# Teaching & Learning Plan

## Inferential Statistics for Proportions

**Leaving Certificate Syllabus**

Project Maths Tionscadal Mata
Development Team

# The Teaching & Learning Plans are structured as follows:

Project Maths Tionscadal Mata Development Team

**Aims** outline what the lesson, or series of lessons, hopes to achieve.

**Prior Knowledge** points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

**Learning Outcomes** outline what a student will be able to do, know and understand having completed the topic.

**Relationship to Syllabus** refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

**Resources Required** lists the resources which will be needed in the teaching and learning of a particular topic.

**Introducing the topic** (in some plans only) outlines an approach to introducing the topic.

**Lesson Interaction** is set out under four sub-headings:

i. **Student Learning Tasks – Teacher Input:** This section focuses on possible lines of inquiry and gives details of the key student tasks and teacher questions which move the lesson forward.

ii. **Student Activities – Possible Responses:** Gives details of possible student reactions and responses and possible misconceptions students may have.

iii. **Teacher's Support and Actions:** Gives details of teacher actions designed to support and scaffold student learning.

iv. **Assessing the Learning:** Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

**Student Activities** linked to the lesson(s) are provided at the end of each plan.

# Teaching & Learning Plans: Inferential Statistics for Proportions

Project Maths Tionscadal Mata
Development Team

## Aims[1]

The aim of this series of lessons is:

- To understand why sampling is important.
- To identify that there is a link between statistics and probability.
- To understand the phrase "inferential statistics".
- To understand the link between the 95% confidence and the empirical rule.
- To recognise how sampling variability influences the use of sample information to make statements about the population.
- To understand what factors must be kept in mind when sample information is used to make statements about the population.
- To apply the idea of a confidence interval.
- To understand that a sample proportion may not be the same as the population proportion.
- To evaluate margin of error for a population proportion.
- To analyse that increasing the sample size decreases the size or radius of the margin of error.
- To observe that doubling the sample size does not halve the size or radius of the margin of error.
- To analyse the idea of hypothesis testing.
- To understand how to conduct a hypothesis test on a population proportion using the margin of error.
- To understand that $\frac{1}{\sqrt{n}}$ formalises the intuitive notion about the size of a 95% confidence interval for a population proportion.
- To apply knowledge and skills relating to statistics to solve problems.
- To use mathematical language, both written and spoken, to communicate understanding effectively.

1 This Teaching & Learning Plan illustrates a number of strategies to support the implementation of *Literacy and Numeracy for Learning and Life: the National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2020* (Department of Education & Skills 2011). Attention to the recommended strategies will be noted at intervals within the Lesson Interaction Section of this Teaching and Learning Plan.

## Prior Knowledge

Students have prior knowledge of:

- Quantifying probabilities from *Teaching and Learning Plan 1: Introduction to Probability*
- Task on Household Sizes from page 2 of the Workshop 10 booklet on www.projectmaths.ie
- The Empirical Rule
- Sampling Variability
- The difference between a population and a sample.
- Simple random sampling
- Describing the shape, centre and spread of distributions
- The Data Handling Cycle.

## Learning Outcomes

As a result of studying this topic, students will be able to:

- Calculate the margin of error for a 95% confidence interval for a population proportion using $\frac{1}{\sqrt{n}}$.
- Make a statement about the population proportion using a 95% confidence interval.
- Conduct a hypothesis test on a population proportion using the margin of error.
- Understand how inferential statistics might be applied in every-day situations.

## Catering for Learner Diversity

In class, the needs of all students, whatever their level of ability level, are equally important. In daily classroom teaching, teachers can cater for different abilities by providing students with different activities and assignments graded according to levels of difficulty so that students can work on exercises that match their progress in learning. Less able students, may engage with the activities in a relatively straightforward way while the more able students should engage in more open-ended and challenging activities.

In interacting with the whole class, teachers can make adjustments to meet the needs of all of the students.

Apart from whole-class teaching, teachers can utilise pair and group work to encourage peer interaction and to facilitate discussion. The use of different grouping arrangements in these lessons should help ensure that the needs of all students are met and that students are encouraged to articulate their mathematics openly and to share their learning.

# Relationship to Leaving Certificate Syllabus

| Sub-Topic | Learning Outcomes | |
|---|---|---|
| Students learn about | *Students working at OL should be able to* | *Students working at HL should be able to* |
| 1.4 Statistical reasoning with an aim to becoming a statistically aware consumer | • discuss populations and samples<br>• decide to what extent conclusions can be generalised | |
| 1.7 Analysing, interpreting and drawing inferences from data | • recognise how sampling variability influences the use of sample information to make statements about the population<br>• use appropriate tools to describe variability drawing inferences about the population from the sample<br>• interpret the analysis and relate the interpretation to the original question<br>• make decisions based on the empirical rule<br>• recognise the concept of a hypothesis test<br>• calculate the margin of error ($\frac{1}{\sqrt{n}}$) for a population proportion*<br>• conduct a hypothesis test on a population proportion using the margin of error | • construct 95% confidence intervals for the population mean from a large sample and for the population proportion, in both cases using z tables |
| * The margin of error referred to here is the maximum value of the radius of the 95% confidence interval. | | |

# Resources Required

*Formulae and Tables*, whiteboards, rulers, *GeoGebra* and calculators.

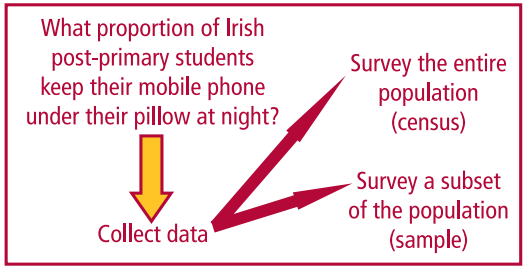
60 yellow unifix cubes, blocks or pieces of card.

140 non-yellow unifix cubes, blocks or pieces of card.

| Lesson Interaction | | | |
|---|---|---|---|
| **Student Learning Tasks: Teacher Input** | **Student Activities: Possible and Expected Responses** | **Teacher's Supports and Actions** | **Checking Understanding** |
| **Section A – Sampling variability and confidence intervals** | | | |
| » In today's lesson we are going to carry out a statistical investigation. From the investigation we would like to answer the following question: "What proportion of Irish post-primary students keep their mobile phone under their pillow at night?" | | » On one half of the board write the question "What proportion of Irish post-primary students keep their mobile phone under their pillow at night?" | |
| » When we say "Irish post-primary students" how many Irish post-primary students do we mean? | • Many.<br>• 50,000 students.<br>• 300,000 students.<br>• All of the post-primary students.<br>• All post-primary students in Ireland. | | » Do students understand that when we say "Irish post-primary students" we mean all of them? |
| » In statistics when we refer to "all" or "everybody", what name do we give to this group?<br>» So we would like to know what proportion of the population of Irish post-primary students keep their mobile phone under their pillow at night? We are interested in answering a question about a population. | • The population. | » Write the word "population" on the board and encourage students to write an explanation of the term in their copybooks. | » Do students understand that, in statistics, the complete set of people/items is known as "the population"?<br>» Do students understand that when we say "Irish post-primary students" we mean the population of Irish post-primary students? |

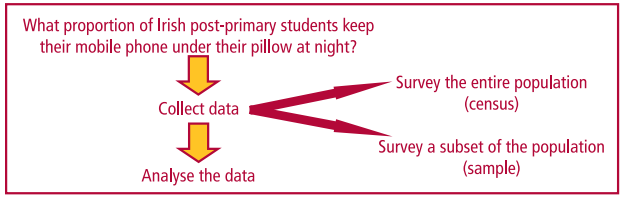
| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » Can anybody suggest how we might go about answering this question? | • We need to survey some people.<br>• We need some data.<br>• We could ask everybody here in the room. | » Write the second stage of the data-handling cycle "Collect Data" on the board. Link it to the first stage by means of an arrow.<br>  | » Can students identify the second stage of the data-handling cycle? |
| » If we were to gather the data ourselves, how many students could we ask? | • We could ask them all.<br>• We could ask some students.<br>• We could take a sample of students.<br>• 100.<br>• 1,000.<br>• All the students in our school. | » Add to the diagram of the data-handling cycle to highlight the two general approaches to gathering data-conducting a census and sampling.<br>  | » Do students understand that, in general, when gathering data you can survey the entire population or a subset of the population? |

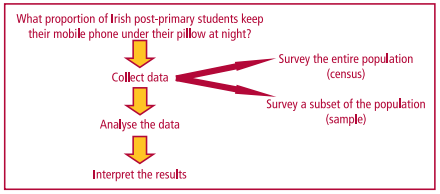
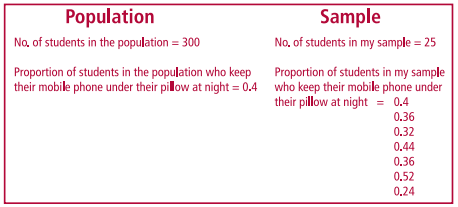
| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » Can you explain why you might choose one approach over the other? | • Asking everybody should provide a more accurate answer.<br>• Asking everybody is expensive and takes a long time.<br>• It wouldn't be possible to ask every post-primary student.<br>• Sampling is faster and cheaper.<br>• If you sample you mightn't get an accurate answer.<br>• When you sample you have to be careful to make sure the sample is representative. | » Add some of the important advantages and disadvantages of sampling vs. conducting a census to the flow chart. | » Do students recognise that there are advantages and disadvantages to both approaches to collecting data?<br>» Can students identify the advantages and disadvantages of each approach to collecting data? |
| » For many reasons you have just discussed, when answering a question in statistics, we usually use data from a sample instead of from the entire population. | | | » Do students recognise that sampling is used in the majority of statistical investigations?<br>» Do students understand why sampling is used in the majority of statistical investigations?<br>» Do students understand that the use of sampling raises the question of how accurate the results of a statistical investigation are? |

<table>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible and Expected Responses</th><th>Teacher's Supports and Actions</th><th>Assessing the Learning</th></tr>
<tr>
<td>» You also pointed out a major disadvantage to sampling – that of accuracy.<br><br>» Because of this, we are going to carry out a small investigation to see if it's possible to use the result from a sample to answer a question about a population.<br><br>» For the investigation, I have created (simulated) my own population of students using coloured counters.<br><br>» There are 300 students (or 300 counters) in my population. Each colour counter represents a different location in which students keep their mobile phone at night.<br><br>» I have purposely set up my population to have a specific proportion of students who keep their mobile phone under their pillow at night.<br><br>» I've written this proportion on a piece of paper in this envelope.<br><br>» We are now going to see if, by choosing a sample from my population, I can find out what this proportion is.</td>
<td></td>
<td>» Show students the container of counters.<br><br>» Distribute a key to each group of students explaining what each colour counter represents.<br><br>
<table>
<tr><th>Where do you keep your mobile phone at night?</th><th>Counter Colour</th></tr>
<tr><td>• under pillow</td><td>yellow</td></tr>
<tr><td>• in your bedroom</td><td>blue</td></tr>
<tr><td>• in another room</td><td>red</td></tr>
<tr><td>• other</td><td>green</td></tr>
</table>
<br>» Show students the envelope with the population proportion sealed in it. Pin it to the board.<br><br>» On one side of the board, write the heading "Population". Underneath it write "No. of students in population = 300" and "Proportion of students in population who keep their mobile phone under their pillow = _____".<br><br>» Encourage each group of students to replicate what's written on the board on their own miniature whiteboard.</td>
<td>» Do students understand that I have created a population so that I can investigate how reliable a sample is in describing a population?<br><br>» Do students understand that each unit of my population is represented by a coloured counter?<br><br>» Do students understand that different colours represent different locations in which students keep their mobile phone?<br><br>» Do students understand that we are going to use the simulated population to see if a sample can be used to determine the population proportion?</td>
</tr>
</table>

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » In turns, I would like each group to choose a simple random sample of 25 students (counters) from the container and calculate the proportion of the sample who keep their mobile phone under their pillow at night.<br><br>» This is stage three of the data-handling cycle – analyse the data. | • Students draw 25 counters from container and record results.<br><br>• Students calculate the proportion of their sample which is yellow. | » Across from the heading "Population" write a second heading "Sample". Underneath it write "Number of students in my sample = 25" and "Proportion of students in my sample who keep their mobile phone under their pillow at night = _____ "<br><br>Population: No. of students in the population = 300. Proportion of students in the population who keep their mobile phone under their pillow at night =<br><br>Sample: No. of students in my sample = 25. Proportion of students in my sample who keep their mobile phone under their pillow at night =<br><br>» Encourage each group of students to replicate what's written on the board on their own miniature whiteboard.<br><br>» Add in the third stage of the data-handling cycle to the flow chart on the board.<br><br>  <br><br>» Circulate to make sure students are completing the task correctly.<br><br>» Encourage students to write their proportion in the appropriate space on their whiteboard. | » Do students understand that they are interested in the proportion of counters which are yellow?<br><br>» Do students understand how to choose a simple random sample?<br><br>» Do students understand how to calculate a proportion? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » Group 1, could you tell me the proportion of students in your sample who keep their mobile phone under their pillow at night?<br><br>» Is there any other way in which this result could be written? | • $\frac{10}{25}$<br><br>**Note:** This is only one of the possible proportions calculated from the sample.<br><br>• $\frac{10}{25}$ or 40% or 0.4.<br><br>• As a fraction or as a decimal or as a percentage. | » Write Group 1's result in the appropriate space on the board in the form in which they reported it.<br><br>» Encourage students to convert Group 1's proportion to different representations.<br><br>» Add the different ways in which this proportion could be written to the board in the appropriate location. | » Do students recognise that a proportion may be written in different ways?<br><br>» Do students understand that fractions, decimals and percentages are equally valid ways of representing a proportion?<br><br>» Can students easily change between the different ways of representing a proportion? |
| » I am now going to use the result from Group 1's sample to make a statement about the population of 300 students. This is the final stage in the data-handling cycle – interpret the results.<br><br>» The proportion of students in the population who keep their mobile phone under their pillow at night is 0.4.<br><br>» Are you happy with this statement? | • Yes.<br><br>• Yes, I got the same result.<br><br>• Well I got a different answer.<br><br>• No, our group got a different proportion.<br><br>• We all got different answers.<br><br>• Why are we using Group 1's answer? | » On the flow-chart showing the data-handling cycle, add in the final step of "Interpret the results".<br>  <br><br><br>» Add Group 1's result to the appropriate location under the heading "Population".<br><br>» Write each group's sample proportion in the correct location under the heading "Sample".<br>  <br> | » Do students recognise that each group got a different sample proportion?<br><br>» Do students recognise that this makes it difficult to make any firm conclusions about a population based on the results of a single sample? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » The fact that we all get different proportions when we sample is known as "sampling variability".<br><br>» Can you explain why we all get different proportions i.e. can you explain why sampling variability occurs? | • Our samples were randomly chosen.<br><br>• We all chose different samples from the container.<br><br>• We chose our samples randomly so you wouldn't expect the answers to be the same. | » On the side of the board write the key term "sampling variability".<br><br>» Encourage students to discuss with each other what sampling variability means and to write the term and its description into their journals. | » Can students explain what sampling variability is?<br><br>» Do students understand why sampling variability occurs?<br><br>» Can students explain why sampling variability occurs? |
| » The aim of this activity was to see if we can use a single sample to determine the proportion of students in a population who keep their mobile phone under their pillow at night.<br><br>» Because I simulated the population, I know what the population proportion is: remember it's sealed in the envelope on the board.<br><br>» Given what we've just discovered, how confident would you be that Group 1's proportion is the same as the population proportion? | • Not very confident.<br>• I'd say it's around the right answer.<br>• Reasonable confident.<br>• I don't think it's likely to be the same. | | |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » Can you explain to me why you're not very confident with Group 1's result? | • Well, it's just one of the possible results we could get.<br>• Because of sampling variability.<br>• Different groups got different values to Group 1.<br>• There's nothing special about Group 1's result.<br>• Maybe our result is the correct one. | » Encourage students to discuss their ideas with each other.<br>» Encourage each group to share their thinking with the other groups in the classroom. | » Do students understand that Group 1's result is only one of the possible answers we can get when we sample a population? |
| » Would you have more confidence in the result from your own group? | • Not really.<br>• All the results are as good as each other.<br>• One of the results is probably correct.<br>• Some of the results are probably closer to the real value than others.<br>• There's no way to know which result is best. | » Encourage students to discuss their ideas with each other.<br>» Encourage each group to share their thinking with the other groups in the classroom. | » Do students understand that while some results are better than others we have no way of knowing which are better? |

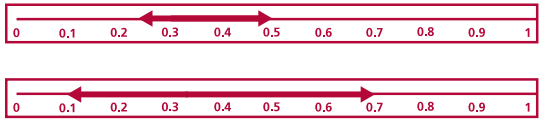
| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » So we agree that we are not very confident using a proportion from a single sample to make conclusions about the population.<br><br>» With this in mind and based on all the information we have up on the board, could you come up with a statement about the population proportion in which you'd have greater confidence? | • The proportion of students in the population who keep their mobile phone under their pillow at night is around 0.4.<br><br>• The proportion of students in the population who keep their mobile phone under their pillow at night is around 0.32.<br><br>• The proportion of students in the population who keep their mobile phone under their pillow at night is somewhere between 0.24 and 0.52.<br><br>• The proportion of students in the population who keep their mobile phone under their pillow at night is the average of all our results. | » Encourage students to come up with a statement about the population by discussing it in groups.<br><br>» Encourage students to justify their statement. | » Do students understand that they cannot assume that their sample proportion is the same as the population proportion?<br><br>» Do students understand that the chance of their sample proportion being equal to the population proportion is low?<br><br>» Do students understand that they can make a more definite statement about the population proportion (i.e. a statement in which they have more confidence) using a range or interval of values? |

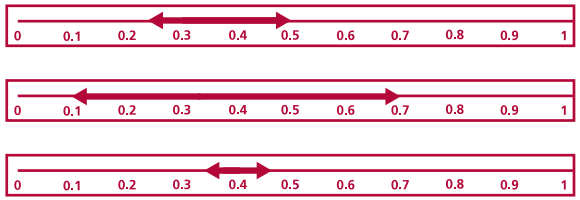
 

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » Can you explain why you have more confidence in a statement which is based on a range of values?<br><br>**Note:** If students answer "Sampling variability", this idea should be discussed with the class. | • It says that the population proportion is around 0.4 not that it's exactly equal to 0.4.<br><br>• It says that the population proportion could be lots of values, not just one.<br><br>• It takes into account the fact that different samples give different answers.<br><br>• Although different groups got different answers, they're all close to each other and the last statement takes this into account.<br><br>• It recognises the existence of sampling variability. | | » Can students explain why they have more confidence in the last statement compared to previous statements? |



| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » Our class has just come up with one of the most important ideas in statistics and that is: the emergence of a range of values when making a statement about a population based on a single sample.<br><br>» By using an interval we can have more confidence that what we are saying about the population is true.<br><br>» Using an interval takes into account the existence of sampling variability. | | » On the board cross off the statement about the population and replace it with the following "The proportion of students in the population who keep their mobile phone under their pillow at night is between 0.24 and 0.52".<br><br>**Population**<br>No. of students in the population = 300<br><br>~~Proportion of students in the population who keep their mobile phone under their pillow at night = 0.4~~<br><br>Proportion of students in the population who keep their mobile phone under their pillow at night is between 0.28 and 0.52.<br><br>**Sample**<br>No. of students in my sample = 25<br><br>Proportion of students in my sample who keep their mobile phone under their pillow at night = 0.4, 0.36, 0.32, 0.44, 0.36, 0.52, 0.24<br><br>» Write the term "confidence interval" on the board.<br><br>» Encourage students to come up with an explanation of what a confidence interval is and to write this into their journal. | » Do students understand that by using an interval I can be more confident that the statement I am making about the population is true?<br><br>» Do students understand what the term "confidence interval" means?<br><br>» Can students verbalise what a "confidence interval" is? |
| » If I were to widen my interval to make the following statement "The proportion of students in my population who keep their mobile phone under their pillow at night is between 0.1 and 0.7 – would you be more or less confident in this statement?<br><br>» Explain your reasoning. | • Less confident (wrong).<br><br>• More confident.<br><br>• More confident because we're including more possible values.<br><br>• More confident because a wider interval means it's more likely to be true. | » Draw this confidence interval on the board and compare it to our original interval.<br><br>  | » Do students understand that a narrower interval affects how confident we are in our statement about the population? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » If I were to make my interval narrower with the following statement "The proportion of students in my population who keep their mobile phone under their pillow at night is between 0.35 and 0.45 – would you be more or less confident in this statement?<br><br>» Explain your reasoning. | • Less confident.<br><br>• Less confident – we're only looking at a small number of the answers we could get.<br><br>• Less confident – there's a smaller chance that we are capturing the real population proportion. | » Add the narrower confidence interval to the diagrams on the board.<br>  | » Do students understand that a narrower interval affects how confident we are in our statement about the population? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » So when we make a statement about a population based on a single sample, how wide should we make our interval? And how confident should we be that the interval captures the population proportion?<br><br>» Luckily for us, statisticians have already decided this. Using the empirical rule, they developed a simple method for creating this interval based on the results of a single sample.<br><br>» If we take the proportion calculated from our single sample and subtract $\frac{1}{\sqrt{sample\ size}}$ from it we get the lower end of the interval.<br><br>» If we take the proportion calculated from our single sample and add $\frac{1}{\sqrt{sample\ size}}$ to it we get the upper end of the interval.<br><br>» When we do this we create an interval for which we can be 95% confident that what we are saying about the population is true.<br><br>» This is known as the 95% confidence interval. | • Students complete the calculation of the confidence interval, for the example on the board. | » Write the expression $\frac{1}{\sqrt{sample\ size}}$ on the board.<br><br>» Go through a sample calcultion on the board using any sample proportion (preferably one which wasn't calculated by any of the groups in the classroom).<br><br>» Encourage students to complete the calculation using their calculators.<br><br>**Population**<br>No. of students in the population = 300<br><br>~~Proportion of students in the population who keep their mobile phone under their pillow at night = 0.4~~<br><br>~~Proportion of students in the population who keep their mobile phone under their pillow at night is between 0.28 and 0.52.~~<br><br>Proportion of students in the population who keep their mobile phone under their pillow at night is between $0.28-\frac{1}{\sqrt{25}}$ and $0.28+\frac{1}{\sqrt{25}}$<br><br>I can be 95% confident<br><br>**Sample**<br>No. of students in my sample = 25<br><br>Proportion of students in my sample who keep their mobile phone under their pillow at night = 0.4, 0.36, 0.32, 0.44, 0.36, 0.52, 0.24 | » Do students understand that while we understand the need for a confidence interval when making statements about a population we have yet to discuss how to construct this interval using a single sample?<br><br>» Are students able to complete the confidence-interval calculation? |

Project Maths Development Team

**Teacher Reflections**

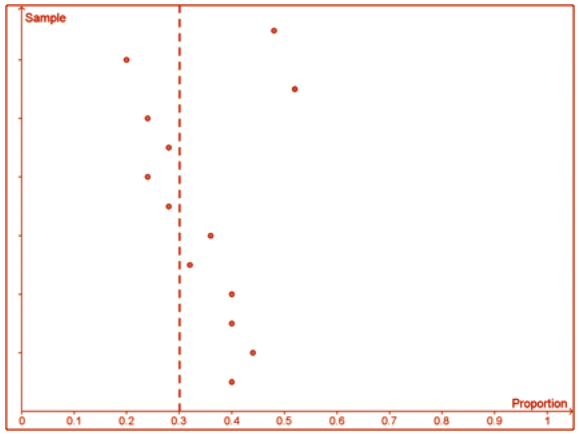
| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » Now, using your own sample proportion, I would like you to create a 95% confidence interval and use this to make a statement about the population. | • Students complete their own confidence – interval calculation. | » Circulate around the room to ensure students understand the task and are completing it correctly.<br><br>» Encourage students to write a statement about the population using their confidence interval.<br><br>» Get each group to write their confidence interval on the board beside their sample proportion. | » Do students understand that they are using their own sample proportion to construct their own confidence interval?<br><br>» Do students know how to construct a 95% confidence interval?<br><br>» Can students use their 95% interval to make a statement about the population? |
| » We introduced the idea of a confidence interval because we realised that, due to sampling variability, we cannot simply use the result from a single sample to make statements about the population.<br><br>» Let's see if our confidence-interval approach has worked i.e. does it allow us to make correct statements about the population?<br><br>» Because I simulated the population we investigated I know what the proportion of students in the population who keep their mobile phone under their pillow at night is: it's 0.3. | | » Open the envelope to reveal the proportion of students in the population who keep their mobile phone under their pillow at night. | |

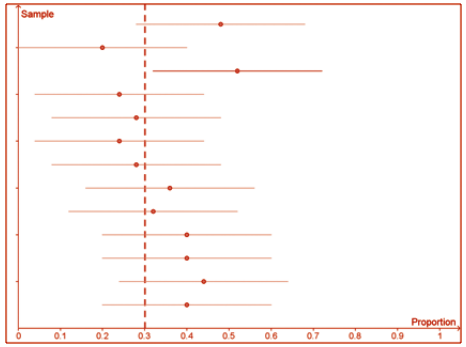
 

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » If you had used your sample proportion only to make a statement about the population, would your statement have been correct? | • No.<br>• It would have been close.<br>• It wouldn't have been too bad.<br><br>**Note:** Because the sample size is 25, it is not possible for a student to get a sample proportion of 0.3 since all the sample proportions must be a multiple of 0.04. | » Highlight each group's result on the board and the fact that none of these equal the population proportion. | » Do students see that using just the sample proportion to make a statement about the population almost certainly leads to an incorrect statement and that if it doesn't it is merely due to chance? |
| » If you use your confidence interval to make a statement about the population, is your statement correct? | • Yes.<br>• Yes the answer lies within my interval.<br>• Not for mine. | » Go through each group's confidence interval on the board and use it to make a statement about the population.<br><br>» Encourage students to answer if each statement about the population is correct or incorrect. | » Do students understand that by creating an interval around our sample proportion we are now able to make a statement about the population which is very likely (95%) to be true? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » Would you expect every group's statement about the population to be true / would you expect every group's confidence interval to contain the true population proportion? | • Yes (incorrect).<br>• No.<br>• Only 95% of the time.<br>• There's a small chance some won't. | | » Do students understand that by creating an interval in this way there is still a chance that the interval will not contain the true population proportion/the statement that they make about the population will not be true?<br>» Do students understand that you would expect your statement about the population to be true only 95% of the time?<br>» Do students understand that there is only a 95% chance of their statement/ confidence interval being correct? |

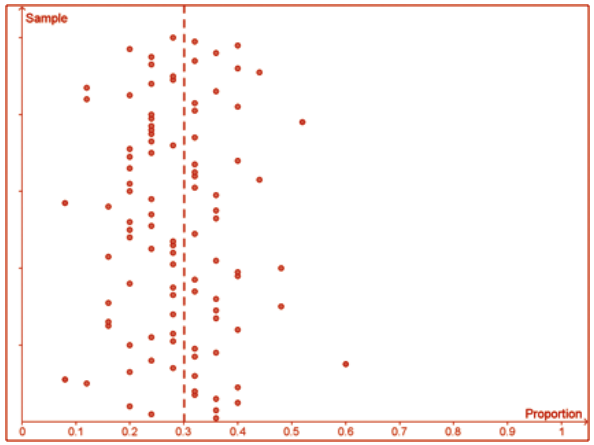
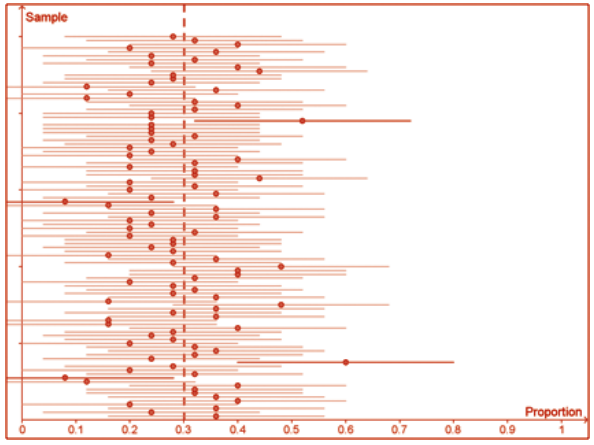
| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » Let's look at this idea using a graphical representation.<br><br>» The dashed line represents the proportion of students in our population who keep their mobile phone under their pillow at night i.e. the line represents what we were trying to find out using our single samples.<br><br>» Do any of the sample proportions match the population proportion?<br><br>» If we had made a statement about the population using our sample proportion would it have been correct? | • No.<br>• Some are close to it.<br>• Most of them are close to it but not a perfect match.<br>• Most are close to it but a few are far away.<br><br>• No.<br>• Some of us would have been close.<br>• Some groups' statements would have been way off. | » Open the *GeoGebra* file "*Confidence Interval. ggb*".<br><br>» Click the button "Show population proportion".<br><br>» Enter one group's sample proportion in the input box labelled "Single Sample Proportion" and click "Return" to plot it.<br><br>» Repeat the previous step to input and plot every group's sample proportion.<br><br>  | » Do students understand that the line represents the population proportion of 0.3?<br><br>» Do students understand that the graph reinforces the notion of sampling variability?<br><br>» Do students understand that none of their sample proportions is equal to the population proportion? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » Does every group's 95% confidence interval cover the same range of values?<br><br>» Can you explain why this is? | • No.<br><br>• Some do but most do not.<br><br>• Because of sampling variability.<br><br>• Because they were all created using a different sample.<br><br>• Because each group's sample proportion was different. | » On the *GeoGebra* file, click the button "Show 95% confidence interval".<br><br>» Encourage students to explain why each group got a different confidence interval.<br><br>  | » Do students understand that each group's confidence interval is different?<br><br>» Can students explain why each group's confidence interval is different? |
| » Does each group's confidence interval contain the population proportion we were looking for?<br><br>» Can you explain how this can be, given that the confidence intervals are all different? | • Yes.<br><br>• Most of them do.<br><br>• 95% of them do.<br><br>• The intervals overlap.<br><br>• Although the intervals are different they capture a lot of the same values. | » Encourage students to point out any intervals which do not capture the population proportion.<br><br>» Encourage students to discuss how different intervals can all capture the population proportion. | » Do students understand that although each group's confidence interval is different they all have a great deal of overlap? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » Would you expect each group's confidence interval to contain the population proportion? | • Yes.<br>• No.<br>• I'd expect most of them to.<br>• It's a 95% confidence interval so I'd expect 95% of the intervals to contain the population proportion. | » Ask students to explain to each other what it means when we say the interval we construct is a 95% confidence interval. | » Do students understand that because we have chosen to construct a 95% confidence interval, sometimes the interval will not include the population proportion? |
| » If each group makes a statement about the population using their confidence interval, will they all be correct? | • Yes.<br>• No.<br>• Most of them will be.<br>• I'd expect 95% of them to be correct in their statement.<br>• There's a 95% chance that each statement will be correct. | » Encourage students to point (on the board) those statements about the population that are correct and those that are not. | » Do students understand that when the 95% confidence interval captures the population proportion the resulting statement about the population will be true?<br>» Do students understand that using this confidence-interval approach to making statements about a population based on a single sample will be correct 95% of the time? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » We have only looked at a small number of the different samples that could have been chosen from the population of 300 students.<br><br>» Using ICT I can very quickly take many more samples and see if what we've learned so far still holds true.<br><br>» If I took 100 different samples how many of these would you expect to be the same as the population proportion?<br><br>» If I took 100 different samples and used the results to construct 100 different 95% confidence intervals, how many of these intervals would you expect to capture the population proportion?<br><br>» How many confidence intervals would you expect not to capture the population proportion? | • Very few.<br><br>• None.<br><br>• There's a tiny chance that one of the sample proportions will be equal to the population proportion.<br><br>• Most of them.<br><br>• Nearly all of them.<br><br>• 95% of them.<br><br>• Hardly any.<br><br>• Less than 5% of them. | » On the *GeoGebra* file click the "Reset" button.<br>» Click on the button "Show population proportion"<br>» Drag the slider "No. of samples" to create additional samples of size 25.<br>  <br><br>» Encourage students to identify any sample proportions which are identical to the population proportion.<br>» Click on the button "Show 95% confidence interval".<br>» Encourage students to identify any confidence interval which does not capture the population proportion.<br>  <br> | » Can students predict how many confidence intervals will capture the population proportion?<br><br>» Do students understand that when we use a 95% confidence interval we would expect our interval to capture the population proportion 95% of the time?<br><br>» Do students understand that by using this approach, we expect the statement we make about the population to be correct 95% of the time? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » Let's take a minute and review what we've just discovered. | | » Encourage students to discuss these questions with each other and to explain their thinking to each other. | » Do students understand that due to sampling variability it is highly unlikely that a result from a sample will match the result for the population? |
| » If I take a sample from a population will the results from the sample match the results for the entire population? | • No.<br>• It's highly unlikely. | » Encourage each group to write an agreed answer to each question on their whiteboards. | » Do students understand that because of this we use a confidence interval when making a statement about a population based on a sample? |
| » Can you explain why this is so? | • Because of sampling variability.<br>• Because you're only looking at a piece of the population. | » Circulate the room to see if students understand the questions. | » Do students understand that the interval we choose to create is a 95% confidence interval? |
| » Given this, is it possible to make a correct statement about the population based on the results from a single sample? | • Yes.<br>• Yes, but not just by using the result from our sample.<br>• We can't say for definite what the result for the population is but we can say that it's likely to be in a certain range.<br>• Yes we can, by using a confidence interval.<br>• Yes, by using a 95% confidence interval. | » Where needed, use appropriate questioning to help students formulate an answer to each question. | » Do students understand that a 95% confidence interval means that there is a 95% chance that the interval captures the correct answer?<br>» Do students understand that by using a 95% confidence interval, there is a 95% chance that the statement we make about the population is true? |
| » When we create a 95% confidence interval, what does this mean? | • It's very likely that the interval we create captures the population proportion.<br>• There's a 95% chance that our interval holds the correct value for the population.<br>• There's a 95% chance that the statement we make about the population is correct. | » Write a summary answer to each question on the board – based on the class's work and encourage students to take note of the questions and answers in their copybooks. | » Do students understand that there is a small (5%) chance that the confidence interval does not capture the correct answer? |
| » Is it possible that when we use a confidence interval, the statement we make about the population will not be true? | • Yes.<br>• Yes, but it's very unlikely.<br>• Yes, but there's only a 5% chance of that happening. | | » Do students understand that sometimes (5% of the time) the statement we make about the population based on the results from a sample will not be true? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » The statements we made on the board take up a lot of space and take a long time to write down. In maths we like to represent variables using symbols or letters, for this very reason. We will do the same here. | | » On the diagram on the whiteboard showing the population and sample information, re-write each statement using the correct notation. | » Do students understand that it makes sense to use short-hand notation to represent the various quantities? |
| » What notation might we use the represent the number of units in our population and in our sample? | • $N$ and $n$. | | |
| » What symbols might we use to represent the population proportion and the sample proportion? | • $P$ and $p$ (incorrect).<br>• $p$ and $\hat{p}$. | | |
| » Could we write the idea of a confidence interval using mathematical notation? | • Yes, using an inequality.<br>• $0.28 - \frac{1}{\sqrt{25}} \leq p \leq 0.28 + \frac{1}{\sqrt{25}}$<br>• $0.28 - 0.2 \leq p \leq 0.28 + 0.2$<br>• $0.08 \leq p \leq 0.48$ | » Use suitable questioning to help students write down the 95% confidence interval as an inequality. | » Can students write down the 95% confidence interval as an inequality? |
| » We've just written down the confidence interval for one specific sample proportion from a sample of size 25. Could we write down the general form of a confidence interval for any sample proportion from any sized sample? | • Yes.<br>• No.<br>• That sounds hard.<br>• $\hat{p} - \frac{1}{\sqrt{n}} \leq p \leq \hat{p} + \frac{1}{\sqrt{n}}$ | » Use suitable questioning to help students write down the general form of a 95% confidence interval for a proportion.<br><br>» Encourage students to record this in their journals. | » Can students write down the general form of a 95% confidence interval for a proportion? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » To construct a 95% confidence interval we add and subtract the same quantity from our sample proportion. What is this quantity?<br><br>» This quantity $\frac{1}{\sqrt{n}}$ is known as the margin of error. Could you explain why it is so called? | • 0.2.<br>• $\frac{1}{\sqrt{25}}$<br>• $\frac{1}{\sqrt{sample\ size}}$<br>• $\frac{1}{\sqrt{n}}$<br>• No.<br>• It takes into account the fact that our sample proportion may not be exactly right.<br>• It allows for some error in relating our sample proportion to the population proportion.<br>• It allows for the fact that the population proportion is probably different to the sample proportion. | » Circle $\frac{1}{\sqrt{n}}$ on the board and label it as the margin of error.<br><br>» Encourage students to write this term into their journals. | » Do students understand that the margin of error for a proportion is $\frac{1}{\sqrt{n}}$ ?<br><br>» Do students understand why this is known as the margin of error? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » We will now apply our newfound knowledge of sampling to another statistical investigation.<br><br>» In groups, I would like you to complete **Section A: Student Activity 1**.<br><br>» Is the proportion of the sample who admits to regularly speeding 0.42?<br><br>» How did you confirm this?<br><br>» Did you think the newspaper headline was fair? Explain your reasoning. | • Yes.<br><br>• Yes, $\frac{210}{500}$ =0.42.<br><br>• Yes, the population proportion mightn't be exactly 42% but it's likely to be close.<br>• No, the proportion of our sample is 42%. We don't know what the proportion is for all lorry drivers.<br>• No, if a different sample had been taken it would have given a different result.<br>• No, this is the result for a single sample, not the entire population.<br>• There is a 95% chance that the population proportion lies between 0.42 - $\frac{1}{\sqrt{500}}$ and 0.42 + $\frac{1}{\sqrt{500}}$. | » Distribute copies of **Section A: Student Activity 1** to all students.<br><br>» Circulate around the room to ensure that students understand what they are supposed to do and are on task.<br><br>» Encourage students to discuss the questions with each other and to explain the reasoning behind their answers.<br><br>» Encourage students to use the relevant notation when writing down their answers.<br><br>» Where needed, use suitable questioning to help students complete the activity. | » Can students apply their understanding of sampling and confidence intervals to complete **Section A: Student Activity 1**?<br><br>» Can students calculate the sample proportion from the information given?<br><br>» Do students recognise that we cannot say the population proportion is equal to the sample proportion because of sampling variability?<br><br>» Do students understand that to make a fair statement about the population, they should use a confidence interval?<br><br>» Can students calculate the 95% confidence interval?<br><br>» Can students calculate the margin of error?<br><br>» Are students comfortable using mathematical notation to represent various quantities? |

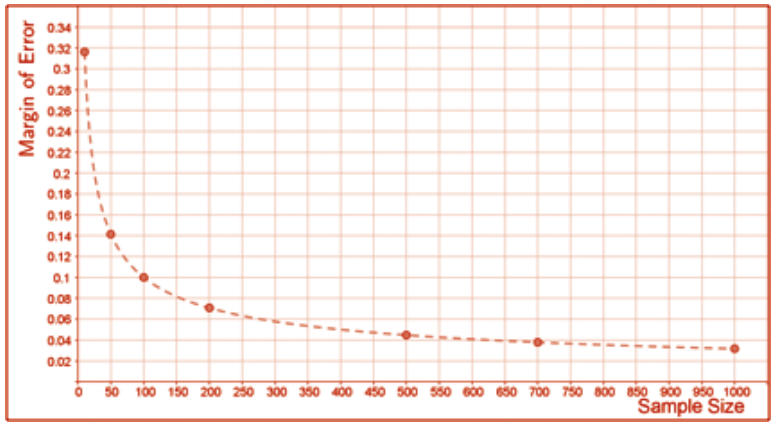
| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » What is the 95% confidence interval for the population proportion? | • $0.42 - \frac{1}{\sqrt{500}} \leq p \leq 0.42 + \frac{1}{\sqrt{500}}$<br>• $0.3753 \leq p \leq 0.4647$ | | |
| » What is the margin of error for our sample population proportion? | • $\frac{1}{\sqrt{500}}$<br>• 0.4472 | | |
| » What statement can you make about the population based on your 95% confidence interval. | • I'm 95% confident that the proportion of all truck drivers who regularly speed is between 0.3753 and 0.4647.<br>• It is most likely that the proportion of all truck drivers who regularly speed is somewhere between 37.53% and 46.47% | | |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » When we calculate the margin of error or the 95% confidence interval we use sample size to do so.<br><br>» It is important to understand how sample size affects the margin of error or how sample size affects the 95% confidence interval. | | | » Can students complete **Section A: Student Activity 2**?<br><br>» Can students calculate the margin of error for a given sample size? |
| » To this end, in groups, I would like you to complete **Section A: Student Activity 2**. | • Students complete **Section A: Student Activity 2**.<br><br>**Note:** This activity could also be given as a homework exercise which would then be reviewed and discussed at the start of the next lesson. | » Distribute **Section A: Student Activity 2** to all students.<br><br>» Circulate around the room to ensure students are completing the task correctly. | » Can students calculate the 95% confidence interval for a given sample size?<br><br>» Can students plot a graph to show the relationship between the margin of error and sample size? |
| » Can you describe what happens to the margin of error as sample size increases? | • The margin of error gets smaller.<br><br>• It decreases. | » Where needed, help students complete the activity by using suitable questioning. | » Can students describe the relationship between the margin of error and sample size? |
| » Can you describe what happens to the 95% confidence interval as the sample size increases? | • It gets narrower.<br><br>• The 95% confidence interval covers a smaller range of values. | | |

Project Maths Development Team

Teacher Reflections

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » Can you explain why this relationship exists between margin of error (or the 95% confidence interval) and sample size? | • If you used a small number of people in your sample they might not be reflective of the general population.<br>• Because of randomness, a small sample could give a result which is very different from the population.<br>• A larger sample is likely to produce a more reliable result.<br>• A large sample is more likely to give a result which is the same as the population.<br>• A small sample could, by chance, give you an answer far away from the population proportion so to capture the population proportion you would need a wide interval. | » Select different groups of students to fill in each row of the table on the board.<br><br>» Ask one group of students to sketch the graph showing the relationship between margin of error and sample size.<br><br>  | » Can students describe the relationship between the 95% confidence interval and sample size?<br>» Can students explain why these relationships exist?<br>» Do students understand how margin of error (or the width of the 95% confidence interval) affects our ability to make useful statements about a population?<br>» Do students understand how sample size affects our ability to make useful statements about the population?<br>» Do students understand the disadvantages to using larger sample sizes? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » If I were to decrease the sample size, what effect would that have on the margin of error (95% confidence interval)? | • The margin of error will increase.<br>• The width of the 95% confidence interval will increase.<br>• The 95% confidence interval will get wider. | » On the board highlight the statement made about the population based on a sample size of 10 to demonstrate how limited the statement is. | |
| » Can you identify an advantage of using a larger sample size when making statements about a population? | • Your statement covers less possible answers.<br>• Your statement is more definite.<br>• Your statement is more useful. | | |
| » Can you identify any disadvantages of using a larger sample size? | • No.<br>• Not really.<br>• It costs more money.<br>• It takes more time. | | |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » We have learned a huge amount about how to use the results from a sample to make statements about a population. | | | |
| » To review the learning, in groups, I would like you to complete **Section A: Student Activity 3**. | • Students complete **Section A: Student Activity 3**. | » Distribute **Section A: Student Activity 3** to all students.<br>» Move around the room to ensure all students understand the task. | » Can all students complete **Section A: Student Activity 3**? |
| » What information would you need to complete this task? | • The size of the sample.<br>• The number of students who said they intended to continue into third-level education. | » If students are having difficulties completing the task, use suitable questioning to guide them on their way. | » Can students describe what information they would need to complete this task? |
| » What would you do with this information? | • Use it to calculate a sample proportion.<br>• Use it to calculate a sample proportion and a 95% confidence interval.<br>• Calculate a 95% confidence interval to take sampling variability into account.<br>• Construct a 95% confidence interval and use it to make a statement about the population. | » Encourage students to write a description of how to use a sample to make a statement about a population into their copybooks. | » Can students describe the process of using the results from a single sample to make a statement about a population?<br>» Can students describe the process of constructing a 95% confidence interval and how to use it to make a suitable statement about a population?<br>» Can students explain why it is important to use a confidence interval when making a statement about a population based on a single sample?<br>» Can students explain what they would need to do to make a more definite statement about the population using a single sample?<br>» Do students understand that a larger sample size enable a more definitive statement about the population? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » What type of statement would you make about the population? | • The proportion of Junior Certificate students who intend going to third level is between $x$ and $y$.<br>• I am 95% confident that the proportion of Junior Certificate students who intend entering third-level education is between $a$ and $b$.<br>• I am 95% confident that: $\hat{p} - \frac{1}{\sqrt{n}} \leq p \leq \hat{p} + \frac{1}{\sqrt{n}}$. | | |
| » What could you do to improve the quality of the statement you make about the population? | • Use a larger sample.<br>• Increase my sample size.<br>• Reduce the margin of error.<br>• Make the 95% confidence interval narrower.<br>• We could increase $n$. | | |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| **Section B – Hypothesis Testing** | | | |
| » We are now going to look at another area of statistics which is important in everyday life and that is determining if claims made by companies, governments, or by anybody are accurate.<br><br>» Could you give me an example of a claim you've seen or heard in the media?<br><br>» Is it important to know if a claim is accurate or not? Explain. | • Taking a certain supplement will help you lose weight.<br>• Different creams can get rid of wrinkles.<br>• Some yoghurts help boost your immune system.<br>• Smoking causes cancer.<br>• Eating fatty foods causes heart disease.<br>• Support for a political party is at a certain level.<br><br>• Not really.<br>• Yes, so that we don't waste our money.<br>• Yes, especially if it's to do with your health. | » Write some examples of claims on the board. | » Do students understand what a claim is?<br><br>» Can students recall examples of claims that they have seen or heard in everyday life?<br><br>» Can students explain the importance of checking a claim? |
| » We would like to decide a fair way to determine if a claim is true or not.<br><br>» We're going to look at a particular type of claim that is a claim about a proportion. | | | |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » There is an example of such a claim in **Section B: Student Activity 1**.<br>» In groups, I would like you to complete this activity. | • Students complete **Section B: Student Activity 1**. | » Distribute **Section B: Student Activity 1** to all students.<br>» Move around the room to make sure all students understand what they are supposed to do. | |
| » To test a claim what is the first thing we need? | • Gather some evidence.<br>• Survey some customers.<br>• Get some data. | » Use suitable questioning strategies to help students who are having difficulty completing the task. | » Do students understand that to test a claim we need to gather data? |
| » In Question 2 of **Section B: Student Activity 1** you are asked to make a statement about satisfaction levels of all of the airline's customers. Is there another word used to describe "all of the airline's customers"? | • The population.<br>• The population of customers. | » Encourage students to discuss each question and to come up with an agreed answer. | » Do students understand that when we refer to all customers we are talking about the population of customers? |
| » If I made the following statement: "The proportion of the population that is satisfied with the service provided by the airline is 64%" would you be happy with it? Explain your reasoning. | • No.<br>• No, 0.64 is the sample proportion, not the population proportion.<br>• No, this is the proportion from a single sample. A different sample could give a different result.<br>• No. Because of sampling variability we cannot say that.<br>• No, the chance of that being true is tiny. | » Encourage students to explain their reasoning to each other. | » Do students understand that we cannot say that the proportion of customers in our sample that is satisfied is unlikely to equal the proportion of customers in the population that is satisfied? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » What statement did you make about satisfaction levels amongst the population of the airline's customers? | • It is very likely that between 0.6084 and 0.6716 of the population are satisfied with the service provided by the airline.<br>• There's a 95% chance that between 0.6084 and 0.6716 of the population are satisfied with the service provided by the airline.<br>• I am 95% confident that between 0.6084 and 0.6716 of the population are satisfied with the service provided by the airline.<br>• $0.6084 \leq p \leq 0.6716$. | » Sketch a proportion line on the board and mark in the sample proportion.<br>» On the same diagram, mark in the airline's claim.<br>» On the same diagram shade in the 95% confidence interval.<br>  | » Can students make a fair statement about the proportion of the population which is satisfied with the airline's service?<br>» Do students recognise the need for a confidence interval when making a statement about the population using a single sample?<br>» Can students construct a 95% confidence interval correctly? |
| » Based on our evidence do you think the airline is correct to claim that 70% of their customers are satisfied with the service they provide? Explain your reasoning. | • No.<br>• Yes. Their claim is close to the result we got.<br>• No. There is a more than a 95% chance that they are wrong.<br>• No. There is a less than 5% chance that their claim is right.<br>• They could be right but it's very unlikely.<br>• No. We know there is a 95% chance that the true population proportion lies between 0.6084 and 0.6716 so their claim of 0.7 is unlikely to be true. | | » Do students recognise that the chance of the true population proportion lying outside the 95% confidence interval is very low?<br>» Do students understand that, based on the 95% confidence interval, it is extremely unlikely that the airline's claim is correct? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » Is it possible that the true proportion of the population that is satisfied is not between 0.6084 and 0.6716? | • Yes.<br>• Yes, but it's very unlikely.<br>• Yes, but the chance of that is less than 5%. | | |
| » When we test a claim we use a 95% confidence interval to determine if the claim is fair or not.<br><br>» If a claim lies outside the 95% confidence interval constructed using our data, we reject the claim. | | | » Do students understand that if a claim lies outside the 95% confidence interval we reject the claim (because it only has a 5% chance or less of being true)? |
| » Does this mean we are rejecting Go Fast Airline's customer-satisfaction claim of 70%? Explain. | • Yes.<br>• Yes, because their claim lies outside our 95% confidence interval.<br>• Yes, because 0.7 lies outside the 95% confidence interval we constructed. | » On the board write "I reject the airline's claim". | » Do students understand that because the airline's claim of 70% lies outside our 95% confidence interval we reject their claim? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » What do you think our conclusion would have been had the company's claim lay within our 95% confidence interval. | • We would have accepted the company's claim.<br>• We'd conclude that the company's claim is true.<br>• We'd conclude that it is reasonable to say that the proportion of all customers that is satisfied is 70%. | | » Do students understand that if the claim lies within the 95% confidence interval our conclusion will be different? |
| » Let's look at **Section B: Student Activity 2** to see if your last statements make sense.<br>» I would like you to complete the activity, working in groups. | • Students complete **Section B: Student Activity 2**. | » Distribute **Section B: Student Activity 2** to all students.<br>» Move around the room to make sure that all students are on task and know what to do.<br>» Help students who are having difficulties with the task using suitable questioning. | |

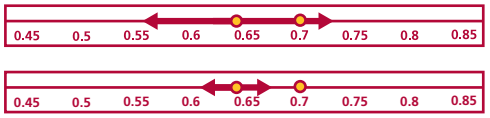
 

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » To test a claim, what is the first thing we need to do? | • Gather some evidence.<br>• Survey some customers.<br>• Get some data. | | » Can students identify that the first thing needed to test a claim is some data? |
| » What statement did you make about satisfaction levels amongst the population of the airline's customers? | • There's a 95% chance that between 0.5584 and 0.7216 of the population are satisfied with the service provided by the airline.<br>• I am 95% confident that the proportion of the population satisfied with the airline's service is between 0.5584 and 0.7216. | » On the board, below the proportion line used in **Section B: Student Activity 1**, draw another proportion line.<br>» On the new proportion line, mark in the airline's claim.<br>» On the new proportion line mark in the sample proportion.<br>» On the new proportion line shade in the 95% confidence interval.<br>  | » Can students construct a 95% confidence interval using the data and use it to make a statement about the population?<br>» Can students make a suitable conclusion regarding the airline's claim? |
| » Based on your data, would you reject the airline's claim of a 70% satisfaction rating? Explain. | • No.<br>• No, because the airline's claim lies within my 95% confidence interval.<br>• No, because given my 95% confidence interval runs from 0.5584 to 0.7216 it is possible that the true proportion of customers satisfied with the service is 70%.<br>• No, I'd accept the claim. | | |
| » Based on your data, would you accept the airline's claim of a 70% satisfaction rating? Explain. | • Yes.<br>• Yes, because the claim lies within my confidence interval. | » Write the conclusion "I accept the airline's claim" on the board. | » Do students naturally use the word "accept" when making a conclusion about a claim which lies within the 95% confidence interval? |

Project Maths Tionscadal Mata Development Team

Teacher Reflections

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » We need to be a little careful here with how we describe our conclusion regarding the company's claim. Let's understand why. | | | |
| » **Section B: Student Activity 1** and **Section B: Student Activity 2** are similar to each other. Can you identify the similarities between the two activities? | » They're both about the same airline.<br>» The claim is the same in both cases.<br>» The sample proportion we calculate is the same (0.64) for both. | | |
| » Even though the sample proportion is the same for both activities our conclusions are very different for each. How can this be? | » Because the confidence intervals are different sizes.<br>» The margin of error is not the same for each.<br>» Because the confidence interval is wider in **Section B: Student Activity 2**, the airline's claim lies within in.<br>» Because the confidence interval is narrower in **Section B: Student Activity 1**, it fails to capture the airline's claim and so we reject the claim. | » Point out that the confidence intervals from **Section B: Student Activity 1** and **Section B: Student Activity 2** have different widths.<br>  | » Do students recognise that the confidence intervals in **Section B: Student Activity 1** and in **Section B: Student Activity 2** are different widths? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » Why is the confidence interval wider in **Section B: Student Activity 2** compared to **Section B: Student Activity 1**? | • Because the sample size is smaller.<br>• Because we didn't sample as many customers.<br>• Because the margin of error gets bigger as the number in your sample gets smaller.<br>• Because the smaller $n$ is, the wider your 95% confidence interval will be. | | » Do students understand what causes the difference in the confidence-interval widths? |
| » In **Section B: Student Activity 2**, the consumer agency had originally planned to sample 1000 customers but only managed to sample 150. If they had sampled 1000 customers do you think the 95% confidence interval would still have captured the airline's claim of 0.7? Explain. | • Probably not.<br>• No, because the 95% confidence interval would have been much narrower.<br>• No, because we would have gotten a similar result to **Section B: Student Activity 2**. | | |
| » With this in mind, for **Section B: Student Activity 2**, can you explain why the airline's claim falls within the 95% confidence interval? Is it because the claim is now true? | • Yes.<br>• Not necessarily.<br>• Maybe.<br>• It's because the confidence interval is now wider.<br>• It's because we had a much smaller sample. | | » Do students understand that the reason we get two different conclusions for **Section B: Student Activity 1** and **Section B: Student Activity 2** is because of the different confidence-interval widths? |
| » When we test a claim and find that it lies within the 95% confidence interval, what could the reason be? | • Because the claim is true.<br>• Because a small sample was used making the 95% confidence interval very wide.<br>• It could be for two reasons: The claim could be true or we mightn't have used a big enough sample to find out that it's false. | | » Do students understand that a claim may lie inside our 95% confidence interval, not because it is true but because the confidence interval is really wide?<br>» Do students understand that a claim may lie inside our 95% confidence interval because we used a small sample to test it? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » For this reason when a claim lies within the 95% confidence interval we don't say we accept the claim, rather we say we fail to reject the claim.<br><br>» While each of these statements may seem to say the same thing, they do not. | | » Draw a line through the statement "I accept the company's claim" and replace it with "I fail to reject the company's claim".<br>» Underline the two possible conclusions when testing a claim "reject" and "fail to reject". | » Do students understand that "accept a claim" and "fail to reject a claim" do not mean the same thing? |
| » If we say "we accept the claim" what does this mean? | • It means the claim is true.<br>• It means we accept the claim to be true. | | » Do students understand why we use the term "fail to reject" instead of "accept" when making a conclusion about a claim? |
| » If we say "we fail to reject the claim" what does this mean? | • It means the claim could be true.<br>• It means the claim could be true or we haven't used a big enough sample to find out. | | |
| » When we are evaluating a claim using a sample what are the two possible conclusions we can make? | • We can reject the claim or we can fail to reject the claim. | | » Do students understand that when making a conclusion about a claim the two options are "reject the claim" or "fail to reject the claim"? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » In groups, I would like you to complete Question 1 of **Section B: Student Activity 3**. | • Students complete Question 1 of **Section B: Student Activity 3**. | » Distribute **Section B: Student Activity 3** to all students. | » Can students test the claim contained in the Activity Sheet? |
| » What was your conclusion about the Department of the Environment's claim? | • I rejected the claim (wrong).<br>• I accepted the claim (incorrect language).<br>• I failed to reject the claim. | » Move around the room, using suitable questioning to test students' understanding of the activity.<br><br>» Draw a proportion line on the board and fill in the claim, the sample proportion and the 95% confidence interval.<br><br>  | » Do students use the correct language when making their conclusion on the claim?<br>» Can students justify their work? |
| » Why did you fail to reject the claim? | • Because the claim lies within our 95% confidence interval.<br>• Because the claim of 0.85 lies within our 95% confidence interval of 0.7984 and 0.8616. | | » Can students justify the use of "fail to reject" when making their conclusion? |
| » Why is it incorrect to say we accept the claim? | • Because the fact that the claim lies within my 95% confidence interval does not necessarily mean it's true.<br>• The existence of the claim within my 95% confidence interval might be due to a small sample and not because the claim is true.<br>• If I took a larger sample, my margin of error would be smaller and the claim might lie outside my 95% confidence interval. | | » Can students explain why using the word "accept" is not correct? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » As with all areas of mathematics, there is formal language to describe the process of testing a claim. | | | |
| » I am going to describe the formal language used and as I do so I would like you to answer Question 2 of **Section B: Student Activity 3**. | • Students answer Question 2 of **Section B: Student Activity 3**. | » On one side of the board write the term "Hypothesis Test". | |
| » The process of testing a claim is known as a hypothesis test. | | | |
| » The claim being made is known as the null hypothesis. The shorthand notation for the null hypothesis is $H_0$. | | | |
| » What is the null hypothesis in **Section B: Student Activity 3**? | • The proportion of Irish households who pay the local property tax is 0.85.<br>• 85% of Irish households pay the local property tax.<br>• $p = 0.85$. | | » Do students understand that the null hypothesis means the claim being tested?<br>» Can students identify the null hypothesis?<br>» Do students understand that the alternative hypothesis is the counter claim? |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » For every claim that's made we can make a counter claim. The counter claim is known as the alternative hypothesis. The shorthand notation for the alternative hypothesis is $H_A$ or $H_1$. | | | |
| » What is the alternative hypothesis in **Section B: Student Activity 3**? | • 85% of Irish households do not pay the local property tax (wrong). | | » Can students identify the alternative hypothesis? |
| » When testing a claim (or carrying out a hypothesis test) it is usual to start off by stating the null hypothesis and the alternative hypothesis. | • The proportion of Irish households that pays the local property tax is not 0.85.<br>• $p \neq 0.85$. | | |

| Student Learning Tasks: Teacher Input | Student Activities: Possible and Expected Responses | Teacher's Supports and Actions | Assessing the Learning |
|---|---|---|---|
| » We have learned a huge amount about testing a claim or about carrying out a hypothesis test.<br><br>» Let's take some time to review what we've learned. | | | |
| » Working in pairs, I would like you to complete Question 1 of **Section B: Student Activity 4**.<br><br>» When you have completed Question 1 of **Section B: Student Activity 4**, I want you to swap your worksheet with the group next to you. | • Students complete Question 1 of **Section B: Student Activity 4**. | » Distribute **Section B: Student Activity 4** to all students.<br><br>» Move around the room to ensure all students are fully engaged and that they understand what they are meant to do. | » Can students accurately describe the steps involved in carrying out a formal hypothesis test?<br><br>» Can students complete a formal hypothesis test, using appropriate language? |
| » When you've received the other group's worksheet, I want you to answer Question 2 of **Section B: Student Activity 4** using their instructions on how to carry out a hypothesis test.<br><br>» When you've answered Question 2, I want you to correct the other group's Question 1 and give them feedback on their description of a hypothesis test. | | » Emphasise that, having swapped worksheets with another group, students must use the other group's instructions to answer Question 2 of **Section B: Student Activity 4**.<br><br>» Encourage each group to give constructive feedback to each other in relation to their description of a hypothesis test. | » Can students critically evaluate other students' description of a hypothesis test? |

# Section A: Student Activity 1

The ***Road Safety Authority of Ireland (Údaras Um Shábháilteacht Ar Bhóithre)*** is interested in how many lorry drivers speed on a regular basis. To answer this question they choose a simple random sample of lorry drivers from the members list of the Irish Road Haulage Association and asked them if they regularly break the speed limit. Of the 500 drivers who replied, 210 stated that they regularly speed.

1. Using suitable calculations, confirm that the proportion of the sample that admit to regularly speeding is 0.42.

2. A national newspaper includes the following headline on its front page: "42% of all Irish lorry drivers admit to regularly speeding". Is this a fair statement? Explain your reasoning.

3. Construct a 95% confidence interval for the proportion of Irish lorry drivers who admit to speeding.

4. Write down the margin of error for this sample.

5. Using your 95% confidence interval, make a statement about all lorry drivers which you consider fair.

# Section A: Student Activity 2

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The government is interested in finding out what proportion of the Irish population is in favour of introducing water metering. To find this out, they commission ***RED C Research and Marketing Ltd.*** to carry out a survey on a sample of the population. RED C find that 0.37 of their sample are in favour of introducing water metering.

1. The table below shows some of sample sizes RED C may have used to calculate the proportion of 0.37. For each sample size presented:

a Calculate the margin of error (correct to three decimal places).

b Construct a 95% confidence interval for the population proportion.

c Shade in the 95% confidence interval on the proportion line.

d Make a statement about the proportion of the Irish population which is in favour of introducing water metering.

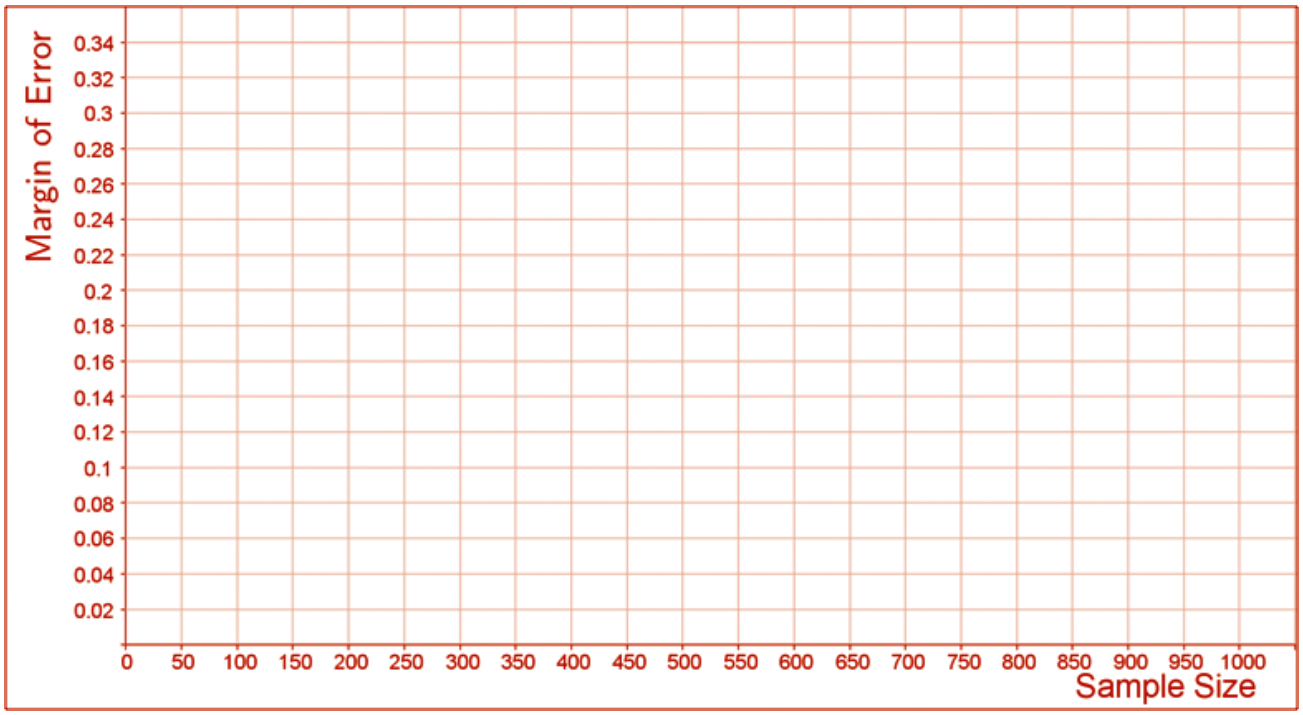
| Sample size | Margin of error | 95% confid-ence interval | The Proportion Line | Statement about the population |
|---|---|---|---|---|
| 10 | | | 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 | I can be 95% confident that between ___ and ___ of the population are in favour of water metering. |
| 50 | | | 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 | |
| 100 | | | 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 | |
| 200 | | | 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 | |
| 500 | | | 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 | |
| 700 | | | 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 | |
| 1000 | | | 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 | |

# Section A: Student Activity 2 (continued)



2. Draw a graph of margin of error versus sample size.




3. By referring to your table and graph complete the following statements:

a As the size of a sample increases, the margin of error _____.

b As the size of a sample increases, the 95% confidence interval _____.

4. By referring to the fourth column of the table, can you see any advantage to using a larger sample size? Explain your reasoning.

# Section A: Student Activity 3

For future planning, ***The Central Application Office (CAO)*** are interested in what proportion of current Junior Certificate students think it likely that they will continue to third-level education after leaving second-level education. Due to cost and time concerns they decide to use a sample of students to help them answer this question.

Describe how the CAO should use a single sample of students to make a fair statement about all students. As part of your description you should consider the following:

a What information would the CAO need?

b What the CAO would do with this information and why would they do this? (Include a description of any calculations needed.)

c The type of statement would the CAO make about the population.

d What the CAO could do to improve the quality of the statement they make about the population?

# Section B: Student Activity 1

***Go Fast Airlines*** advertise that 70% of their customers are satisfied with the service they provide. A consumer agency wants to determine if the company's claim is true or not.

1. Describe how the consumer agency could go about determining if the claim is true or not?

2. The consumer agency survey 1,000 customers and find that 640 of these are satisfied with the service provided by the airline. Using this information make a fair statement about satisfaction levels amongst all of the airline's customers.

3. Based on your answer to Question 2, do you think the airline's claim of a 70% satisfaction rating is correct? Explain your reasoning.

# Section B: Student Activity 2

***Go Fast Airlines*** advertise that 70% of their customers are satisfied with the service they provide. A consumer agency wants to determine if the company's claim is true or not.

1. Describe how the consumer agency could go about determining if the claim is true or not?

2. The consumer agency plan to survey 1000 customers to help them test this claim, however due to time and money constraints they only manage to survey 150 customers. Of these, 96 say that they are satisfied with the service provided by the airline. Use this information to make a fair statement about satisfaction levels amongst the population of the airline's customers.

3. Based on your answer to Question 2 what would you conclude about the airline's claim.

# Section B: Student Activity 3

***The Department of the Environment, Community and Local Government (Roinne Comhshaoil, Pobail agus Rialtais Áitiúil)*** claims that 85% of households now pay the local property tax (LPT). An opposition party commissions research to test the validity of this claim. This research finds that, out of a sample of 1,000 households surveyed, 830 confirm that they pay the LPT.

1. Based on the evidence presented, what would you conclude about The Department of the Environment's claim?

2. Fill in each of the following terms for the above claim.

| | |
|---|---|
| Null Hypothesis | |
| Alternative Hypothesis | |

# Section B: Student Activity 4

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1. In the space below, write a brief, step-by-step guide of how to carry out a hypothesis test. As part of this you should include explanations of why you carry out each step.

| |
|---|
| Step 1 |
| Step 2 |
| Step 3 |
| |
| |
| |

# Section B: Student Activity 4 (continued)



2. Using the above instructions, answer the following question: Manchester United claims that 90% of fans were in favour of replacing David Moyes as manager. The Manchester United fanzine, www.rednews.com carries out a survey of 2,000 of its members, 1,700 of whom state that they were in favour of replacing David Moyes.

   Conduct a hypothesis test of Manchester United's claim using the evidence presented.

| |
|---|
| Step 1 |
| Step 2 |
| Step 3 |
| |
| |
| |