Co-ordinate Geometry, The Line

The equations that will be given in the Formulae and Tables book are:

The Line:

Slope: $= \frac{rise}{run}$

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

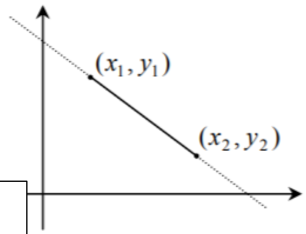
Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Use when given slope and a point on the line or two points on the line.

Equation of a line: $y - y_1 = m(x - x_1)$ or $y = mx + c$

Use when given slope and y-intercept



Area of a triangle with one vertex at the origin $= \frac{1}{2}|x_1 y_2 - x_2 y_1|$

Point dividing a line segment in the ratio $a : b$ $= \left(\frac{bx_1 + ax_2}{a + b}, \frac{by_1 + ay_2}{a + b} \right)$

Distance from a point to a line $= \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

Angles between two line of slopes m_1 and m_2 : $tan\theta = \pm \frac{m_1 - m_2}{1 + m_1 m_2}$

All of the above formulae are given in the Formulae and Tables Booklet. However, there is some other key information that you must be aware of:

Slopes:

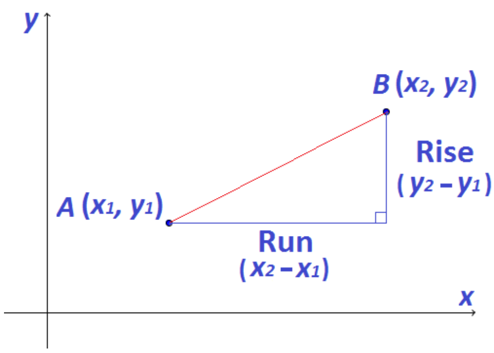
The **slope** or gradient of a line is a value that describes both the direction and the steepness of the line.

The given diagram shows the points $A(x_1, y_1)$ and B (x_2, y_2)

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}}$$

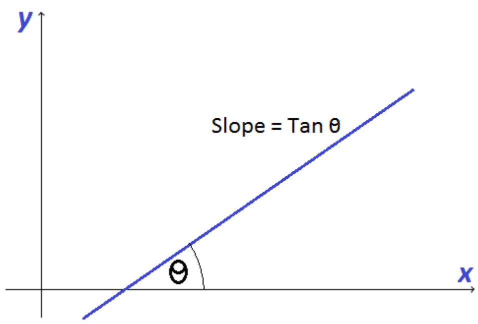
The slope, m, of the line passing through the points (x_1, y_1) and (x_2, y_2) is

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$



The slope of a line is also defined as the tangent of the angle between the line and the positive sense of the *x*-axis.

Slope = Tan θ



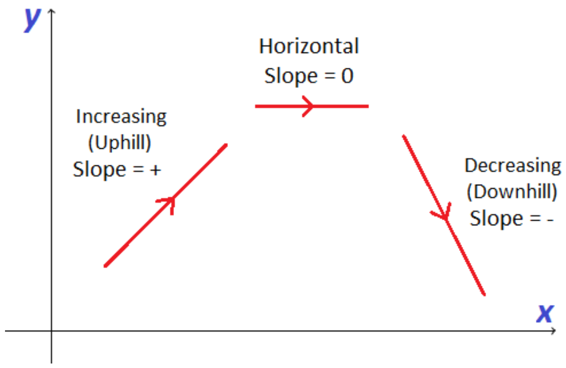
Positive and Negative Slopes

Graphs are read from left to right.

If a line is increasing (going uphill), it has a positive slope.

A horizontal line has a slope of zero.

If a line is decreasing (going downhill), it has a negative slope.



Parallel Lines

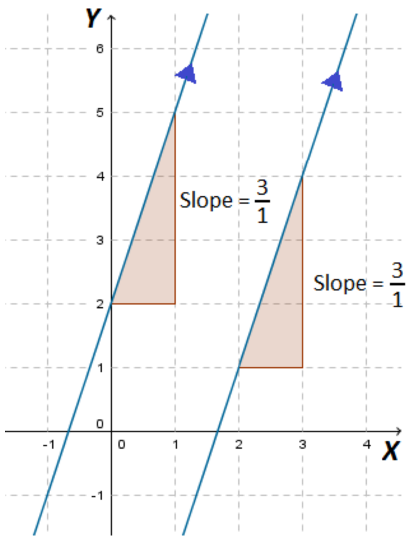
If two lines are parallel if their slopes are equal.

$$\text{If } l_1 || l_2 \text{ then } m_1 = m_2$$

In the given diagram, the two lines are parallel, therefore their slopes are equal:

$$\frac{3}{1} = \frac{3}{1}$$

If two lines are parallel, their slopes are equal.



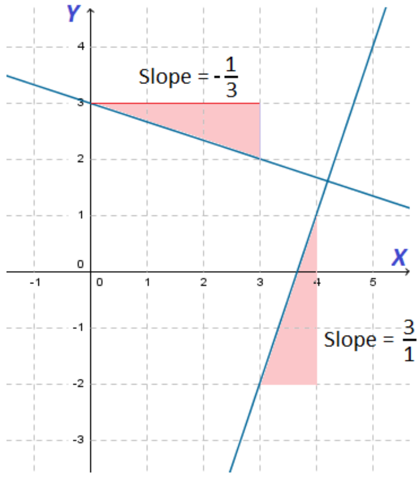
Perpendicular Lines

If two lines are perpendicular, when we multiply their slopes we get -1.

$$\text{If } l_1 \perp l_2 \text{ then } (m_1)(m_2) = -1$$

In the given diagram, the two lines are perpendicular, therefore the product of the slopes is:

$$-\frac{1}{3} \times \frac{3}{1} = -\frac{3}{3} = -1$$



If two lines are perpendicular, the product of their slopes is -1.

1. **Note:** If we know the slope of a line and we need to find the slope of a line perpendicular to it, we turn the given slope upside down and change the sign.

2015 Paper 2 Question 3

(a) The co-ordinates of two points are $A(4, -1)$ and $B(7, t)$.

The line $l_1 : 3x - 4y - 12 = 0$ is perpendicular to AB. Find the value of t.

$$\text{Slope } AB = \frac{t+1}{7-4} = \frac{t+1}{3} \qquad \text{Slope } l_1 = \frac{3}{4}$$

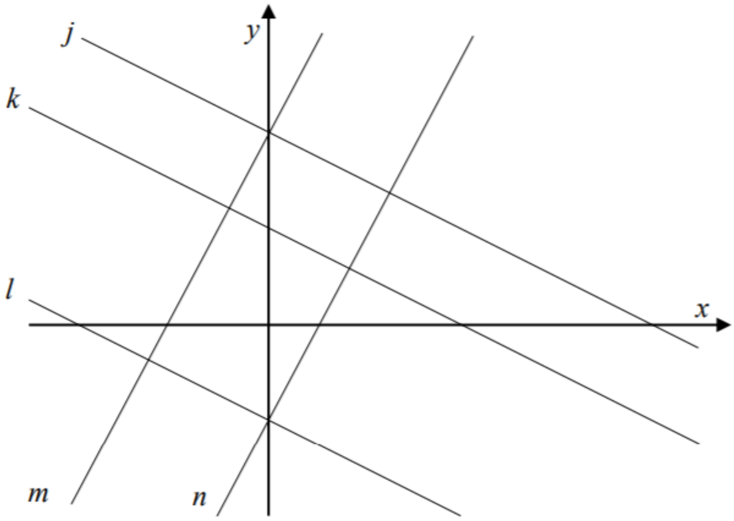
$$AB \perp l_1 \Rightarrow \frac{t+1}{3} \times \frac{3}{4} = -1 \Rightarrow t+1 = -4 \Rightarrow t = -5$$

Since they are perpendicular, we find each slope and then use m_1 x m_2 = -1

When using the equation $\boldsymbol{y = mx + c}$, m = the slope and c = y-intercept

2011 Paper 2 Question 3

In the co-ordinate diagram shown, the lines *j*, *k*, and *l* are parallel, and so are the lines *m* and *n*. The equations of four of the five lines are given in the table below.



Equation	Line
$x+2y=-4$	
$2x-y=-4$	
$x+2y=8$	
$2x-y=2$	

Rearrange each equation so that it's in the form $y = mx + c$

$x+2y=-4 \Rightarrow \quad y=-\frac{1}{2}x-2 \quad \rightarrow l$

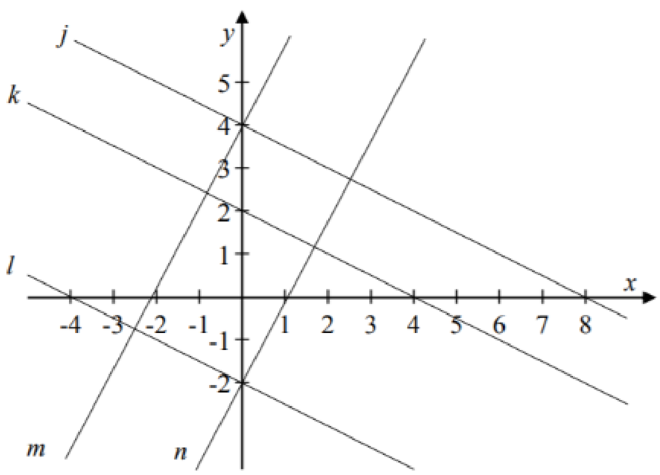
$2x-y=-4 \Rightarrow \quad y=2x+4 \quad \rightarrow m$

$x+2y=8 \Rightarrow \quad y=-\frac{1}{2}x+4 \quad \rightarrow j$

$2x-y=2 \Rightarrow \quad y=2x-2 \quad \rightarrow n$

(a) Complete the table, by matching four of the lines to their equations.

(b) Hence, insert scales on the *x*-axis and *y*-axis.



Points on a Line:

Substitute the coordinates of the point in for *x* and *y* in the equation of the line.
If the equation then balances, the point is on the line (LHS = RHS).
If the equation does not balance, the point is not on the line (LHS ≠ RHS).

2018 Paper 2 Question 5

The line $m: 2x + 3y + 1 = 0$ is parallel to the line $n: 2x + 3y - 51 = 0$.

(a) Verify that $A(-2, 1)$ is on m.

(b) Find the coordinates of B, the point on the line n closest to A, as shown below.

(b) Find the coordinates of B, the point on the line n closest to A, as shown below.

Here we must find the equation of the line *AB* and then find where the lines *AB* and *n* intersect.

Slope of m or $n = \frac{-2}{3}$

To find a slope that is perpendicular to another slope, turn the first slope upside down and change the sign.

Slope of AB is $\frac{3}{2}$ and $(-2, 1)$ is on AB

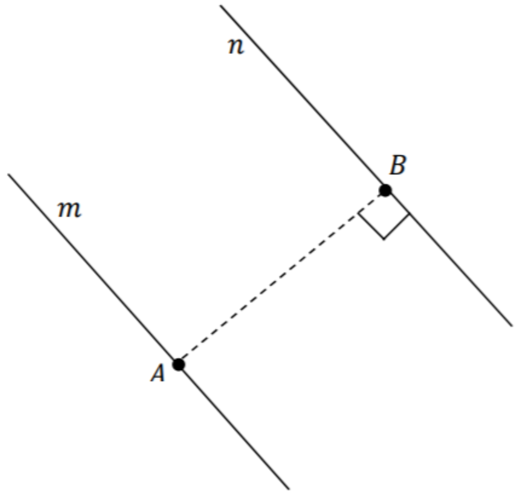
$$y - 1 = \frac{3}{2}(x - (-2))$$

equation of AB is $3x - 2y + 8 = 0$

Solve for (x, y) between

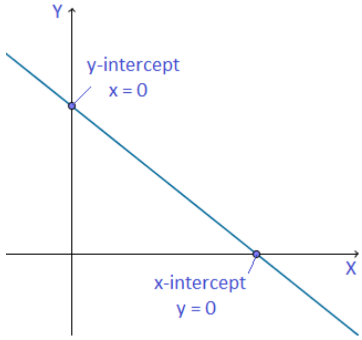
$3x - 2y + 8 = 0$ and $2x + 3y - 51 = 0$

$n \cap AB = (6, 13) = B$



The point of intersection is found using simultaneous equations.

Sketching Lines:



To sketch any line we need any **two points** on the line.

The easiest points to find are the *x* and *y*- intercepts:

To find the point where a line cuts the ***x*-axis we let *y* = 0**

To find the point where a line cuts the ***y*-axis we let *x* = 0**

To find the point of intersection of two lines we use **simultaneous equations**

Equation of a line

To find the equation of a line, we need a point on the line (x_1, y_1) and the slope of the line, m.

The equation of a line can then be found by using the formula:

$$y - y_1 = m(x - x_1)$$

It is good practice to write the completed equation of the line in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{R}$.

2019 Paper 2 Question 2

(a) The line p makes an intercept on the x-axis at $(a, 0)$ and on the y-axis at $(0, b)$, where $a, b \neq 0$.
Show that the equation of p can be written as $\frac{x}{a} + \frac{y}{b} = 1$.

$m = \frac{b-0}{0-a} = \frac{-b}{a}$

$y - y_1 = m(x - x_1)$

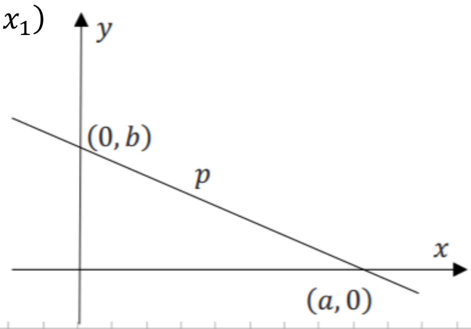
$y - 0 = \frac{-b}{a}(x - a)$

$ay = -bx + ab$

$bx + ay = ab$

Now divide across by ab

$\frac{x}{a} + \frac{y}{b} = 1$



(b) The line l has a slope m, and contains the point $A(6,0)$.

(i) Write the equation of the line l in terms of m.

$$y - y_1 = m(x - x_1)$$

$y - 0 = m(x - 6)$ or $y = m(x - 6)$

Or

$$y = mx - 6m$$

Or

$$y = mx + c$$

$\therefore 0 = 6m + c \;\Rightarrow\; c = -6m$

To find the slope when given the equation of the line

Method 1:

When an equation of a line is written in the form $ax + by + c = 0$, with all terms on the left-hand side, then

$$\text{Slope} = -\frac{\text{Number in front of } x}{\text{Number in front of } y}$$

For a line in the form $\boldsymbol{ax + by + c = 0}$

$$\text{Slope} = -\frac{a}{b}$$

Method 2:

When an equation of a line is written in the form $y = mx + c$,
with the y on its own on the left-hand side, then

$$\text{Slope} = m = \text{number in front of the } x$$

For a line in the form $\boldsymbol{y = mx + c}$

$$\text{Slope} = m$$

y-intercept = c

The ***y*-intercept** is the point where the line crosses the y-axis.

Finding the equation of Parallel and Perpendicular lines

To find the equation of a line parallel or perpendicular to a given line:

1) Find the slope of the given line.
2) Find the slope of the required parallel or perpendicular line
3) Use the given point and the new slope to find the equation of the required line

Remember:
* Parallel lines have equal slopes
* The product of the slopes of perpendicular lines equals -1.

A line parallel to $ax + by + c = 0$ will be in the form $ax + by + d = 0$

A line perpendicular to $ax + by + c = 0$ will be in the form $bx - ay + d = 0$

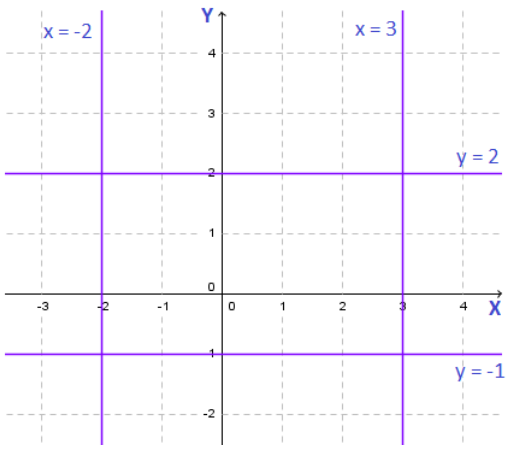
Lines that are parallel to the axes

Some lines are parallel to the *x*-axis (horizontal) and some lines are parallel to the *y*-axis.

$x = 0$ is the equation of the *y*-axis.
$y = 0$ is the equation of the *x*-axis.

The line $x = a$ is vertical and passes through the value a on the *x*-axis.

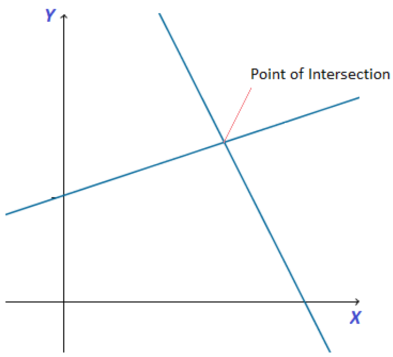
The line $y = b$ is horizontal and passes through the value b on the *y*-axis.



Point of Intersection of Two Lines

To find the point of intersection of two lines you could graph the lines on the coordinated plane and read off the point of intersection.

However, it can be quicker and possibly more accurate to find the point of intersection using Algebra. To do this, write the equation of each line in the form $ax + by = c$, where $a, b, c \in \mathbb{R}$. Solve these two equations simultaneously, to find a value for x and a value for y. The point of intersection is then (x, y)

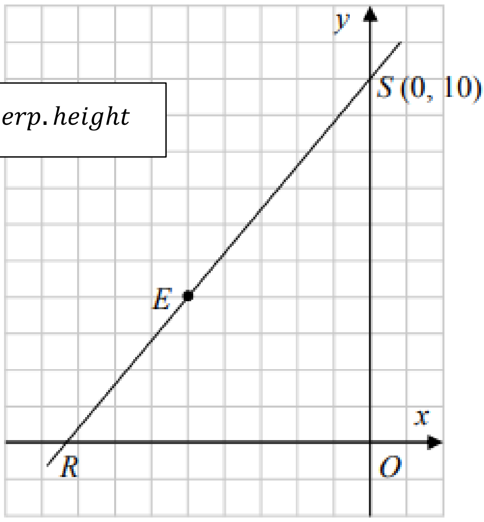


Use your knowledge of solving simultaneous equations from Algebra. Remember that there are two methods for solving simultaneous equations:
Method 1: Elimination method
Method 2: Substitution method

Remember that the symbol $\cap$ means intersection.

2016 Paper 2 Question 5

The line RS cuts the x-axis at the point R and the y-axis at the point $S(0, 10)$, as shown. The area of the triangle ROS, where O is the origin, is $\frac{125}{3}$.



Area of a Triangle = $\frac{1}{2} \times base \times perp.height$

(a) Find the co-ordinates of R.

Area $ROS = \frac{1}{2}|RO|.|OS| = \frac{125}{3}$

$\Rightarrow \frac{1}{2}|RO|(10) = \frac{125}{3}$

$\Rightarrow |RO| = \frac{25}{3}$

$R\left(-\frac{25}{3}, 0\right)$

(b) Show that the point $E(-5, 4)$ is on the line RS.

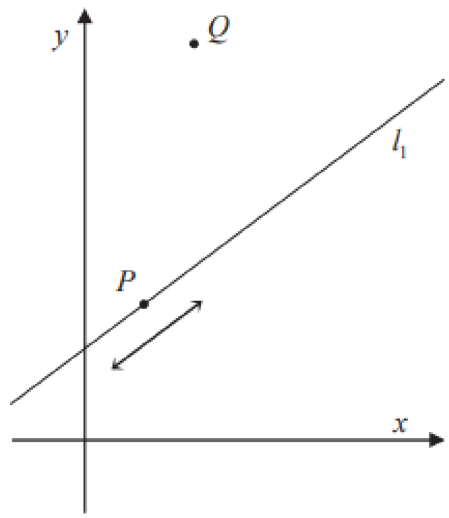
Find the equation of the line first and then check if E is on the line.

RS: $y-10=\frac{6}{5}(x-0) \Rightarrow 6x-5y+50=0$

$6(-5)-5(4)+50=-30-20+50=0 \Rightarrow (-5, 4) \in RS$

2014 Sample Paper 2 Question 2

(a) Show that, for all $k \in \mathbb{R}$, the point $P(4k-2, 3k+1)$ lies on the line $l_1 : 3x - 4y + 10 = 0$.



If $(x, y) = (4k-2, 3k+1)$ then

$$\begin{aligned} 3x - 4y + 10 &= 3(4k-2) - 4(3k+1) + 10 \\ &= 12k - 6 - 12k - 4 + 10 \\ &= 0 \end{aligned}$$

So the equation of l_1 is satisfied. Therefore $(4k-2, 3k+1)$ lies on l_1.

(b) The line l_2 passes through P and is perpendicular to l_1. Find the equation of l_2, in terms of k.

$3x - 4y + 10 = 0$

$-4y = -3x - 10$

$y = \frac{3}{4}x + \frac{5}{2}$

Therefore the slope of l_1 is $\frac{3}{4}$. Therefore l_2 has a slope of $-\frac{4}{3}$ and a point of $(4k - 2, 3k + 1)$. So it has equation:

$y - (3k + 1) = -\frac{4}{3}(x - (4k - 2))$

$3y - 3(3k + 1) = -4(x - (4k - 2))$

Resulting in: $4x + 3y - 25k + 5 = 0$

(c) Find the value of k for which l_2 passes through the point $Q(3, 11)$.

The equation of l_2 is

$$4x + 3y - 25k + 5 = 0.$$

Now $(3, 11)$ lies on l_2 if and only if $4(3) + 3(11) - 25k + 5 = 0 \Leftrightarrow 25k = 50 \Leftrightarrow k = 2$. So the $k = 2$ is the required value.

(d) Hence, or otherwise, find the co-ordinates of the foot of the perpendicular from Q to l_1.

When $k = 2$ the equation of l_2 is

$$4x + 3y - 45 = 0.$$

Solving the equations:

$$\begin{aligned} 1.\ 3x - 4y + 10 &= 0 \\ 2.\ 4x + 3y - 45 &= 0 \end{aligned}$$

$$\begin{aligned} 1.\ 12x - 16y + 40 &= 0 \\ 2.\ 12x + 9y - 135 &= 0 \end{aligned}$$

$$-25y + 175 = 0.$$

Therefore $25y = 175$ and $y = \frac{175}{25} = 7$.

Now $3x - 4(7) + 10 = 0 \Leftrightarrow 3x = 4(7) - 10 = 18 \Leftrightarrow x = 6$. So the foot of the perpendicular from Q to l_1 has co-ordinates $(6, 7)$.

Area of a Triangle given its vertices:

Find the area of a triangle with vertices (1, 5), (-3, 1) and (3, -5). $\frac{1}{2}|x_1y_2 - x_2y_1|$

In order to find the area of this triangle we must **translate** one of the points to the origin, (0, 0) and the other points must follow the same translation.

In this case we take 1 from each *x*-value and 5 from each *y*-value.

Let (1, 5) → (0, 0)

(-3, 1) → (-4, -4)

(3, -5) → (2, -10)

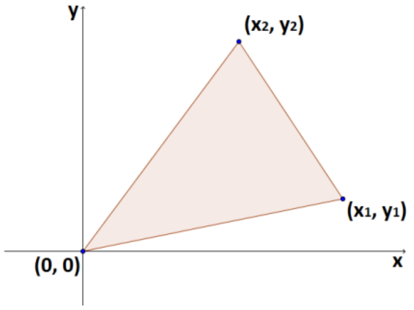
Area of a triangle $= \frac{1}{2}|x_1y_2 - x_2y_1|$

$= \frac{1}{2}|(-4)(-10) - (2)(-4)|$

$= \frac{1}{2}|40 + 8|$

$= \frac{1}{2}|48|$

= 24 square units

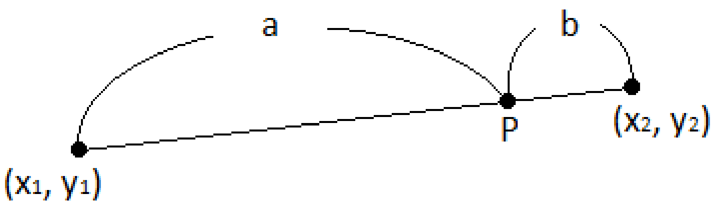


Division of a Line segment internally in the ratio of a:b:

In the given diagram, the point P divides the line segment [AB] in the ratio a : b.

The coordinates of P are given by the formula:

$$P = \left(\frac{bx_1 + ax_2}{b + a}, \frac{by_1 + ay_2}{b + a}\right)$$



Ex. 1 Find the coordinates of the point which divides the line segment *A*(-1, 3) and *B*(4, -2) internally in the ratio of *a*:*b* = 2:1.

$$\left(\frac{ax_2+bx_1}{a+b}, \frac{ay_2+by_1}{a+b}\right)$$

A(-1, 3) and *B*(4, -2)

(x_1, y_1) (x_2, y_2)

$= \left(\frac{ax_2+bx_1}{a+b}, \frac{ay_2+by_1}{a+b}\right)$

$= \left(\frac{(2)(4)+(1)(-1)}{2+1}, \frac{(2)(-2)+(1)(3)}{2+1}\right)$

$= \left(\frac{8-1}{3}, \frac{-4+3}{3}\right)$

$= \left(\frac{7}{3}, \frac{-1}{3}\right)$

Ex. 2

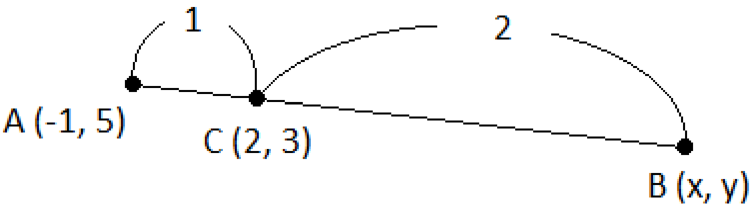
A (-1, 5) and *B* (*x*, *y*) are two points on a plane. The point *C* (2, 3) divides [*AB*] in the ratio 1 : 2.

Find the values of *x* and *y*.

Solution

Method 1 – using the formula

Sketch the line segment:



A (-1, 5) = (x_1, y_1) *B* (*x*, *y*) = (x_2, y_2)

1 = *a* 2 = *b*

$$P = \left(\frac{bx_1 + ax_2}{b + a}, \frac{by_1 + ay_2}{b + a}\right)$$

$$(2, 3) = \left(\frac{(2)(-1) + (1)(x)}{2 + 1}, \frac{(2)(5) + (1)(y)}{2 + 1}\right)$$

$$(2, 3) = \left(\frac{-2 + x}{3}, \frac{10 + y}{3}\right)$$

Equate the coordinates:

x coordinates:	*y* coordinates
$2 = \frac{-2 + x}{3}$	$3 = \frac{10 + y}{3}$
$6 = -2 + x$	$9 = 10 + y$
$8 = x$	$-1 = y$

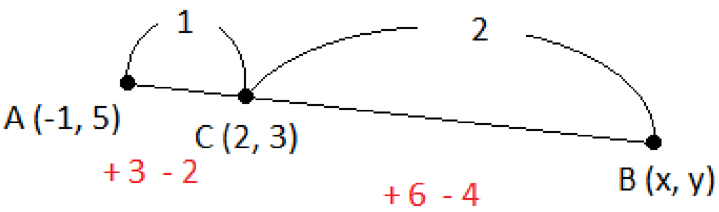
Therefore, *B* (8, -1)

Method 2 – using translations

The translation to go from A to C is: Add 3 to x coordinate and subtract 2 from y coordinate

Using the ratios we can see that the translation from C to B is twice that of the translation from A to C.

Therefore, the translation to go from C to B is: Add 6 to x coordinate and subtract 4 from y coordinate



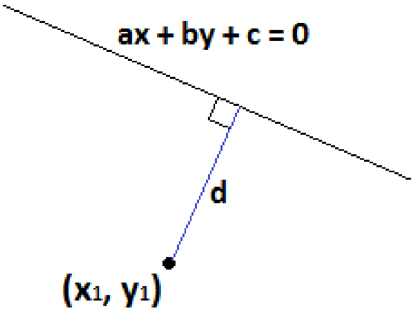
Translations are seen in transformation geometry.

$$C(2, 3) \rightarrow (2 + 6, 3 - 4) \rightarrow B(8, -1)$$

Perpendicular Distance Formula:

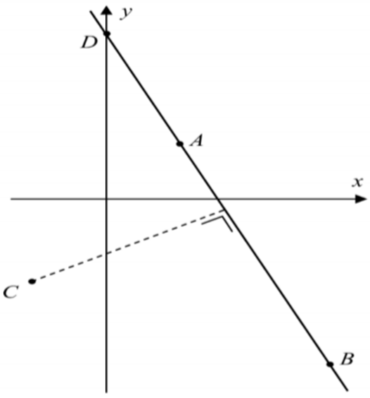
The perpendicular distance, d, from the point (x_1, y_1) to the line $ax + by + c = 0$ is given by the formula

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$



2012 Sample Paper

The co-ordinates of three points A, B, and C are: $A(2, 2)$, $B(6, -6)$, $C(-2, -3)$.
(See diagram on facing page.)



(a) Find the equation of AB.

Here $(x_1, y_1) = (2, 2)$ and $(x_2, y_2) = (6, -6)$. First find the slope of AB

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 2}{6 - 2} = \frac{-8}{4} = -2$$

Now use the equation of a line formula

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 2 &= -2(x - 2) \\ y - 2 &= -2x + 4 \\ 2x + y - 6 &= 0 \end{aligned}$$

(c) Find the perpendicular distance from C to AB.

Using the perpendicular distance formula:

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

From the equation of AB we have $a = 2, b = 1, c = -6$ and from the coordinates of C we have $x_1 = -2, y_1 = -3$. Then the perpendicular distance between C and AB is

$$\frac{|2(-2) + 1(-3) - 6|}{\sqrt{2^2 + 1^2}} = \frac{|-4 - 3 - 6|}{\sqrt{4 + 1}} = \frac{|-13|}{\sqrt{5}} = \frac{13}{\sqrt{5}}$$

Example

Find the equations of the two lines that are parallel to the line 3*x* + 2*y* + 9 = 0 and $\sqrt{13}$ units from it.

Solution

Any line parallel to the line 3*x* + 2*y* + 9 = 0 will be of the form 3*x* + 2*y* + *k* = 0.

Find a point on the line 3*x* + 2*y* + 9 = 0:

Let *y* = 0

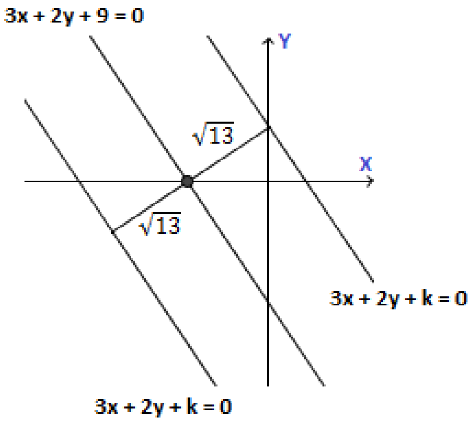
3*x* + 2(0) + 9 = 0

3*x* + 9 = 0

3*x* = - 9

x = - 3

The point (-3, 0) is on the line



The distance from the point (-3, 0) and the line

3*x* + 2*y* + *k* = 0 is $\sqrt{13}$.

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$\sqrt{13} = \frac{|(3)(-3) + (2)(0) + k|}{\sqrt{(3)^2 + (2)^2}}$$

$$\sqrt{13} = \frac{|-9 + 0 + k|}{\sqrt{9 + 4}}$$

$$\sqrt{13} = \frac{|-9 + k|}{\sqrt{13}}$$

$$(\sqrt{13})(\sqrt{13}) = |-9 + k|$$

$$13 = |-9 + k|$$

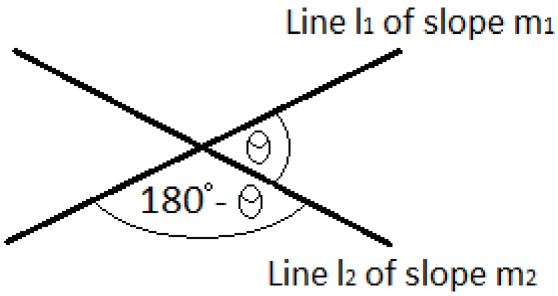
$$-13 = -9 + k \qquad 13 = -9 + k$$

$$-4 = k \qquad 22 = k$$

Therefore, the lines are:

3*x* + 2*y* – 4 = 0   3*x* + 2*y* + 22 = 0

Finding the angle between two lines:



To find the acute angle, θ, between two lines, we need the slopes, m_1 and m_2, of the two lines.

Then

$$\text{Tan } \theta = \pm \frac{m_1 - m_2}{1 + m_1 m_2}$$

- To find the acute angle, let
$$\text{Tan } \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

The angle between perpendicular lines is 90°. Since Tan 90° is undefined this formula does not work for perpendicular lines.

- To find the obtuse angle between the lines:

  Obtuse angle = 180° - Acute angle

2013 Paper 2 Question 3

The equations of six lines are given:

Line	Equation
h	$x = 3 - y$
i	$2x - 4y = 3$
k	$y = -\frac{1}{4}(2x - 7)$
l	$4x - 2y - 5 = 0$
m	$x + \sqrt{3}y - 10 = 0$
n	$\sqrt{3}x + y - 10 = 0$

(b) Find the acute angle between the lines m and n.

Slope of m: $m_1 = -\frac{1}{\sqrt{3}}$

Slope of n: $m_2 = -\sqrt{3}$

For m_1: $y = -\frac{1}{\sqrt{3}}x + \frac{10}{\sqrt{3}}$

For m_2: $y = -\sqrt{3}x + 10$

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2} = \pm\frac{-\frac{1}{\sqrt{3}} + \sqrt{3}}{1 - \frac{1}{\sqrt{3}}\left(-\sqrt{3}\right)} = \pm\frac{\frac{-1+3}{\sqrt{3}}}{1+1} = \pm\frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \theta = 30°$$

Points of Interest: By construction and geometrically

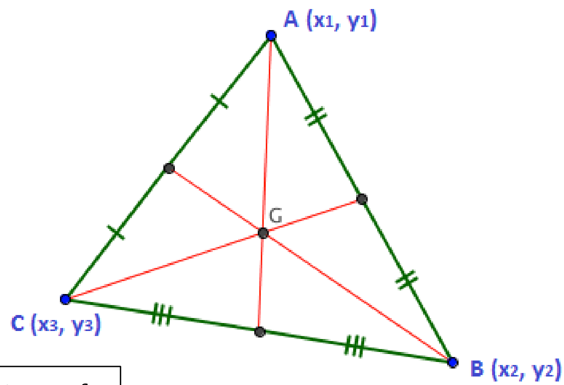
Three of the constructions from your study of Synthetic Geometry are to construct the Centroid, Circumcentre and the Orthocentre of a triangle. It is important to understand the link between these geometrical constructions and the graphing of a triangle on the coordinated plane. You should be able to use your knowledge of coordinate geometry to find the coordinates of any of these points.

In geometry, three or more lines in a plane are said to be **concurrent** if they all intersect at a single point.

The Centroid

The **median** of a triangle is the line drawn from one vertex to the midpoint of the opposite side.

The three medians of a triangle intersect at a point called the Centroid. The centroid is the centre of gravity of the triangle and it cuts the medians in the ration 2:1.
The coordinates of the Centroid are found by finding the average of the three vertices.

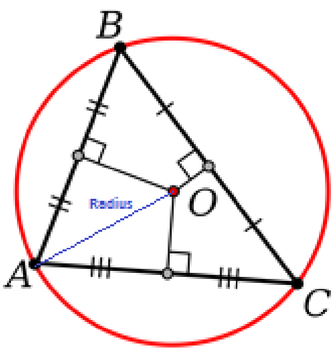


If A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are the vertices of a triangle, then the coordinates of the centroid, G are:

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

The Circumcentre

The perpendicular bisectors of the sides of a triangle, known as the **mediators**, intersect at a point called the Circumcentre.



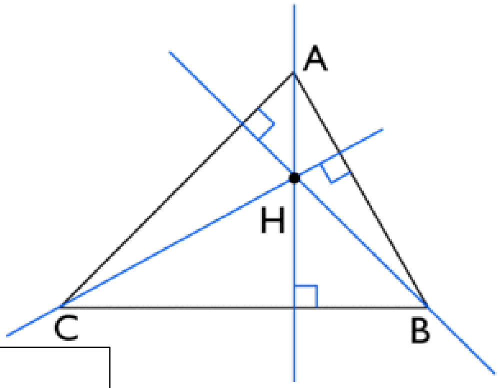
This point is the centre of the circumcircle, which is a circle that passes through the three vertices of the triangle.

The line segment from the circumcentre to any one of the vertices of the triangle is the radius of the circumcircle.

The Orthocentre

An **altitude** is a line which passes through a vertex of a triangle and is perpendicular to the opposite side.

The point of intersection of the three altitudes of a triangle is called the Orthocentre of the triangle.



The orthocentre is:
* Inside an acute triangle
* Outside of an obtuse triangle
* At the right-angled vertex of a right-angled triangle

It's important to be aware that the circumcentre could be outside the triangle; this is perfectly normal and can be encountered at times. Students are intended to have encountered this while exploring it in class.

The following question came up in a pre-paper for the pilot schools in 2010:

Q. State the condition(s) under which the circumcentre of a triangle will lie **inside** the triangle and justify your answer.

A. The circumcentre of the circle will lie within the triangle if all the angles are acute.

- You should be aware that if the triangle is right angled, the circumcentre is on the hypotenuse of the triangle.

- If the triangle is obtuse, the circumcentre lies outside the triangle.

2016 Paper 2 Question 1

The points $A(6, -2)$, $B(5, 3)$ and $C(-3, 4)$ are shown on the diagram.

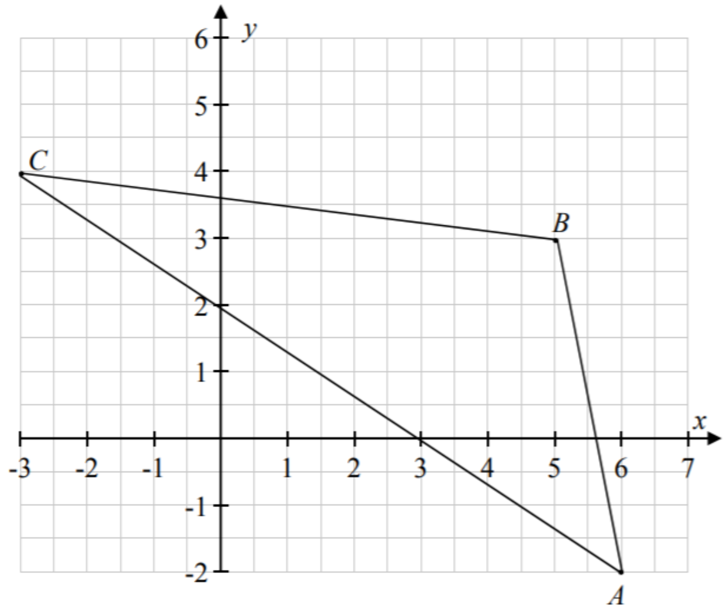
(a) Find the equation of the line through B which is perpendicular to AC.

$$\text{Slope } AC = -\frac{2}{3}$$

$$\text{perp. slope} = \frac{3}{2}$$

$$y - 3 = \frac{3}{2}(x - 5)$$

$$3x - 2y = 9$$



(b) Use your answer to part **(a)** above to find the co-ordinates of the orthocentre of the triangle ABC.

Point of intersection of the altitudes

$$\text{Slope } AB = \frac{3+2}{5-6} = -\frac{5}{1}$$

$$\text{perp. slope} = \frac{1}{5}$$

$$y - 4 = \frac{1}{5}(x + 3)$$

$$x - 5y + 23 = 0$$

Orthocentre:

$$3x - 2y = 9 \cap x - 5y = -23$$

$$\Rightarrow y = 6 \quad x = 7$$

$$(7, 6)$$

Use simultaneous equations to find where the two lines intersect.

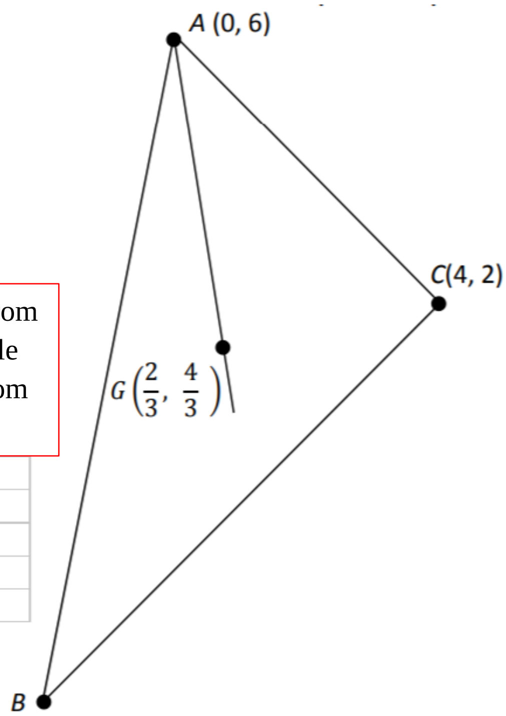
2017 Paper 2 Question 3

ABC is a triangle where the co-ordinates of *A* and *C* are (0, 6) and (4, 2) respectively.

$G\left(\frac{2}{3}, \frac{4}{3}\right)$ is the centroid of the triangle *ABC*.

AG intersects *BC* at the point *P*.

$|AG| : |GP| = 2 : 1$.



(a) Find the co-ordinates of *P*.

$$A(0,6) \rightarrow G\left(\frac{2}{3}, \frac{4}{3}\right)$$

$$\rightarrow P\left(\frac{2}{3}+\frac{1}{2}\left(\frac{2}{3}\right), \frac{4}{3}+\frac{1}{2}\left(\frac{-14}{3}\right)\right)$$

$$= \left(\frac{3}{3}, -\frac{3}{3}\right)$$

$$P = (1\,, -1)$$

The distance from A to G is double the distance from G to P.

(b) Find the co-ordinates of *B*.

$$C(4,2) \rightarrow P(1,\ -1\,) \rightarrow B(1-3, -1-3)$$

$$= (-2,\ -4\,)$$

$$B(x,y) \rightarrow \left(\frac{4+x}{2}, \frac{2+y}{2}\right) = (1, -1)$$

$$x = -2, \quad y = -4$$

$$B = (-2\,, -4)$$

(c) Prove that *C* is the orthocentre of the triangle *ABC*.

$$AC\ \perp\ BC$$

$$AC = \frac{2-6}{4-0} = -1$$

$$BC = \frac{2+4}{4+2} = 1$$

$$-1 \times 1 = -1$$

lines are perpendicular

If *C* is the orthocentre then *AC* will be perpendicular to *BC*.

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$y - 3 = \frac{3}{2}(x - 5)$

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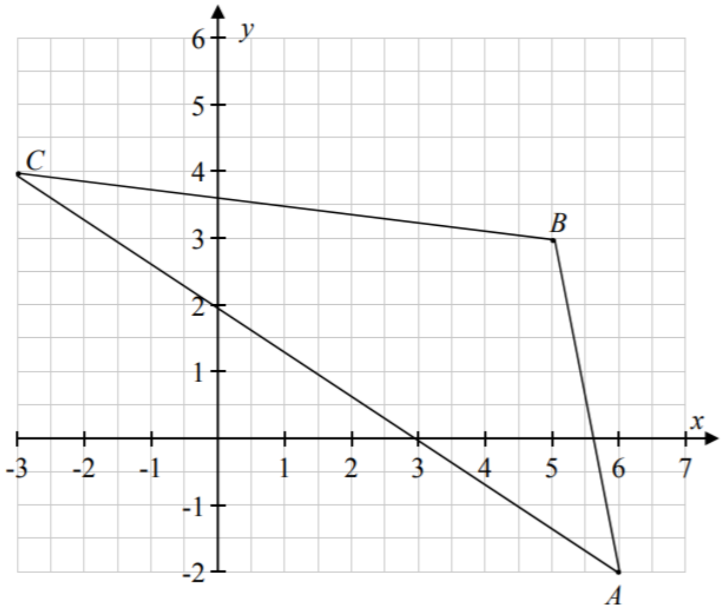

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$(7, 6)$