Module 5 – Activities & Problems

5.1 **(a)** The following formula relates to the binomial distribution.

$$P(X=r)=\binom{n}{r}p^r q^{n-r}$$

(i) Describe the type of experiment that results in a random variable that has a binomial distribution.

(ii) State what each of the letters p, q, n and r represents in the formula above.

(b) In a certain type of archery competition, Laura hits the target with an average of two out of every three shots.
The shots are independent of each other.
During one such competition, she has ten shots at the target.

(i) Find the probability that Laura hits the target exactly nine times.
Give your answer correct to three decimal places.

(ii) Find the probability that Laura hits the target fewer than nine times.
Give your answer correct to three decimal places.

[NCCA, Pre Paper, Q5]

5.2 For data that is symmetrically distributed and bell shaped (similar to a normal distribution), approximately what proportion of observations lies within one standard deviation of the mean according to the empirical rule?
What proportion lies within two standard deviations of the mean? What proportion lies within three standard deviations of the mean?

5.3 **(a)** Using the tables find $P(z \leq 2\cdot 11)$.

(b) Using the tables find $P(z \geq 2\cdot 43)$.

(c) Using the tables find $P(z \leq -1\cdot 73)$.

(d) Using the tables find $P(-1\cdot 96 \leq z \leq 1\cdot 96)$.

(e) Using the tables find $P(-1\cdot 44 \leq z \leq 1\cdot 23)$.

5.4 The distribution of scores in a statistics exam is normally distrubed with a mean of 45 and a standard deviation of 4.
You receive a mark of 49. What is the probability of someone scoring higher than you?
What percentage of people score above the mean but lower than you?
[NCE – MSTL, Example, Pg. 73]

5.5 The average age of a person getting married for the first time in the U.S. is 26 years.
Assume the ages have a normal distribution with a standard deviation of 4 years.

(a) What is the probability that a person getting married is younger than 23 years?

(b) 90% of people get married before what age?

[NCE – MSTL, Q5, Pg. 74]

5.6 An athlete finds that in the long jump his distances form a normal distribution with mean 6.1 m and standard deviation 0.03 m.

(a) Calculate the probability that he will jump more than 6.17m on a given occasion.

(b) What distance can he expect to exceed once in 500 jumps.

[NCE – MSTL, Q8, Pg. 74]

5.7 A survey is carried out on 400 randomly selected students in Munster and the result is that 60% are in favour of Project Maths. The confidence level is cited as 95%.

(i) Calculate the margin of error.

(ii) A similar Survey was carried out in Leinster among 400 randomly selected students to see if there was any appreciable difference between support for Project Maths in the Munster and Leinster Area, and the results show that 45% of students were in favour of Project Maths. State the Null Hypothesis and would you accept or reject the Null Hypothesis according to this survey? Give a reason for your conclusion.

Module 5 – Solutions

5.1 **(a)** **(i)** Where only two outcomes are possible, i.e. success or failure.

(ii) p is the probability of success, q is the probability of failure, n is the number of trials
r is the number of successes for n trials.

(b) **(i)** $P(\text{hit}) = \frac{2}{3}$ $P(\text{miss}) = \frac{1}{3}$ $n = 10$

$$P(9 \text{ hits}) = \binom{10}{9}\left(\frac{2}{3}\right)^9\left(\frac{1}{3}\right)^1 = \frac{10 \times 2^9}{3^{10}} = 0.0867$$

(ii) $P(10 \text{ hits}) = \left(\frac{2}{3}\right)^{10} = 0.0173$

$P(< 9) = 1 - [P(9) + P(10)]$

$P(< 9) = 1 - [0.0867 + 0.0173]$

$P(< 9) = 0.896$

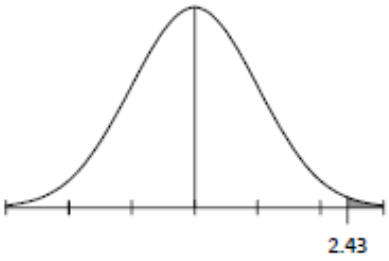
Thus fewer than 9 times $\Rightarrow$ probability = 0.896

5.2 68%, 95%, 99.7%

5.3 **(a)**

$P(z \leq 2.11) = 0.9826$ (98.26%)

(b)

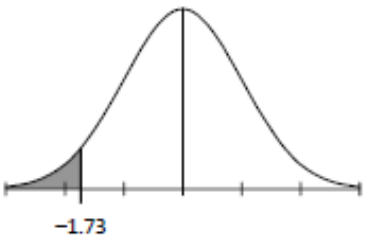


$P(z \geq 2.43) = 1 - P(z \leq 2.43)$

$P(z \geq 2.43) = 1 - 0.9925$

$P(z \geq 2.43) = 0.0075$ (0.75%)

(c)



$P(z \leq -1.73) = P(z \geq 1.73)$

$P(z \leq -1.73) = 1 - P(z \leq 1.73)$

$P(z \leq -1.73) = 1 - 0.9582$

$P(z \leq -1.73) = 0.0418$ (4.18%)

(d)

$P(-1.96 \leq z \leq 1.96)$

$P(-1.96 \leq z) = P(z \geq -1.96) = 1 - P(z \leq 1.96)$

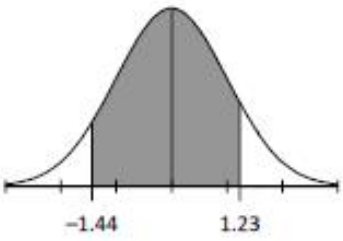
$P(-1.96 \leq z \leq 1.96)$ = Area to the left of 1.96
− Area to the left of −1.96

$P(-1.96 \leq z \leq 1.96) = P(z \leq 1.96) - [1 - P(z \leq 1.96)]$

$P(-1.96 \leq z \leq 1.96) = 0.9750 - [1 - 0.9750]$

$P(-1.96 \leq z \leq 1.96) = 0.95$ (95%)

(e)



$P(-1.44 \le z \le 1.23)$

$P(-1.44 \le z) = P(z \ge -1.44) = 1 - P(z \le 1.44)$

$P(-1.44 \le z \le 1.23)$ = Area to the left of 1.23

−Area to the left of −1.44

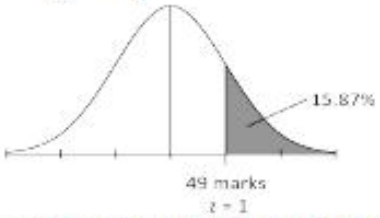
$P(-1.96 \le z \le 1.96) = P(z \le 1.23) - [1 - P(z \le 1.44)]$

$P(-1.96 \le z \le 1.96) = 0.8907 - [1 - 0.9251]$

$P(-1.96 \le z \le 1.96) = 0.8158$ (81.58%)

5.4

$$z = \frac{x-\mu}{\sigma} = \frac{49-45}{4} = 1$$



$\Rightarrow$ The probability of scoring above 1 in the standard normal distribution is $1 - 0.8413 = 0.1587$.

The percentage of people scoring above the mean is 50%.

The percentage of people scoring higher than 49 is approx. 16%.

The percentage of people scoring above the mean but lower than 49 is $50 - 16 = 34\%$.

5.5

(a)

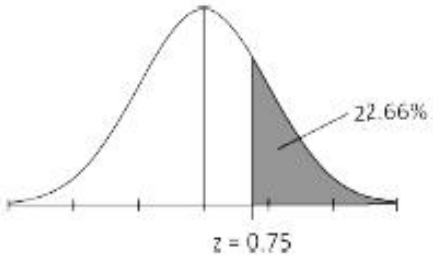
$$z = \frac{x-\mu}{\sigma} = \frac{23-26}{4} = -0.75$$

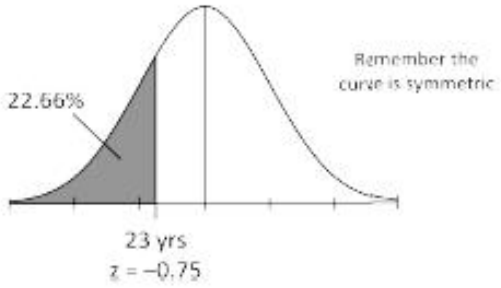
Can only look up postive values in tables

$P(Z < -0.75)$

$P(Z > 0.75) = 1 - 0.7734 = 0.2266$

22.66% chance of getting married younger than 23.





(b)

$90\% = 0.90 \approx 0.8997$ [closest in tables]

$z = 1.28$

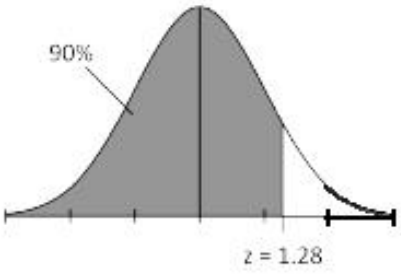
$$z = \frac{x-\mu}{\sigma}$$

$$1.28 = \frac{x-26}{4}$$

$(4)(1.28) = x - 26$

$5.12 + 26 = x$

31.12 years = x

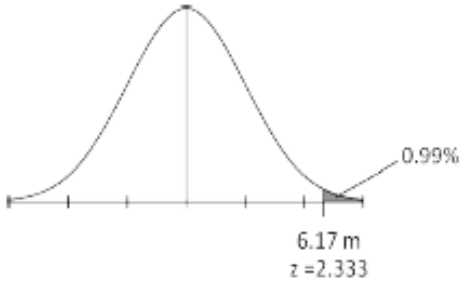


5.6 **(a)** $z = \frac{x-\mu}{\sigma} = \frac{6.17-6.1}{0.03} = 2.333$

$P(Z \leq 2.33) = 0.9901$

$P(Z > 2.33) = 1 - 0.9901 = 0.0099$

0.99% chance



6.17 m

$z = 2.333$

(b) 1 in 500 = 0.002 [0.2% of Jumps]

$99.8\% \Rightarrow z = 2.88$

$z = \frac{x-\mu}{\sigma}$

$2.88 = \frac{x-6.1}{0.03}$

$(2.88)(0.03) = x - 6.1$

$0.0864 + 6.1 = x$

$6.186\text{ m} = x$

$z = 2.88$

5.7 **(i)** 5% = margin of error.

(ii) Null hypothesis : There is no difference in the attitude of Leinster students to PM.

According to the results of the survey we fail to accept the null hypothesis as 45% is outside the margin of error of the results for Munster which is from 55% to 65%.