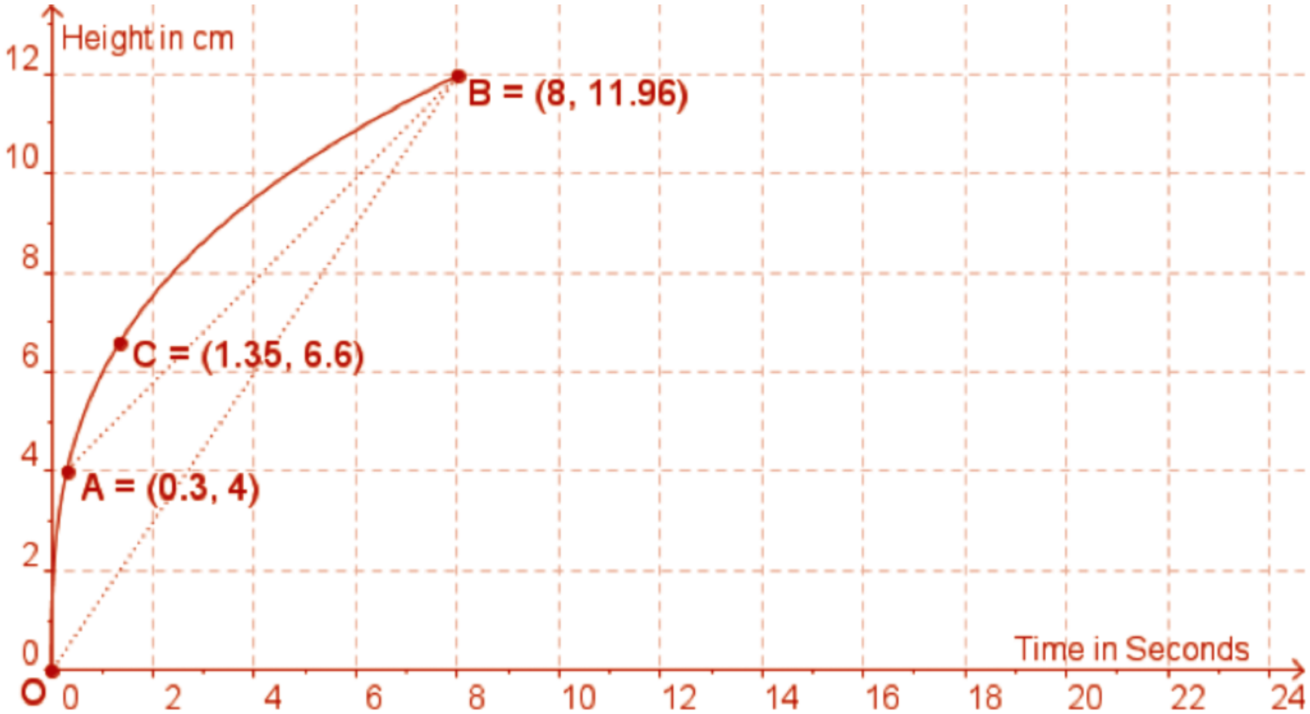
(i) Using $m = \frac{y_2 - y_1}{x_2 - x_1}$ or rise over run, find the slope of [AB]

(ii) At what rate is the height of the water rising at any time between 0 and 24 seconds? Explain your reasoning.

(iii) At what rate is the height of the water rising at point C (after 16 seconds)? Explain your reasoning.

Section A: Student Activity 3, part 2

5. Graph of cylinder B being filled



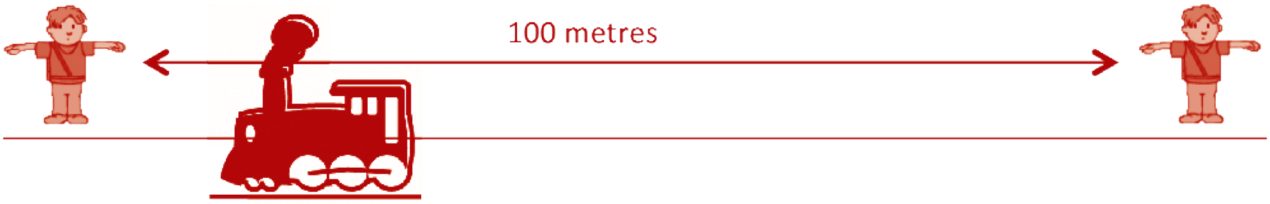
(i) Using $m = \frac{y_2 - y_1}{x_2 - x_1}$ or rise over run, find the slope of [AB].

(ii) Using $m = \frac{y_2 - y_1}{x_2 - x_1}$, find the slope of the line [OB], correct to two decimal places.

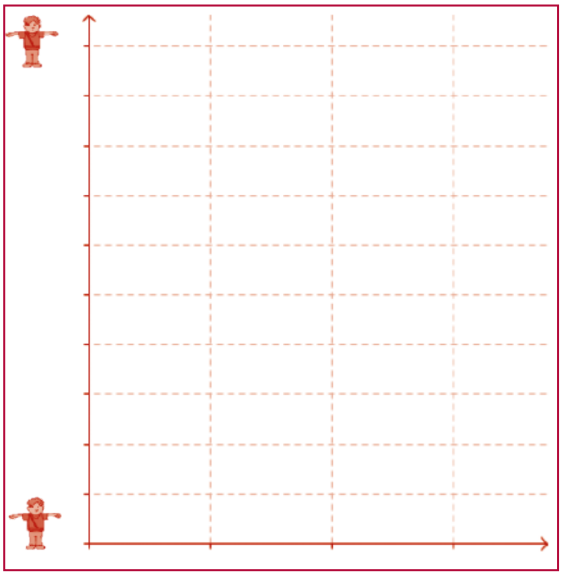
(iii) Can you use the slope of [AB] or the slope of [OB] to find the slope of the graph at the at the point C? Give a reason for your answer.

Section A: Student Activity 4

Some Transition Year students decide to carry out an experiment on constant speed. They have a class discussion on where they might see a model for constant speed. They decide that if they go to a train station and choose a train that is not scheduled to stop there, that the train will most likely pass them at a constant speed. Two students from the class arrange to stand 100 metres apart at either end of the platform and time the train between these two positions.



1. The two students stand 100 metres apart and discover that it takes 3 seconds for the front of the train to travel between the two positions. Draw a graph to represent how the distance changes with time during these 3 seconds. Let the position of the first student be at the origin of the graph and put the independent variable on the horizontal axis.



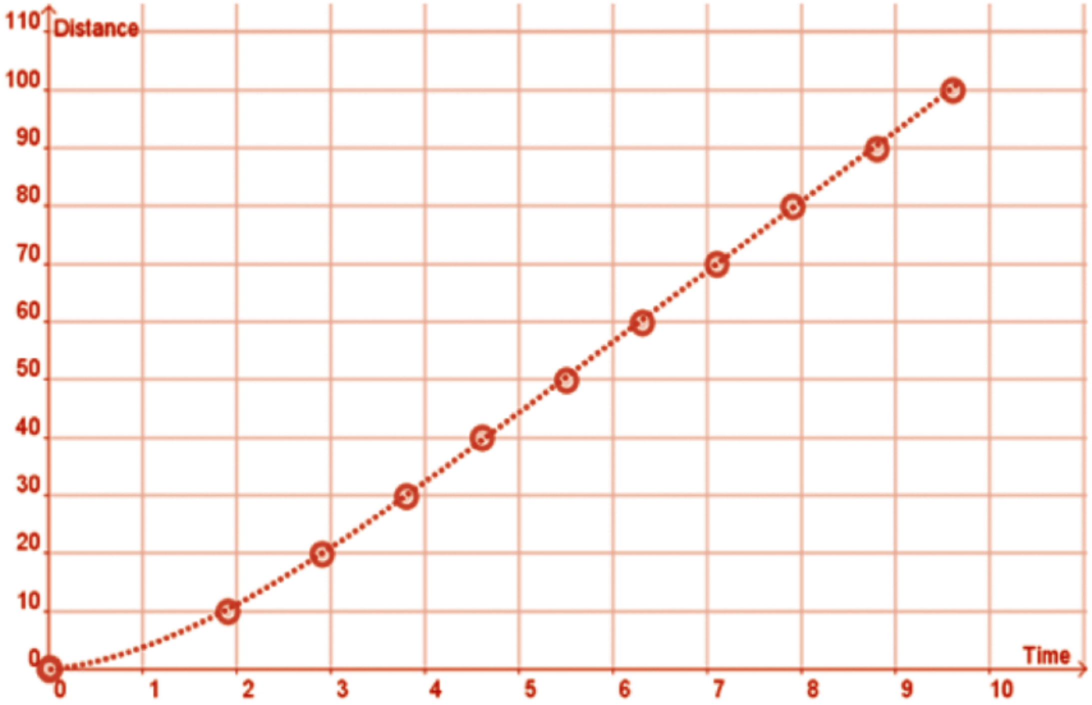
2. At what speed does the train pass the two students in km/hour?

3. The teacher was standing half way between the students during the experiment to supervise. At what speed did the train pass the teacher? Give a reason for your answer.

Section A: Student Activity 5

In the 2009 World Championships in Berlin, Usain Bolt set the World Record for the Men's 100m sprint, running it in 9.58 seconds. Below is a table of Usain Bolt's split times every 10 metres during the race.

Distance (m)	0	10	20	30	40	50	60	70	80	90	100
Time (s)	0	1.89	2.88	3.78	4.64	5.47	6.29	7.10	7.92	8.75	9.58



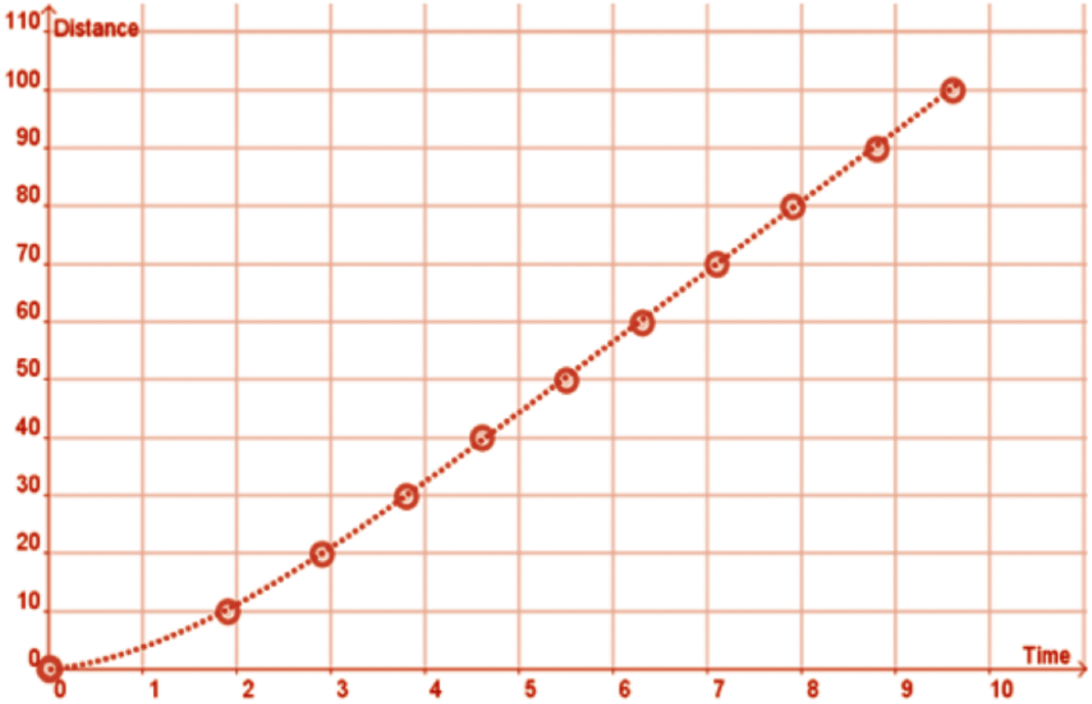
1. How fast do you think Usain Bolt ran during the race? Give your answer correct to 2 decimal places in m/sec.

2. Do you think he ran at this speed throughout the whole race? Give two reasons for your answer.

3. What do you think your answer for Question 1 represents?

Section A: Student Activity 5

4. (i) Using a ruler, join the points (0,0) and (9.58,100) on the graph below.



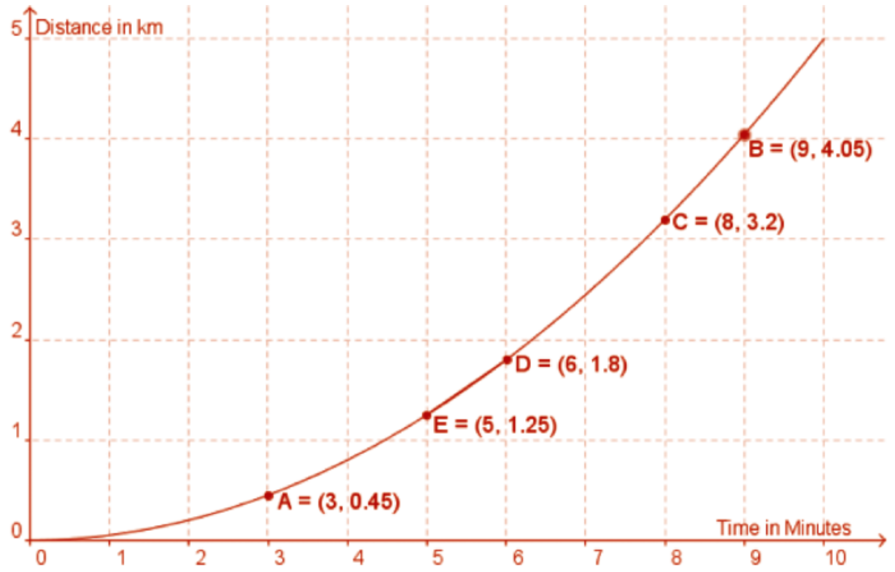
(ii) Find the slope of this line.

(iii) The line that joins (0,0) to (9.58,100) has a special name. It is called a secant line to the above curve. What observation can you make about the slope of this secant line?

5. How do you think we could calculate Usain's speed at precisely 1 second into the race?

Section A: Student Activity 6, part 1

Below is a distance-time graph of the first ten minutes of a warm-up cycle by Olympic Gold medallist Victoria Pendleton.



1. Over these 10 minutes, what is Victoria Pendleton's average speed in km/min?

2. The coach wants to know what her speed is at exactly 3 minutes during this warm-up. To help answer this question do the following:

(i) Using your ruler, draw in the secants [AB], [AC], [AD], [AE].

(ii) Fill in the following table. Answers correct to 2 decimal places.

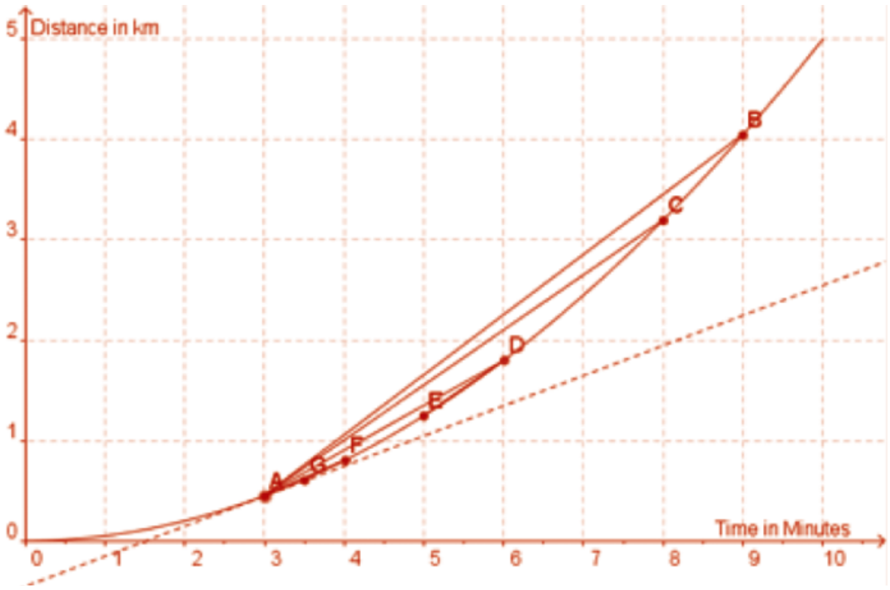
Slope of Secant [AB]=	Average speed between A and B =
Slope of Secant [AC]=	Average speed between A and C =
Slope of Secant [AD]=	Average speed between A and D =
Slope of Secant [AE]=	Average speed between A and E =

3. The slope of which secant is the nearest estimate to Victoria's speed after exactly 3 minutes?

4. How might you find a better estimate for Victoria's speed after 3 minutes?

Section A: Student Activity 6, part 2

Below is a distance-time graph of the first ten minutes of a warm-up cycle by Olympic Gold medallist Victoria Pendleton.



1. Average rates of change are found using $m = \frac{y_2 - y_1}{x_2 - x_1}$ on a curve.

   The denominator $x_2 - x_1$ tells us the length of the interval of x values or in other words the length of the 'run' when using 'rise over run' to find the slope.

   Fill in the following table.

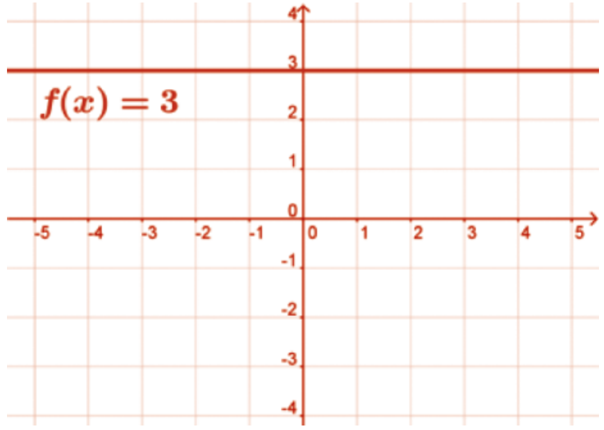
The interval of x values for secant [AB] =	6
The interval of x values for secant [AC] =	
The interval of x values for secant [AD] =	
The interval of x values for secant [AE] =	
The interval of x values for secant [AG] =	

2. What interval of x values would give us the secant whose slope is closest to the tangent at the point A?

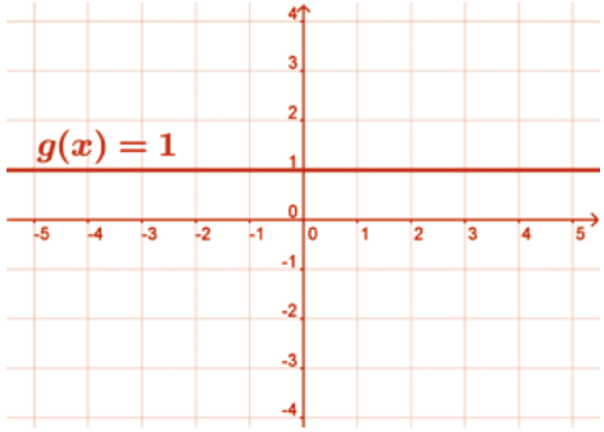
Section B: Student Activity 1

Linear Functions and their Slope Functions

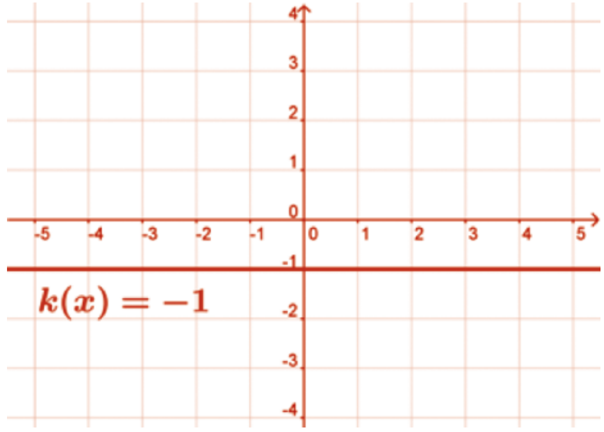
A



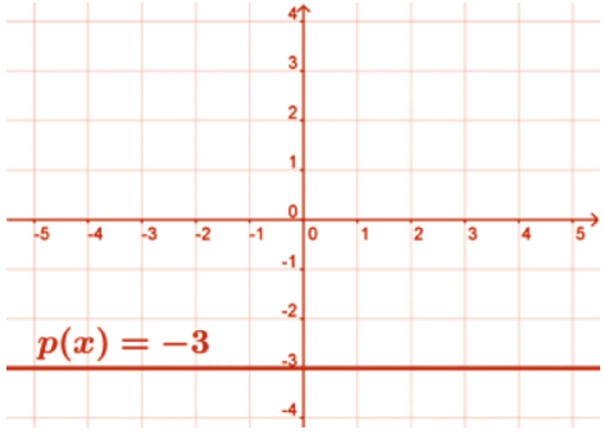
B



C



D

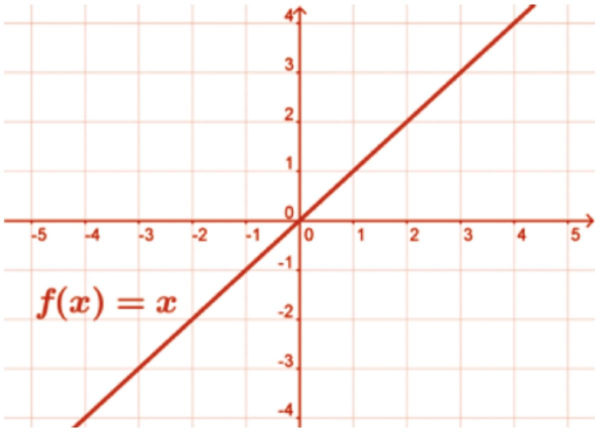


Function	Slope (Rate of change) when x is:							Slope Function
	-3	-2	-1	0	1	2	3	
$f(x) = 3$								$f'(x) =$
$g(x) = 1$								$g'(x) =$
$k(x) = -1$								$k'(x) =$
$p(x) = -3$								$p'(x) =$
$q(x) = 95$								$q'(x) =$
$f(x) = c$								$f'(x) =$
Complete the following: The rate of change of a constant is								

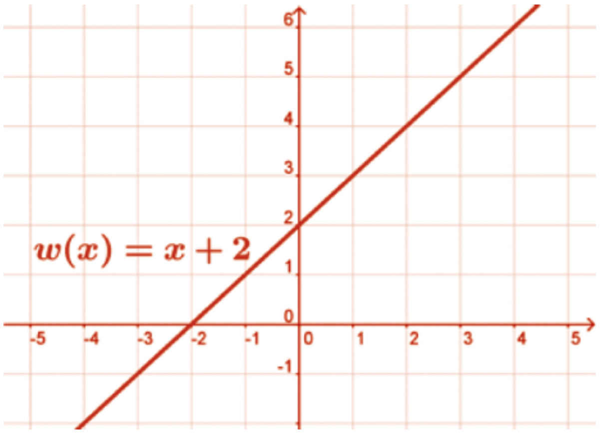
Section B: Student Activity 2

Linear Functions and their Slope Functions

A



B



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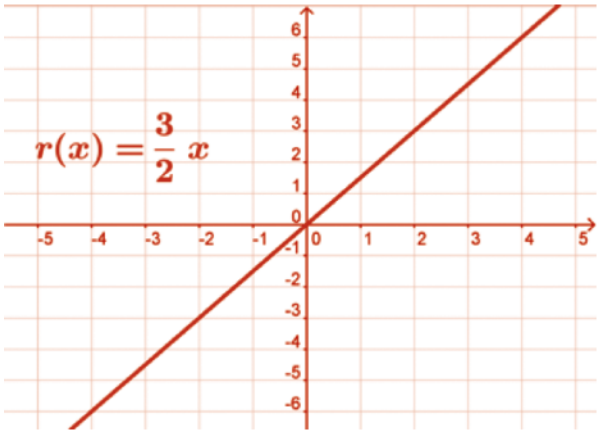

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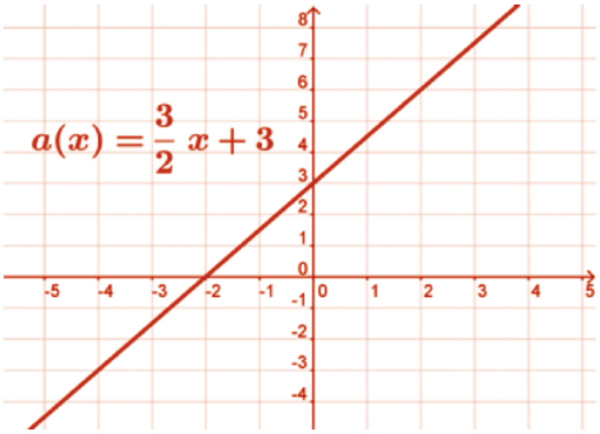
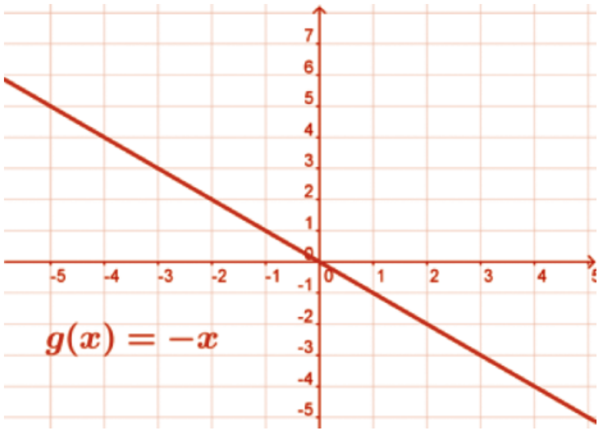
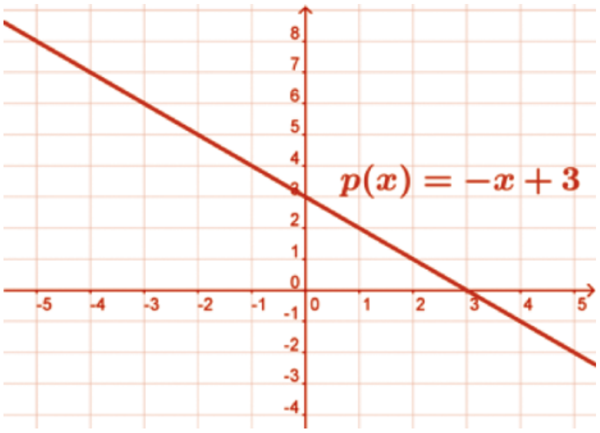


E



F



Section B: Student Activity 2

Linear Functions and their Slope Functions

Function	Slope (Rate of change) when x is:							Slope Function
	-3	-2	-1	0	1	2	3	
$f(x) = x$								$f'(x) =$
$w(x) = x + 2$								$w'(x) =$
$r(x) = \frac{3}{2}x$								$r'(x) =$
$a(x) = \frac{3}{2}x + 3$								$a'(x) =$
$g(x) = -x$								
$p(x) = -x + 3$								
$n(x) =$								
$q(x) =$								
$f(x) = nx$								$f'(x) =$
$f(x) = nx + c$								$f'(x) =$

Complete the following:

The rate of change of a function $f(x) = nx$ is ________________________________

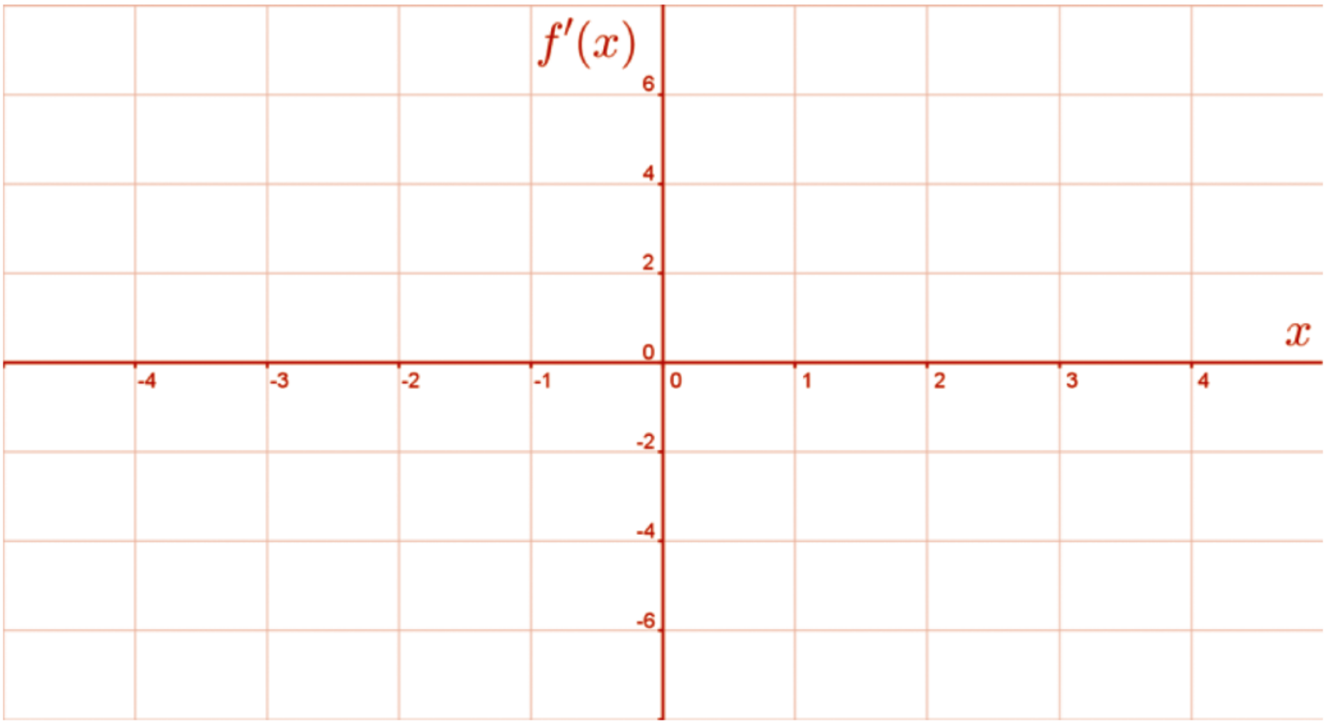
The rate of change of a function $f(x) = nx + c$ is ________________________________

Section B: Student Activity 3, A

Quadratic Functions and their associated Slope Functions $f(x) = x^2$

1. Using the appropriate *GeoGebra* file or the graph on the next page complete the table by filling in the slopes of the tangents to the function at points A – G.
2. Investigate the pattern of the slopes by completing the Change column in the table.
3. Graph the slopes of the tangent (as a function of x) in the space provided.
4. Using your pattern-analysis skills, write down the slope function ($f'(x)$).

Point	x	Slope of Tangent $f'(x)$	Change of $f'(x)$
A	-3		
B	-2		
C	-1		
D	0		
E	1		
F	2		
G	3		
The slope function $f'(x)$ is			

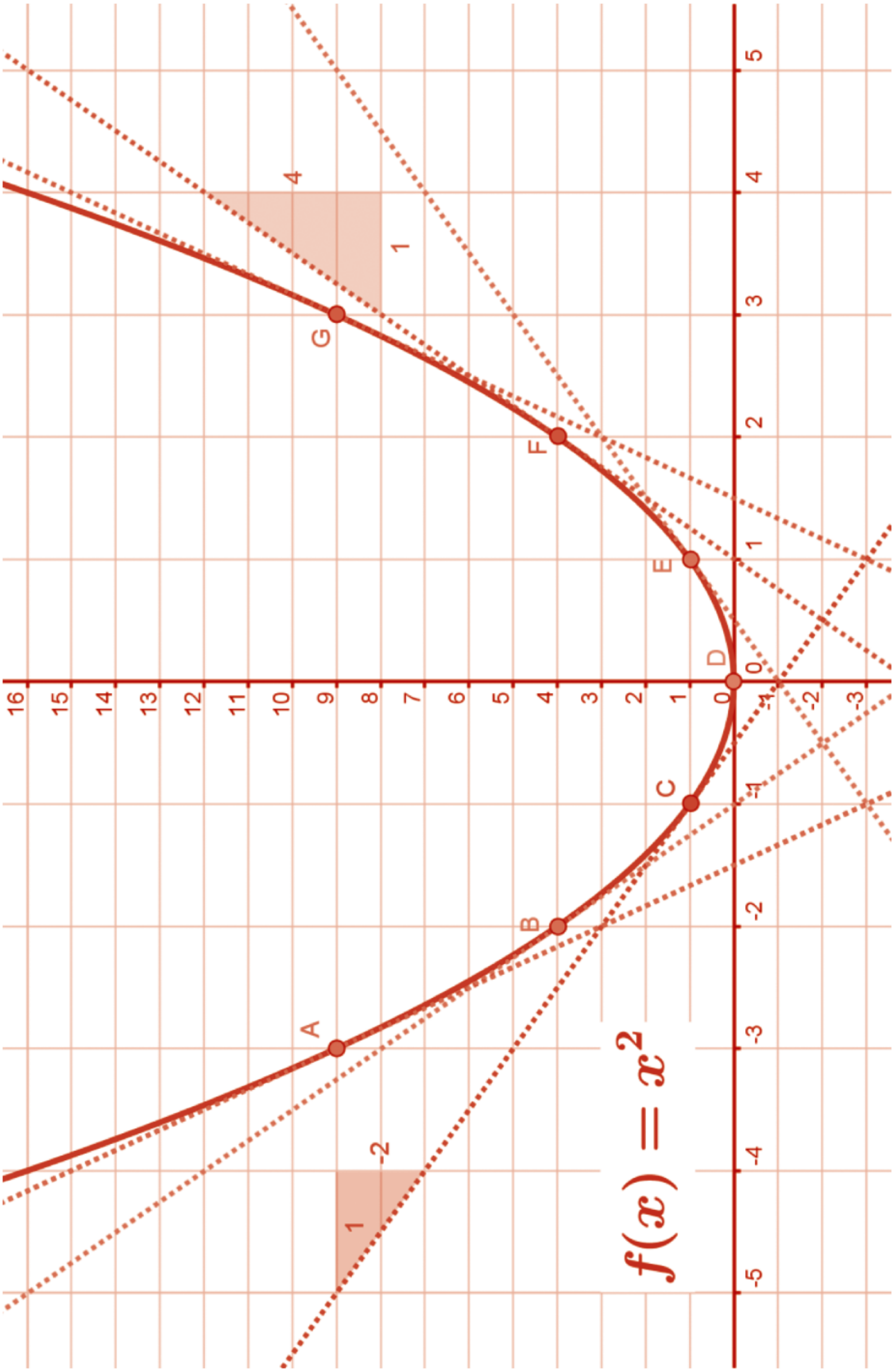


Slope of $f'(x)$:

y-intercept of $f'(x)$:

Equation of $f'(x)$:

Section B: Student Activity 3, A

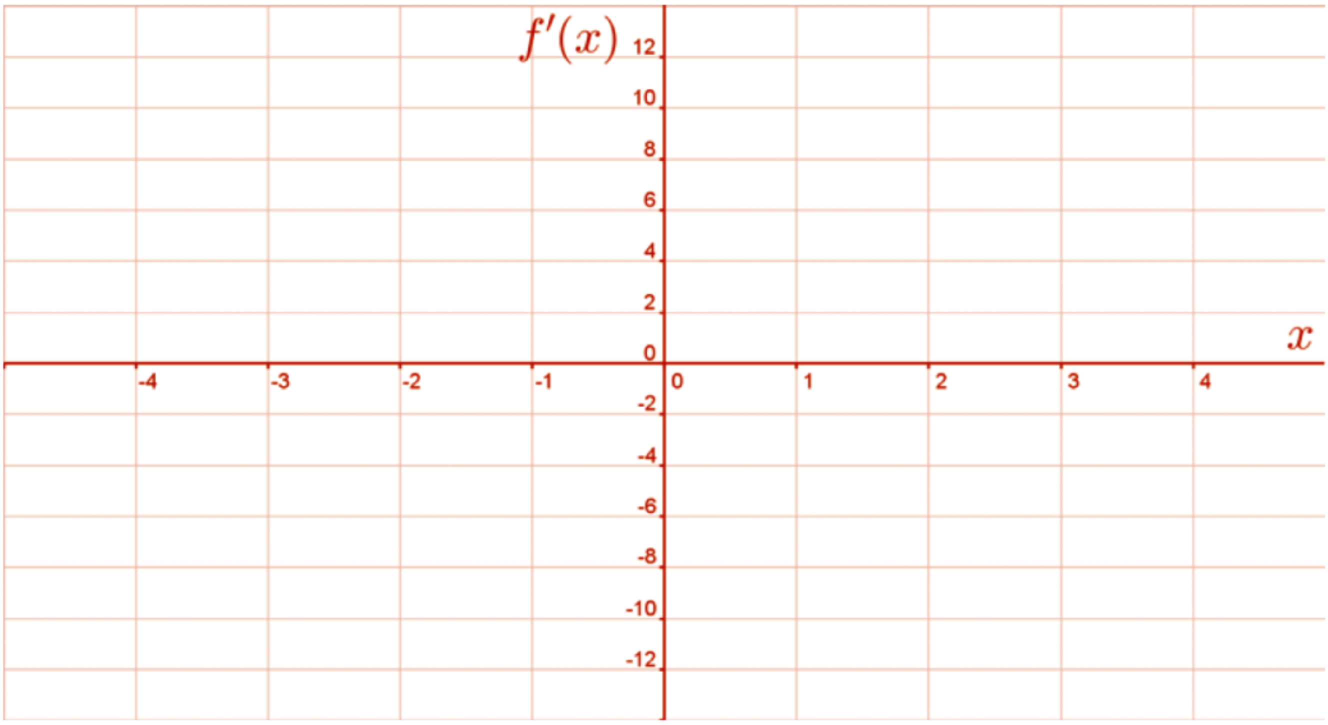


Section B: Student Activity 3, B

Quadratic Functions and their associated Slope Functions $f(x) = 2x^2$

1. Using the appropriate *GeoGebra* file or the graph on the next page complete the table by filling in the slopes of the tangents to the function at points A – G.
2. Investigate the pattern of the slopes by completing the Change column in the table.
3. Graph the slopes of the tangent (as a function of x) in the space provided.
4. Using your pattern-analysis skills, write down the slope function ($f'(x)$).

Point	x	Slope of Tangent $f'(x)$	Change of $f'(x)$
A	-3		
B	-2		
C	-1		
D	0		
E	1		
F	2		
G	3		
The slope function $f'(x)$ is			

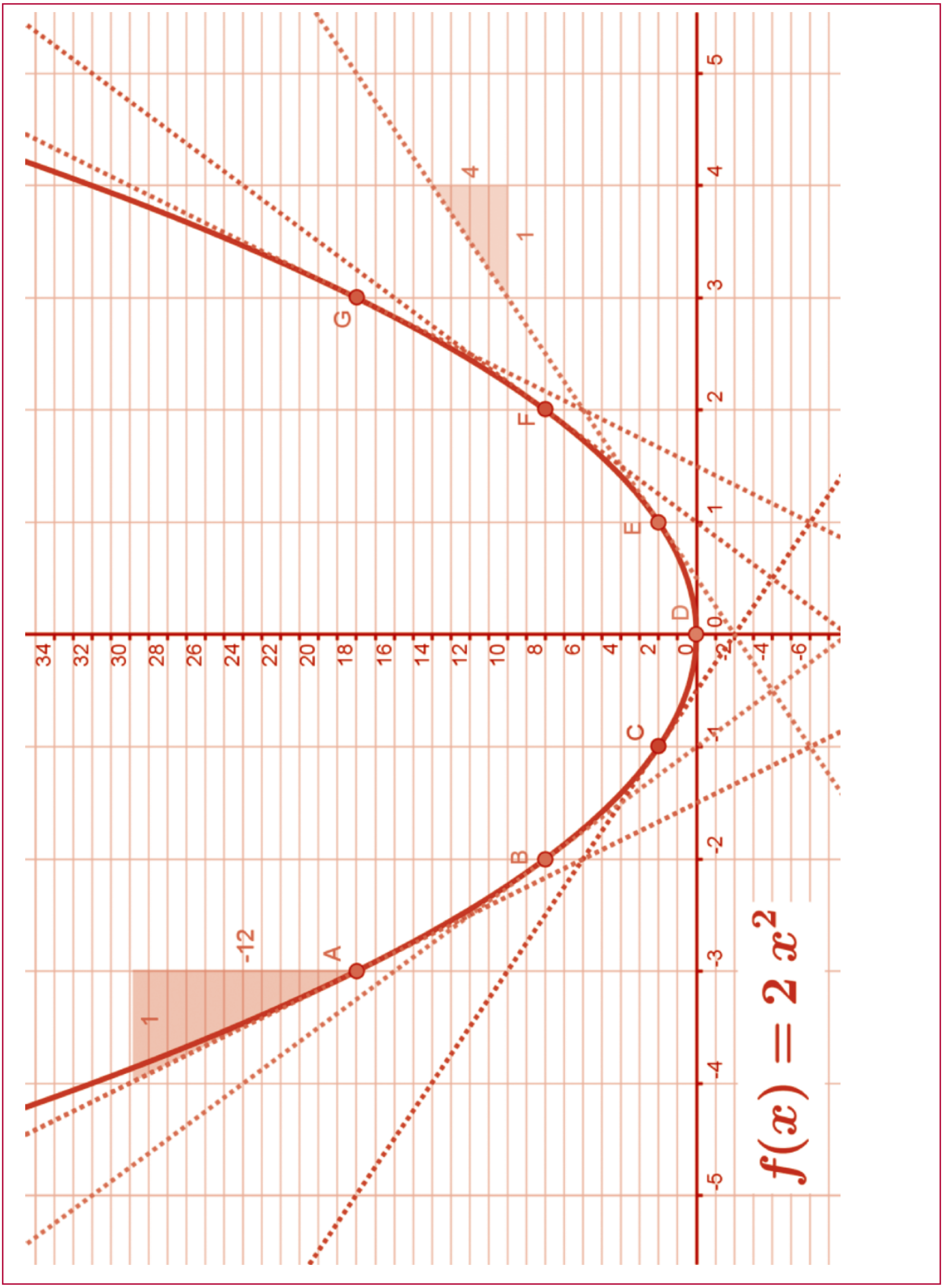


Slope of $f'(x)$:

y-intercept of $f'(x)$:

Equation of $f'(x)$:

Section B: Student Activity 3, B



Section B: Student Activity 3, C

Quadratic Functions and their associated Slope Functions $f(x) = 3x^2$

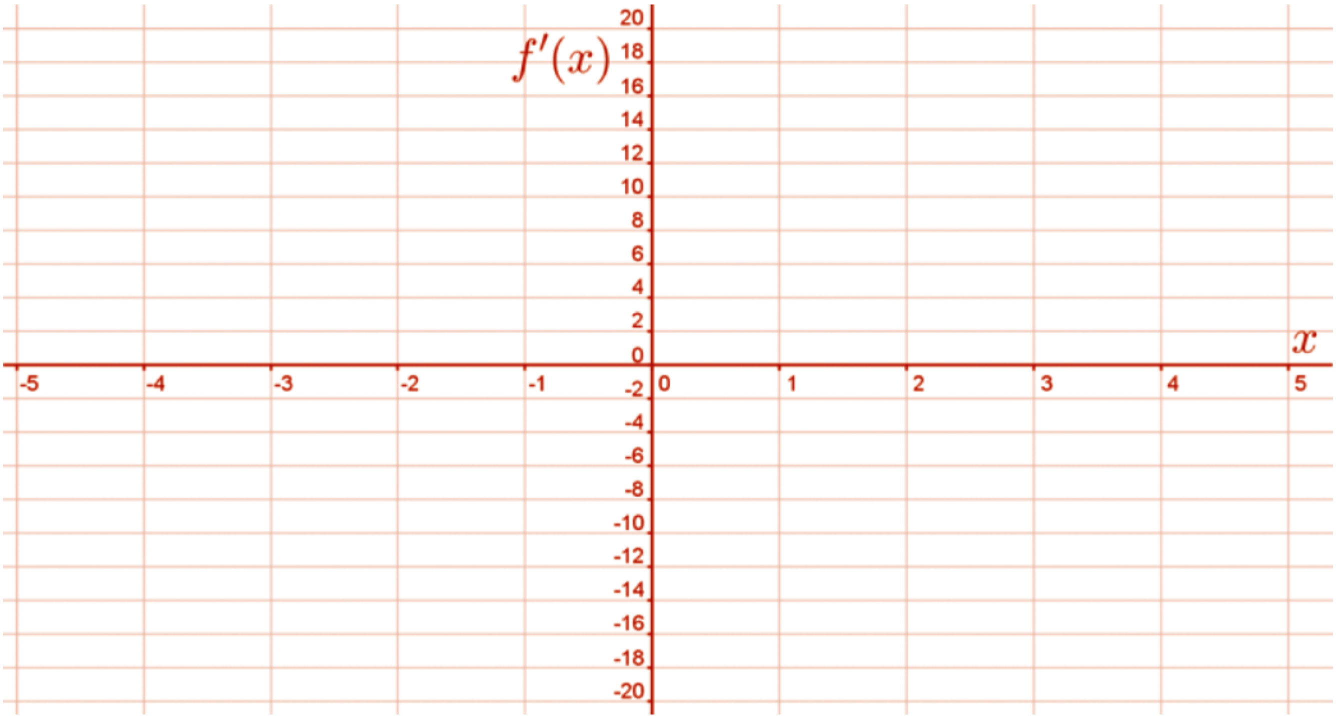
1 Using the appropriate *GeoGebra* file or the graph on the next page complete the table by filling in the slopes of the tangents to the function at points A – G.

2 Investigate the pattern of the slopes by completing the Change column in the table.

3 Graph the slopes of the tangent (as a function of x) in the space provided.

4 Using your pattern-analysis skills, write down the slope function ($f'(x)$).

Point	x	Slope of Tangent $f'(x)$	Change of $f'(x)$
A	-3		
B	-2		
C	-1		
D	0		
E	1		
F	2		
G	3		
The slope function $f'(x)$ is			

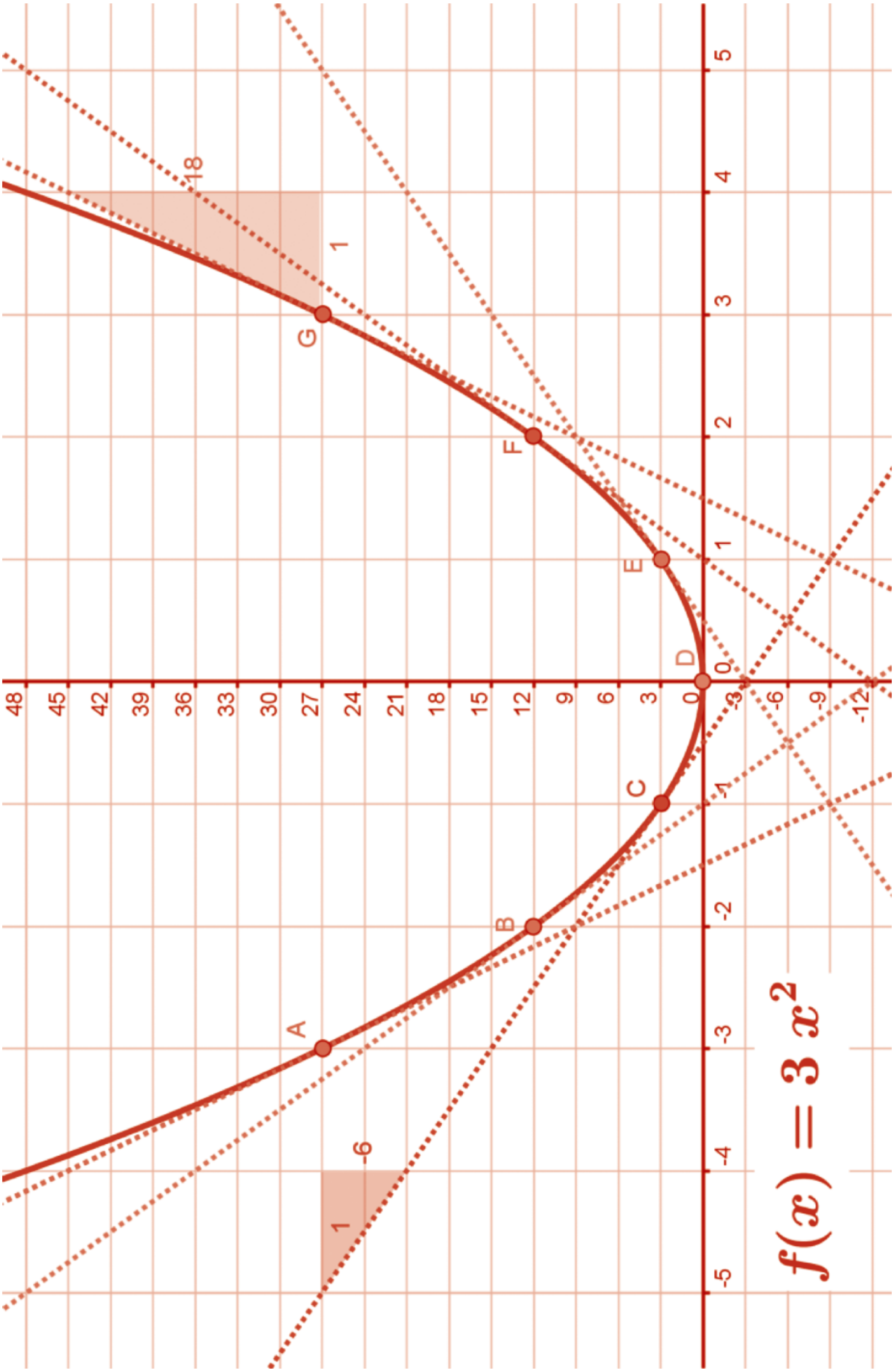


Slope of $f'(x)$:

y-intercept of $f'(x)$:

Equation of $f'(x)$:

Section B: Student Activity 3, C



Section B: Student Activity 3, D

Quadratic Functions and their associated Slope Functions $f(x) = 4x^2$

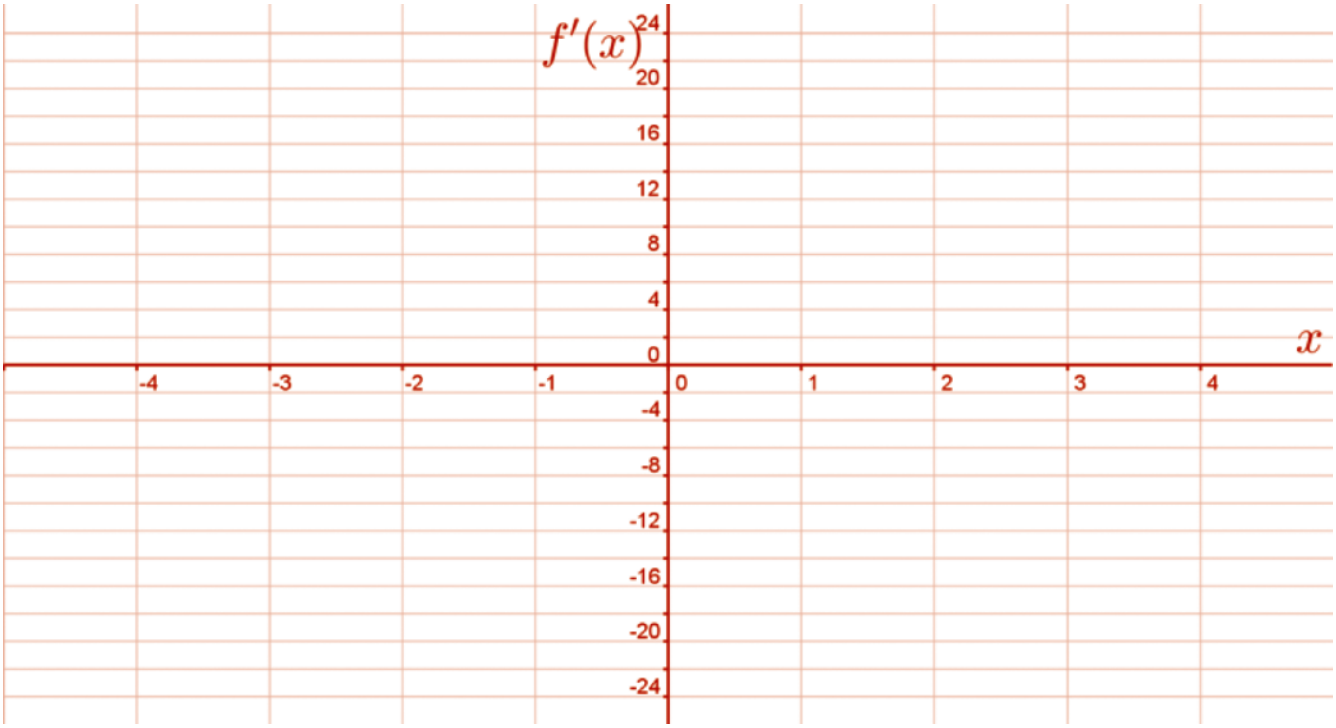
1 Using the appropriate *GeoGebra* file or the graph on the next page complete the table by filling in the slopes of the tangents to the function at points A – G.

2 Investigate the pattern of the slopes by completing the Change column in the table.

3 Graph the slopes of the tangent (as a function of x) in the space provided.

4 Using your pattern-analysis skills, write down the slope function ($f'(x)$).

Point	x	Slope of Tangent $f'(x)$	Change of $f'(x)$
A	-3		
B	-2		
C	-1		
D	0		
E	1		
F	2		
G	3		
The slope function $f'(x)$ is			

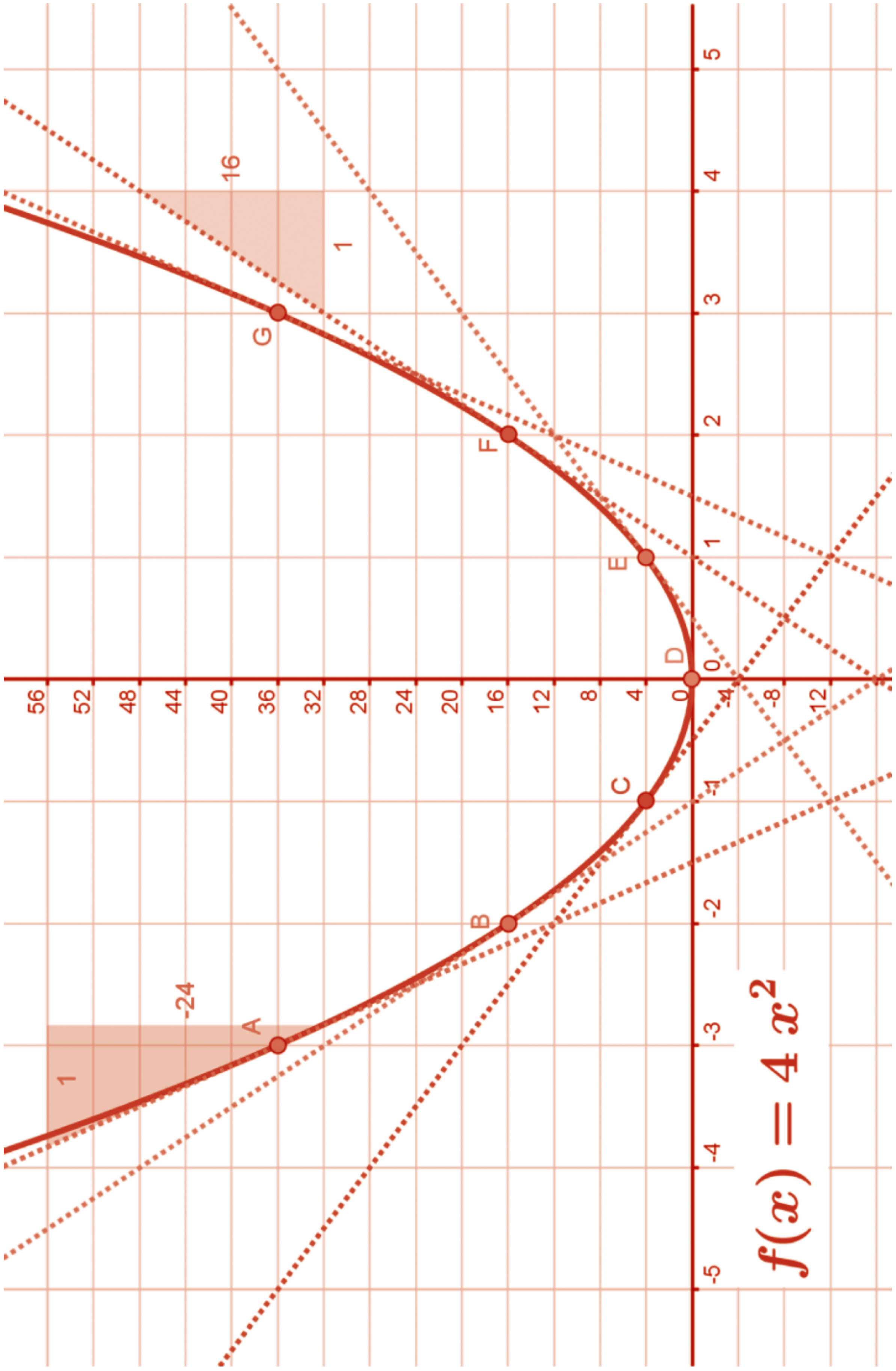


Slope of $f'(x)$:

y-intercept of $f'(x)$:

Equation of $f'(x)$:

Section B: Student Activity 3, D



Section B: Student Activity 3, E

Quadratic Functions and their associated Slope Functions

Conclusion

1 Complete the table below and explain your findings.

Function $f(x)$	Slope Function $f'(x)$
$f(x) = x^2$	
$f(x) = 2x^2$	
$f(x) = 3x^2$	
$f(x) = 4x^2$	
$f(x) = 5x^2$	
$f(x) = ax^2$	

Explanation (in words).

To find the derivative (slope function) of a quadratic function…

Section B: Student Activity 4, A

Cubic Functions and their associated Slope Functions $f(x) = x^3$

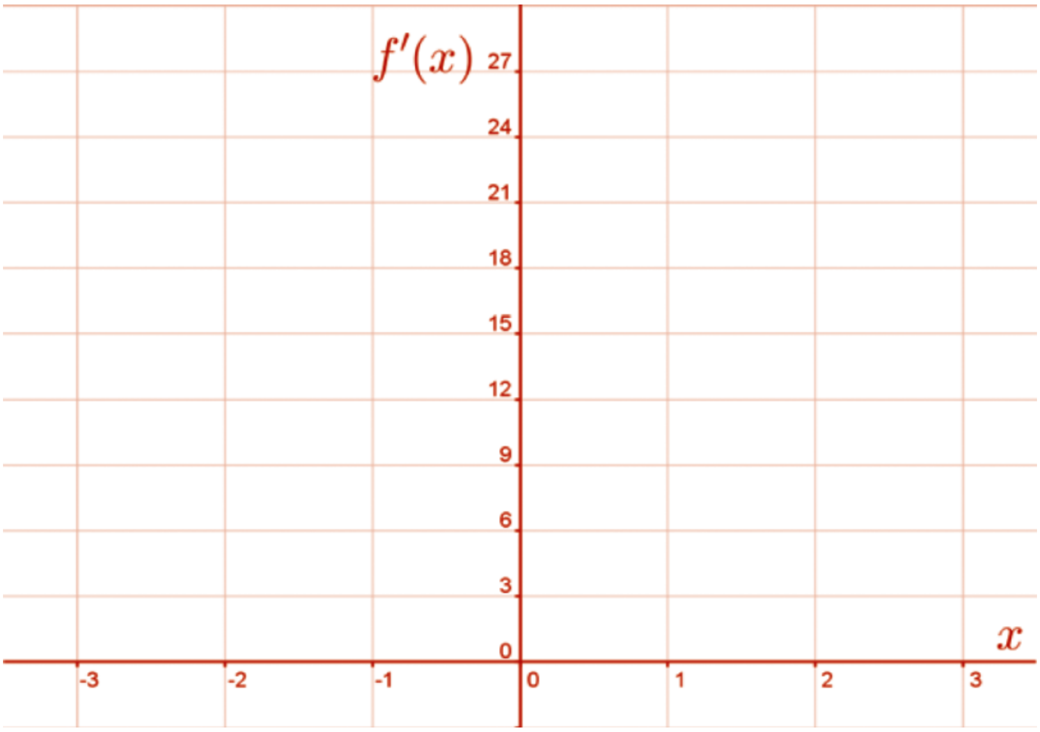
1 Using the appropriate *GeoGebra* file or the graph on the next page complete the table by filling in the slopes of the tangents to the function at points A–G.

2. Investigate the pattern of the slopes by completing the Change column in the table.

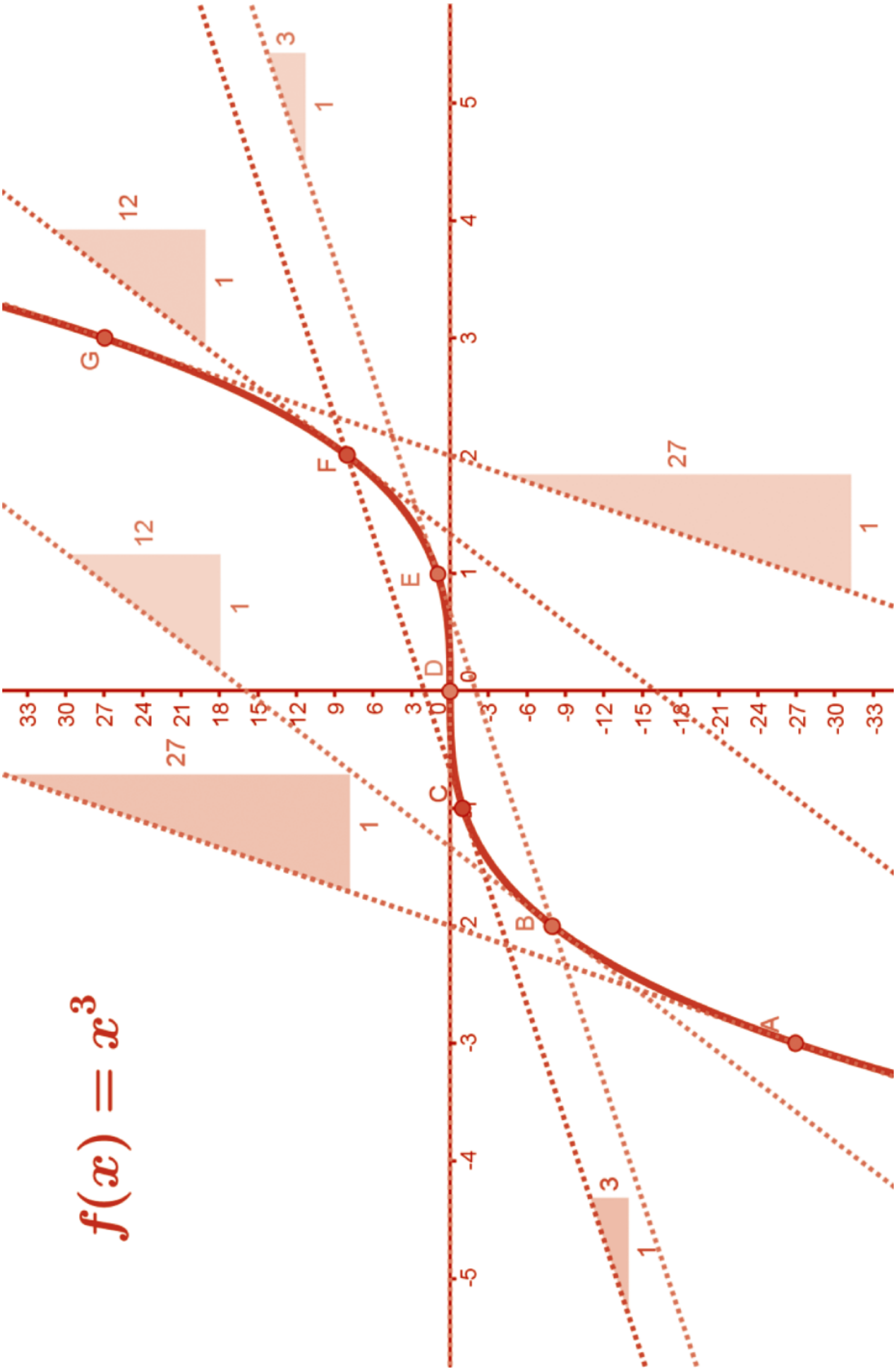
3. Graph the slopes of the tangent (as a function of x) in the space provided.

4. Using your pattern-analysis skills, write down the slope function ($f'(x)$).

Point	x	Slope of Tangent $f'(x)$	Change of $f'(x)$	Change of Change of $f'(x)$
A	-3			
B	-2			
C	-1			
D	0			
E	1			
F	2			
G	3			
The slope function $f'(x)$ is				



Section B: Student Activity 4, A



Section B: Student Activity 4, B

Cubic Functions and their associated Slope Functions $f(x) = 2x^3$

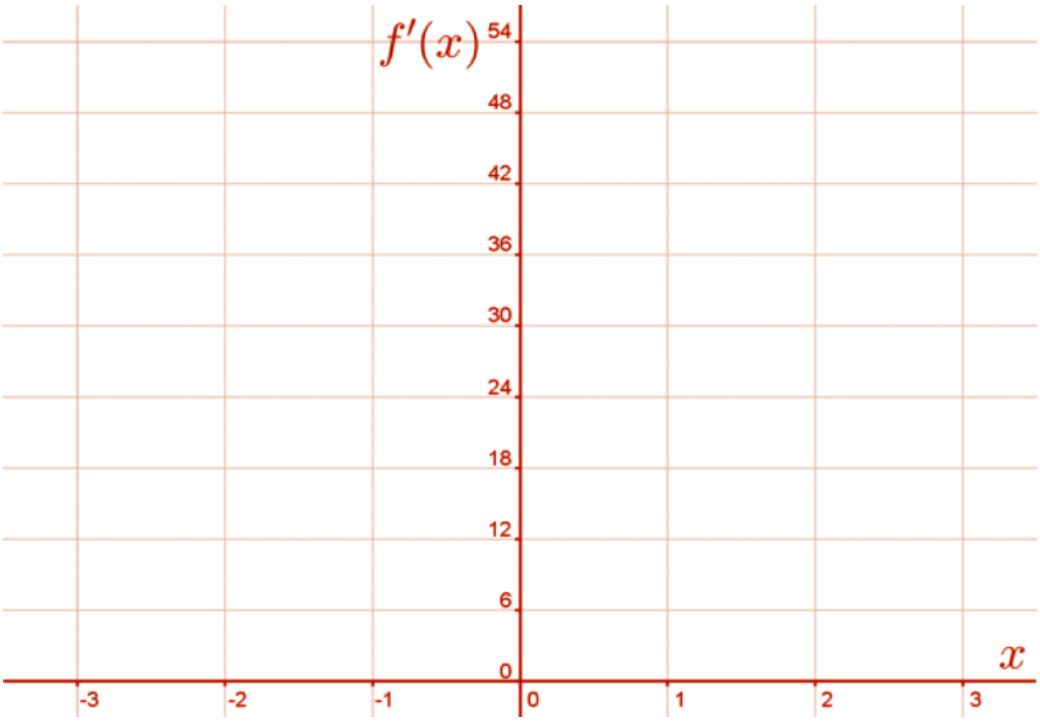
1 Using the appropriate *GeoGebra* file or the graph on the next page complete the table by filling in the slopes of the tangents to the function at points A–G.

2. Investigate the pattern of the slopes by completing the Change column in the table.

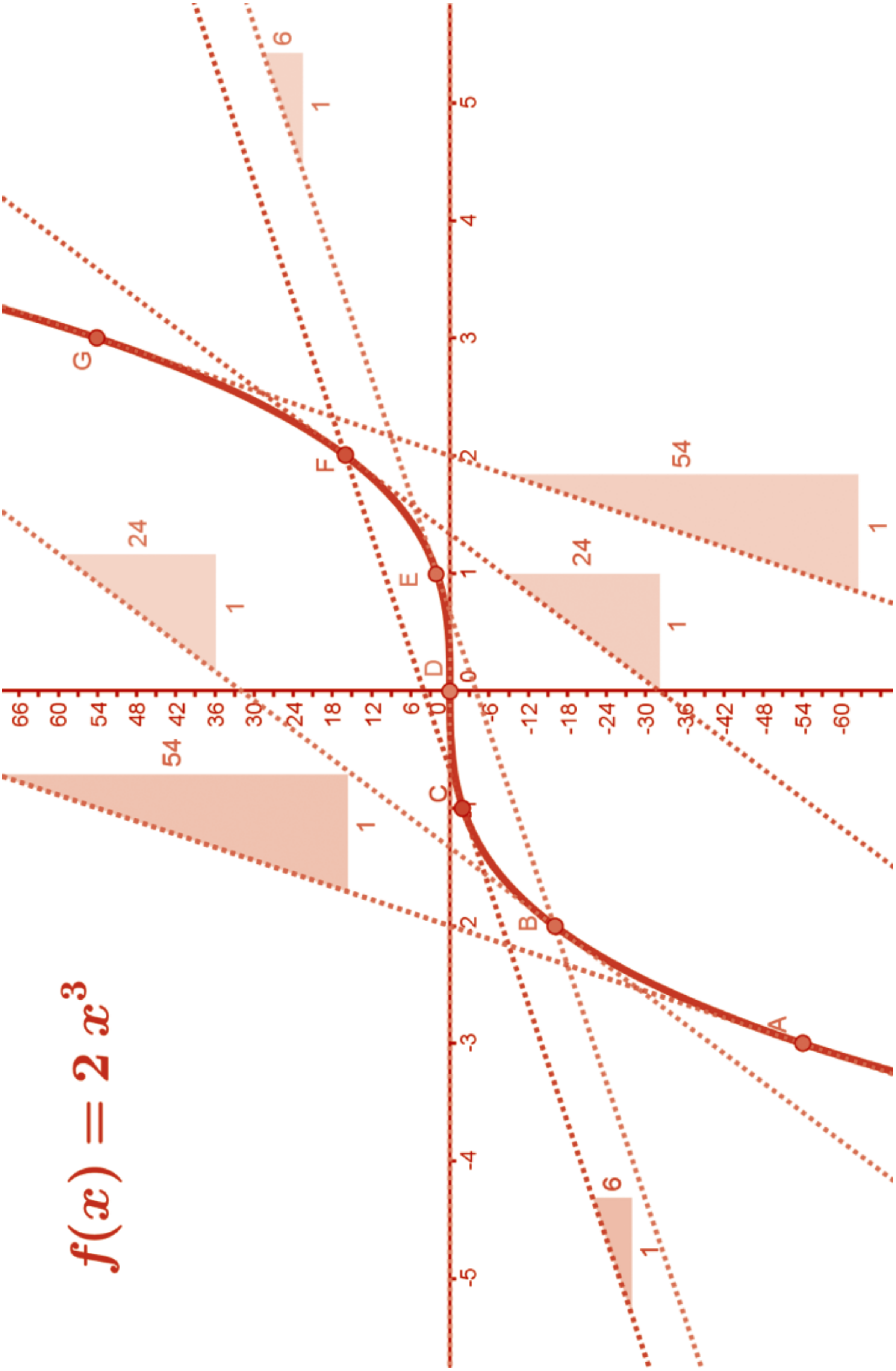
3. Graph the slopes of the tangent (as a function of x) in the space provided.

4. Using your pattern-analysis skills, write down the slope function ($f'(x)$).

Point	x	Slope of Tangent $f'(x)$	Change of $f'(x)$	Change of Change of $f'(x)$
A	-3			
B	-2			
C	-1			
D	0			
E	1			
F	2			
G	3			
The slope function $f'(x)$ is				



Section B: Student Activity 4, B



Section B: Student Activity 4, C

Cubic Functions and their associated slope functions

Conclusion

1 Complete the table below and explain your findings.

Function $f(x)$	Slope Function $f'(x)$
$f(x) = x^3$	
$f(x) = 2x^3$	
$f(x) = ax^3$	

Explanation (in words).

To find the derivative (slope function) of a cubic function…

Have you seen this approach to finding the slope function before? Explain

Section B: Student Activity 5

Finding the slope function of a polynomial

1 Based on the results of activities Section B : Student Activities 1-4, complete the following table:

Function $f(x)$	Slope Function $f'(x)$
$f(x) = 3$	$f'(x) =$
$k(x) = 1$	$k'(x) =$
$p(x) = -1$	$p'(x) =$
$d(x) = -3$	$d'(x) =$
$f(x) = x$	$f'(x) =$
$g(x) = {}^3/_2\, x$	$g'(x) =$
$h(x) = -x$	$h'(x) =$
$f(x) = x^2$	$f'(x) =$
$a(x) = 2x^2$	$a'(x) =$
$f(x) = 3x^2$	$f'(x) =$
$g(x) = 4x^2$	$g'(x) =$
$k(x) = x^3$	$k'(x) =$
$p(x) = 2x^3$	$p'(x) =$

2. Can you summarise how you might find the slope function of any given polynomial function?

Section B: Student Activity 5

Finding the slope function of a polynomial

3. Using this approach write down the slope functions of the following functions

Function $f(x)$	Slope Function $f'(x)$
$f(x) = 5x$	$f'(x) =$
$g(x) = 4x^3$	$g'(x) =$
$h(x) = -9x$	$h'(x) =$
$k(x) = -6x^2$	$k'(x) =$
$f(x) = 0.5x^2$	$f'(x) =$
$w(x) = 5x^4$	$w'(x) =$
$g(x) = -3x^7$	$g'(x) =$
$r(x) = x^{-2}$	$r'(x) =$
$k(x) = x^{-1}$	$k'(x) =$
$f(x) = 2x^0$	$f'(x) =$
$z(x) = {}^{1}/_{x}$	$z'(x) =$
$a(x) = \sqrt{x}$	$a'(x) =$
$g(x) = {}^{2}/_{x^{3}}$	$g'(x) =$
$f(x) = ax^n$	$f'(x) =$

Section B: Student Activity 6, A

Co-polynomial functions and their associated Slope Functions

$f(x) = x^2 - x - 6$

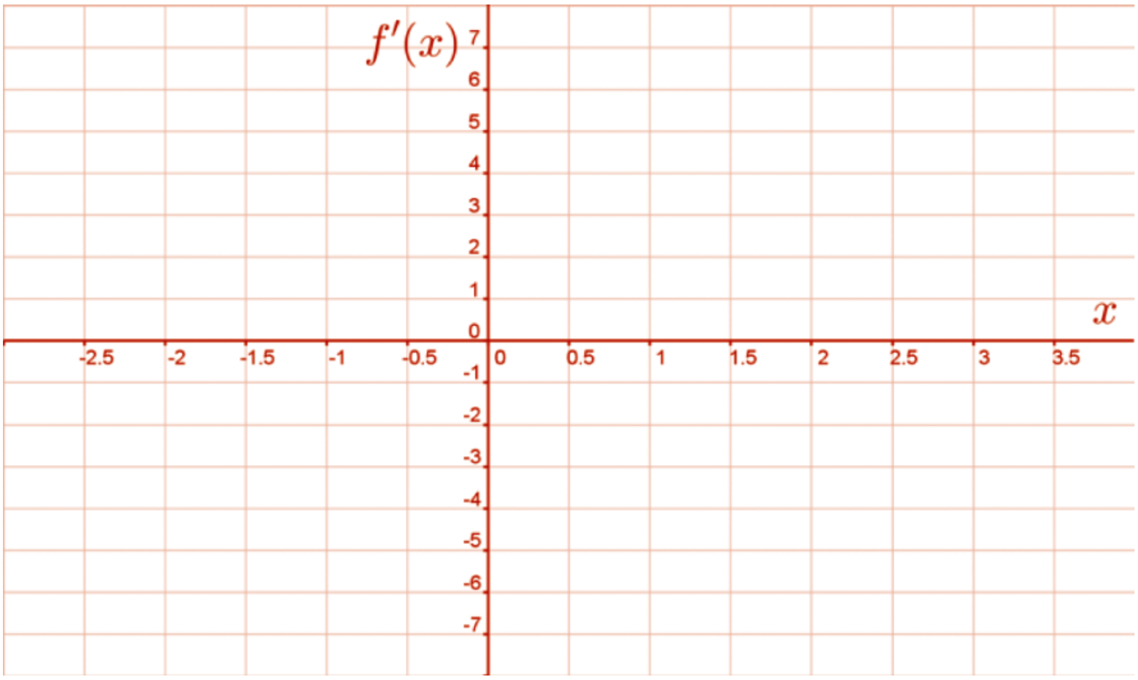
1 Using the appropriate *GeoGebra* file or the graph on the next page complete the table by filling in the slopes of the tangents to the function at points A–G.

2. Investigate the pattern of the slopes by completing the Change column in the table.

3. Graph the slopes of the tangent (as a function of x) in the space provided.

4. Using your pattern-analysis skills, write down the slope function ($f'(x)$).

Point	x	Slope of Tangent $f'(x)$	Change of $f'(x)$
A	-3		
B	-2		
C	-1		
D	0		
E	1		
F	2		
G	3		
The slope function $f'(x)$ is			

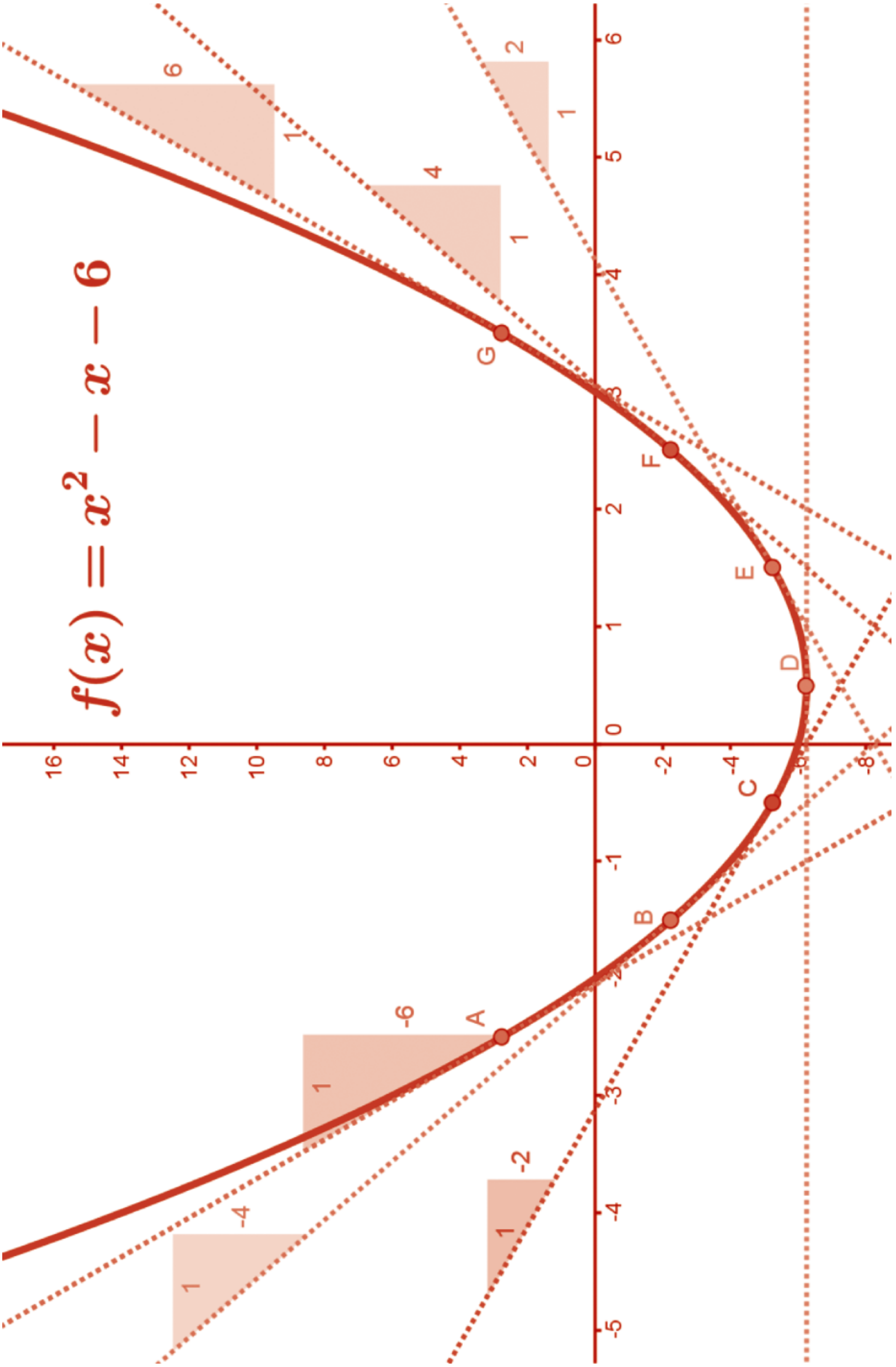


Slope of $f'(x)$:

y-intercept of $f'(x)$:

Equation of $f'(x)$:

Section B: Student Activity 6, A



Section B: Student Activity 6, B

Co-polynomial functions and their associated Slope Functions

$f(x) = x^2 + 3x + 4$

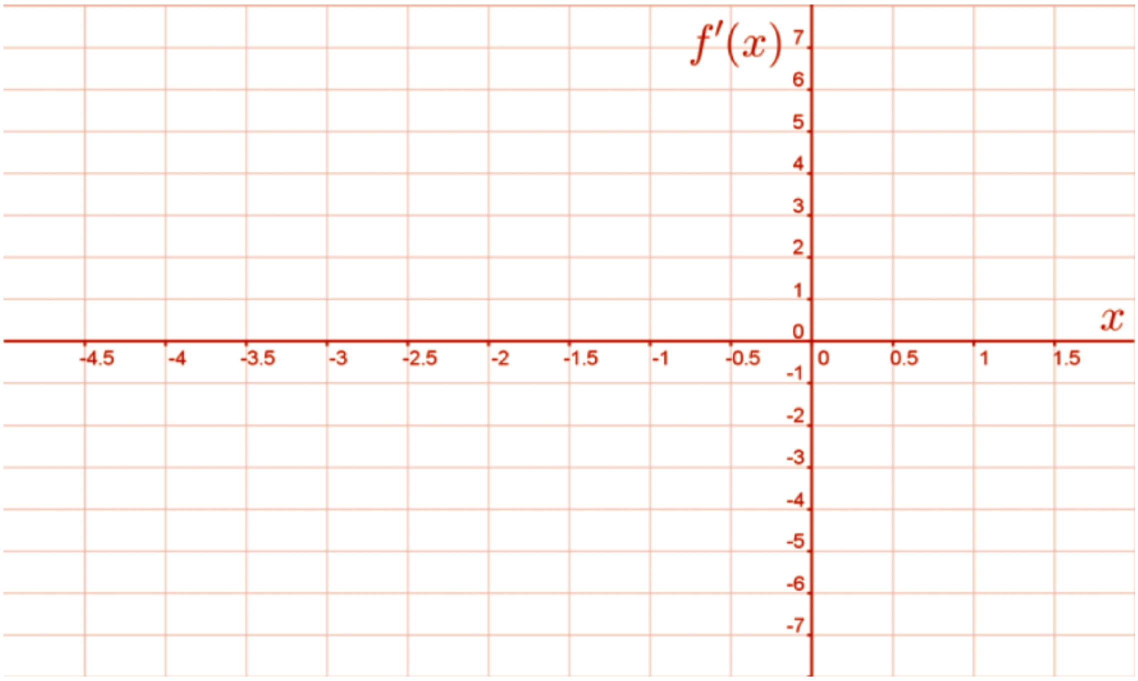
1 Using the appropriate *GeoGebra* file or the graph on the next page complete the table by filling in the slopes of the tangents to the function at points A–G.

2. Investigate the pattern of the slopes by completing the Change column in the table.

3. Graph the slopes of the tangent (as a function of x) in the space provided.

4. Using your pattern-analysis skills, write down the slope function ($f'(x)$).

Point	x	Slope of Tangent $f'(x)$	Change of $f'(x)$
A	-3		
B	-2		
C	-1		
D	0		
E	1		
F	2		
G	3		
The slope function $f'(x)$ is			

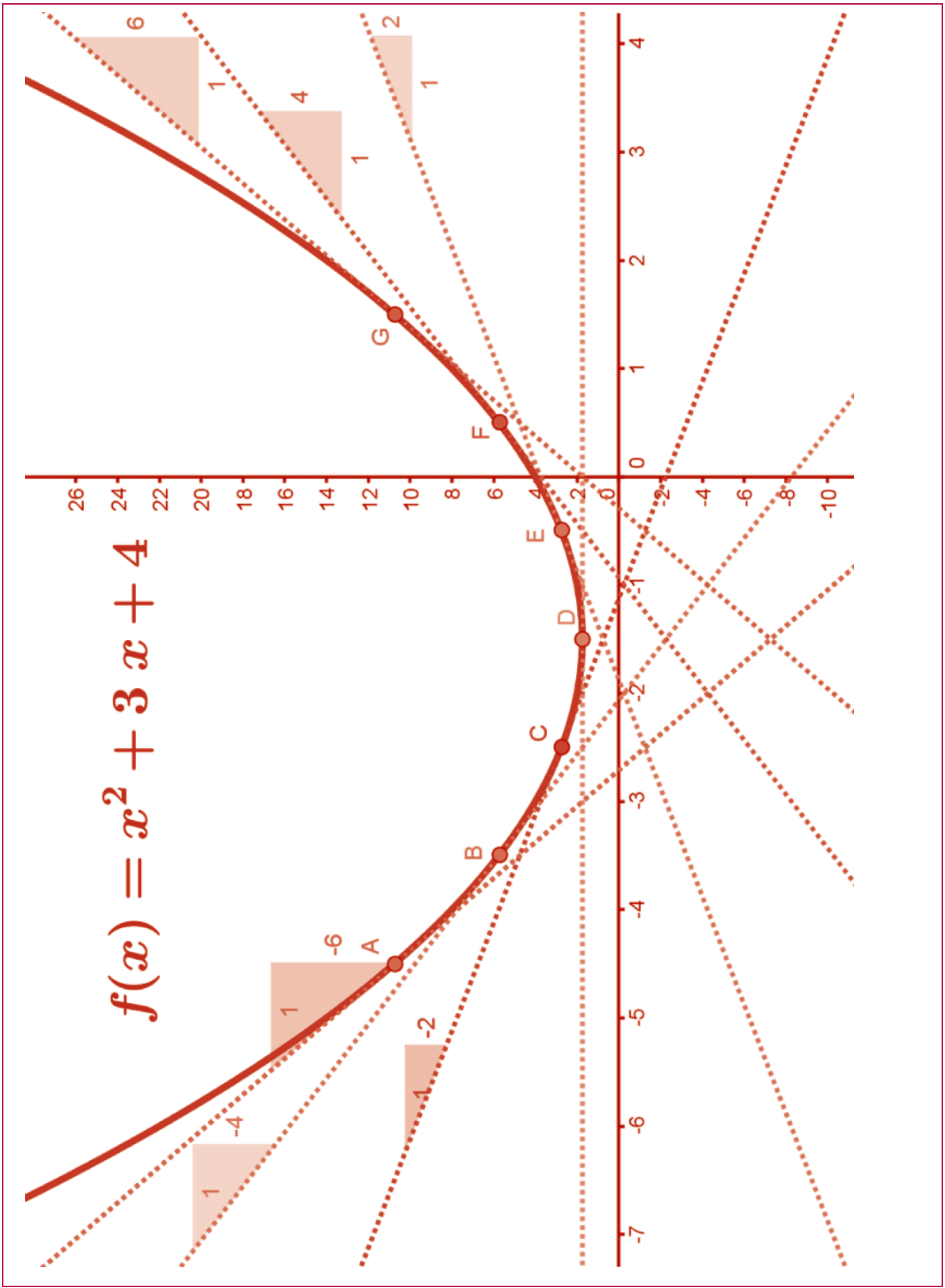


Slope of $f'(x)$:

y-intercept of $f'(x)$:

Equation of $f'(x)$:

Section B: Student Activity 6, B



Section B: Student Activity 6, C

Co-polynomial functions and their associated Slope Functions Conclusion

1 Complete the table below and explain your findings.

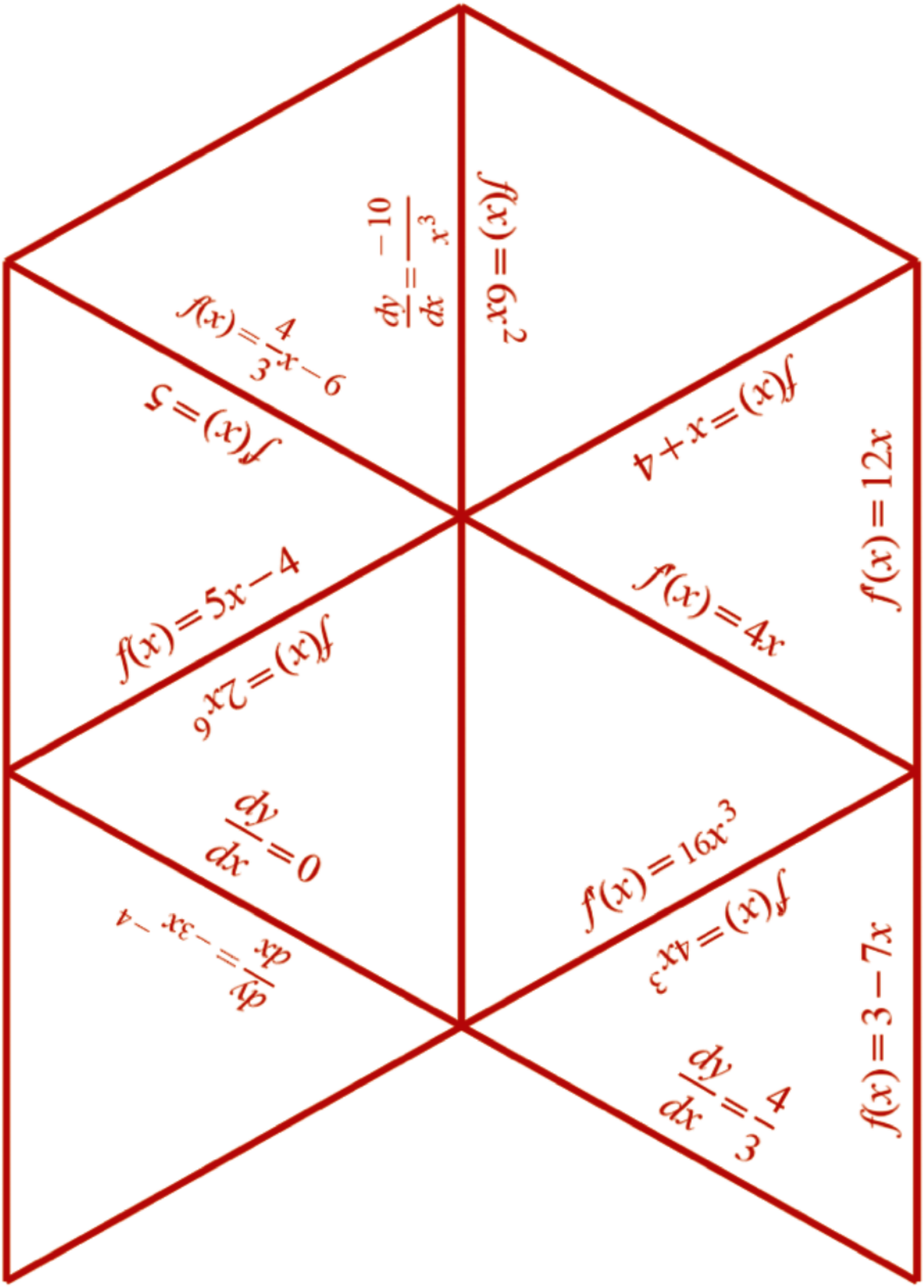
Function $f(x)$	Slope Function $f'(x)$
$f(x) = x^2 - x - 6$	
$f(x) = x^2 + 3x + 4$	
$f(x) = x^2 - 4x - 5$	
$f(x) = 3x^2 - 5x$	
$f(x) = 2x^2 + 3x - 2$	
$f(x) = ax^2 + bx + c$	

Explanation (in words)

To find the derivative (slope function) of any function of the form
$f(x) = g(x) + h(x) + k(x)$...

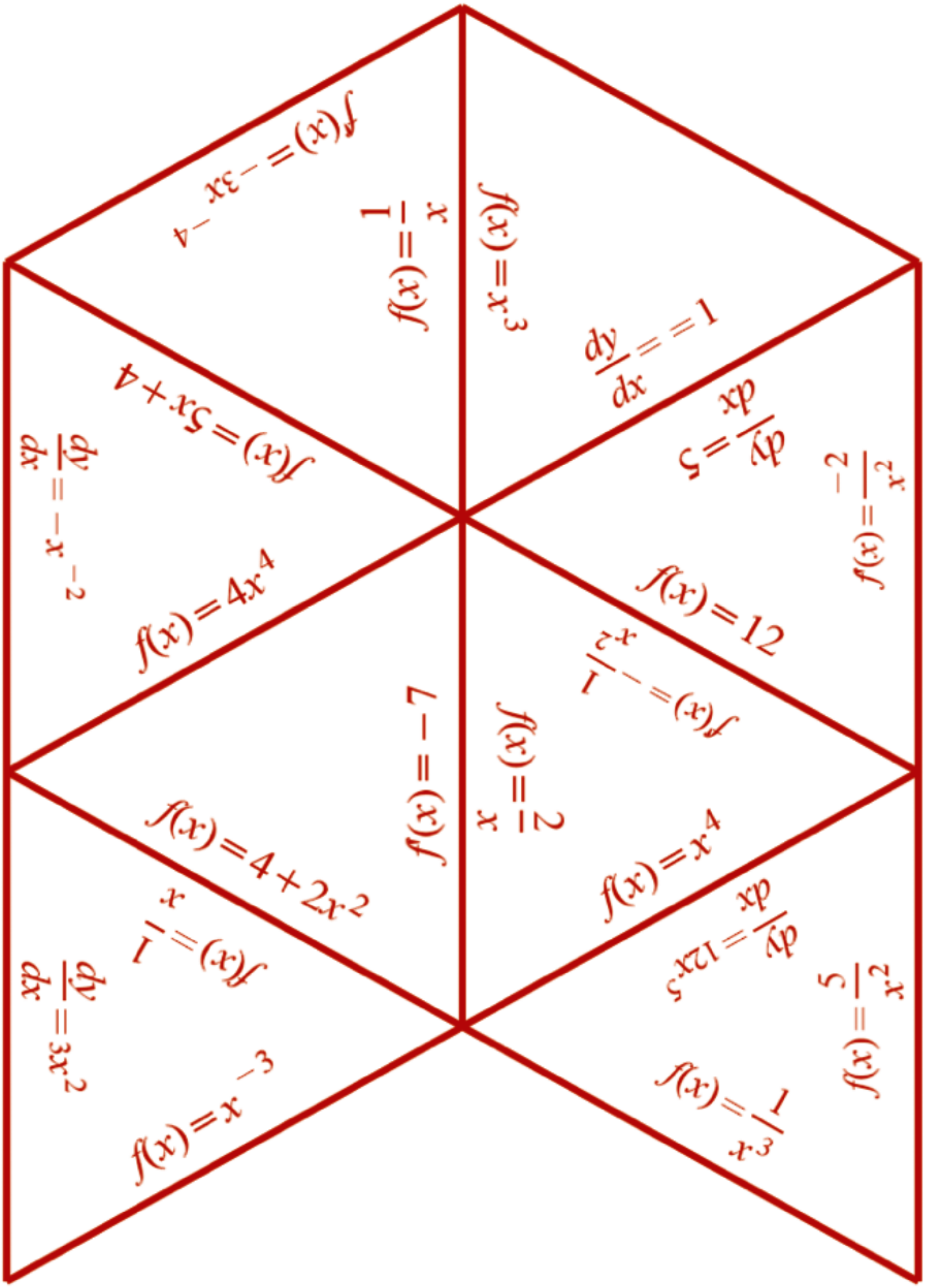
Section B: Student Activity 7

Summary of how to find slope functions Tarsia



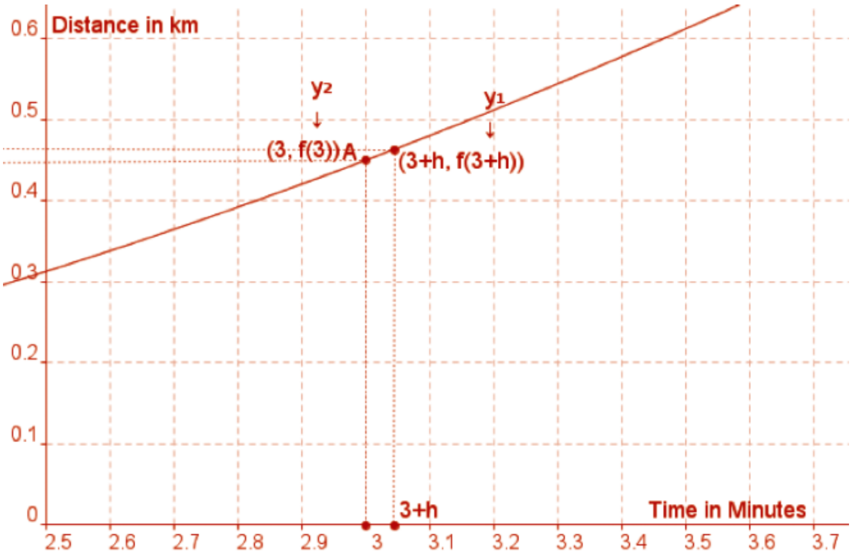
Section B: Student Activity 7

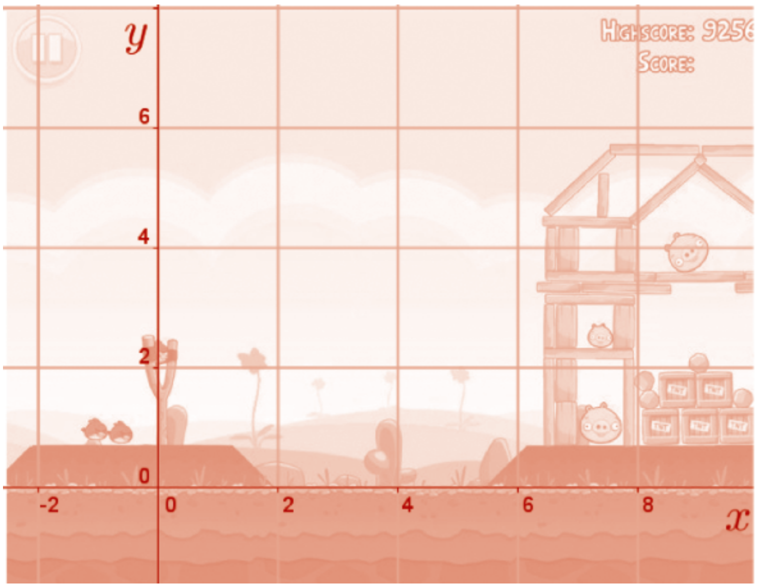
Summary of how to find slope functions Tarsia



Appendix 1

Introducing h as the interval of x values that approach 0





Slope of Secant = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{4.084 - 2.588}{2.94 - 1.56} = \frac{1.496}{1.38} = 1.084$

Average Rate of Change = $\frac{f(a+h) - f(a)}{h} = \frac{4.084 - 2.588}{2.94 - 1.56} = \frac{1.496}{1.38} = 1.084$

Continous Rate of Change = $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = 1.36$

Slope of Tangent = 1.36