Problem Solving

Content Course 4.1 & 4.2

1

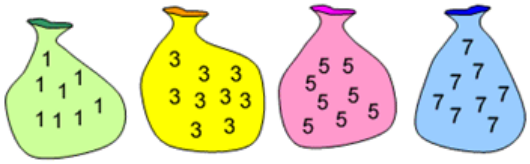
Name: _____

School: _____

Probem Solving Activities & Questions

Activity 1

Four bags contain a large number of 1's, 3's, 5's and 7's.



Pick any 10 numbers from the bag so that their sum equals 37.
Justify your solution.

Activity 2 [Exploring Numbers]

(i) Write down as many properties of even and odd numbers that you can.
Try to discover (in words)...

- **(a)** A rule to represent every even number.
- **(b)** A rule to represent every odd number.
- **(c)** The outcome when two even numbers are added.
- **(d)** The outcome when two odd numbers are added.
- **(e)** The outcome when two even numbers are multiplied.
- **(f)** The outcome when two odd numbers are multiplied.

(ii) Now,
Prove the outcomes for **(c)** to **(f)** above.

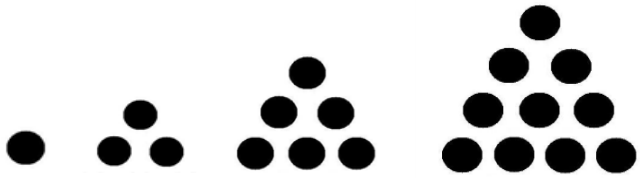
(iii) Show, by using an example, that a square with side an odd number in length has an odd area.
Now, ***Prove*** this is always true

Activity 3

Prove that a square with side an odd number in length, must have an odd area.

Activity 4 [Triangular Numbers]

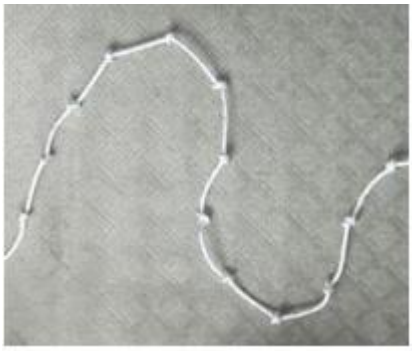
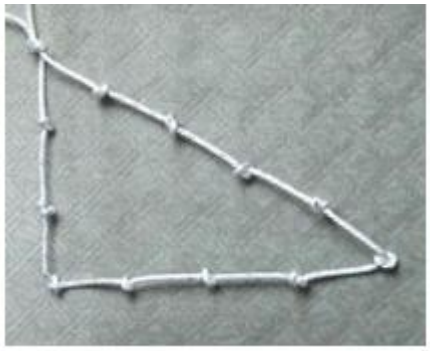
This pattern below continues indefinitely.



(a) Can you find a rule to define the number of circles in any given triangle?

(b) What seems to happen if any two consecutive triangle numbers are added?
Can you prove this to be true?

Activity 5 [Linking our Thinking]

(a) How many different triangles have a perimeter of 12 units?

(b) What kinds of triangles are they?

(c) Explain how you determined this.

(d) Explain what have you discovered.

Activity 6 [Pythagorean Triples]

(a) Show that a ***Pythagorean Triple*** cannot consist of three odd numbers.

(b) Can you now prove that this is always the case?

(c) Prove that there are infinitely many Pythagorean triples.

Activity 7 [Estimation of π]

Find the ratio of the areas of the circles in diagram **(A)** and **(B)**.

Diagram A.

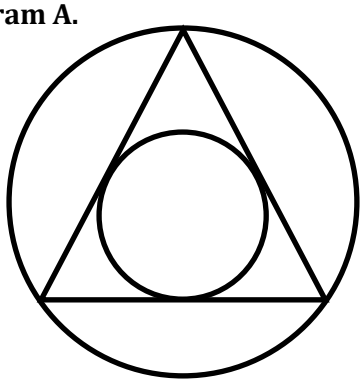
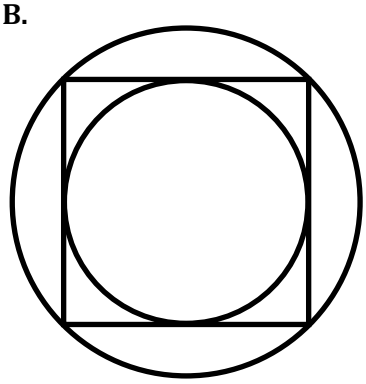


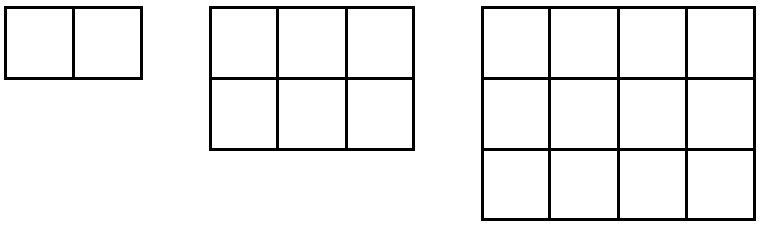
Diagram B.



Can you now get a better approximation of π?

Activity 8 [Growing Rectangles]

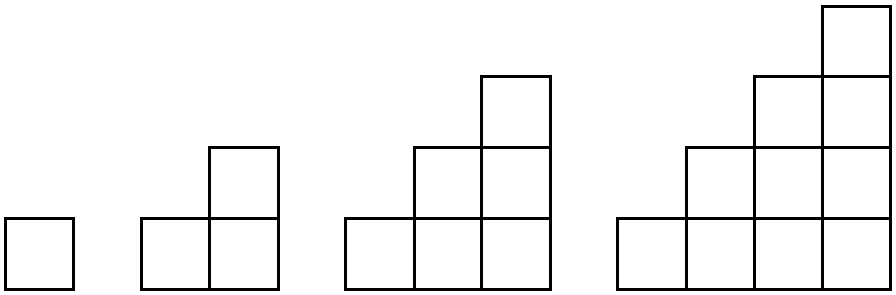
Complete the next two patterns in this sequence of rectangles.



Look at the pattern of growing rectangles. Make a table for the number of tiles in each rectangle for rectangles of height from 1 to 10. Make observations about the values in the table.

Activity 9 [Staircase Towers]

Look at the staircases below. Make a table representing the relationship between the total number of tiles and the number of towers. Make observations about the values in the table. What would a graph look like? Would it be linear? How do you know? Make a graph to check your prediction.



Activity 10 [Perfect Squares]

The table below gives the first thirty two non-negative integers arranged in rows of four.

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31

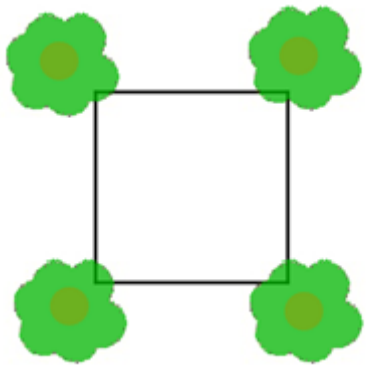
(a) Prove that a perfect square is always a multiple of four or one more than a multiple of four.

(b) Show that the sum of two squares is never three more than a multiple of four.

Activity 11 [Car Park Space]

A client wishes to double the area of his car park. He insists on retaining a square shape and as many of the trees as possible.

What is the best solution?



What is the ratio of the length of the side of the original car park to the length of its diagonal?

Did the ratio remain the same when the area of the car park was doubled?
Justify your reasoning.

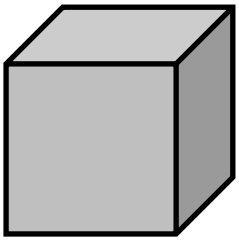
What is the ratio of the length of the side of the original car park to the length of the side of the new car park?

Activity 12 [Photocopying Paper]

A paper manufacturer wants to design a rectangular piece of paper with a special property.
He wants the ratio of the long side to the short side to be such that , if he was to fold the long side of the piece of paper the fold line and the other side will be in the same ratio as the original.
What does he have to do?

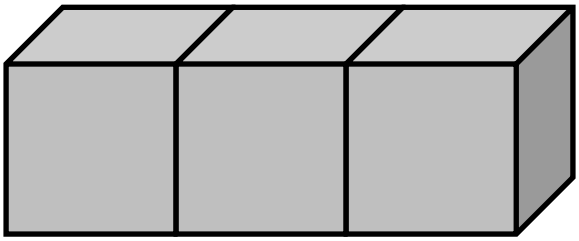
Activity 13 [Nets]

(a) **(i)** What is the surface area of a cube with edge length 5 cm?



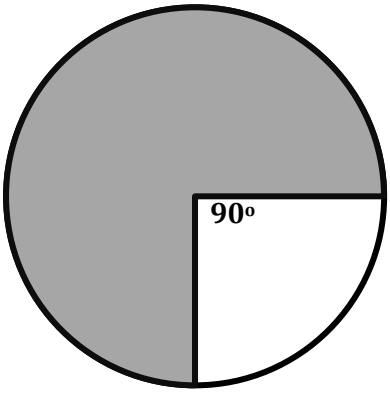
(ii) Three such cubes are joined together side by side and wrapped with paper (no excess paper is used, i.e. no overlap).

1. What is the area of the paper needed?



2. Draw a net for the shape of one cube.

(b) **(i)** The shaded region is cut from a circular piece of plastic of radius 10 cm (as shown below). What shape is this the net of?



(ii) If the curved surface area of this shape is 60π cm^2.
How much liquid would shape this hold?

Activity 14 [Design a Pencil Case]

Design a cylindrical pencil case using fabric the same size as A4 paper. Try to make the pencil case as large as possible.

Extension Question

An average pencil has a height of 15 cm and a width of 1 cm.
What is the maximum number of pencils that will fit in your pencil case that you have designed?

Appendix A: Further Problem Solving Activities and Questions.

Activity A1 [Exploration]

An ant is crawling in a straight line from one corner of a table to the opposite corner. He bumps into a cube of sugar. He decides to climb over it and then continues along his intended route. How much did the detour add to his journey?

Activity A2 [Exploration]

Mrs. Smith has twin girls. While on a day out, they find a bubble gum machine. The machine sells red bubble-gum and blue-bubble gum. Each girl wants a piece of gum, but as they are twins, they must have the SAME colour gum.

If each turn on the bubble-gum machine costs 10 cents for one piece of gum, what is the maximum amount of money Mrs Smith must spend in order to ensure both girls have the same colour gum?

Possible extensions:
If Mr Byrne arrived with his 3 sons (triplets), and they each wanted a piece of gum, which had to be the same colour, what would be his maximum cost to ensure this happened?

What if the bubble gum machine sold 3 different colours of gum, e.g. red, blue and green? How would this effect the outcome of each case?

Appendix B – Student Hand-outs for Problem Solving Strategies/Toolkits[1]

Understand

Tell the problem in your own words.

Plan

How will you solve the problem?

Try

Show how you solved the problem.

Check

Does your answer make sense? Are your calculations correct?

Reflect

Explain how you solved the problem.

What was easy/hard about solving the problem?

[1] May be copied for classroom use. © 2008 by Joy Bronston Schackow and Susan O'Connell from *Introduction to Problem Solving: Grades 6–8* (Heinemann: Portsmouth, NH).

Solving Problems Step-by-Step

1. What's the **Question?**	
What do you want to find out?	What do you already know?
2. Make a **Plan.**	3. **Try** your plan.
What strategy will you use to find the answer? What do you predict the answer might be?	Show your work.
4. **Check** your work.	
Does your answer make sense? Are your calculations correct?	
5. **Reflect** about solving the problem.	
Explain how you got your answer.	

Problem-Solving K-W-P-L

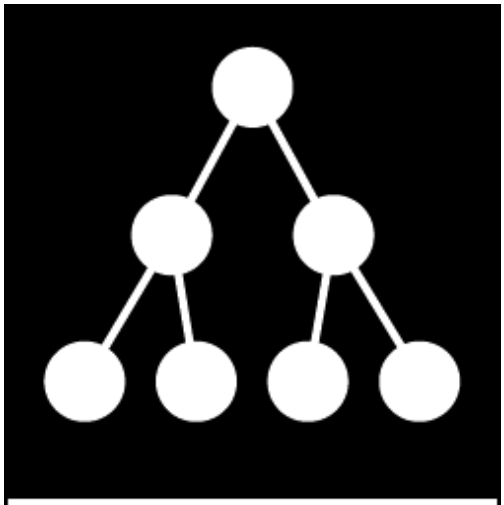
What I **Know**	What I **Want** to Find Out	What I **Plan** to Do	What I **Learned**

Problem-Solving K-W-P-L

What I **Know**	What I **Want** to Find Out	What I **Plan** to Do	What I **Learned**
		What I **Predict**	

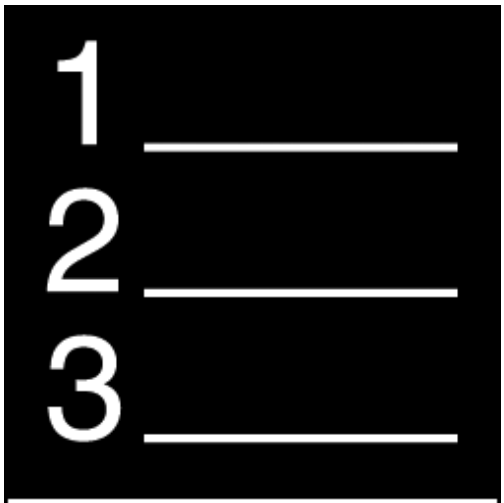
Strategy Icons

Choose an Operation



Draw a Picture or Diagram

Guess, Check, and Revise

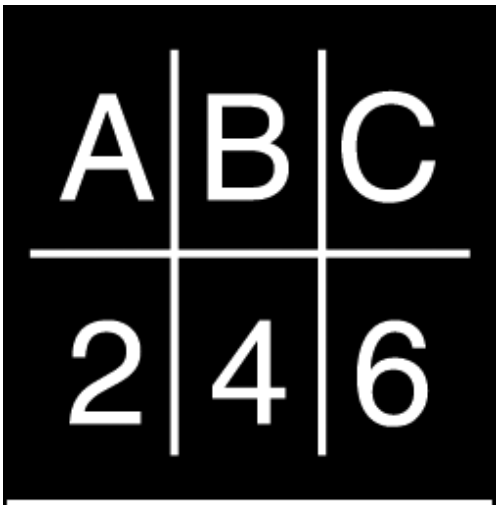


Make an Organized List

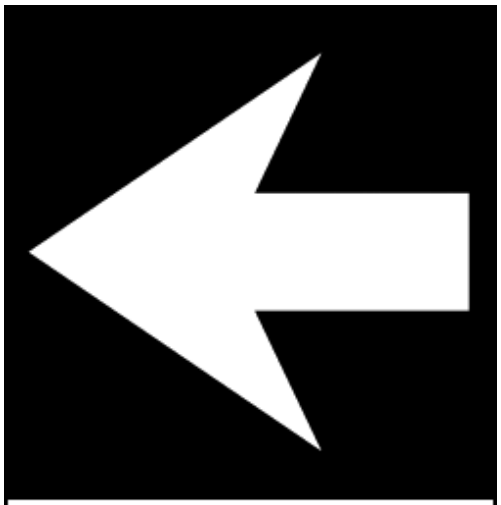
Strategy Icons

Find a Pattern

Use Logical Reasoning



Make a Table



Work Backward

Problem-Solving Bookmark

Steps for Solving Problems

Here are some things to think about as you work on solving the problem.

Question
- What do I know?
- What do I want to find out?

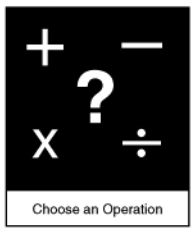
Plan
- What's my plan?
- Does this remind me of other problems I've solved?

Try
- Does my plan work?
- What else can I try?

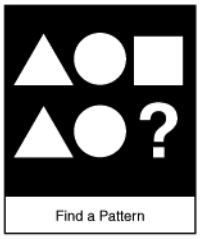
Check
- Does my answer make sense?
- Is it close to what I predicted?

Reflect
- What have I learned?

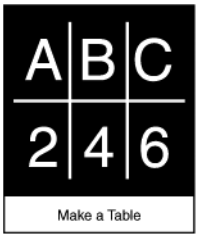
What Strategy Will You Use?



Choose an Operation

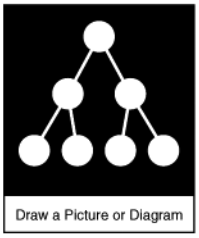


Find a Pattern



Make a Table

Make an Organized List


Draw a Picture or Diagram

Guess, Check, and Revise


Use Logical Reasoning


Work Backward

fold here
and
glue sides
together

Holistic Rubric/ Marking Scheme for Scoring Problem Solving

Expected Student Outcomes:

Students will be able to

1. select and use an appropriate strategy
2. calculate a correct answer
3. explain their strategy for solving the problem

Problem-Solving Rubric:

4 — arrived at a correct answer; used an appropriate strategy; adequately explained answer

3 — used an appropriate strategy; calculated a correct answer but could not explain the strategy; or adequately explained the strategy but did not calculate a correct answer

2 — used an appropriate strategy; did not find a correct answer; could not explain the strategy

1 — attempted to solve the problem, but completely incorrect in attempt

0 — no attempt/blank

Name _____ Date _____

Evaluating My Problem Solving

Carefully look over your corrected paper and answer the questions below.

I used a strategy that made sense. yes no

I got the correct answer. yes no

I explained my strategy. yes no

My score was _____.

Next time I will improve my work by

_____

_____

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Reflecting on Problem Solving

Reflections will help teachers identify students' feelings about problem solving as well as their perceived strengths and weaknesses and their confusions about problem-solving lessons. Students might answer one or several prompts each week in a problem-solving journal. Encouraging students to look back over past entries will help them monitor their growth as problem solvers. Try the following:

When a problem is hard, I . . .
The easiest part of problem solving is . . .
The hardest part of problem solving is . . .
I'm confused about . . .
Now I understand . . .
The most important thing I learned today is . . .
I can use what I learned today when I . . .
When I don't know what to do I . . .
I discovered that . . .
When I work with a partner I feel . . .
When I work with a group I feel . . .
I need help on . . .
I'm glad I know how to . . .
I get frustrated when . . .
Something I learned today that will be very useful is . . .
The strategy I am best at is . . .
The strategy that I'm confused about is . . .
If I could hear one lesson over again it would be . . .

Problem-Solving Group Observation Checklist

Group members:

_____

_____

_____

Key:
(+) behavior observed
(–) negative/opposing behavior observed
(n/a) no opportunity for observing this behavior

_____ All groups members were involved in the task.

_____ Group members were able to verbalize the problem in their own words.

_____ Group members helped one another understand the problem and possible solutions.

_____ Group members were able to decide on a reasonable plan for solving the problem.

_____ Group members worked together to execute their plan.

_____ Group members checked the reasonableness and accuracy of their solution.

_____ Group members were able to explain the process they used to solve the problem.

Additional notes:

Do your students ever get "stuck" when solving math problems? Does he or she get frustrated and want to quit? One important lesson in problem solving is to keep trying, even if an answer is not found after the first few tries. You can help your students develop strategies to get "unstuck." Following are some ideas that might help:

Jot Down Ideas
Jot down a plan for how you will be solving the problem. You might list the important information or draw a diagram of the problem to get you started.

Restate the Problem in Your Own Words
Are you unsure how to begin? Reread the problem and then state it in your own words. You need to understand the problem before you can go any further.

Cross Off Unnecessary Information
Is the problem confusing, containing too much data? Reread the problem and cross out the unnecessary data to simplify the problem.

Substitute Simpler Numbers
Does the problem contain large numbers or fractions or decimals that are confusing you? Substitute simpler numbers for the confusing numbers and then figure out how to solve the problem. Once you know how the problem should be solved, just plug the more complicated numbers back into the problem and repeat the process to solve it.

Take a Break
Are you too frustrated to go on? Take a break for a few minutes. Think about or do something else. Then return to the problem refreshed and ready to begin again.

Use a Manipulative
Use everyday objects (paper clips, toothpicks, pennies) to represent the items in the problem. Act out the problem with the manipulatives.

Talk the Problem Through
Talk out loud to yourself or to someone else. Explain the problem and what you think you should do. Listen to yourself as you talk to see if it makes sense.

Think of a Similar Problem
Does this problem remind you of another that you've solved? How did you solve that one? Try that strategy. Does it work here?

Try a Different Strategy
What you're doing doesn't seem to be working. Try something else. Is there a different strategy that you think might work? Try it and see.

Give Yourself a Pep Talk
Think of a problem you solved by sticking with it. Remember a time when you were frustrated but kept on trying until you found the answer. Remind yourself that you can do it! Model these ideas for your students. Show him or her how to keep calm, think a problem through, and try again. Your encouragement of their efforts and your suggestions for ways to get themselves "unstuck" will help them grow into successful problem solvers.