Functions and different types of functions

A function

A B

- One to one or many to one
- Not all elements of the codomain must be included in range
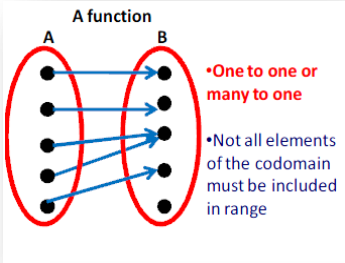
A relation is a **function** if for every *x* **in the domain** there is **exactly** one *y* in the codomain.

A **vertical line** through any element of the domain should intersect the graph of the function exactly once.
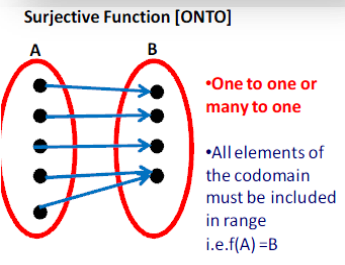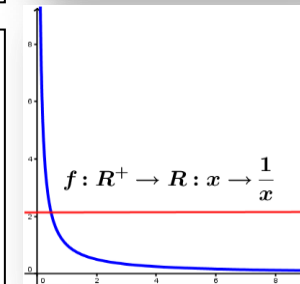
(*one to one or many to one but not all the Bs have to be busy*)

$$f : R \rightarrow R : x \rightarrow x^2$$

Injective Function

A B

Injective
- One to one only
- Not all elements of the codomain must be included in range
- If f(*x*) = f(*y*) then x=y
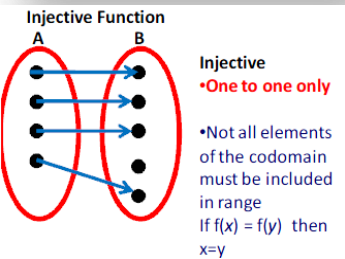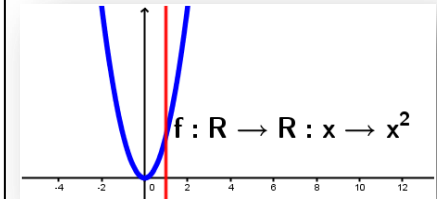
A function is **injective** if for every *y* **in the codomain B** there is **at most** one *x* in the domain.

A **horizontal line** should intersect the graph of the function at most once (i.e.not at all or once).
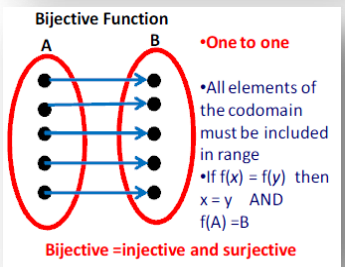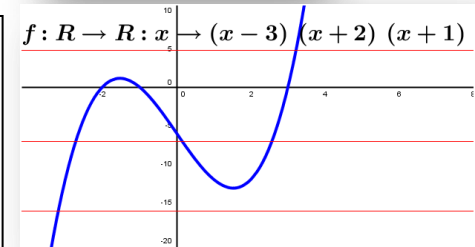
(*one to one only but not all the Bs have to be busy*)

$$f : R^+ \rightarrow R : x \rightarrow \frac{1}{x}$$

Surjective Function [ONTO]

A B

- One to one or many to one
- All elements of the codomain must be included in range
  i.e.f(A) =B

A function is **surjective** if for every *y* **in the codomain B** there is **at least** one *x* in the domain.

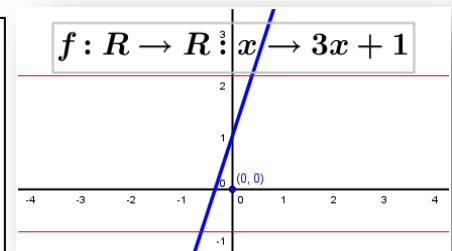A **horizontal line** intersects the graph of the function at least once (i.e.once or more). The range and the codomain are identical.

(*one to one or many to one and all the Bs must be busy*)

$$f : R \rightarrow R : x \rightarrow (x-3)(x+2)(x+1)$$

Bijective Function

A B

- One to one
- All elements of the codomain must be included in range
- If f(*x*) = f(*y*) then x = y   AND
  f(A) =B

Bijective =injective and surjective

A function is **bijective** if for every *y* in the codomain there is **exactly** one *x* in the domain.

A horizontal line through any element of the range should intersect the graph of the function exactly once. (*one to one only and all the Bs must be busy*).

$$f : R \rightarrow R : x \rightarrow 3x + 1$$

> **Question 5**
>
> A is the closed interval $[0,5]$. That is, $A = \{x \mid 0 \le x \le 5, x \in \mathbb{R}\}$.
> The function f is defined on A by:
> $$f : A \to \mathbb{R} : x \mapsto x^3 - 5x^2 + 3x + 5.$$
>
> **(a)** Find the maximum and minimum values of f.

$y = x^3 - 5x^2 + 3x + 5$

$\dfrac{dy}{dx} = 3x^2 - 10x + 3$

Solve $\dfrac{dy}{dx} = 0$

$3x^2 - 10x + 3 = 0$

$(3x - 1)(x - 3) = 0$

$x = \dfrac{1}{3}, x = 3$

$x = \dfrac{1}{3}, y = 5.48$
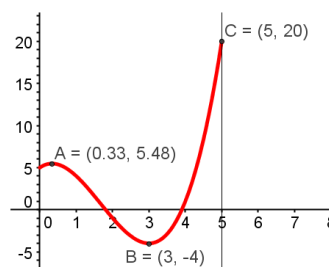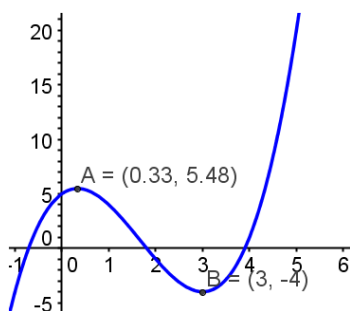
$x = 3, y = -4$

$\dfrac{dy}{dx} = 3x^2 - 10x + 3$

$\dfrac{d^2 y}{dx^2} = 6x - 10$

$\left(\dfrac{d^2 y}{dx^2}\right)_{x=\frac{1}{3}} = -8 < 0 \Rightarrow x = \dfrac{1}{3}$ is a local maximum

$\left(\dfrac{d^2 y}{dx^2}\right)_{x=3} = 8 > 0 \Rightarrow x = 3$ is a local minimum

In the interval $[0,5]$ the minimum value of f is -4 at $x = 3$

In the interval $[0,5]$ the maximum value of f is 20 at $x = 5$

(b) State whether f is injective. Give a reason for your answer.

f is not injective as for every y in the codomain there can be more than one x in the domain as shown by the intersection of two of the horizontal red lines below with the graph. There are for example three values of x for which $f(x) = 5$.