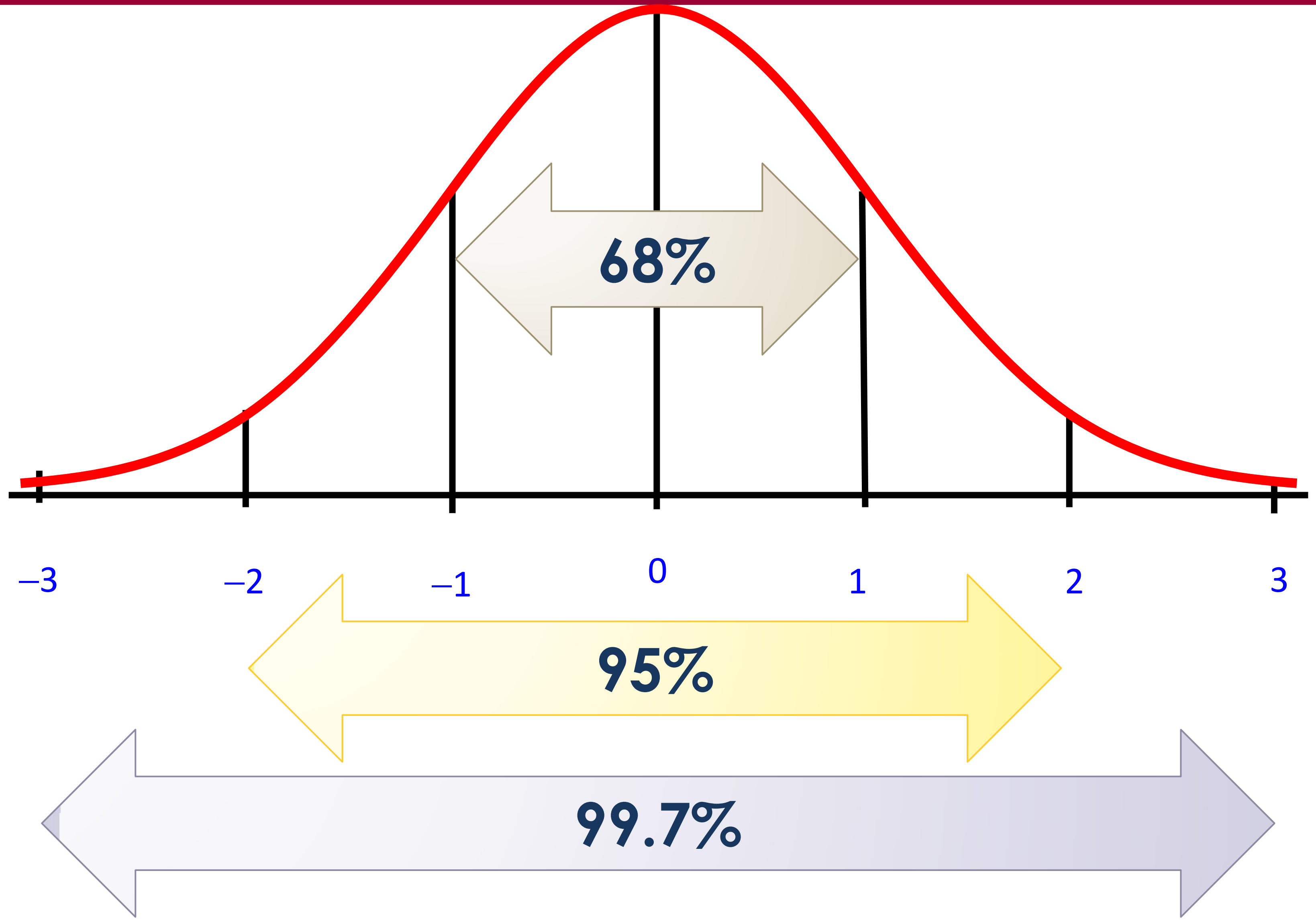
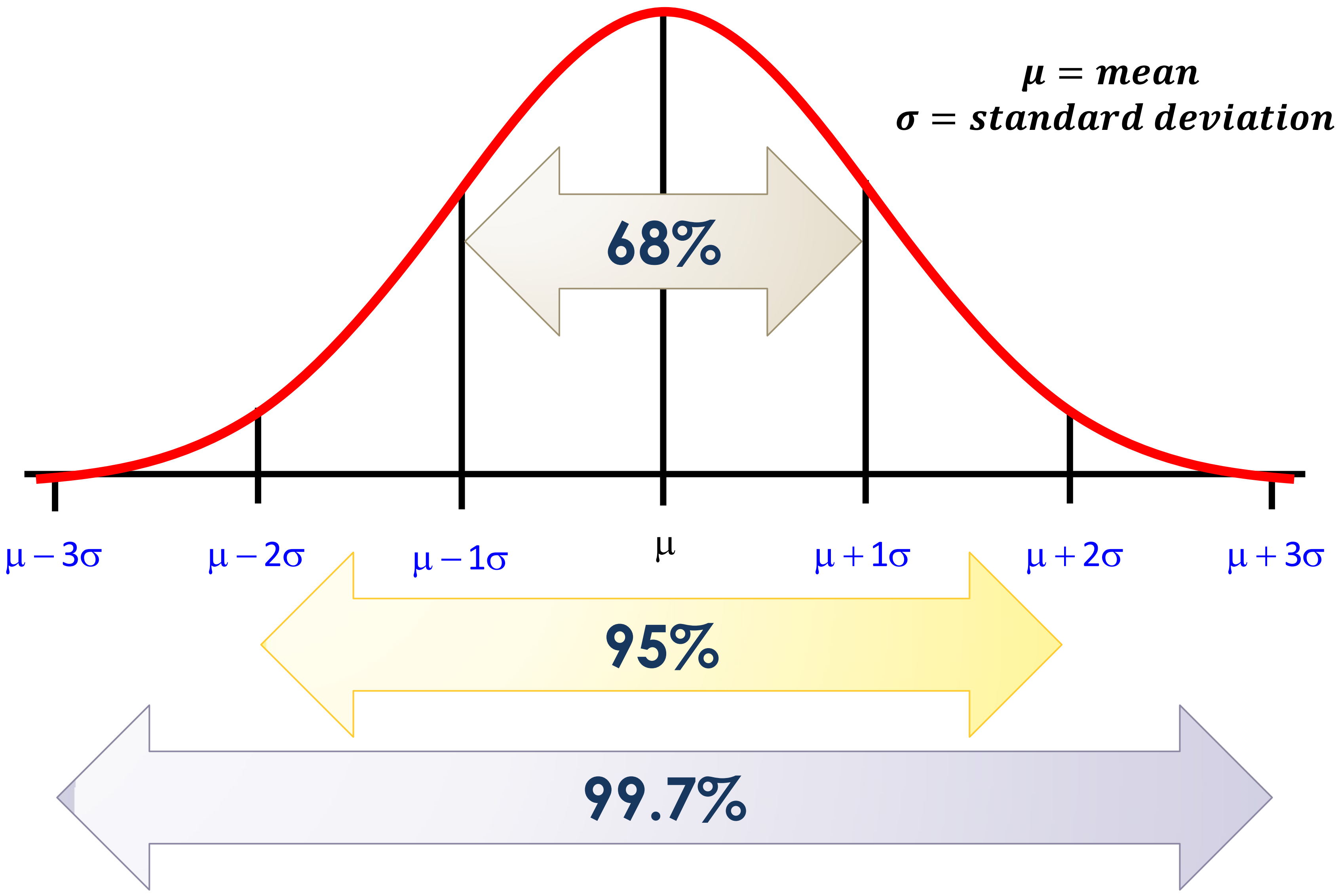
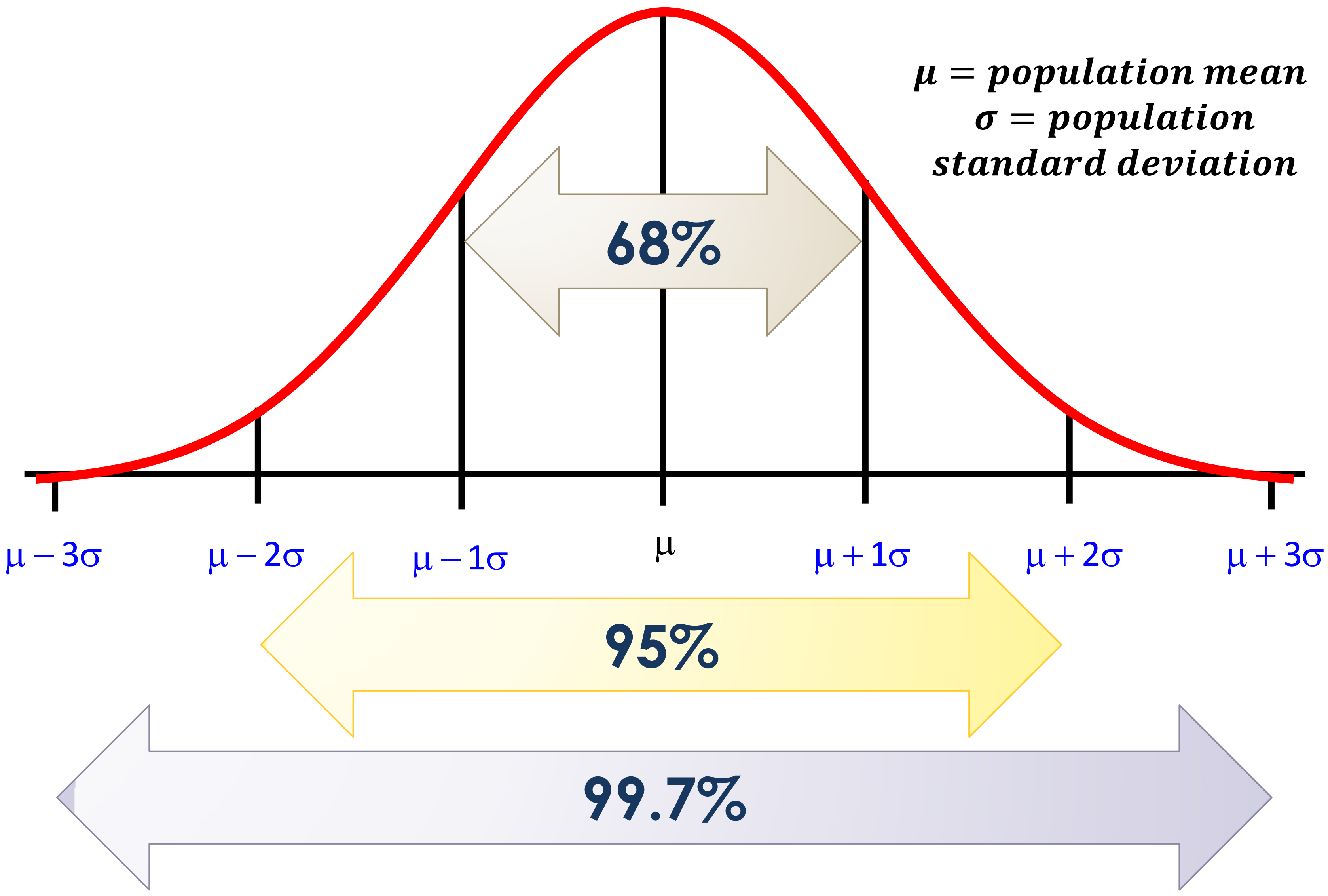
Making Decisions based on the Empirical Rule (Standard Normal Curve)



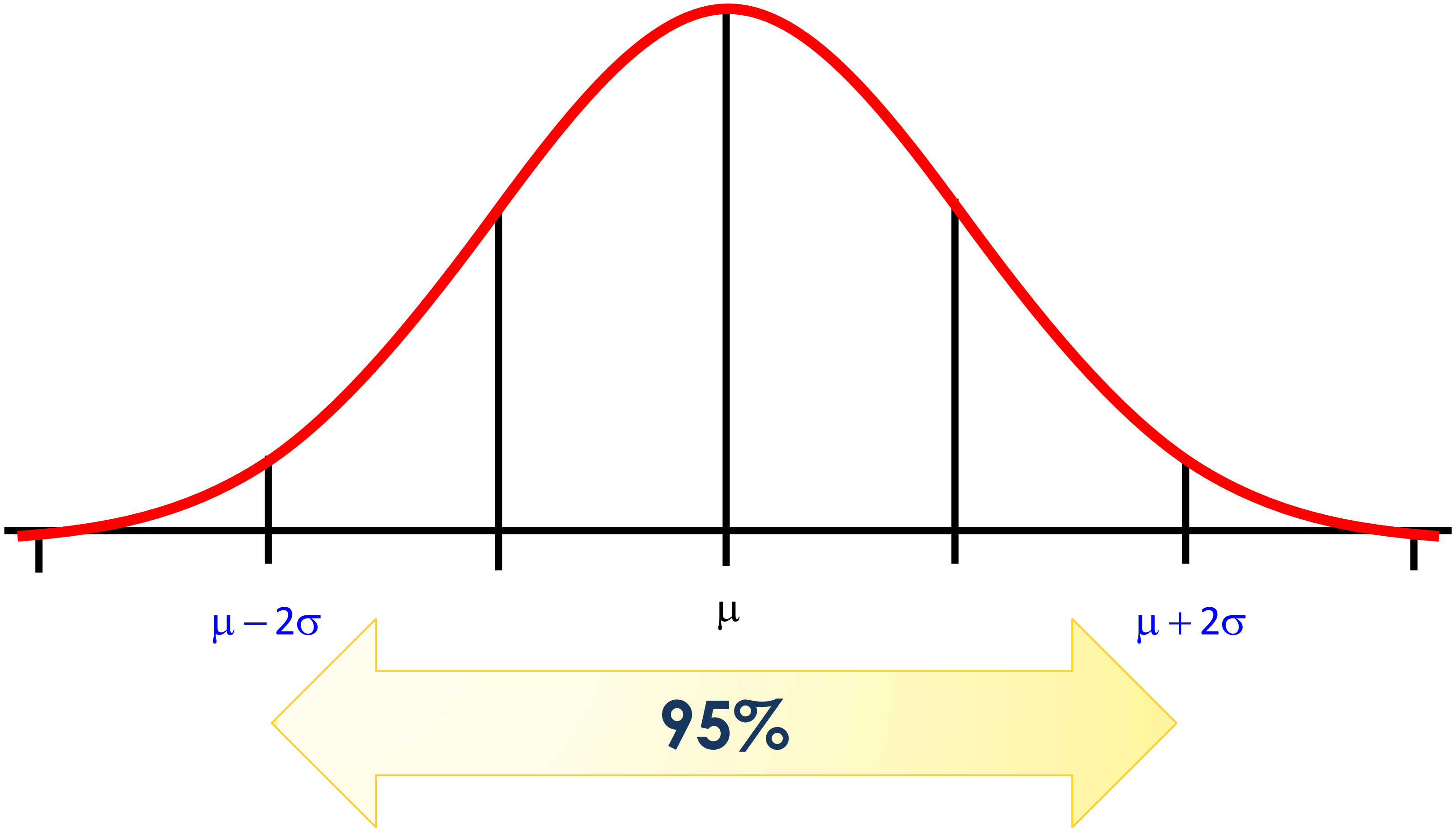
Empirical Rule



Empirical Rule



Most Important for Inferential Stats on our Syllabus



95% of normal data lies within 2 standard deviations of the mean

Example 1

IQ scores are normally distributed with a mean of 100 and a standard deviation of 15.

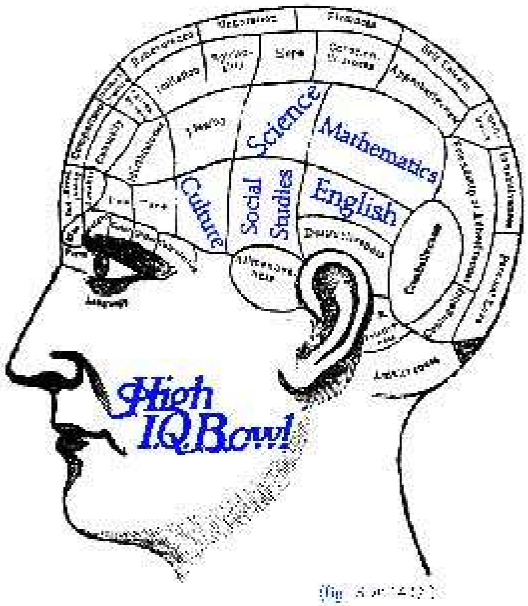
Use the Empirical Rule to show that 95% of IQ scores in the population are between 70 and 130.

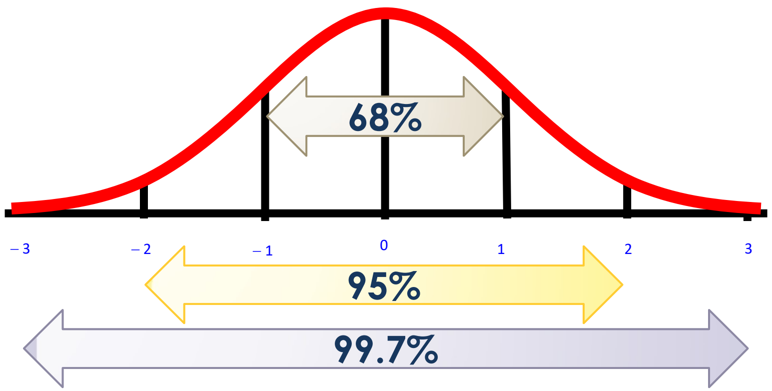
Solution

95% of the IQ scores are within ± 2 standard deviations of the mean.

$$100 + 2(15) = 100 + 30 = 130$$

$$100 - 2(15) = 100 - 30 = 70$$





Example 2

The number of sandwiches sold by a shop from 12 noon to 2 pm each day is normally distributed. The mean of the distribution was 42.6 sandwiches and a standard deviation of 8.2.
Use the Empirical Rule to identify the range of values around the mean that includes 68% of the sale numbers.

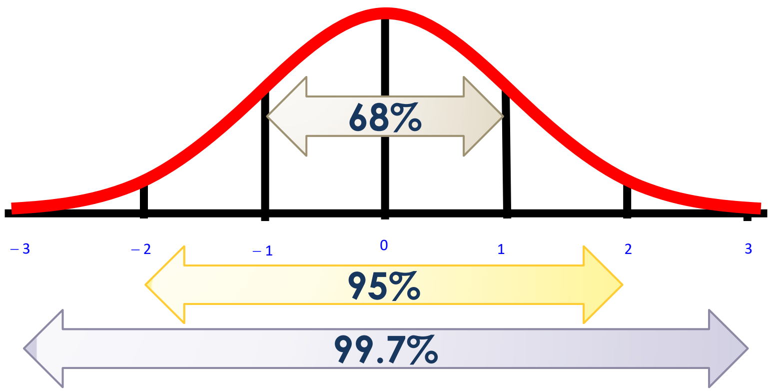
Solution

68% of the sales are within ± 1 standard deviations of the mean .

$$42.6 + 1(8.2) = 42.6 + 8.2 = 50.8$$

$$42.6 - 1(8.2) = 42.6 - 8.2 = 34.4$$

Solution: 68% of the sale are between 34.4 and 50.8 sandwiches.



Question

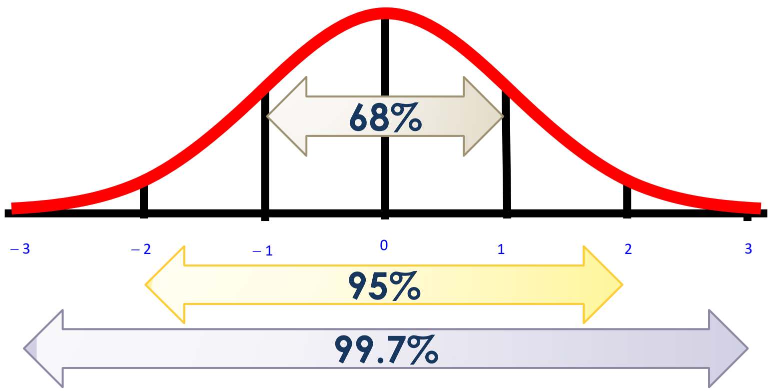
The table below shows the prices charged per room of 40 B&B houses in Galway.

Race - Week B&B prices per room (€)									
56	75	60	70	80	70	50	90	80	75
75	50	75	50	70	60	65	60	50	70
84	70	70	60	60	70	70	70	40	60
70	80	60	65	55	50	70	80	50	55

(i) Calculate, correct to one decimal place, the mean and standard deviation of the data.

(ii) Show that the emperical rule holds true for 1 standard deviation around the mean.

(iii) Show that the emperical rule holds true for 2 standard deviations around the mean.



Solution

(i) Using calculator : Mean = 65.5, SD =11.2

(ii) Upper Range = Mean + 1(Standard Deviation) = 76.7

Lower Range = Mean - 1(Standard Deviation) = 54.4

Of the forty houses 13(68.05%) charge between €54.40 and €76.70

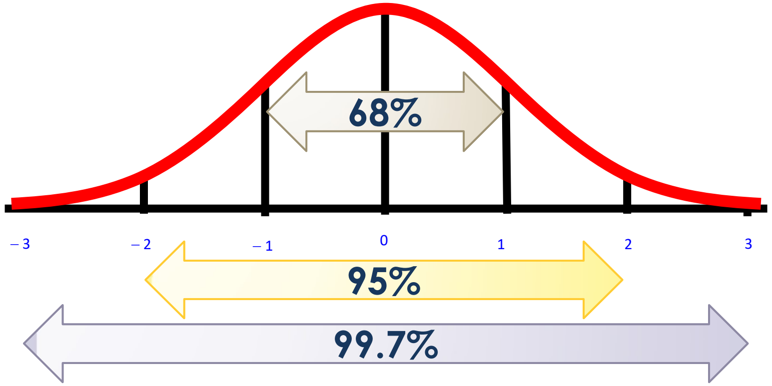
Therefore aprox 68% of the prices lie between 1 standard deviation of the mean.

(iii) Upper Range = Mean + 2(Standard Deviation) = 87.9

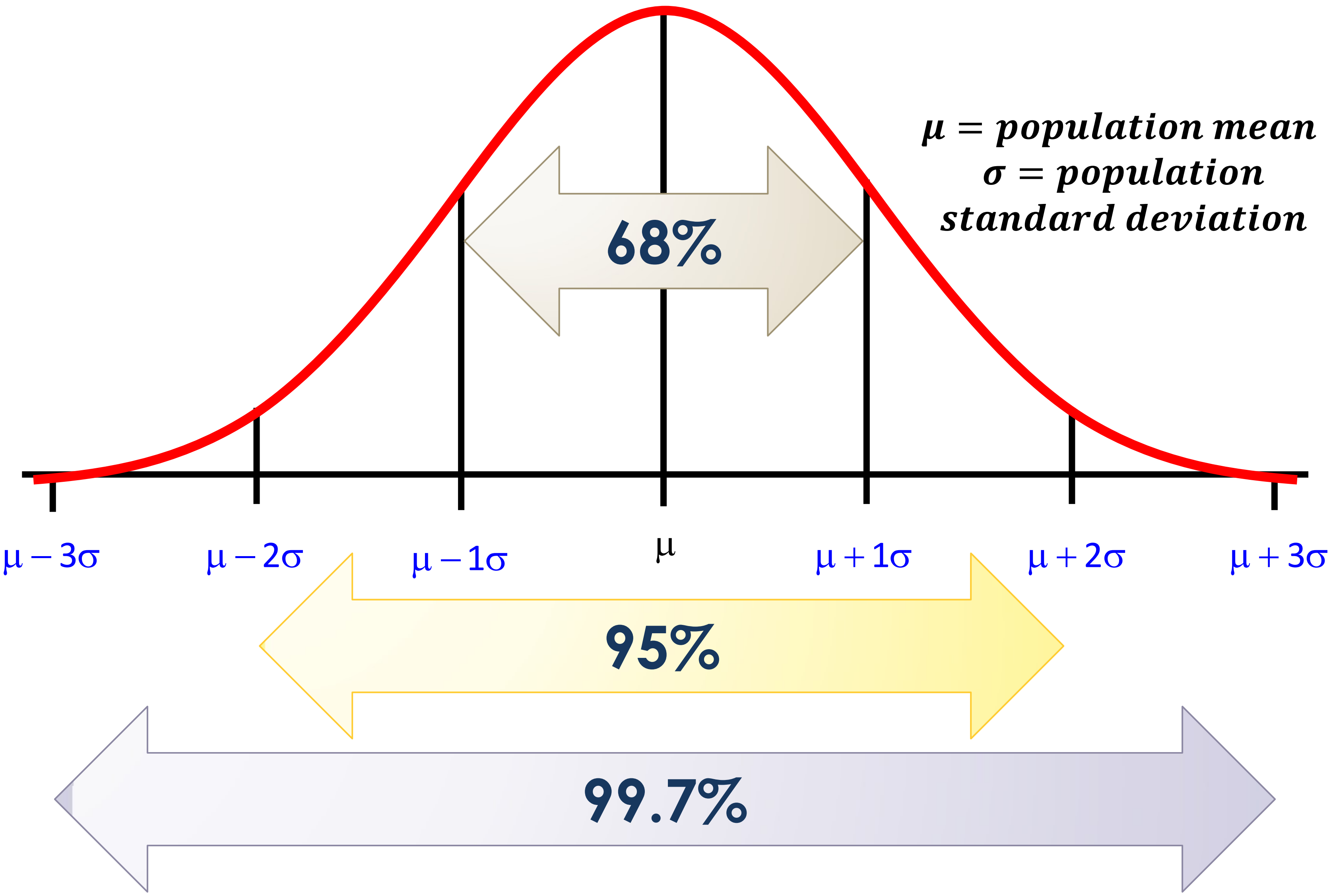
Lower Range = Mean - 2(Standard Deviation) = 43.1

Of the forty houses 38 (95%) charge between €43.10 and €87.90

Therefore aprox 95% of the prices lie between 2 standard deviations of the mean.



Empirical Rule



What about 1·5 std devs or 0·8 std devs?

Normal Distribution to Standard Normal Distribution

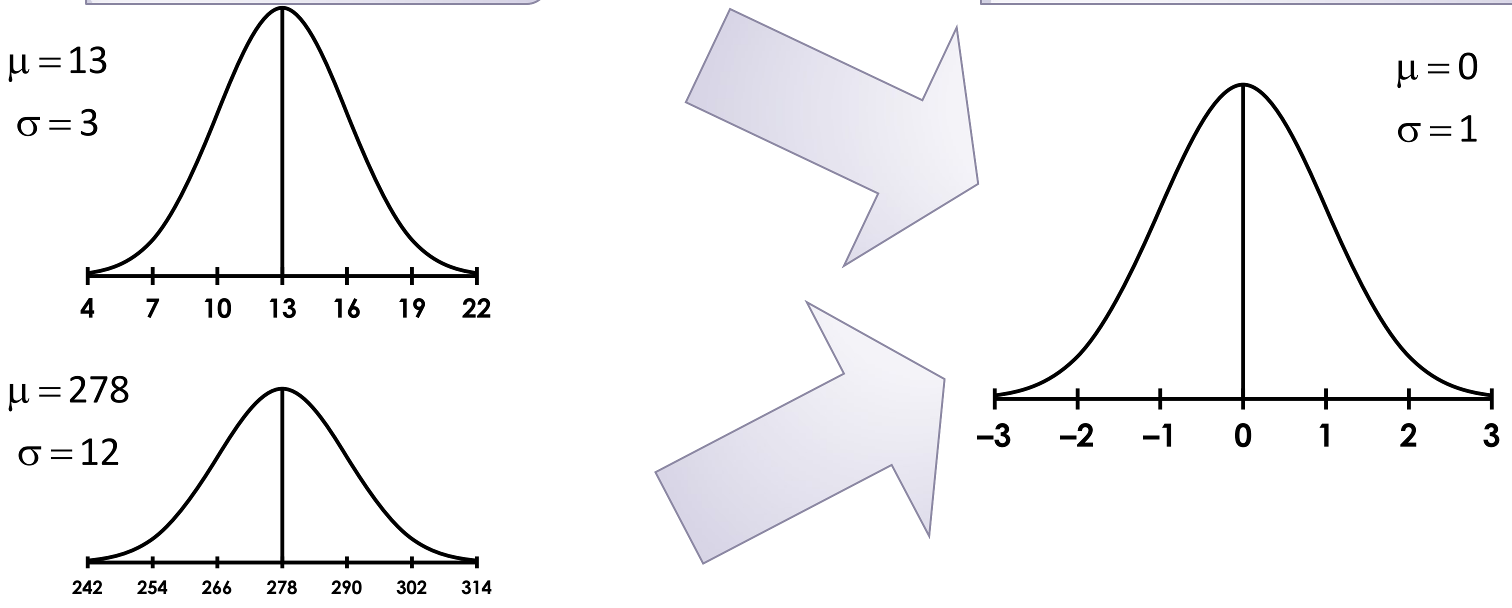
Different sets of data have different means and standard deviations but any that are normally distributed have the same bell-shaped normal distribution type of curves.

Normal Distribution Curve → Standard Normal Curve

In order to avoid unnecessary calculations and graphing the scale of a Normal Distribution curve is converted to a standard scale called the z score or standard unit scale.

Normal Distributions

Standard Normal Distribution



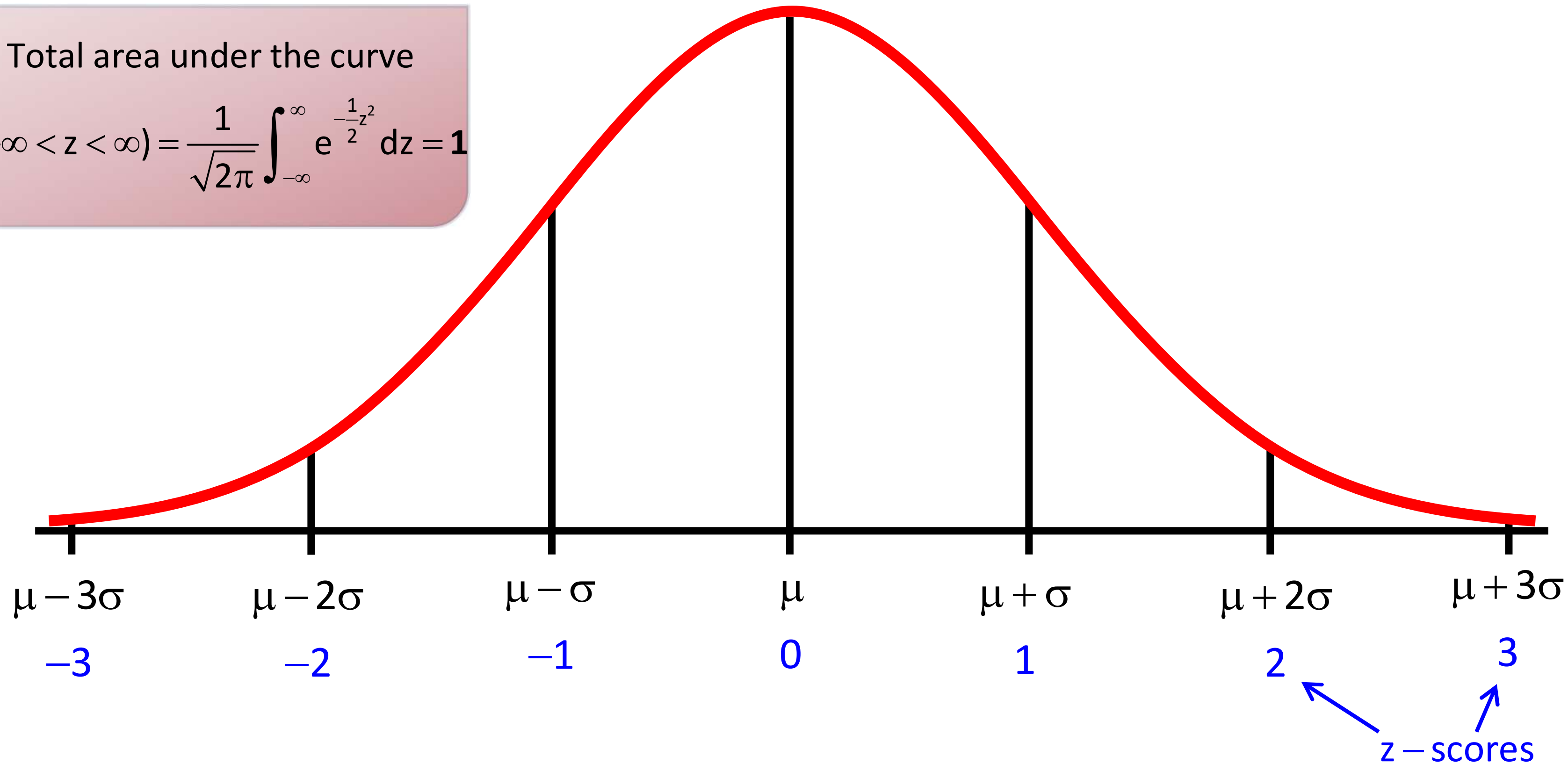
Standard Normal Distribution

If $\mu = 0$ and $\sigma = 1$ we would plot $\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$

This graph gives the Standard Normal Graph with a standardised scale.

Total area under the curve

$$P(-\infty < z < \infty) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2} dz = \mathbf{1}$$



The area between the Standard Normal Curve

and the $z -$ axis between $-\infty$ and $+\infty$ is 1.

Standard Units (z – scores)

$$z = \frac{x - \mu}{\sigma}$$

x is a data point

μ is the population mean

σ is the standard deviation of the population

z – scores define the position of a score in relation to the mean using the standard deviation as a unit of measurement.

z – scores are very useful for comparing data points in different distributions.

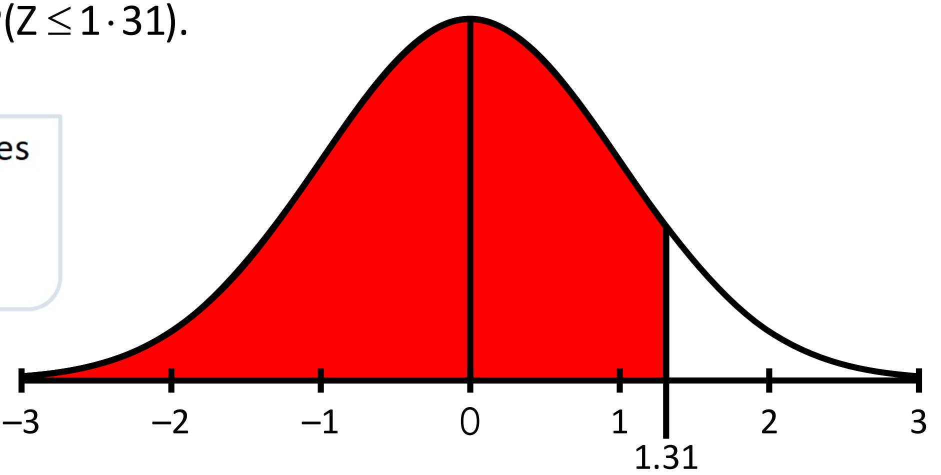
The z – score is the number of standard deviations by which the score departs from the mean.
This standardises the distribution.

Reading z – values From Tables
Example 1

Using the tables find $P(Z \le 1 \cdot 31)$.

For a given z, the table gives

$$P(Z \le z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}t} dt$$



Pg. 36

Pg. 37

$P(Z \le 1 \cdot 31)$ can be read from the tables directly

an dáileadh normalach (ar lean) — normal distribution (continued)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.1	0.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	0.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	0.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	0.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	0.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	0.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	0.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633

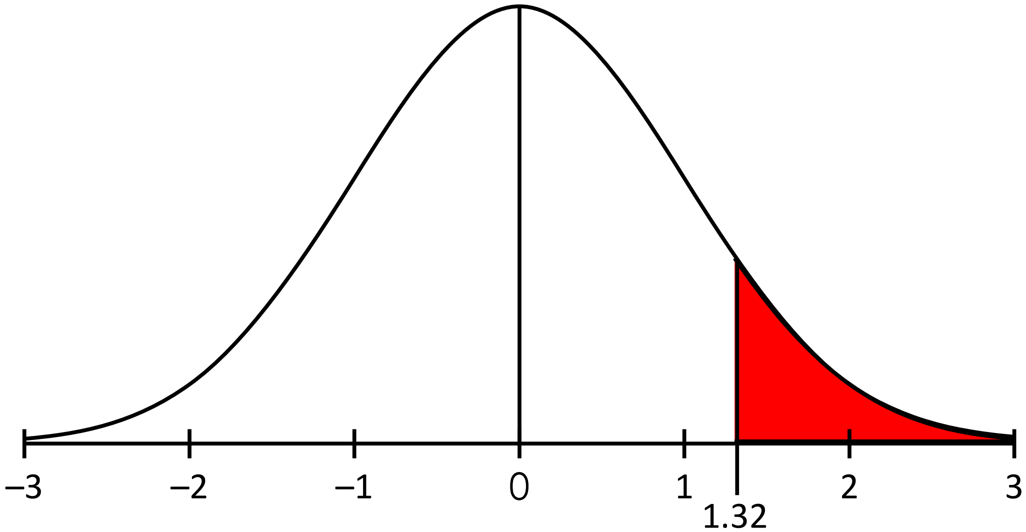
$P(Z \le 1 \cdot 31) = 0 \cdot 9049 = 90.49\%$

Example 2

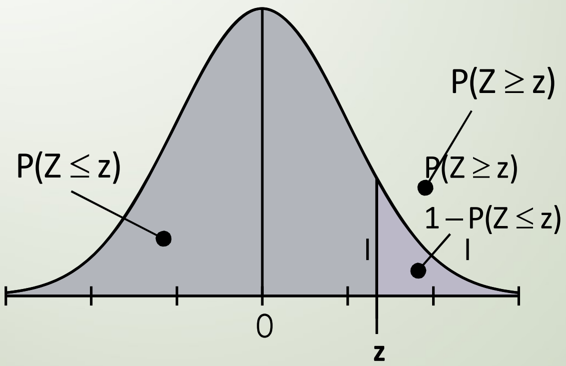
Pg. 36

Pg. 37

Using the tables find $P(Z \geq 1 \cdot 32)$



The table only gives value to the left of z, but the fact that the total area under the curve equals 1, allows us to use, $P(Z \geq z) = 1 - P(Z \leq z)$



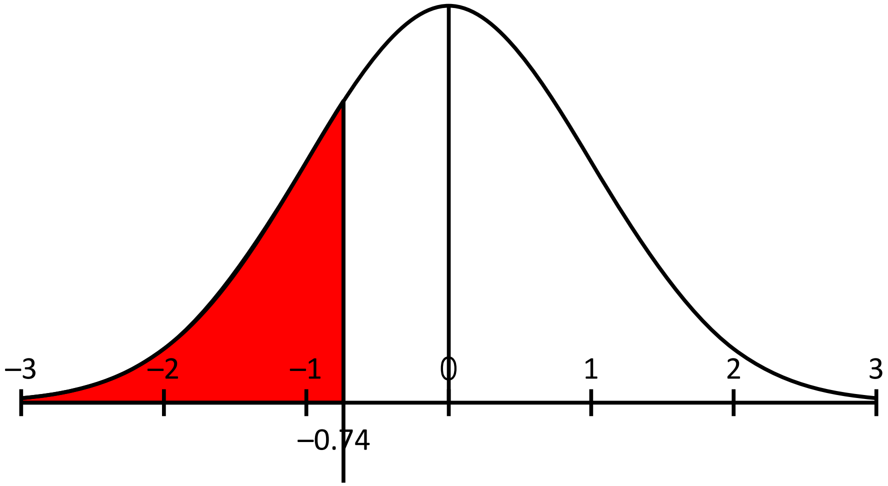
$P(Z \geq z)$ is equal to $1 - P(Z \leq z)$

$P(Z \geq 1 \cdot 32) = 1 - P(Z \leq 1 \cdot 32)$

$P(Z \geq 1 \cdot 32) = 1 - 0 \cdot 9066 = 0 \cdot 0934 = 9.34\%$

Example 3

Using the tables find $P(Z \le -0 \cdot 74)$.



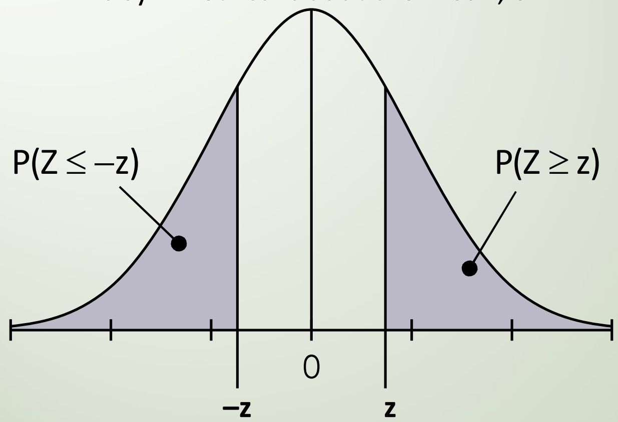
The tables only work for positive values but as the curve is symmetrical about $z = 0$

$$P(Z \le -0 \cdot 74) = P(Z \ge 0 \cdot 74)$$

$$P(Z \le -0 \cdot 74) = 1 - P(Z \le 0 \cdot 74)$$

$$P(Z \le -0 \cdot 74) = 1 - 0 \cdot 7704 = 0 \cdot 2296 = 22.96\%$$

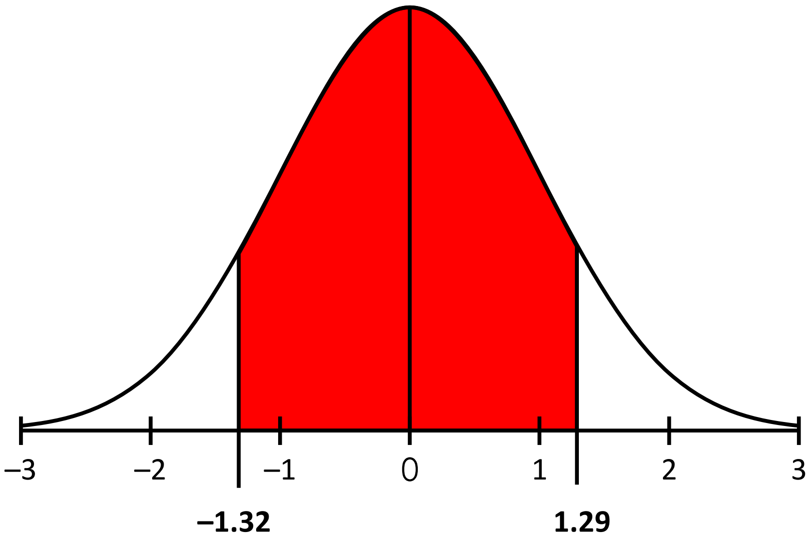
Both areas are the same and hence both probabilities are equal as the curve is symmetrical about the mean, 0.

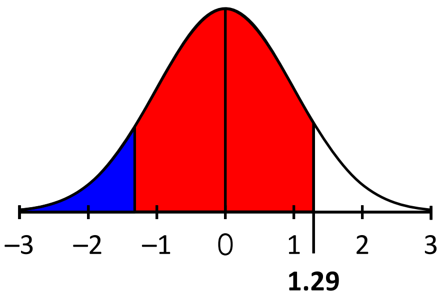


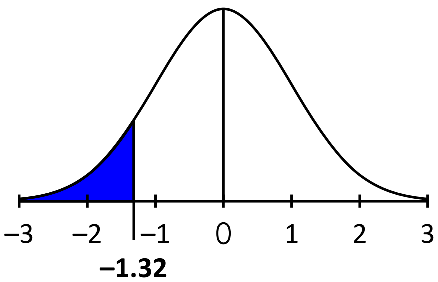
Pg. 36
Pg. 37

Example 4

Using the tables find $P(-1\cdot 32 \le z \le 1\cdot 29)$







$$P(-1\cdot 32 \le z \le 1\cdot 29) = \text{Area to the Left of } 1\cdot 29 - \text{ Area to the left of } -1.32$$

$$= P(z \le 1\cdot 29) - \left[1 - P(z \le 1\cdot 32)\right]$$

$$= 0\cdot 9015 - [1 - 0\cdot 9066] = 0\cdot 8081 = 80.81\%$$

Pg. 36
Pg. 37

Question 1

The amounts due on a mobile phone bill in Ireland are normally distributed with a mean of €53 and a standard deviation of €15. If a monthly phone bill is chosen at random, find the probability that the amount due is between €47 and €74.

Question 1: Solution

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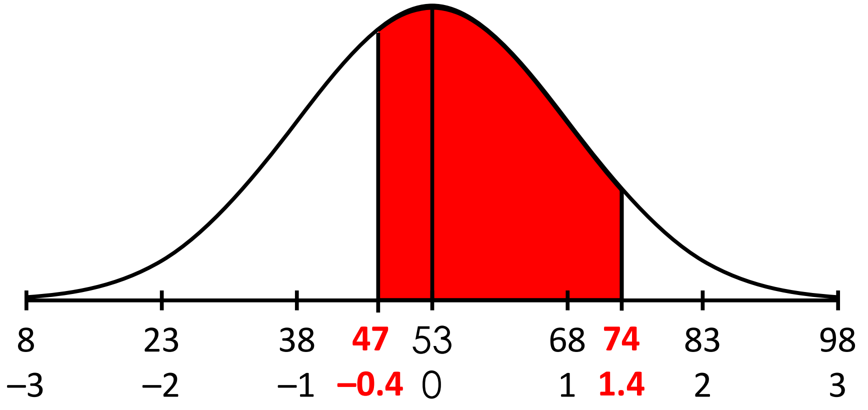
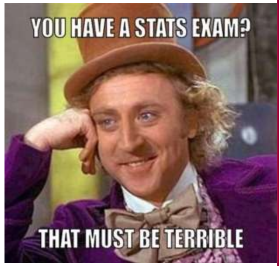


Question 2



The mean percentage achieved by a student in a statistic exam is 60%.

The standard deviation of the exam marks is 10%.

(i) What is the probability that a randomly selected student scores above 80%?

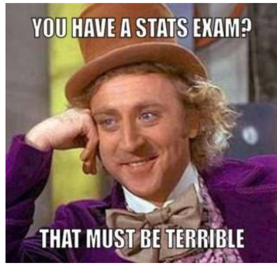
(ii) What is the probability that a randomly selected student scores below 45%?

(iii) What is the probability that a randomly selected student scores between 50% and 75%?

(iv) Suppose you were sitting this exam and you are offered a prize for getting a mark which is greater than 90% of all the other students sitting the exam?

What percentage would you need to get in the exam to win the prize?

Question 2: Solution



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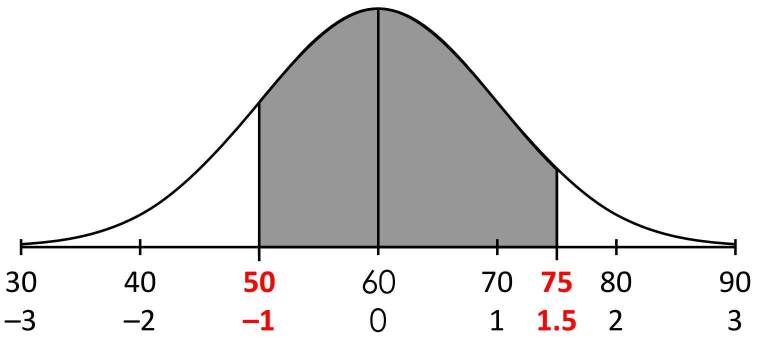
Question 2: Solution

(iii)

$$z_1 = \frac{x-\mu}{\sigma} \qquad z_2 = \frac{x-\mu}{\sigma}$$

$$z_1 = \frac{50-60}{10} \qquad z_2 = \frac{75-60}{10}$$

$$z_1 = -1 \qquad z_2 = 1.5$$



$$P(-1 < Z < 1\cdot5) = P(Z \le 1\cdot5) - \left[1 - P(Z \le 1)\right]$$

$$P(-1 < Z < 1\cdot5) = 0.9332 - [1 - 0.8413]$$

$$P(-1 < Z < 1\cdot5) = 0.7745$$

(iv) From the tables an answer for an area of 90% $(0.9) = 1.28 \Rightarrow Z = 1.28$

$$z = \frac{x-\mu}{\sigma}$$

$$1.28 = \frac{x-60}{10} \Rightarrow x = 72.8 \text{ marks}$$