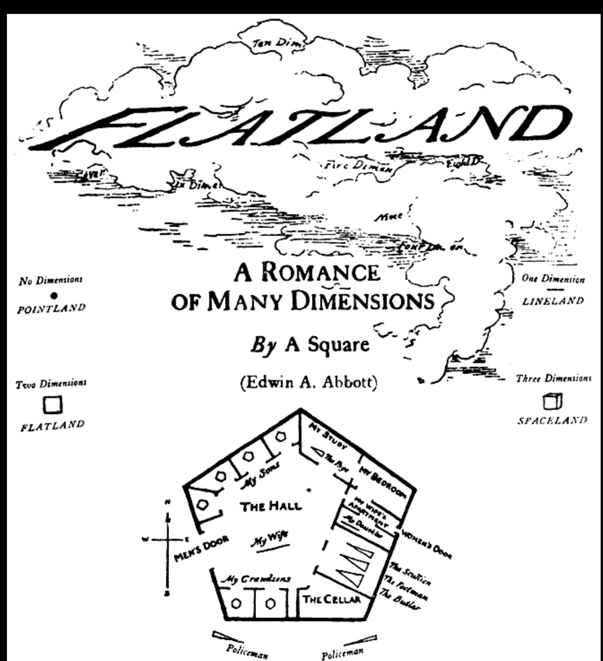
GEOMETRY ENRICHMENT FOR

Flatland – the Movie



Hypercubes

Cross-sections

Interior angles of regular polygons

Flatland

Flatland, A Journey of Many Dimensions is a short movie based on the novel, *Flatland, A Romance of Many Dimensions* by Edwin Abbott Abbott which was published in 1884. *Flatland* (the movie) is the story of a two-dimensional world in which some of the inhabitants are introduced to the third dimension. This animated video includes visually-appealing settings, drama, and geometry. It challenges viewers to question the limitations of reality, perhaps to consider the possibility of higher dimensions.

Flatland is a fun and creative adjunct to geometry instruction. This activity package includes the following geometry enrichment components:

1. Viewing of *Flatland* and discussion of dimensions in space
2. Exploration of hypercubes (four-dimensional cubes)
3. Cross-sectioning of spheres and cubes and identifying the shapes of the cross-sections - this portion aligns with CCSS 7G.3 and G-GMD.4
4. Determining the sum of the interior angles of polygons and the measurement of each angle in a regular polygon

Higher-level thinking makes this a perfect activity for a gifted/talented math class. It has also been used with middle school Math Club students who enjoyed both the movie and some lively discussions afterward.

NOTE: Special materials are needed for these activities.

1. **You will need to obtain a copy of the video, *Flatland, A Journey of Many Dimensions.*** It can be purchased from Amazon.com. Note that this math activity is based on the 2007/2008 version, popularly called "Flatland – the Movie" by Flat World Productions, starring Martin Sheen. Another movie, "Flatland the Film" is different and may not support the activities here.

   Alternately, check your local library. It may be available there for short-term use.

2. **Play-doh® or similar soft modeling compound and plastic knives will be needed.** These materials will be used by the students for demonstrating cross-sections of solid figures. Substitute dental floss or fishing line for the knives, if desired.

PREPARATION

1. Obtain the video, *Flatland, A Journey of Many Dimensions.*

2. Obtain modeling compound and plastic knives, enough for each student in the class. Small, individual (party favor-size) containers of Play-doh® work well.

3. Preview the youtube video (http://www.youtube.com/watch?v=9DoSqeJNG74) and the learner.org tutorial (http://www.learner.org/courses/mathilluminated/interactives/dimension/) to determine which (or both) you wish to use to explain and demonstrate hypercube (see Hypercube activity).

4. Also preview the interactive tutorial, Cross Sections, from Annenberg Learner for the Cross-section activity: http://www.learner.org/courses/learningmath/geometry/session9/part_c/

5. Photocopy worksheets, 1 per student.

<u>Video – *Flatland, A Journey of Many Dimensions*</u>

1. Show the *Flatland* video. The video runs for approximately 36 minutes.

2. Discuss dimensions. Ask students to identify the dimensions of a 2-D figure (length and width). Identify the third dimension (depth or height).

3. Discuss why it is difficult to think about a 4^{th} dimension, relating it to the movie characters' difficulty in understanding a 3^{rd} dimension. It requires unconventional thinking, or "thinking outside the box" (pun intended).

Hypercube

The movie, Flatland, ends with images of a hypercube (4-dimensional cube). This enrichment activity explores a fourth dimension through demonstration of the concept of the hypercube.

1. Show one or both of these videos.
   a. This is an interactive tutorial that uses analogies from one, two, and three dimensions to lead to an intuitive understanding of a fourth dimension. http://www.learner.org/courses/mathilluminated/interactives/dimension/

   b. A 7:44 video with narration as the presenter models the construction of a hypercube and shows different 3-D forms of a hypercube. The presenter's final statement, "Sometimes the most difficult problems can only be tackled when you leave your mind wide open," reflects the final thought in the movie, "Your mind has grasped what your eyes could not see, and your imagination has changed your world forever."

      http://www.youtube.com/watch?v=9DoSqeJNG74

2. In *Flatland*, Arthur Square states, "Powers in arithmetic could translate to geometrical dimensions." Discuss the correlation between the dimensions and the number of points (or vertices) that make up the figures below and complete the table together as a class:
   (Note: The table is enlarged on a separate page to enable projection onto a screen or whiteboard.)

Dimensions	0	1	2	3	4
Figure	Point	Line segment	Square	Cube	Hypercube
# of points/vertices	1	2	4	8	
Power of 2	2^0	2^1	2^2	2^3	

Relate the number of vertices determined for the hypercube with the drawing of the hypercube in either of the videos above.

3. Introduce the word *tesseract.*

Tesseract, another name for a hypercube, is the four-dimensional analog of the cube. The tesseract is to the cube as the cube is to the square. The tesseract is composed of 8 cubes with 3 to an edge. It has 16 vertices (table, above) and 32 edges.

Table enlarged for projection

Dimensions	0	1	2	3	4
Figure	Point	Line segment	Square	Cube	Hypercube
# of points/vertices	1	2	4	8	
Power of 2	2^0	2^1	2^2	2^3	

ANSWER KEY

Dimensions	0	1	2	3	4
Figure	Point	Line segment	Square	Cube	Hypercube
# of points/vertices	1	2	4	8	16
Power of 2	2^0	2^1	2^2	2^3	2^4

Cross-sections of three-dimensional figures

Students will slice a 3-D figure in different ways – horizontally, vertically, and diagonally, and will draw the different 2-D cross-sections that result from the various cuts. This activity supports CCSS 7.G.3 and G-GMD.4.

1. Pose the following questions and discuss the concept of cross-section:
   a. How was Arthur Square seen when he entered Lineland?
      (answer: as a line)
   b. How did Arthur see Spherius when the sphere entered Arthur's home in Flatland? You may need to remind students that Arthur's home is a two-dimensional plane.
      (answer: as a circle)

2. Distribute modeling compound, plastic knives, and worksheets.

3. Demonstrate the activity by forming a 3-D shape with modeling dough and cutting through it. Review the terms horizontal, vertical, and diagonal by showing those directions on your model.

4. Explain that a cross-section is a two-dimensional shape that results from a cut through a solid and that it is different from a face of the figure (which is also a 2-D shape). The shape of a cross-section will depend on the way the 3-D shape was cut.

5. Each student will make a sphere with modeling dough, make a horizontal cut through the sphere, and sketch the two-dimensional shape of the cross-section. This will be repeated twice, once with a vertical cut, and once with a diagonal cut. The three cross-sections will be compared.

6. Each student will form a cube with modeling dough and repeat the process of cutting through the cube three times and sketching the two-dimensional shape of each cross-section. The same three cuts will be made: horizontal, vertical, and diagonal.

7. Each student will re-form a cube with the modeling dough and experiment with <u>different</u> cuts through the cube (e.g. cutting off a corner, or any diagonal cut that differs from the one already done). They will sketch the resulting cross-sections and name the 2-D shapes.

8. Discuss the results of the various cross-sections made on the cube. Select students to draw their cross-sections on the board. Possible shapes that could result from cutting through the cube include square, rectangle, pentagon, hexagon, non-rectangular parallelogram, and equilateral, isosceles or scalene triangle.

9. Use the interactive tutorial, Cross Sections, from Annenberg Learner to wrap up this portion of the activity: http://www.learner.org/courses/learningmath/geometry/session9/part_c/

    a. Use the *Interactive Activity* to model different cuts through the cube and name and discuss the different cross-sections that are formed.
    b. Using *Problem C1* on the tutorial as a guide, ask students if (and how) they had formed the different cross-sections listed. Click on "Solutions" to display explanations on how the different cross-sections are formed.

Name ______________________________________ **Date** ________________

Flatland (the movie)

CROSS-SECTIONS OF SOLID FIGURES

Cross-section: a plane surface (2-dimensional) formed by cutting across a solid figure. Cross-sections can be made by making a cut that is vertical, horizontal, or diagonal across the solid.

1. Form a sphere with modelling dough. Cut through the sphere horizontally. Sketch the shape of the cut surface.

2. Re-form the dough into a sphere. Cut through the sphere vertically. Sketch the shape of the cut surface.

3. Re-form the dough into a sphere. Cut through the sphere diagonally. Sketch the shape of the cut surface.

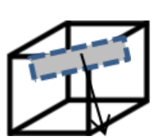
4. Make a statement about the cross-sections of a sphere, describing any similarities or differences.

5. Form the modeling dough into a cube. Remember, all sides of a cube are squares of the same size. Make a horizontal cut through the cube. Sketch the shape of the cut surface.

6. Re-form the dough into a cube. Make a vertical cut through the cube. Sketch the shape of the cut surface.

7. Re-form the dough into a cube. Make a diagonal cut from one vertex to the opposite vertex, cutting through the center of the cube. Sketch the shape of the cut surface.

8. Time to play experiment. Re-form the dough into a cube. Make a straight cut through the cube in any direction. Sketch **both** the cube with a line showing how it was cut, **and** the shape of the cut surface. Name the shape of the cross-section, if possible.
   Repeat this as many times as possible, sketching each cut and the cross-section for each different shape. Try to make as many different shapes of cross-sections as you can. You may use a separate sheet of paper.
   Example:



cross section →

ANSWER KEY

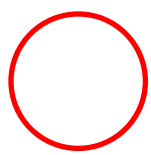
Name ______________________________ **Date** ______________

Flatland **(the movie)**

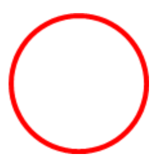
CROSS-SECTIONS OF SOLID FIGURES

Cross-section: a plane surface (2-dimensional) formed by cutting across a solid figure. Cross-sections can be made by making a cut that is vertical, horizontal, or diagonal across the solid.

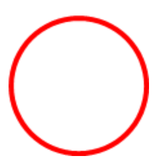
1. Form a sphere with modelling dough. Cut through the sphere horizontally. Sketch the shape of the cut surface.



2. Re-form the dough into a sphere. Cut through the sphere vertically. Sketch the shape of the cut surface.



3. Re-form the dough into a sphere. Cut through the sphere diagonally. Sketch the shape of the cut surface.

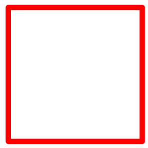


4. Make a statement about the cross-sections of a sphere, describing any similarities or differences.

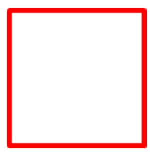
   All of the cross-sections of a sphere are circles.

ANSWER KEY

5. Form the modeling dough into a cube. Remember, all sides of a cube are squares of the same size. Make a horizontal cut through the cube. Sketch the shape of the cut surface.

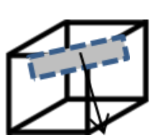


6. Re-form the dough into a cube. Make a vertical cut through the cube. Sketch the shape of the cut surface.



7. Re-form the dough into a cube. Make a diagonal cut from one vertex to the opposite vertex, cutting through the center of the cube. Sketch the shape of the cut surface.

8. Time to play experiment. Re-form the dough into a cube. Make a straight cut through the cube in any direction.   Sketch **both** the cube with a line showing how it was cut, **and** the shape of the cut surface. Name the shape of the cross-section, if possible.
Repeat this as many times as possible, sketching each cut and the cross-section for each different shape. Try to make as many different shapes of cross-sections as you can.   You may use a separate sheet of paper.
Example:

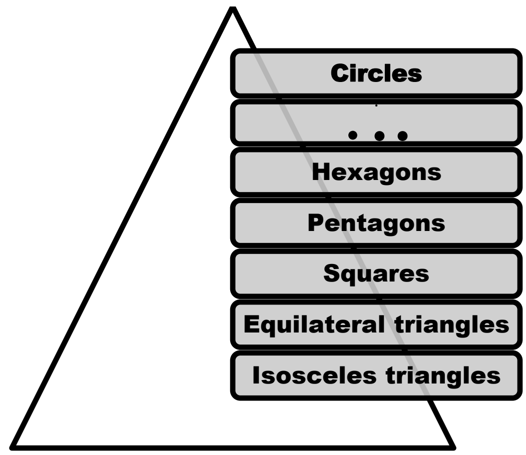


cross section →

ANSWERS WILL VARY

Interior Angles of Polygons

1. Recall the social hierarchy of Flatland and relate it to the Circle Axiom of the Day, ***"Configuration Makes the Man."***



Social Hierarchy of Flatland

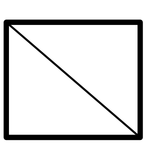
   In Flatland, the more sides a figure has, the greater its angles and the smarter it is believed to be.

2. Distribute worksheets.

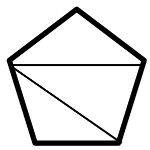
3. Introduce the determination of the sum of interior angles of a polygon by asking the students to recall prior knowledge about triangles and squares.

   Triangle – sum of interior angles = 180^o
   Square – sum of interior angles = 360^o

4. On the board or overhead, show the square as being the sum of two triangles, each containing 180^o



Divide a pentagon into three triangles by drawing line segments from one vertex to each non-adjacent vertex. Three triangles = $180^o \bullet 3 = 540^o$



5 Assign students to complete the worksheet for figures up to 10 sides.

6. Discuss the students' thoughts on a possible formula for determining the sum of the interior angles for a polygon of any number of sides, *n.*

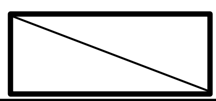
7 Discuss the meaning of the term, "regular polygon" (all sides of equal length, all angles of equal measure).

Name ______________________________________ **Date** ________________

Flatland **(the movie)**

INTERIOR ANGLES OF POLYGONS

1. Complete the following table. The first two rows have been done for you.

# of Sides	Name of polygon	Polygon divided into triangles from one vertex	# triangles	# triangles • 180^{o} = sum of interior angles
3	Triangle		1	180^{o}
4	Square or rectangle		2	360^{o}
5				
6				
7				
8				
9				
10				

2. Look for a pattern in the table above and write a formula for the sum of the interior angles of a polygon with ***n*** sides.

3. Define “regular” polygon.

4. Write a formula for the measurement of one interior angle in a regular polygon with ***n*** sides.

5. What is the measurement of an interior angle of a regular dodecagon (12-sided polygon)?

6. What is the measurement of an interior angle of a regular hexadecagon (16-sided polygon)?

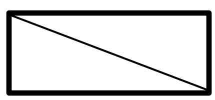
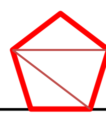
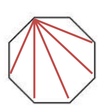
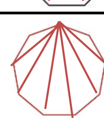
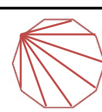
7. Can you use the formula to determine the measurement of an interior angle of any polygon?

Name ANSWER KEY Date

Flatland **(the movie)**

INTERIOR ANGLES OF POLYGONS

1. Complete the following table. The first two rows have been done for you.

# of Sides	Name of polygon	Polygon divided into triangles from one vertex	# triangles	# triangles • 180° = sum of interior angles
3	Triangle		1	180°
4	Square or rectangle		2	360°
5	PENTAGON		3	540°
6	HEXAGON		4	720°
7	HEPTAGON		5	900°
8	OCTAGON		6	1080°
9	NONAGON		7	1260°
10	DECAGON		8	1440°

ANSWER KEY

2. Look for a pattern in the table above and write a formula for the sum of the interior angles of a polygon with ***n*** sides.

   Sum of interior angles = $(n-2) * 180^o$ where *n* is the number of sides

3. Define “regular” polygon.

   A polygon with all sides of equal length and all angles of equal measure

4. Write a formula for the measurement of one interior angle in a regular polygon with ***n*** sides.

   Interior angle of regular polygon = $\frac{(n-2)*180^o}{n}$

   where *n* is the number of sides

5. What is the measurement of an interior angle of a regular dodecagon (12-sided polygon)?

   144^o

6. What is the measurement of an interior angle of a regular hexadecagon (16-sided polygon)?

   157.5^o

7. Can you use the formula to determine the measurement of an interior angle of any polygon?

   No, the formula can only be used when all of the angles are the same measurement, so the formula can only be used with regular polygons .

FINAL DISCUSSION

Wrap up the study of *Flatland* with a discussion of the movie and the activities. Use student responses to guide future lessons. Some questions that can be posed in a final discussion include:

1. Did you like the movie, *Flatland*? Why or why not?
2. Which character in the movie are you most like? Why?
3. Which character in the movie was your favorite? Explain why.
4. What did you learn from watching the movie?

Questions 5 and 6 refer to the following activities – hypercube, cross-sections, and interior angles of polygons.

5. What did you like or dislike about the activities?
6. What did you learn from the activities?

(Answers will vary.)

Thank you for purchasing this product. I would love to hear how your used it and if you and your students enjoyed it. I appreciate feedback on my products. If you have any concerns or problems, please send me a question and I will address it promptly.

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