



Place Value, Decimals & Percentages: Teacher's Manual

A Guide to Teaching and Learning
in Irish Primary Schools



This manual has been designed by members of the Professional Development Service for Teachers. Its sole purpose is to enhance teaching and learning in Irish primary schools and will be mediated to practising teachers in the professional development setting. Thereafter it will be available as a free downloadable resource on www.pdst.ie for use in the classroom. This resource is strictly the intellectual property of PDST and it is not intended that it be made commercially available through publishers. All ideas, suggestions and activities remain the intellectual property of the authors (all ideas and activities that were sourced elsewhere and are not those of the authors are acknowledged throughout the manual).

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Aim of the Guide

The aim of this resource is to assist teachers in teaching the strand units of Place Value (1st to 6th class), Decimals (3rd to 6th class) and Percentages (5th and 6th class). These strand units are not applicable for infant classes. The resource is intended to complement and support the implementation of the Primary School Mathematics Curriculum (PSMC) rather than replace it. By providing additional guidance in the teaching and learning of place value, decimals and percentages, this resource attempts to illuminate an instructional framework for enhancing mathematical thinking. This instructional framework advocates methods of eliciting, supporting and extending higher-order mathematics skills such as reasoning; communicating and expressing; integrating and connecting; and applying and problem solving. Although, this resource highlights the Number strand, this instructional framework can be used for all strands and strand units of the PSMC.

Possible Resources

The following resources may be useful in developing and consolidating a number of concepts in place value, decimals and percentages. This is not an exhaustive list and other resources may also be suitable.

Table 1.1 Possible resources

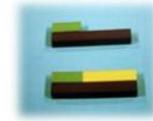
| | |
|---|--|
| dienes blocks (base 10 materials) | dice |
| cuisenaire rods | fraction, decimal, percentage walls |
| fraction bars | pie fraction sets |
| class number lines (clothes line and pegs style), table top number lines | playing cards |
| empty number lines | dominoes |
| counting sticks | notation boards |
| 5 frame, 10 frame | place value chart/template |
| arrow cards | calculators |
| number line (with and without numbers) | abacus |
| 100 square (with and without numbers) | place value houses |
| 99 square | hundredths disc |
| number fans | 10 x 10 grid paper |
| digit cards | dotted paper |
| counting bead string | decimal place mat |

The approach taken to place value, decimals and percentages in this manual lends itself ideally to differentiated teaching and learning. The approach advocates moving from the concrete to the pictorial to the abstract, based on the needs of individual pupils. For this reason the approach is also ideal for use in the learning support, resource and special class settings. Furthermore, it advocates the linear, area and set models for decimals and percentages.

The area model may appeal to spatial learners.



The linear model is compatible with logical and spatial learning styles.



Finally the set model allows for tangible and kinaesthetic learning experiences.



All models allow for the use of manipulatives and concrete materials and transfer to both the pictorial and the abstract representations. This myriad of learning experiences for the development of the same concept means that different learning styles and abilities are catered for as well as providing repeated opportunities to consolidate learning in a fun and interactive way. The learning trajectory is incremental as are the three stages of concrete, pictorial and abstract. As in all good teaching and learning environments the pupil dictates their starting point and the rate at which they move along the trajectory. Teachers in the multi-class context may find the trajectory helpful in this regard. Finally, the instructional framework advocates a differentiated approach to questioning as a fundamental mode of assessment. Examples of various levels of questioning are evident throughout the activities in this manual.

Estimation

Estimation plays a significant role in mathematics as it leads pupils to decide if their answer is mathematically sound. It is important to differentiate between an *estimate* and an *answer*. It is often beneficial for pupils to investigate how sensible their guess / estimate is (for example, is it the right amount of tens / units; if it is a subtraction sum, should the estimate be bigger than the starting number;

etc.). Outlined in the Mathematics Teachers Guidelines (pages 32 – 34) are the following estimation strategies:

- Front-end strategy
- Clustering strategy
- Rounding strategy
- Special numbers strategy

Linkage

Although this guide focuses on three strand units (place value, decimals and percentages) of one strand (number), it is intended that the links to other strands, strand units and subjects would be made where applicable. Some examples of the possible linkage of fractions within the maths curriculum can be seen in Table 1.2 for first and second classes, Table 1.3 for third class, Table 1.4 for fourth class; and Table 1.5 for fifth and sixth classes.

Table 1.2 Possible linkage of place value across the maths curriculum (first and second classes)

| Class Level | Strand | Strand Unit | Objective |
|-----------------------------------|--------|-------------|--|
| 1 st & 2 nd | Number | Place Value | <ul style="list-style-type: none"> • Explore, identify and record place value 0-99 • Explore, identify and record place value 0-199 |

| Class Level | Strand | Strand Unit | Objective |
|-----------------------------------|---------|----------------------------|--|
| 1 st & 2 nd | Number | Counting & Numeration | <ul style="list-style-type: none"> • Count the number of objects in a set • Read, write and order numerals, 0-99 (0-199) • Estimate the number of objects in a set 0-20 |
| | Number | Comparing & Ordering | <ul style="list-style-type: none"> • Compare equivalent and non-equivalent sets 0-20 • Order sets of objects by number • Use the language of ordinal number |
| | Number | Operations | <ul style="list-style-type: none"> • Add numbers without and with renaming within 99 • Explore and discuss repeated addition and group counting • Subtract numbers without renaming within 99 |
| | Algebra | Extending & Using Patterns | <ul style="list-style-type: none"> • Recognise pattern (and predict subsequent numbers) • Explore and use patterns in addition facts • Understand the use of a frame to show the presence of an unknown number |

Table 1.3 Possible linkage of place value and decimals across the maths curriculum (third class)

| Class Level | Strand | Strand Units | Objective |
|-----------------|--------|-------------------------|--|
| 3 rd | Number | Place Value Decimals | <ul style="list-style-type: none"> • Explore, identify and record place value 0-999 • Read, write and order 3 digit numbers • Round whole numbers to the nearest ten or hundred • Explore, express and identify place value in decimal numbers to one place (tenths) • Count, compare and order decimals • Solve problems involving decimals |

| Class Level | Strand | Strand Unit | Objective |
|-----------------|----------|-----------------------------|--|
| 3 rd | Number | Operations | <ul style="list-style-type: none"> • Add and subtract, with and without renaming, within 999 • Develop an understanding of place value as repeated addition and vice versa |
| | Number | Fractions | <ul style="list-style-type: none"> • Compare and order fractions with appropriate denominators and position on the number line • Solve and complete practical tasks and problems involving fractions |
| | Algebra | Number Patterns & Sequences | <ul style="list-style-type: none"> • Explore, recognise and record patterns in number, 0-999 • Explore, extend and describe (explain rule for) sequences |
| | Measures | Length | <ul style="list-style-type: none"> • Solve and complete practical tasks and problems involving the addition and subtraction of units of length (m, cm) |
| | Measures | Weight | <ul style="list-style-type: none"> • Solve and complete practical tasks and problems involving the addition and subtraction of units of weight (kg, g) |
| | Measures | Capacity | <ul style="list-style-type: none"> • Solve and complete practical tasks and problems involving the addition and subtraction of units of capacity (l, ml) |

Table 1.4 Possible linkage of place value and decimals across the maths curriculum (fourth class)

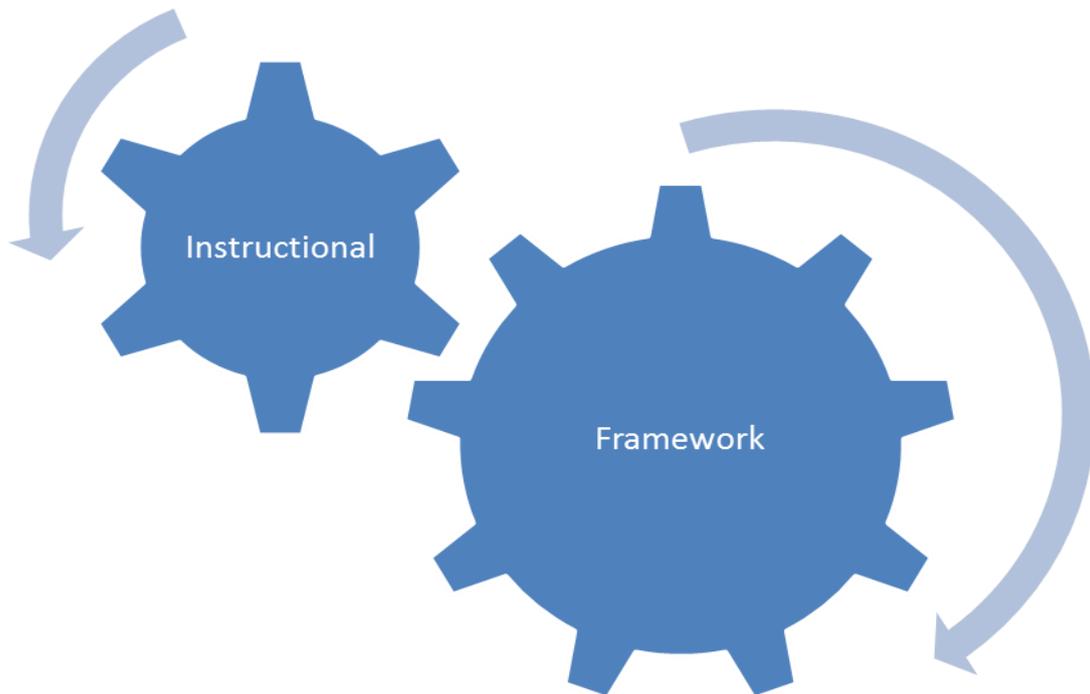
| Class Level | Strand | Strand Units | Objective |
|-----------------|--------|-------------------------|--|
| 4 th | Number | Place Value Decimals | <ul style="list-style-type: none"> • Explore, identify and record place value 0-9999 • Read, write and order 4 digit numbers • Round whole numbers to the nearest thousand • Explore, express and identify place value in decimal numbers to two places (tenths and hundredths) • Make, order, compare and count decimals • Add and subtract whole numbers and decimals to 2 places • Multiply and divide decimals up to 2 places by a single-digit whole number • Solve problems involving decimals |

| Class Level | Strand | Strand Unit | Objective |
|-----------------|----------|-----------------------------|--|
| 4 th | Number | Operations | <ul style="list-style-type: none"> • Add and subtract, with and without renaming, within 9999 • Develop an understanding of place value as repeated addition and vice versa • Use a calculator to check estimates |
| | Number | Fractions | <ul style="list-style-type: none"> • Compare and order fractions with appropriate denominators and position on the number line • Solve and complete practical tasks and problems involving fractions |
| | Algebra | Number Patterns & Sequences | <ul style="list-style-type: none"> • Explore, recognise and record patterns in number, 0-9999 • Explore, extend and describe (explain rule for) sequences |
| | Measures | Length | <ul style="list-style-type: none"> • Rename units of length using decimal and fraction form • Solve and complete practical tasks and problems involving the addition and subtraction of units of length (m, cm) |
| | Measures | Weight | <ul style="list-style-type: none"> • Rename units of weight using decimal and fraction form • Solve and complete practical tasks and problems involving the addition and subtraction of units of weight (kg, g) |
| | Measures | Capacity | <ul style="list-style-type: none"> • Rename units of length using decimal and fraction form • Solve and complete practical tasks and problems involving the addition and subtraction of units of capacity (l, ml) |
| | Measures | Money | <ul style="list-style-type: none"> • Rename units of money as euro or cents and record using € and decimal point |

Table 1.5 Possible linkage of place value decimals and percentages across the maths curriculum (fifth and sixth classes)

| Class Level | Strand | Strand Units | Objective |
|-----------------------------------|--------|--|---|
| 5 th & 6 th | Number | Place Value Decimals Percentages | <ul style="list-style-type: none"> Round whole numbers and round decimals to nearest whole number (to one, two or three decimal places) Express tenths, hundredths and thousandths as fractions and decimals Compare and order fractions, percentages and decimals Explore and calculate simple interest, profit, loss, VAT |

| Class Level | Strand | Strand Unit | Objective |
|-----------------------------------|----------|----------------------------------|---|
| 5 th & 6 th | Number | Operations | <ul style="list-style-type: none"> Add and subtract whole numbers and decimals (to three decimal places) with and without a calculator Multiply a decimal (up to three places) by a whole number, without and with a calculator Multiply a decimal by a decimal, without and with a calculator Divide a three-digit number (four-digit number) by a two-digit number, without and with a calculator Divide a decimal number by a whole number, without and with a calculator |
| | Number | Fractions | <ul style="list-style-type: none"> Express improper fractions as mixed numbers and vice versa and position them on the number line |
| | Measures | Length | <ul style="list-style-type: none"> Rename units of length (express results as fractions and decimal fractions of appropriate metric units) |
| | Measures | Weight | <ul style="list-style-type: none"> Rename measures of weight (express results as fractions and decimals of appropriate metric units) |
| | Measures | Capacity | <ul style="list-style-type: none"> Rename units of capacity (express results as fractions and decimal fractions of appropriate metric units) |
| | Measures | Money | <ul style="list-style-type: none"> Compare 'value for money' using unitary method Explore value for money (percentage discounts, VAT added, etc.) |
| | Data | Representing & Interpreting Data | <ul style="list-style-type: none"> Compile and use simple data sets Use data sets to solve problems |
| | Data | Chance | <ul style="list-style-type: none"> Estimate the likelihood of occurrence of events; order on a scale from 0 to 100%, 0 to 1 |



Instructional Strategies

Table 1.4 on the following page illustrates a framework for advancing mathematical thinking. Although it does not explicitly refer to concrete materials or manipulatives, the use of these are often a prerequisite for developing mathematical thinking and can be used as a stimulus for this type of classroom discourse.

Table 1.4 Strategies for Supporting and Developing Mathematical Thinking

| Eliciting | Supporting | Extending |
|--|--|---|
| <p>Facilitates pupils' responding</p> <p>Elicits many solution methods for one problem from the entire class <i>e.g. "Who did it another way?; did anyone do it differently?; did someone do it in a different way to X?; is there another way of doing it?"</i></p> <p>Waits for pupils' descriptions of solution methods and encourages elaboration</p> <p>Creates a safe environment for mathematical thinking <i>e.g. all efforts are valued and errors are used as learning points</i></p> <p>Promotes collaborative problem solving</p> <p>Orchestrates classroom discussions</p> <p>Uses pupils explanations for lesson's content</p> <p>Identifies ideas and methods that need to be shared publicly <i>e.g. "John could you share your method with all of us; Mary has an interesting idea which I think would be useful for us to hear."</i></p> | <p>Supports describer's thinking</p> <p>Reminds pupils of conceptually similar problem situations</p> <p>Directs group help for an individual student through collective group responsibility</p> <p>Assists individual pupils in clarifying their own solution methods</p> <p>Supports listeners' thinking</p> <p>Provides teacher-led instant replays <i>e.g. "Harry suggests that ...; So what you did was ...; So you think that ..."</i></p> <p>Demonstrates teacher-selected solution methods without endorsing the adoption of a particular method <i>e.g. "I have an idea ...; How about ...?; Would it work if we ...?; Could we ...?"</i></p> <p>Supports describer's and listeners' thinking</p> <p>Records representation of each solution method on the board</p> <p>Asks a different student to explain a peer's method <i>e.g. revoicing (see footnote on page 8)</i></p> | <p>Maintains high standards and expectations for all pupils</p> <p>Asks all pupils to attempt to solve difficult problems and to try various solution methods</p> <p>Encourages mathematical reflection</p> <p>Facilitates development of mathematical skills as outlined in the PSMC for each class level <i>e.g. reasoning, hypothesising, justifying, etc.</i></p> <p>Promotes use of learning logs by all pupils <i>e.g. see Appendix A for a sample learning log</i></p> <p>Goes beyond initial solution methods</p> <p>Pushes individual pupils to try alternative solution methods for one problem situation</p> <p>Encourages pupils to critically analyse and evaluate solution methods <i>e.g. by asking themselves "are there other ways of solving this?; which is the most efficient way?; which way is easiest to understand and why?"</i></p> <p>Encourages pupils to articulate, justify and refine mathematical thinking <i>Revoicing can also be used here</i></p> <p>Uses pupils' responses, questions, and problems as core lesson including student-generated problems</p> <p>Cultivates love of challenge</p> |

This is adapted from Fraivillig, Murphy and Fuson's (1999) Advancing Pupils' Mathematical Thinking (ACT) framework.

Classroom Culture

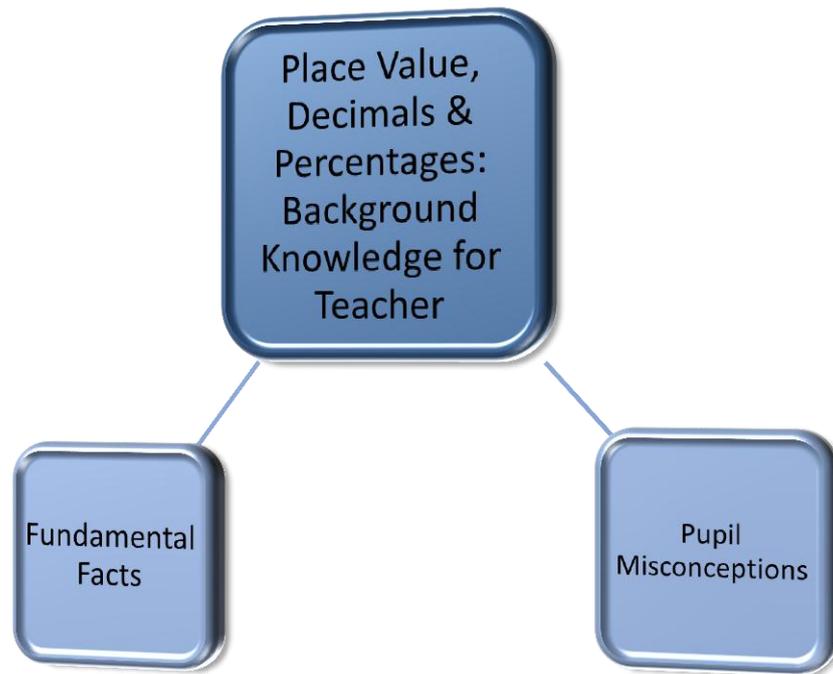
Creating and maintaining the correct classroom culture is a pre-requisite for developing and enhancing mathematical thinking. This requires the teacher to:

- cultivate a ‘have ago’ attitude where all contributions are valued;
- emphasise the importance of the process and experimenting with various methods;
- facilitate collaborative learning through whole-class, pair and group work;
- praise effort;
- encourage pupils to share their ideas and solutions with others;
- recognise that he/she is not the sole validator of knowledge in the mathematics lesson;
- ask probing questions (see Appendix B for a list of sample questions and sample teacher language);
- expect pupils to grapple with deep mathematical content;
- value understanding over ‘quick-fix’ answers; and
- use revoicing¹ (reformulation of ideas) as a tool for clarifying and extending thinking.

In this type of classroom pupils are expected to:

- share ideas and solutions but also be willing to listen to those of others; and
- take responsibility for their own understanding but also that of others.

¹ Revoicing is ‘the reporting, repeating, expanding or reformulating a student’s contribution so as to articulate presupposed information, emphasise particular aspects of the explanation, disambiguate terminology, align students with positions in an argument or attribute motivational states to students’ (Forman & Larreamendy-Joerns, 1998, p. 106).



PLACE VALUE: BACKGROUND KNOWLEDGE FOR TEACHERS

Fundamental Facts about Place Value

1. We use a decimal place value system (based on ten) where there are different symbols for the natural numbers 1 to 9, and a symbol for 0 which is used as a placeholder.²
2. There are patterns in the way that numbers are formed, for example, each decade has a symbolic pattern that is reflective of the symbols 1 to 9 (e.g. 21, 31, 41, 51, 61, 71, 81, 91, etc.).
3. The position of digits in numbers determines what they represent. The lowest value digits are on the right and the value of each digit is 10 times the value of the digits on its immediate right.³
4. 0 has a double function. Sometimes it represents ‘nothing’ and so represents no objects and as such makes no difference when it is added to another number. At other times it represents a placeholder, for example, in the number 208 it indicates that there are no tens and forces the 2 into the hundred place.⁴
5. For most numbers we say them in the order that they are written, for example, 29 (twenty-nine), 76 (seventy-six), 81 (eighty-one), etc. The exceptions to this are the numbers between 11 and 19. 11 and 12 are unusual names in themselves. Then 13 (thirteen), 14 (fourteen), 15 (fifteen), 16 (sixteen), 17 (seventeen), 18 (eighteen) and 19 (nineteen) are said in the opposite order to the way that they are written.
6. One million is the product of 1,000 multiplied by 1,000. This can also be represented as 10^6 because it is 10^3 multiplied by 10^3 .
7. One billion is usually the product of 1000 multiplied by a 1,000,000. This can also be represented as 10^9 . This is the meaning of a billion in the US and this is how it is used on the international money markets; however, in the UK it originally meant a million million which is 10^{12} .⁵
8. The latter - a million multiplied by a million or 10^{12} - is referred to as one trillion in the US. This is the most common usage of a trillion. One trillion can mean 10^{18} in the UK.⁶

Possible Pupil Misconceptions involving Place Value

- Some pupils have difficulty in understanding the different conventions which are necessary to say/read the numbers between 11 and 19. Much discussion will be necessary for these pupils in relation to the names of numbers.

² Suggate, Davis & Goulding (2010)

³ Van de Walle (2007); Suggate, Davis & Goulding (2010)

⁴ Suggate, Davis & Goulding (2010)

⁵ Suggate, Davis & Goulding (2010)

⁶ Suggate, Davis & Goulding (2010)

- Pupils often have difficulty saying and reading numbers beyond the thousands. Therefore, they will need a lot of opportunities to practice saying and reading large numbers. Often in reading large numbers it is useful to use a comma or a space to separate the thousands and the millions, for example, 97845327 is more difficult to read than 97,845,327 or 97 845 327 as 97 million, 845 thousand and 327.

DECIMALS: BACKGROUND KNOWLEDGE FOR TEACHERS

Fundamental Facts about Decimals

1. Decimals or decimal fractions are another way of representing numbers which are not whole numbers. It is another way of writing fractions.
2. We use the Hindu-Arabic number system which has a decimal base or base 10 system. This base 10 system extends infinitely in two directions.
3. This simply means that in the oral format, we start with the largest part of the number and precede to the smaller parts, which each are 10 times smaller than the previous part, for example, four hundred and seventy eight means four lots of hundreds, seven lots of tens and eight units.
4. Similarly, in the written format, each column is 10 times larger than the one on its right. This is the 10-to-1 relationship between the values of any two adjacent positions⁷. Therefore, we can extend the number system to include numbers (or parts) that are less than one, for example, tenths, hundredths, thousandths, etc.
5. The decimal point separates the whole number from fractional parts or parts of a whole.
6. The decimal point is placed to the right of the unit column and is used to indicate the border line between the whole numbers and the numbers (parts) less than one. The numbers to the immediate right of the decimal point represent the parts. The role of the decimal point is to indicate the unit position (to its left).
7. Fractions are either finite or infinite repeating decimals. It is possible to change any fraction into a decimal and vice versa; however, some fractions like $\frac{1}{3}$ are never ending (0.333333 with the 3 repeated indefinitely) so are called infinite repeating decimals or recurring decimals. Dots are placed above the first and last of the digits to be repeated.

Possible Pupil Misconceptions involving Decimals

- Sometimes, pupils read 28.297 as 'twenty eight point two hundred and ninety seven'; this is incorrect and may cause confusion for pupils because .297 does not represent two hundred and ninety seven. It represents $\frac{2}{10}$ and $\frac{9}{100}$ and $\frac{7}{1000}$. Therefore, it is important that pupils read decimals correctly, for example, 'twenty eight point two nine seven' or 'twenty eight and two hundred and ninety seven thousandths.
- It may be confusing for pupils to grasp that $\frac{3}{4}$ is the same amount as 0.75 because taken at face value the numbers are very different.

⁷ Van de Walle (2007)

- In relation to the position of the decimal point, this can be confusing for pupils considering that it will depend on what has been chosen to represent the 'unit'. In other words, any quantity can be represented differently depending on what is chosen as the ones piece or unit. For example, when considering quantities of chocolate (1 packet of chocolate bars with 10 bars in each packet, 6 chocolate bars with 10 squares of chocolate in each, and 2 squares of chocolate) the decimal point is dependent on the 'unit' which I choose as my ones piece:
 - 1.62 packets
 - 16.2 bars
 - 162 squares.

Pupils may find it difficult to understand that these amounts all represent the same quantity of chocolate. Another useful example of this can be found in the monetary system. Any of the monetary subsets can be designated as the 'unit' thus influencing where the decimal point will be placed.

| Hundreds (euro) | Tens (euro) | Ones (euro) | Tenths (euro) | Hundredths (euro) |
|-----------------|-------------|-------------|---------------|-------------------|
| 3 | 8 | 9 | 7 | 5 |

This amount of money can be written in various formats depending on the 'unit' chosen, for example:

- 38975 cents or 38975.0 cents
- 389.75 euro
- 0.38975 thousands of euro
- One of the most common misconceptions for pupils is that when multiplying decimals the answer will always be bigger and when dividing decimals the answer will always be smaller (as with whole numbers). Thus, pupils will need a lot of practice multiplying and dividing decimals less than 1⁸ and subsequently discussing and justifying the results.
- There are also some pupil misconceptions when comparing and ordering decimals:⁹
 - a) **Longer is Larger:** pupils select the number with the most digits as largest as they are applying whole number ideas to decimals, for example, 1.732 is larger than 2.1.

⁸ Suggate et al (2010)

⁹ Van de Walle, Karp, Bay-Williams (2013)

- b) **Shorter is larger:** a pupil thinks 0.3 is bigger than 0.95 because a tenth is larger than a hundredth. They believe that as the digits to the right represent very small numbers, shorter numbers are bigger.
- c) **Internal zero:** pupils can be confused by an internal zero, thinking that 0.56 is less than 0.087. Here pupils are not considering the zero as a place holder.
- d) **Less than zero:** pupils think 0.45 is less than 0 because they consider 0 a whole number positioned to the left of the decimal point.
- e) **Reciprocal thinking:** pupils incorrectly associate 0.4 as representing $\frac{1}{4}$ and 0.6 as representing $\frac{1}{6}$ therefore concluding that 0.4 is larger.

PERCENTAGES: BACKGROUND KNOWLEDGE FOR TEACHERS

Fundamental Facts about Percentages

1. Percent is another name for hundredths so percentages are hundredths and similar to fractions and decimals, they are another way of writing fractional parts.
2. They are different to fractions and decimals in that they always give the number of parts out of 100.
3. In simple terms, percentages are ratios whose denominator is 100¹⁰. Therefore, one way of thinking about percentages is that they can be used for comparison, for example, to ascertain the discount that I will get in the sale in comparison to the full original price. Resulting from this need to compare, percentages have a fixed denominator which is 100. Therefore, it is important to know what 100% refers to each time.
4. Another way of looking at percentages is to think of them as operators, that is, the percentage tells me to take a particular action. If I need to find the VAT of an object or the discount on an item, I will use the numerator and denominator in the percentage to complete an operation.

Possible Pupil Misconceptions involving Percentages

- Finding the original price instead of the actual price and vice versa.
- Finding the average percentage of things which have different quantities (this can't be done).

¹⁰ Deboys & Pitt (1979)

Place Value Learning Trajectory Level A¹¹

| Trajectory Levels | Concept | Developmental Experiences | | |
|-------------------|--|--|---|---|
| | | Concrete | Pictorial | Abstract |
| | Level A.1 Explore, identify and record place value 0-99 |  <u>Linear</u> <u>Set</u> |  <u>Linear</u> <u>Set</u> |  |
| | Level A.2 Explore, identify and record place value 0-199 |  |  |  |
| | | <i>As for Learning Experiences in Level A.1</i> | | |

¹¹ This level is generally aligned with the 1st and 2nd class objectives for Place Value

Place Value and Decimals Learning Trajectory Level B¹²

| Trajectory Levels | Concept | Developmental Experiences  | | |
|--|---|--|---|---|
| | | Concrete  | Pictorial  | Abstract  |
|  | Level B.1 Explore, identify and record place value 0-999 | <i>As for Learning Experiences in Level A.1</i> | | |
| | Level B.2 Read, write and order 3 digit numbers |  |  |  |
| | Level B.3 Round whole numbers to the nearest ten or hundred |  |  |  |
| | Level B.4 Explore, express and identify place value in decimal numbers to one place (tenths) |  |  |  |
| | Level B.5 Count, compare and order decimals |  <u>Linear</u> <ul style="list-style-type: none"> • calibrated lines e.g. metre stick and measuring jug (using post-its to order and compare) |  <u>Linear</u> <ul style="list-style-type: none"> • empty number lines (initially with number cards to 1 place) |  <ul style="list-style-type: none"> • benchmarks • estimation |
| | Level B.6 Solve problems involving decimals |  |  |  |

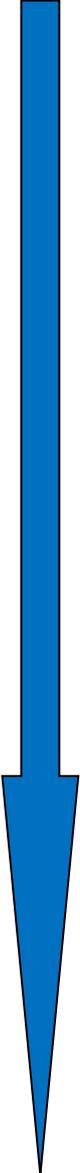
¹² This level is generally aligned with the 3rd class objectives for Place Value and Decimals

Place Value, Decimals and Percentages Learning Trajectory Level C¹³

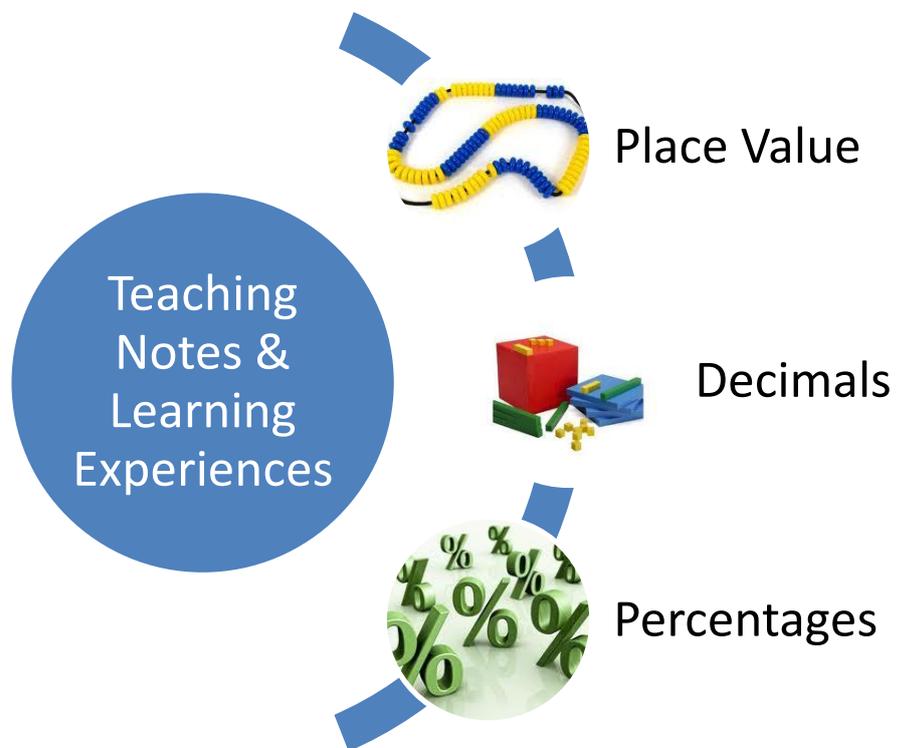
| Trajectory Levels | Concept | Developmental Experiences | | |
|--|--|---------------------------|---------------------------------------|--------------|
| | | Concrete | Pictorial | Abstract |
| | Level C.1 Explore, identify and record place value 0-9999 | | | |
| | As for Learning Experiences in Level A.1 | | | |
| | Level C.2 Read, write and order 4 digit numbers | | | |
| | As for Learning Experiences in Level B.2 | | | |
| | Level C.3 Round whole numbers to the nearest thousand | | | |
| | As for Learning Experiences in Level B.3 | | | |
| | Level C.4 Explore, express and identify place value in decimal numbers to two places (tenths and hundredths) | | | |
| | As for Learning Experiences in Level B.4 | | | |
| | Level C.5 Make, order, compare and count decimals | | | |
| As for Learning Experiences in Level B.5 | | | | |
| Level C.6 Add and subtract whole numbers and decimals to 2 places | | | • estimation and rounding-off | |
| Level C.7 Multiply and divide decimals up to 2 places by a single-digit whole numb | | | | |
| Level C.6 Solve problems involving decimals | | | | |
| As for Learning Experiences in Level B.6 | | | | |

¹³ This level is generally aligned with the 4th class objectives for Place Value and Decimals

Place Value, Decimals and Percentages Learning Trajectory Level D¹⁴

| Trajectory Levels | Concept | Developmental Experiences  | | |
|--|--|--|--|--|
| | | Concrete  | Pictorial  | Abstract  |
|  | Level D.1 Round whole numbers and round decimals to nearest whole number (to one, two or three decimal places) | As for Learning Experiences in Level B.3 | | |
| | Level D.2 Express tenths, hundredths and thousandths as fractions and decimals |  |  |  |
| | Level D.3 Compare and order fractions, percentages and decimals |  <u>Linear</u> <ul style="list-style-type: none"> • calibrated lines e.g. metre stick and measuring jug (using post-its to order and compare) |  <u>Linear</u> <ul style="list-style-type: none"> • empty number lines • percentage bar • benchmark charts |  <ul style="list-style-type: none"> • benchmarks • estimation exchange whole unit for 100% |
| | Level D.4 Explore and calculate simple interest, profit, loss, VAT |  |  |  <ul style="list-style-type: none"> • link percentages with fractions e.g. 80% of X? |

¹⁴ This level is generally aligned with the 5th and 6th class objectives for Place Value, Decimals and Percentages



Teaching Notes

Base Ten

In our number system we exchange in tens. This is known as base- ten. Pupils however, should be given ample opportunity to exchange in lots of different bases. If pupils are given the experience of exchanging in 2s, 3s, 4s, etc. they will then have a variety of examples from which they can begin to grapple with the concept of place value.

Concrete materials¹⁵

As is explicit in the learning trajectory all mathematical concepts should be taught moving from the concrete to pictorial and finally to the abstract. Outlined below are suggestions regarding the type of concrete materials that should be used in the teaching of place value.

Group-able models

Models that most clearly reflect the relationship of ones, tens, and hundreds are those for which the tens can actually be made or grouped from the single pieces. As pupils become more familiar with these models, collections of tens can be made by pupils and kept as ready-made tens, for example, lollipop sticks pre-bundled.

Pre-grouped models

These are models that are pre-grouped and cannot be taken apart. They should be introduced after pupils have worked with group-able models. Pre-grouped models are an efficient way to model large numbers. Base ten blocks and dienes blocks are examples of pre-grouped models.

Non-proportional models

These are models where the ten is not physically ten times bigger than the one. Money is an example of a non-proportional model. The non-proportional model **should not** be used to introduce place value. It can however, be used when pupils already have a conceptual understanding of place value and need some additional reinforcement.

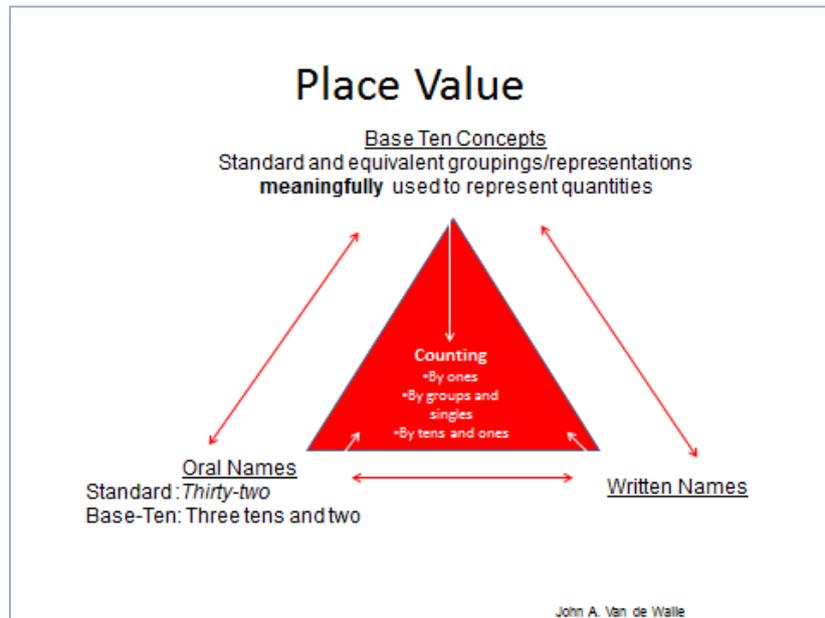


Regardless of the type of concrete materials pupils use, the connection of the manipulation of materials with the language pattern 'one of these is ten of those' is critical in the understanding of place value.

¹⁵ Van de Walle, Karp, Bay-Williams (2013)

Writing numbers

The way numbers are written, that is, ones on the right, tens to the left of the ones, and so on needs to be introduced as children grapple with grouping and exchanging in tens. Activities need to be designed so that pupils physically associate a tens and ones grouping with the oral name for the number and the written name for the number (See below 11.3 Van de Walle, p.195)



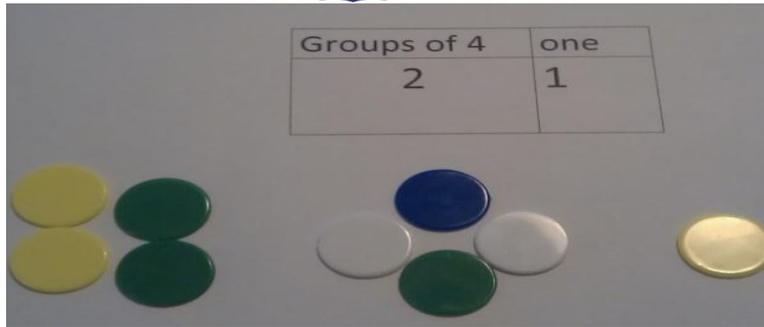
Sample Learning Experiences

Pre Base Ten Work¹⁶

Grouping

Extending the work done in Early Mathematical Activities and using a variety of materials pupils are introduced to the idea of grouping. Teacher hands out nine counters to each pair. Pupils are asked to group the counters in 4s. Then they record their findings both pictorially and symbolically as illustrated below.

¹⁶ Deboys and Pitt (1979)



Initially it is best to group in only one number per day. As the pupils become more familiar with the task it can be extended to include grouping a given number in several ways, for example, grouping 9 counters in 2s, 3s, 5s as well as 4s.



You have 23 counters, how many ways can you group them?

How many groups of 5, 6, 7, 10 can you make?

How many ones have you left each time?

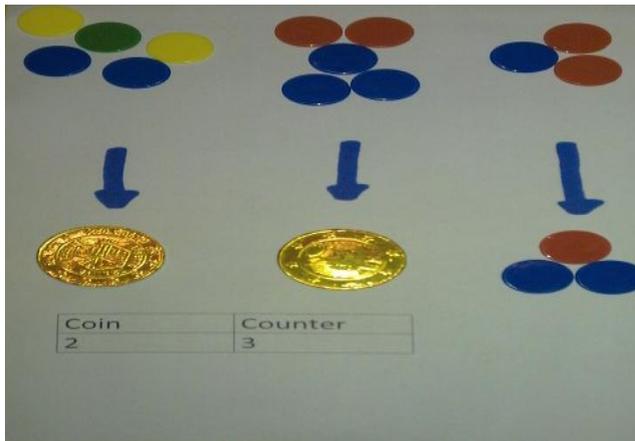
Pupils can also record these tasks on a recording sheet (a sample recording sheet is shown below).

| | |
|---------------------|-------------|
| Groups of 5 | Ones |
| | |
| Groups of 6 | Ones |
| | |
| Groups of 7 | Ones |
| | |
| Groups of 10 | Ones |
| | |

Grouping and exchanging

Single exchange

Pupils are now ready to progress to single exchange. For example, pupils can now exchange five counters for one golden coin. See illustration below.¹⁷



Second exchange

After pupils have a sound understanding of single exchange, second exchange can be introduced. For example:

- 3 counters can be exchanged for 1 golden coin
- 2 golden coins can be exchanged for 1 bar of chocolate

See illustration below.



¹⁷ ICT link is

http://nlvm.usu.edu/en/nav/frames_asid_209_g_1_t_1.html?open=activities&from=category_g_1_t_1.html

It is critical that pupils get adequate time and opportunity to engage meaningfully in second exchange. This is a complex activity and pupils will need lots of experience doing second exchange with various bases. It will also enhance their understanding if they exchange for different items, for example, using base 4, pupils could exchange:

- 4 counters for an eraser
- 4 erasers for a pencil

Pupils can also decide themselves what the ‘exchange rate’ is to be in the task (depending on the base being explored).

Consolidation Activity

Ring is King

To play this exchange game you will need:

- First grouping: buttons
- Second grouping: bottle tops
- Third grouping: straws
- Fourth grouping: rings

A dice labelled as follows (depending on the quantity of your groups)

| | |
|-----------------------|------------------|
| Grouping in 3s | 1, 2, 1, 2, 1, 2 |
| Grouping in 4s | 1, 2, 3, 1, 2, 3 |
| Grouping in 5s | 1, 2, 3, 3, 4, 4 |

This game can be played in pairs. First choose the grouping, for example, 3 and pick the appropriate dice (1, 2, 1, 2, 1, 2). Pupils take turns throwing the dice and collecting the corresponding number of buttons. Once they have collected three they exchange them for a bottle top and then continue to collect buttons. As soon as they have collected three bottle tops they exchange for a straw and as soon as they have three straws they exchange for a ring. First to exchange for a ring wins the game.

Introducing Base Ten

Place Value

The understanding of place value requires pupils to group by tens (the base ten concepts). This requires procedural knowledge regarding how these groups are recorded in our place-value system. This takes time...

Find a collection of items that pupils might be interested in counting, for example, sweets in a jar, crayons in the class, etc. Ensure that the collection is countable and is somewhere between 50 and 200.¹⁸



*Estimate how many straws are in this box.
Discuss with your partner ways in which you can count the straws.
Is there an easier way than counting by ones?
Did anyone do it another way?
What is the fastest way to count these straws?*

Teachers should try out the various counting suggestions taken in feedback. Through modelling counting in ones, twos, and threes the teacher then poses a question to prompt a faster way of counting. Hopefully a pupil will suggest grouping in tens.



Pupils may count ten, twenty, thirty, thirty one, thirty two but may not fully recognise the thirty-two-ness of the quantity. To connect the count-by-ten method with their understood method of counting by ones the pupils need to count both ways and discuss why they get the same answer.

Grouping in tens and recording the count¹⁹

How many tens?

Prepare bags of counters, unfixed cubes, tooth picks, buttons, beans etc. and hand out a record sheet similar to the one shown below. Pupils work in pairs and begin with one collection. The collection is grouped in as many tens as possible and the result recorded. Once pupils have completed the task they can trade their bag for a different collection.

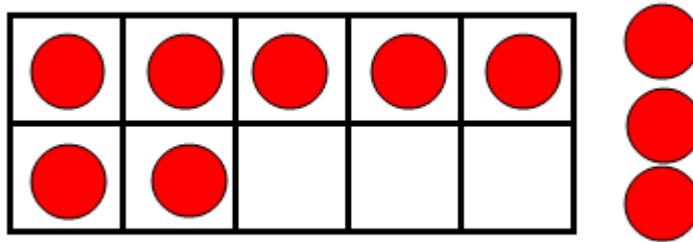
¹⁸ Van de Walle, Karp & Bay-Williams (2013)

¹⁹ Van de Walle, Karp, Bay-Williams (2013)

Further Exploration of Base Ten

Ten frames

Ten frames help pupils to form mental images in their heads and to associate these images with the corresponding numbers. Ten frames help pupils to organise numbers into chunks of five or ten – which helps match the base ten number system. For example, the most efficient way of seeing the 7 with a ten-frame is one column of 5 dots and another of 2 dots. This facilitates pupils to think in terms of equivalence, that is, thinking of 7 as $5+2$. When introducing pupils to ten frames, it is a good idea to begin with familiar patterns like those that appear on dice or dominoes.



*Show me seven on your ten-frame.
Did anyone represent it in another way?
Is there one way that is better than the other?
Explain why.*

Extend questioning by asking:



How many more do you need to fill your ten frame?

Fill the tens

In this activity the pupils begin with the number word (that is 'twenty-three'), then they fill the ten frames with counters accordingly and identify the groups and ones.

| | | | | |
|--|--|--|--|--|
| | | | | |
| | | | | |

| | | | | |
|--|--|--|--|--|
| | | | | |
| | | | | |

| | | | | |
|--|--|--|--|--|
| | | | | |
| | | | | |

Tens _____ Ones _____

Double ten frames

Here pupils are required to see the teens as the set of numbers that come after ten. By using double ten frames pupils are helped to see the teen numbers as 'ten and some more'. Teacher distributes a bag of counters to each pair of pupils. The bag should contain any number of counters between 11 and 19.



*Name the number shown on your ten frames.
Choose arrow cards and or digit cards to represent the number.
Record the number.*

Work on ten frames can be extended by using multiple ten frames. Pupils should represent the numbers in a variety of ways. Initially, this should be done incrementally:

1. concretely in the ten frame
2. with arrow cards (see section below)
3. with digit cards or number fans.

The connection between concrete representations and numerals will form the basis for an effective understanding of place value.

Ten one mats²⁰

Many educationalists suggest that using a ten-one mat to introduce the tens place will greatly increase pupils understanding of place value. Use of the mat prompts pupils to look for tens and to regroup, as necessary, to represent two digit numbers. Pupils can represent the numbers concretely followed by the use of arrow cards and finally digit cards. It is important when recording the number that the tens are recorded in the tens column and the ones in the one column. This is a fundamental step in helping pupils learn how our base ten system works.

²⁰ Accessed from www.mathwire.com

| tens | ones | | | | | | | | | | |
|------|---|--|--|--|--|--|--|--|--|--|--|
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Arrow Cards



Arrow cards

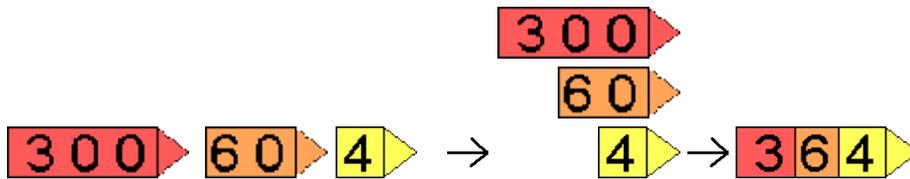
Arrow cards should be introduced to pupils using concrete materials. This will give the pupils the opportunity to see and feel the value of each number.

Arrow cards²¹ are a set of place value cards with an 'arrow' or point on the right hand side. Pupils can organise the cards horizontally or vertically to represent different numbers. By placing a 60 card on top of a 300 card and then a 4 card on top of a 60 card, pupils can see that the cards show the number 364 where the 3 stands for hundreds, the 6 for tens and the 4 for ones. Arrow cards provide pupils with a picture image of what we write when we record numbers with more than one digit.

ICT Opportunities

[Link: Place Value Cards with Abacus](#)

²¹ ICT link is <http://www.wmnet.org.uk/resources/gordon/Abacus.swf>



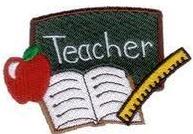
Listed below are a few activities that can be used with arrow cards to develop pupils' concepts of :

- number;
- number order;
- relationships between numbers; and
- place value.

Show me

Start with a series of 'show me' activities in the suggested order:

1. 7, 70, 700, etc. (same digit in a different place)
2. 11, 25, 46, 77, etc. (two-digit numbers)
3. 328, 752, 927, etc. (three-digit and four-digit numbers)
4. 307, 7,089, etc. (0 as a placeholder)



Show me 47
Seán can you tell me how you built your number?
Did anyone do it another way?
Can anyone think of another name for forty?
Represent this number in another way using any material you like.
Record your thinking in your learning log.

Similar but not the same



*Show me 63.
Show me 36.
Explain why even though both numbers have the same digits they are not the same.
Show me all the numbers you can build with the digits 456.*

More or less

Show me 540, now show me 10 more; 100 less; 1,000 more, etc.

Compose and decompose

Have pupils not only compose numbers but also decompose numbers with arrow cards.



*Show me 732 with arrow cards.
Here is 525. Discover how it is made up.*

Pupils may decompose 525 in a number of ways, for example:

- 5 hundreds + 2 tens + 5 ones
- 52 tens + 5 ones
- 525 ones
- 4 hundreds + 12 tens + 5 ones
- 4 hundreds + 10 tens and 25 ones
- etc.

Consolidation Activities

Place Value Cards²²

On-line place value cards with abacus.

ICT Opportunities

[On-line Place Value Cards](#)

²² ICT link at <http://www.wmnet.org.uk/resources/gordon/Abacus.swf>

Number fans

Numbers fans are a logical progression from working with arrow cards. Here pupils are required to represent the number without the scaffold of a hundred, ten or one card. The number fan is a valuable assessment tool for the teacher – at a glance a teacher can identify if a pupil is having difficulty ordering numbers correctly. Outlined below are a few sample activities for number fans.



Make the number

This could be 1/2/3 digits, depending on the class level being targeted.

Number before/after

Show the number that comes before/after a given number.

Show me the answer

This could cover any one of the four operations or even a combination of two.

Multiples

Can the pupils give a number that is a multiple of 2, 5, 10, etc.

Biggest number

Select target digits and ask the pupils to show the "biggest three digit number", "the biggest three digit number you can make with your partner", "the biggest two digit number that does not use a 9".

Zero

Think about numbers that contain zeros, say these numbers and encourage the pupils to make them (talk about the place value that a zero holds in the middle or end of numbers).

99 Square (or 100 Square)



The 100 square and the 99 square show the order of the numbers and the patterns and relationships that exists in the base ten system.

A 99 square and 100 square²³ as shown below are very important teaching tools in the development of place value. Through pupil self-discovery and teacher questioning pupils can discover numerous patterns.

ICT Opportunities

[Link: Interactive Hundred Square](#)



*Look at the first column what do you notice?
 Look at any row and “count from left to right “what do you notice?
 Did anyone else notice anything different?*

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 |
| 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 |
| 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 |
| 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 |
| 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 |
| 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 |
| 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 |

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|-----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

Listed below are a few activities that can be used with the 99 and 100 squares to develop pupil’s concept of number, number order, relationships between numbers and place value. The following activities use the 100 square and can also be used with a 99 square.

²³ ICT Link at <http://www.primarygames.co.uk/pg2/splat/splatsq100.html>

Ordering digits

This activity can be done using a hundreds pocket chart with the digit cards removed or a blank 100 square. Randomly distribute all the digit cards to the class. Counting either forwards from 1 or backwards from 100 have pupils place the cards on the hundreds chart in the correct order whenever they have the next digit.

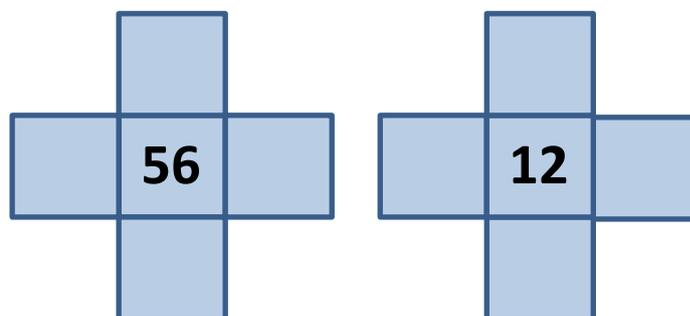


Missing number

Remove some numbers from the 100 square. Pupils replace them and explain why they chose that particular number. You can remove random numbers, sequences of numbers or even all numbers.

Neighbours number

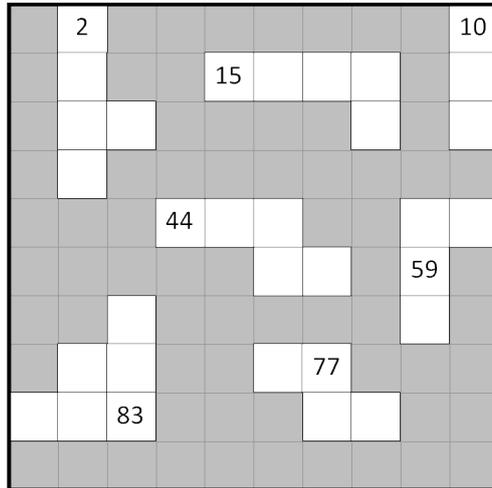
Begin with a blank 100 square. Highlight a number and ask pupils to identify the neighbouring numbers. Pupils should try several of these activities. Teacher questioning is very important for this activity. The questions posed should enable pupils to identify and discuss the relationships and patterns between numbers.



*What do you notice about the number to the left/right/above/below?
What do you notice about the numbers on the diagonal?*

Fill in the Blanks

In this activity, pupils need to write numerals as well as order them. Create worksheets or a laminated version for pupils to complete with a white board marker. Create a hundreds chart that has sections that need to be filled in. Provide some numerals to guide pupils.²⁴



Guess My Number

Pupils are given a 99/100 square. Explain to pupils that you have written down a number between 1 and 100 and that they are going to try to work out what number it is by asking questions about it. As each pupil asks a question the teacher answers it and the pupils can cross out any numbers that they now know are not the correct number. For example, if a pupil asks, “Is it a multiple of ten?” and the teacher says it is not, the pupils can cross out all the multiples of ten. Pupils should be encouraged to ask questions that reflect a range of understanding of numbers, for example, “Is it in the twenties?”; “Is it an even number?”; “Is it greater than 50?” As the pupils become familiar with this activity an extra challenge can be added where pupils record their questions. The pupil with the least amount of questions needed to identify the number is the winner. As an extension, pupils could be given a blank 99/100 square on which they must write the numbers which have been eliminated.

²⁴ ICT link at <http://www.oswego.org/ocsd-web/games/DogBone/gamebone.html>

Hundreds Chart Jigsaw

Give pupils a blank hundreds chart. Cut up another hundreds chart in to several pieces to create a jigsaw and ask pupils to put the hundreds chart back together. To make the task more difficult, omit some pieces.²⁵

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|
| 1 | 2 | 3 | 4 | 5 | 6 | | 8 | 9 | 10 |
| | | 13 | 14 | 15 | 16 | | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | | | | 38 | | |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| | | | 54 | 55 | 56 | 57 | 58 | | |
| | 62 | 63 | 64 | 65 | 66 | 67 | | 69 | 70 |
| | 72 | 73 | | 75 | 76 | 77 | | 79 | 80 |
| | 82 | | | 85 | | 87 | 88 | 89 | 90 |
| | | 93 | 94 | 95 | | | | | |

ICT Opportunities

[Link: Hundred Square Jigsaw](#)

Consolidation Activities

Beat My Number

Pupils can do this activity in pairs. Each pair needs a blank laminated hundreds chart, a set of digit cards placed face down and two different coloured whiteboard markers. Pupils take turns picking up two digit cards to create a two digit number then place this number in the correct position on the blank hundreds chart. They get one point for every number they create, two points for any number greater than 50, any number with a 0 in the units place and three points for any number with the same digit in the units and tens place, for example, 11, 22, 33, 44.

²⁵ ICT link at <http://nrich.maths.org/5572>

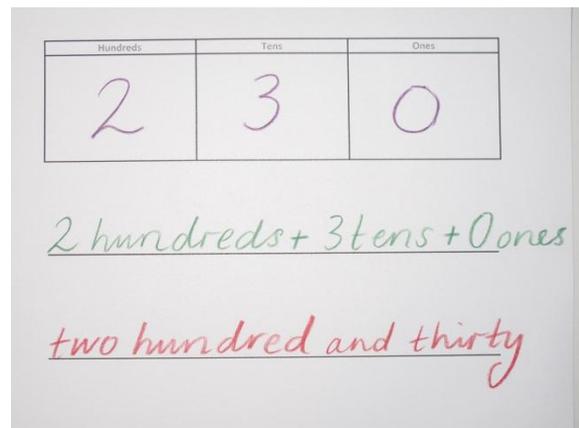
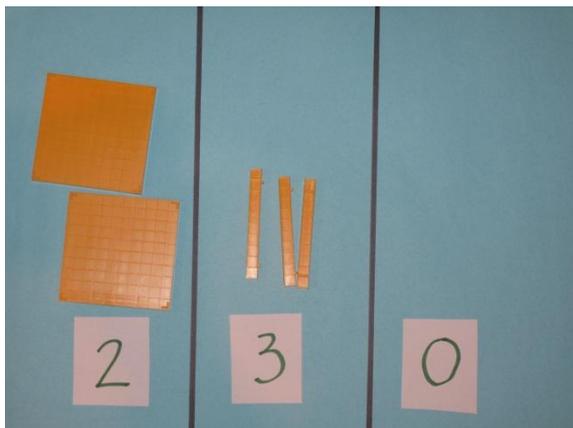
Level B.2

Read, write and order three digit numbers

Teaching Notes

Place mat

Learning to read and write three digit number names (as with two digit number names) should be initially accompanied by the use of base ten materials. The convention that we use for writing numbers (ones on the right, tens to the left, hundreds to the left of tens and so on) should be linked with concrete materials and numerals should be represented alongside them in the correct column. Initially, a place mat (notation board) may be useful to help pupils order their concrete materials as hundreds, tens and units. Pupils then represent the number by placing the appropriate digits in the correct columns. Finally, pupils should record the number using words and symbols.

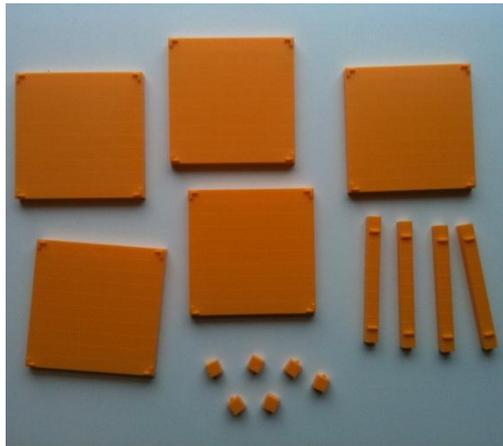


Sample Learning Experiences

Read and Write Three Digit Numbers

Recording in base ten language

Pupils should be taught to say both the base ten name as well as the standard name for any number. For example, the number 546 in **base ten language** is five hundreds, four tens and six ones or $500 + 40 + 6$. Initially the teacher displays an arrangement of base ten materials to the class, a visualiser might be used for this.



1. Pupils write this number down in 'base ten language' – the words and the numbers.
2. Pupils then write the number in standard form.
3. Teacher changes one piece (the hundred, the ten or one), for example, change 546 to 576, pupils now write this new number in base ten language (words and numbers) and in standard form.
4. Pupils now work in groups. One pupil constructs a number using base ten materials. The other pupils complete steps 1, 2 and 3 above to represent the new number.²⁶



Pupils may have difficulty with numbers involving no tens, for example, 506, particularly when writing them. The use of base ten language, as used in the activities above, can be helpful here.

Number callout

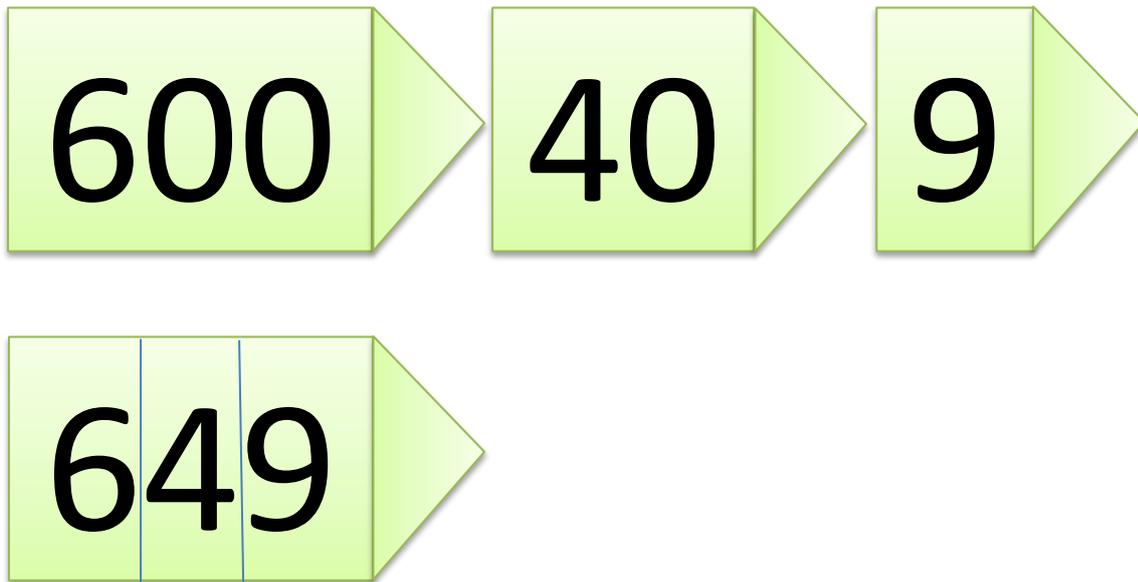
This activity is an extension of the activity above as numbers must be written without the support of base ten materials.

1. Teacher calls out a number using standard names
2. Pupils model the number using base ten materials
3. Pupils write the number in base ten and standard form.
4. Pupils can then work with a partner to practice the same activity.

²⁶ ICT link at http://nlvm.usu.edu/en/nav/frames_asid_152_g_1_t_1.html?from=category_g_1_t_1.html

Say the number

This activity helps pupils to identify and read three digit numbers. Arrow cards²⁷ are useful as they express numbers in their expanded form, for example, the number 649 is expressed on three individual cards as 600, 40 and 9. For this game, each pair needs a set of arrow cards. All hundreds, tens and ones cards are placed face down and the first player turns over a hundred, tens and one's card and builds the number using their three cards. They say both the expanded version of the number, for example, "600 plus 40 plus 9" as well as the standard number, for example, "649".



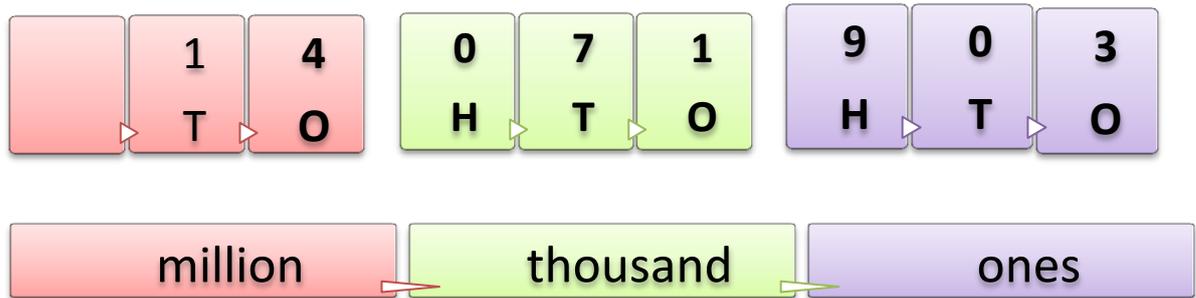
When extending this activity to four digit numbers, pupils could use digit cards to build the number using a 'place value house graphic organiser' (see 'Place Value Houses') instead of arrow cards.

Convention for Reading Large Numbers



To read large numbers, begin on the right and mark the number off in triples using commas. These triples are then read from left to right, stopping at the end of each to say the unit name i.e. billion, million, thousand. When reading and writing large numbers, if pupils are taught to read numbers like 008 as eight or 072 as seventy two, they will be able to ignore leading zeros in each triple when they arise. (Van de Walle et al, 2013)

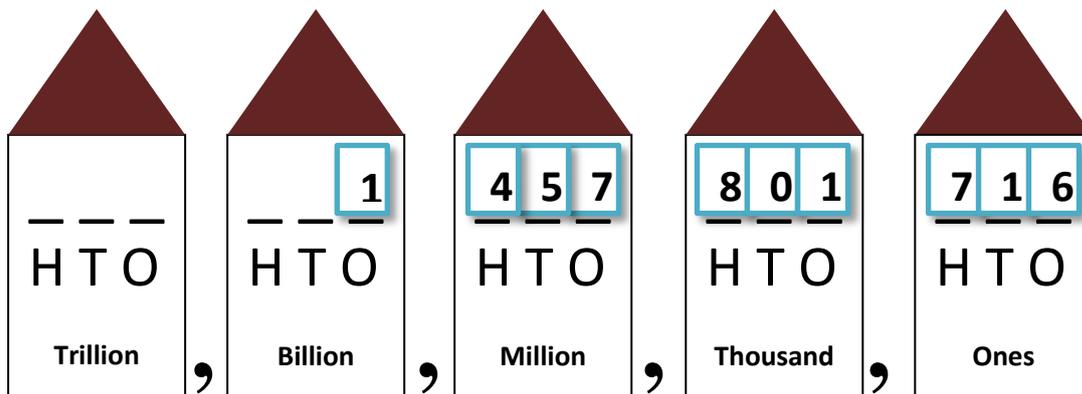
²⁷Arrow cards are advocated by many educationalists including Gattegno (1988) and Montessori (1912/1964)



“Fourteen million, seventy one thousand, nine hundred and three.”²⁸

Place value houses

This place value graphic helps pupils to develop their ability to read large numbers. It is only appropriate when pupils can confidently read three digit numbers. The teacher will have to make the graphic below and will need digit cards. This activity is done initially as a whole class and then in pairs.



This number would read as “One *billion*, four hundred and fifty seven *million*, eight hundred and one *thousand*, seven hundred and sixteen.

²⁸ Van de Walle, Karp & Bay-Williams (2013)

Common Misconception

Many pupils focus on the value of digits in a number rather than the value determined by the digit's place value. For example, when asked to order the following numbers from largest to smallest: 910

1,001 901 99 109 190 999 some pupils might answer

999 99 910 901 190 109 1001

They believe that three nines in 999 must make it bigger than 1,001 which only have zeros and ones in it. Writing out numbers using a place value table can help.



| Thousands | Hundreds | Tens | Units |
|-----------|----------|------|-------|
| 1 | 0 | 0 | 9 |
| | 9 | 9 | 9 |

Which number is bigger?

Initially, pupils can use ordering tasks to express their knowledge of place value. This activity is enhanced through the use of concrete materials used alongside digit cards. Ask children to order two three digit numbers by saying which is bigger and which is smaller. Ask pupils to justify their answer using base ten materials.



“251 is bigger because it has 5 more tens than 201”

Place your number

Each pair of pupils will need an empty number line with benchmarks set from 0 to 100 or 0 to 1,000 and a set of digit cards placed face down on the desk. The first pupil turns over two or three single digit cards and places these in any order to make a two or three digit number. They then estimate where on the number line it belongs. Their partner then has a turn and must place their number on the number line in relation to the other numbers already present. This continues until all digit cards have been used.

Ordering digits on a human number line

The teacher places a rope on the ground which serves as an empty number line. Two pupils throw a dice three times to create two three digit numbers. These numbers become the minimum and maximum values on the human number line and the pupils stand at the beginning and end of the number line accordingly. These numbers are written on A4 paper and held at chest height for all to see. The teacher chooses another pupil to write out a secret three digit number (which is not revealed) and stands on the point on the number line where they believe their number should be placed. Other pupils guess the value of the number based on the position the pupil is standing in.

The Thousands Chart

For this activity pupils work in groups of three or four. Each group will need several blank hundreds charts. Pupils work together to create a 1 to 1,000 chart. Pupils discuss how this might be done. They will need to tape 10 hundreds charts together to form a long strip. Groups will have to negotiate how this will be done and how tasks will be divided amongst the group.

Consolidation Activities

Nice or Nasty (2 - 5 players)

Pupils will need a pack of playing cards with picture cards removed, paper and pencils. The ace card is used for number 1. Each pupil draws three card sized boxes side by side on paper. The first pupil chooses three cards from the top of the face down deck and decides on which order to place them in their three boxes. The pupils must decide before beginning, whether the aim of the game will be to make the biggest or smallest number possible. Each player must read out their three digit number as it is made. Four cards can be used to extend the activity.²⁹

ICT Opportunities

[Link: Football Numbers](#)

²⁹ ICT link at

http://www.learnalberta.ca/content/me3us/flash/lessonLauncher.html?lesson=lessons/05/m3_05_00_x.swf

In the ‘nasty’ version of the game, players can opt to place a card on their own board or an opponent’s. When playing with a group, this version involves a lot of strategy as pupils aim to prevent others from winning a round.



Race to 100 (2 players)

Each pair will need a dice, paper and pencil. The aim of this game is to make 100 before your partner does. Pupils take turn rolling the dice and can choose to either take the value shown on the dice e.g. 3, or to take ten times the value e.g. 30. Pupils keep adding their own scores until one reaches exactly 100. Alternatively, pupils could start at 100 and subtract their scores to see who can reach 0 first.

Target Boards (whole class then in pairs)

Target boards are a useful consolidation tool as they allow for a variety of skills to be consolidated and assessed. Encourage pupils to ask open questions, for example ‘Can you add two numbers to get a number with 3 in the hundreds place?’ As opposed to only closed questions, ‘What is the biggest number on the board?’ Teachers can easily make target boards to be used repeatedly on the interactive whiteboard. Once the teacher has successfully modelled asking target board questions, target boards can be printed and pupils can work in pairs to ask and answer questions. As an extension, pupils can be asked to fill in their own blank target board then devise questions for a partner.

| | | | |
|------------|------------|------------|------------|
| 614 | 390 | 223 | 739 |
| 425 | 462 | 999 | 201 |
| 672 | 820 | 255 | 119 |
| 927 | 346 | 466 | 327 |



*Find a three digit number with 5 tens.
Which row of numbers has the most tens?
Find a number that has 2 hundreds and 23 ones.
Find a number with a 7 in the ones place.
Find a number that is 3 tens bigger than 225.
How many numbers can you find with a zero place holder?
Add or subtract two numbers to get a number with 3 in the hundreds place.
How many numbers can you find with an even number in the hundreds place?*



Digit Change (whole class then in pairs)

For this activity each pupil will need a calculator. A three digit number is entered into the calculator, for example, 547. A single digit must be changed without changing the other two. If we wanted to change 547 to 527, the four needs to be changed to a 2. Numbers are changed by adding or subtracting the correct amount. In this example, 20 must be subtracted from 547 to get 527. It is important that pupils write down and or discuss the thinking behind their solution.

Teaching Notes



Estimating or Guessing?

Estimation involves reasoning, understanding and possibly computation while a guess can be made with no supporting evidence. Therefore, it is important to avoid using the term 'guess' when estimating.

Rounding is a useful skill as it aids computational estimation. Estimation is particularly important as it is a practical skill applied to every-day numeracy. If you were trying to figure out the approximate cost of buying 64 paving stones at 28 euro each i.e. 64×28 , you could make a sound estimation by multiplying 60 and 30. Before pupils can apply computational estimation, they must learn to round to the nearest ten or hundred.

Sample Learning Experiences

Rounding using concrete materials

Pupils use concrete materials to decide what ten or hundred is nearest to a given number. Some of the materials that can be used for exploring rounding to the nearest ten or hundred are shown below.

Hundred beads

100 bead strings are marked off in groups of tens and are a useful visual for rounding up or down. Multiple 10 bead strings can be used to round to the nearest 10. You can see 22 beads would round down to 20 and 19 beads would round up to 20.



In the beadstring shown on the visualiser are we closer to twenty or thirty?

We can say that 22 can be rounded down to twenty.

In the beadstring below is 19 closer to ten or to twenty?

Explain how you know.

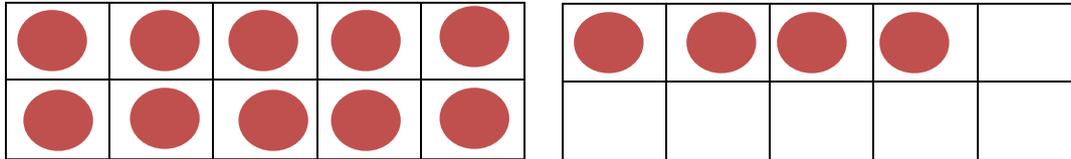
We can say that we can round 19 up to twenty.

We are going to tidy the beads away now can you record your findings in your learning log.



Ten frames

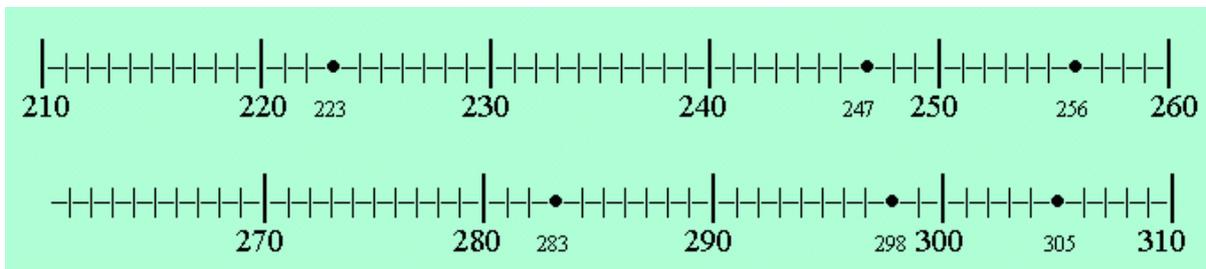
Multiple ten frames and multiple hundred squares can be used as other concrete resources in the exploration of rounding up or down to the nearest 10 or 100.



*Round this number (14) to the nearest ten.
Justify your answer.*

Number lines³⁰

Both empty and filled number lines are a useful way for pupils to explore rounding up and down to the nearest ten or hundred. Listed below are a few suggestions.



Is 223 closer to 220 or 230?

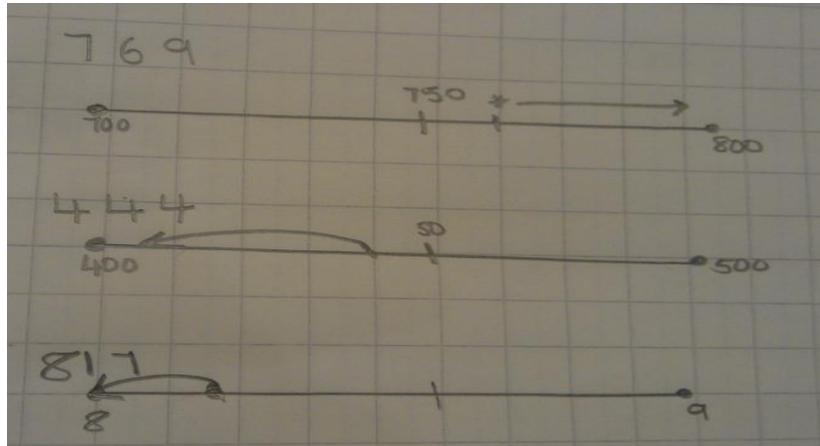
Is 247 closer to 240 or 250?

ICT Opportunities

[Link: Interactive Number Line](#)

³⁰ ICT link at <http://www.topmarks.co.uk/PlayPop.aspx?f=NumberLinev5>

An empty number line is a great resource. Pupils can estimate markings to reach an answer.



Is 769 closer to 700 or 800?

Is 444 closer to 400 or 500?

Is 817 closer to 800 or 900?



*What did you and your partner notice when you rounded up?/rounded down?
Did anyone else notice anything different?
Mary revoice for me what Seán said.
Explore if this will work for other numbers.*

Test it out

| Number | Explain | Round to |
|--------|---------|----------|
| 37 | | |
| 198 | | |
| 999 | | |
| 23 | | |

At this stage pupils should have discovered the convention for themselves. See blue box below.



REMEMBER
less than five, leave it alone,
five or more, round up.

Practicing mental strategies

Once pupils are comfortable with rounding they can apply it to mental maths problems. For example, the following type of problem could provide the basis of discussion around different strategies and their effectiveness.



*If you had €100, could you afford to buy these three toys?
Estimate their cost.
Explain how you got this answer.*



€22



€66



€33

I rounded 22 down to 20, 66 up to 70 and 33 down, so $€20 + €70 + €30 = €120$, so it is over €100

Consolidation Activities

Let's Round and Estimate

| | | | |
|------------------------------------|----------------------------------|---------------------------------|----------------------------------|
| a. $148 + 43$ $150 + 40 = 190.$ | b. $678 + 45$ ___ + ___ = ___ | c. $89 + 56$ ___ + ___ = ___ | d. $344 + 34$ ___ + ___ = ___ |
| e. $237 + 52$ | f. $66 + 42$ | g. $23 + 98$ | h. $458 + 31$ |
| i. $178 - 43$ | j. $278 - 56$ | k. $873 - 98$ | l. $771 - 37$ |

Hundreds Cards

| | | | | |
|------------|------------|------------|------------|------------|
| 100 | 200 | 300 | 400 | 500 |
| 600 | 700 | 800 | 900 | |

Cut out the hundreds cards and place in a pile face down between the players. Turn over one of the hundreds cards and deal 3 playing cards to each player. The first player to arrange their three cards so they round to the hundreds number wins the round. Ask the pupils to read their number out loud.

Discuss which player, if any, rounds to the hundreds card. The winner takes 3 new playing cards from the pile, and keeps the hundreds cards as a record of the win. If no one wins the round then keep your playing cards but turn over a new hundreds card. If both players can round to the hundreds card then the player whose number is closest to the hundreds card wins. The game continues until all the hundreds cards have been used. The winner is the player who wins the most rounds.

Teaching Notes



Note on Money!

Although it is an important application of decimals, money is not recommended as an initial model for decimals. Money is a two-place system and is non-proportional. Numbers like 5.6 or 7.2943 can cause confusion to pupils in the early stages. Initially, pupil contact with decimals should be more flexible and so money is not recommended.

Place Value

A thorough knowledge of the number system and, in particular, a robust understanding of the concept of place value is necessary before pupils can begin to understand decimals. It is an extension of place value and a sound understanding of division by 10 will assist pupils³¹. Pupils should be encouraged to group and exchange as they did with whole numbers.

Base Ten Materials

Pupils should get ample opportunities to use base-10 materials to develop and consolidate their understanding of decimals. These activities should be grounded in discussion, justification and reasoning.

Link with Measures

Within the Measures strand, particularly the strand units of length and capacity, there are ample opportunities for pupils to explore and experiment with decimals. This can lead on to decimal calculations which are based on real-life experiences and which also have a concrete representation. For example, the difference between the capacities of varying containers could be estimated; then explored using containers and calibrated tools; and finally, calculated using decimals. The results of calculations involving measures can then be placed on the number line to consolidate understanding. Furthermore, recording either calculations or practical tasks with measures can involve two or three places of decimals as tenths, hundredths and thousandths (such as 1millimetre as 0.001 litres or 1cm as 0.01m) as necessary³².

³¹ Deboys and Pitt (1979)

³² Anghileri (2007)

Concrete Exploration

Decimals can be explored concretely in a number of ways including the linear and area models:

Linear Model³³ - metre stick

Each decimetre (10cm) is one tenth, each centimetre is one-hundredth and each millimetre is one-thousandth. Any number-line split into 10 or 100 parts can be used as a model for tenths or hundredths.

Area Model

Introduce decimals by using dienes blocks as illustrated below. Teachers will use these concrete materials to introduce the concept of decimals, that is, the idea of **ten times smaller**.

Discussion

A critical question arises when considering fractions and decimals - 'is there ever a smallest piece?'. This question deserves careful consideration and discussion with pupils. The ultimate goal of the discussion is for pupils to become aware that the 10-to-1 relationship extends infinitely in two directions. Teacher generates discussion emphasising that each time the dienes blocks are getting ten times smaller. Now the teacher asks:



How can we make the one block ten times smaller?

Teacher and pupils discuss that in order to make the ones block ten times smaller they break it into ten equal parts. This cannot be demonstrated concretely using the one from the dienes blocks so the teacher can represent it in many different ways, two of which are outlined below.

³³ Van de Walle, Karp & Bay-Williams (2013)

Concrete Exploration³⁴



ICT Opportunities

[Link: Base Ten Decimal Blocks](#)

Teacher shows the hundred, ten and ones block and asks pupils what number it represents.



*Compare the hundred and the ten blocks with your partner.
Record what you notice.
Did anyone notice anything else?
Now compare the ten and the ones.
Record what you notice.
Did anyone notice anything else?
What do you notice when you compare all three.
Is there ever a smallest piece?*



*Using the equipment that we have (dienes blocks), how can we represent a one, a tenth and a hundredth?
Remember the relationship between the values of the blocks (10 times smaller/bigger).*



Dienes for Decimals

While exploring decimals with dienes blocks, the value attributed to the flat is one, the long is tenths and the small is hundredths.

The following activities can be used to explore the concept of decimals.³⁵

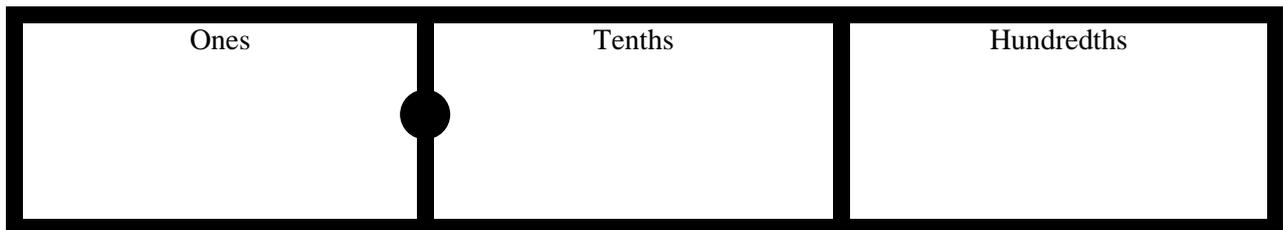
Win a One

This game can be used to reinforce and consolidate the decimal values attributed to the dienes blocks.

Pupils have one game board and a collection of base ten materials in the centre.



Ones, tenths and hundredths



Each player takes a turn to throw a 9-sided die. The number on the dice tells the players how many hundredths they may take from the centre. When they have ten shorts they must trade them for a long (1 tenth). When they have ten longs they trade them for a flat (a whole or one). The first player with a flat wins the game.

How to Play 'Lose a Whole'



Ones, tenths and hundredths

The same as above but you just start with a one (a flat) and subtract the totals shown by the die. First person with nothing left is the winner.

ICT Opportunities

[Link: Interactive Fraction/Decimal Wall](#)

³⁵ ICT link at <http://www.visnos.com/demos/fraction-wall>

1. Draw a rectangle on the board and split it into ten equal sections. Ask pupils how each section can be labelled. Write $\frac{1}{10}$ in each section.

| | | | | | | | | | |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $\frac{1}{10}$ |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|

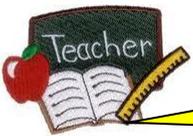
Next colour some of the rectangles, for example:

| | | | | | | | | | |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $\frac{1}{10}$ |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|

Then ask the pupils what fraction of the rectangle has been coloured, that is: $\frac{3}{10}$

2. Explain that $\frac{3}{10}$ can be recorded in a similar way that we used to record place value and ask pupils to fill the place value mat below. Lots of examples can be explored including the following; hundreds, tens, ones and tenths for example 3 hundred, 5 tens, six ones and 4 tenths.

| Tens | Ones | $\frac{1}{10}$ |
|------|------|----------------|
| 0 | 0 | 3 |



*When I look at my place value mat I see that I have no tens and no ones and that I have 3 tenths.
If I take away my place mat how can I record this number? (003)*

3. Give pupils a chance to discuss their ideas and then formally introduce the convention of the decimal point.



Decimal point

The decimal point is always placed to the right of the unit position. In other words the decimal point separates the whole numbers from the fractional parts. The whole numbers are located to the left of the decimal point and the decimal fractions to the right.



Reading and Saying Decimals

We should read and say decimals in a way that supports pupil's understanding. For example, it may be useful to say five and two tenths as well as five point two. In this way, pupils are helped to make connections between decimals and fraction.

When they hear two tenths they will see $\frac{2}{10}$ and 0.2

Learning Experiences from Level A.1

Learning experiences from level A.1 can be used to further develop and extend the concept of decimals. Whole and decimal fraction numbers can be used instead of solely whole numbers. This can be done moving from the concrete experience

1. to concrete experiences and arrow cards
2. to number fans
3. to digit cards.

Consolidation Activities

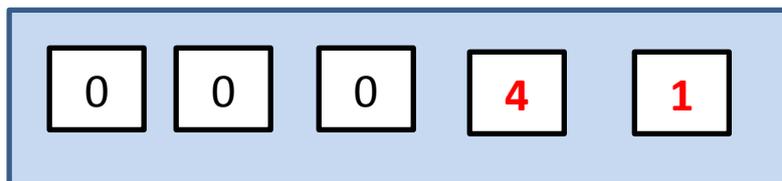
Car odometer

Car odometers show tenths of a kilometre in colour. Ask pupils to record various distances on an odometer, for example,

4.1

10.6

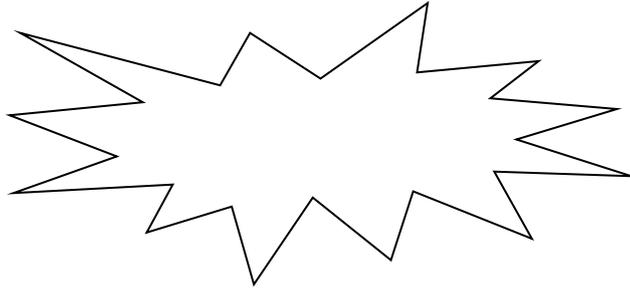
325.3



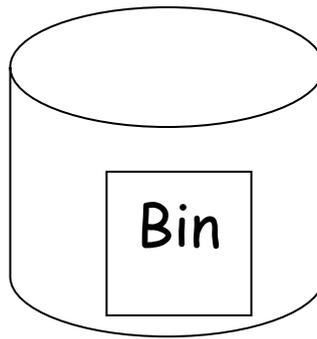
Sweets

Packets of sweets containing 10 sweets may be used where 1 sweet represents 0.1. Record 56 sweets as 5.6 (5 full packets and six sweets).

Place Value Game³⁶



| | ● | $\frac{1}{10}$ | $\frac{1}{100}$ | $\frac{1}{1000}$ |
|--|---|----------------|-----------------|------------------|
| | ● | | | |
| | ● | | | |
| | ● | | | |



Each player gets a place mat and bin. Get as near to the target number as you can, without going over. You can only use each box and the bin once!!! Take target number from bag and write into the star shape. Roll the die and decide what box to put the digit into. (What value to give it?) Take turns rolling the die. Roll the die another four times until all boxes are filled. The winner is the player with the number that is nearest but not bigger than the target number.

³⁶ This game can be adapted to suit just tenths, tenths and hundredths or tenths, hundredths and thousandths.

Level B.5

Make, order, compare and count decimals

Counting Decimals

See Appendix A for some sample counting activities.

Make, Order and Compare Decimals



REMEMBER

All of the following tasks should make specific reference to pupils' fraction knowledge so that decimals and fractions can be used interchangeably by pupils.

Linear Model

Paper Strips

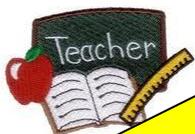
In pairs, pupils use paper strips of various lengths to represent decimals.



*Show me 0.6 on 2 different lengths of paper.
What is 0.6 as a fraction? How can you be sure?
Explain how you did it.
Is there any other way to do it?
Did someone do it in a different way to Mary?
Compare your 0.6 strips to your partner's 0.6 strips.*

This can be repeated for other decimals. Due to the nature of paper strips it may be unwieldy to show hundredths in this way, therefore this activity is most suited to tenths, for example, 0.8, 0.3, 0.4, 0.9, etc.

Give pupils 4 equal length paper strips. Ask them to represent 0.75 on one strip, 0.5 on another strip, 0.25 on another strip, and 0.1 on the final strip.



*Order your decimal paper strips from smallest to largest.
Which is the smallest?
Which is the largest?
Explain why this is.
John, can you explain what Mary said in your own words?
Share something that you notice about these decimals.
Does anyone have a conjecture about the relationship between any of these decimals?
How could we prove this conjecture?*

String

The above activities can also be completed using lengths of string and something to mark the string, for example, blue tack, paper clips, pegs, markers. Here the focus could be on estimation (pupils estimate where to mark the decimal rather than folding).

Cuisenaire Rods

Each group of pupils have a box of Cuisenaire rods that they use to solve problems like the following:

- What decimal of the brown rod is the red rod?
- If the purple rod is 0.66 which rod is the whole?

Pupils can also design their own questions.³⁷

Empty Number Line

Getting pupils to place decimals on an empty number line or to sequence them in order of size will help to establish an understanding of decimals. The distances between any two numbers on the number line will need to be discussed. This requires deep thinking about the relative size of decimals in addition to requiring estimation skills because the space between the numbers has meaning. Initially, this task should involve number cards with only one place of decimals, for example, 0.7, 2.3, 1.5, 0.1 and 0.9³⁸.



Calibrated Models

Subsequently, numbers with two places of decimals (hundredths) can be addressed but will require a number line which is calibrated in one-hundredths such as a metre stick showing centimetres or a measuring jug marked in millimetres³⁹. The numbers could be placed in the appropriate place using post-its.

³⁷ Clarke, Roche & Mitchell (2008)

³⁸ Anghileri (2007)

³⁹ Anghileri (2007)

In addition to linear models, pupils need experience of identifying, comparing and ordering decimals in various area models.

Geoboards⁴⁰

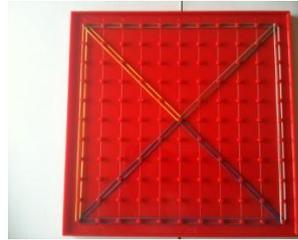
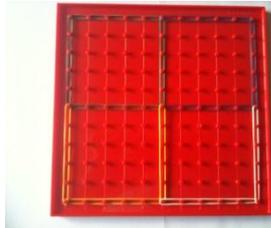
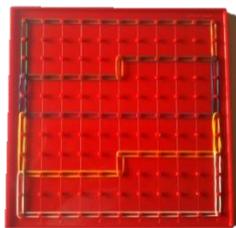
Revision of the geoboard work from the Level B.1 of the Fractions: Teacher’s Manual (PDST) can be used here to identify equivalence between fractions and decimals. It is easier for pupils to identify and display fractional parts on the geoboard. These fractions can then be linked to their decimal equivalents.

ICT Opportunities

[Link: Interactive Geoboard](#)



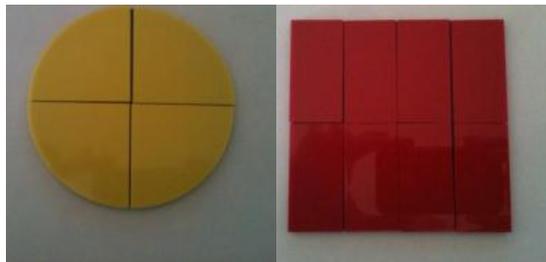
*Show me a quarter/eighth/tenth of the whole geoboard.
 What is its decimal equivalent?
 What other decimal fractions can you represent on the geoboard?
 Record your findings in your learning log.*



These geoboards illustrate equal-sized quarters of a unit (0.25).

Pie or Rectangular Pieces

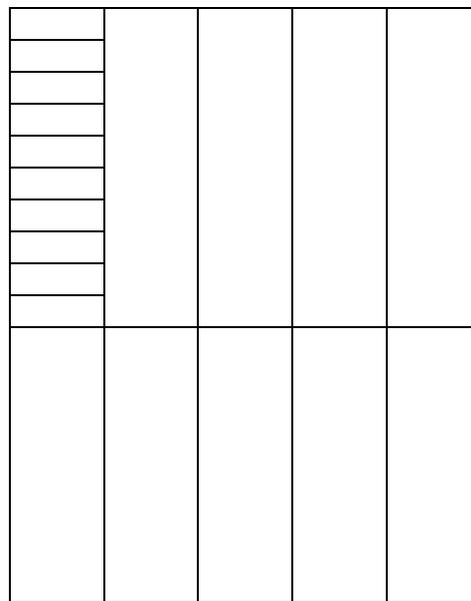
Pie pieces can be used for identifying, comparing and ordering decimals.



⁴⁰ ICT link at <http://www.mathplayground.com/geoboard.html>

Decimal Fraction Mat

Provide copies of the decimal fraction mat enlarged to A3 size. Get the pupils to cut out a large piece (one-tenth) and place it on an uncut mat. Discuss how big the piece is compared to the whole mat (one-tenth the size). If extending this activity to hundredths and thousandths, get the students to cut out the next largest piece and place it on an uncut mat. Discuss the size of this piece compared to the whole (one-hundredth). Continue this process to progressively smaller pieces (one-thousandth and one ten-thousandth). Get the students to try to cut a ten-thousandth into 10 equal pieces. Discuss the size of each of these tiny pieces compared to the original mat (one hundred-thousandth). Discuss how continued cutting would produce millionths to hundred-billionths. Emphasise that, with each division by 10, the pieces decrease in relative size very rapidly. See Appendix B for a sample decimal fraction mat.



Set Model

Extending the work done in Early Mathematical Activities (sorting, classifying, partitioning) sets of 10 can be sorted for various attributes to decimal parts. In the example below a set of transport vehicles are used and can be sorted in various ways including colour, wheels, type of vehicle, flies/doesn't fly, etc.



Other sorting materials (groups of 10) can also be used here, for example, attribute blocks, random collections, beads, buttons⁴¹. Elicit from the pupils what decimal of the whole set each subset makes, for example, what decimal fraction do the helicopters represent?



*What decimal fraction do the helicopters represent?
What decimal fraction is yellow?
What subset represents three tenths?*

Decimal Pairs Task

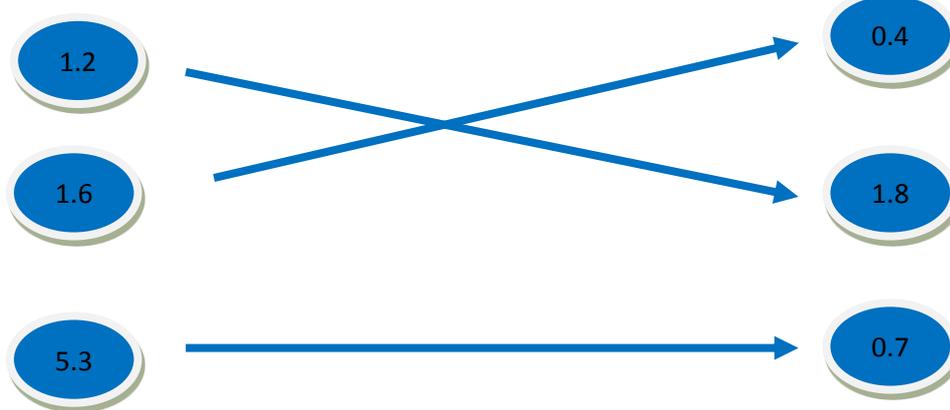
Provide pupils with eight pairs of decimals and ask them to decide, for each pair, which decimal is larger. It is important that pupils give you a rationale for this. The comparison should be made in their heads. It is not a pen and paper exercise. This activity should encourage the use of estimation and rounding. Benchmarks may also be useful here.

| Decimal Pairs | Which Decimal is larger? Record your strategy. |
|----------------|--|
| 0.375 or 0.875 | |
| 0.5 or 0.625 | |
| 0.75 or 0.8 | |
| 0.5 or 0.66 | |
| 1.125 or 1.5 | |
| 0.375 or 0.333 | |
| 0.25 or 0.2 | |
| 0.75 or 0.66 | |

⁴¹ Refer to Ready, Set, Go Maths manual for ideas regarding sorting materials.

Matching Decimals⁴²

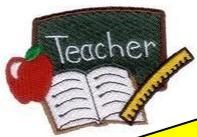
It may be useful for pupils to be able to match decimals which add to make a whole number. This is similar to the ‘story of 10’ in whole numbers where recognising familiar number relationships can be helpful for pupils when calculating (for example, $6 + 4$; $9 + 1$).



Consolidation Activities

First to Last

Give pupils 4 decimal numbers which fall between 2 consecutive whole numbers, for example, 2.3, 2.32, 2.327 and 2.03.



Estimate your answer ordering from the smallest to the largest.

*Use concrete materials to support your answer.
Record and explain your thinking in your learning log.*

How Close Can You Get?

Pupils name a decimal between 3 and 4, on each turn the next pupil names a decimal that is even closer to 4.

⁴² ICT link at

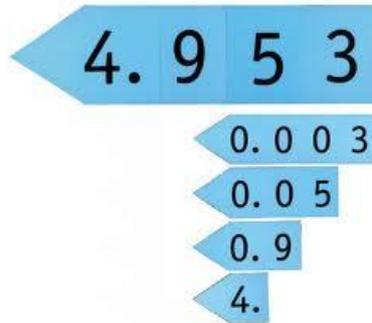
http://nlvm.usu.edu/en/nav/frames_asid_187_g_4_t_1.html?open=instructions&from=category_g_4_t_1.html

Decimal Empty Number Line

Give pupils a series of two to four decimal numbers and have them order them using the decimal empty number line.

Beat the Clock

Provide pupils with ten decimal arrow cards. In pairs, time the pupils as pupils order them from largest to smallest. This can be differentiated to include tenths, tenths and hundredths, tenths, hundredths and thousandths.



Decimal Cards⁴³



⁴³ ICT link at

http://www.wmnet.org.uk/wmnet/custom/files_uploaded/uploaded_resources/853/PVcardsdecimalv2.swf

Level B.6

Solve problems involving decimals

Teaching Notes

To fully explore problem solving it is necessary for pupils to experience a variety of problems. Word problems are not the only type of problem. Include some, and if possible, all of the types of problems listed below.

- Word problems
- Practical tasks
- Open-ended investigations
- Closed questions
- Puzzles
- Games
- Projects
- Mathematical trails.
- Missing/Contradictory/Surplus data

It is important to remember that classroom discourse, that is, interactions that occur throughout the lesson, are vital for meaningful problem solving. Such discourse develops higher order thinking skills while pupils are learning and formalising mathematical concepts⁴⁴. The purpose of this discourse is not for pupils to tell their answers to the teacher who then validates them but rather for the teacher to facilitate a discussion among pupils where processes, strategies and mathematical thinking can be expressed in a supportive environment. The instructional framework outlines the type of language that the teacher can employ to support pupil learning in this way.

⁴⁴ Cited in Van de Walle (2013, p. 42)

Using the learning log in problem solving⁴⁵

Writing as part of a maths lesson can improve pupil learning and understanding, aids reflection, is a useful rehearsal for discussion and provides a lasting record for assessment purposes. With some consideration, prompts for learning log entries can be problem solving in nature. Log entries can be written or oral. Oral entries can be recorded using a range of recording devices including free software such as audacity <http://audacity.sourceforge.net/> . In addition to paper records, the following free ICT tools allow pupils to write, edit, share with peers and submit work electronically either in class or as homework:

Text Editing (real-time, collaborative writing tools)

- Google docs and spreadsheets (<http://docs.google.com>)
- Synchroedit (www.synchroedit.com)
- Zoho Writer (<http://zoho.com>)

Wikis (free, asynchronous, collaborative website creation tools)

- PBworks (<http://pbworks.com/education>)
- Wikispaces (<http://www.wikispaces.com/>)
- Wikidot (<http://www.wikidot.com/>)

Blogging Tools

- Blogger (www.blogger.com)
- Tumbler (www.tumblr.com)
- WordPress(<http://wordpress.com>)

These tools can be used for homework as well as in class.⁴⁶



⁴⁵ Van de Walle, (2013, p.46)

⁴⁶ Adapted from Emry, Lewis and Morfett (2006)

Sample Learning Experiences

A variety of problems may include the following:

Find your match to make one⁴⁷

| | | | |
|-------|-----|-------|------|
| 2.5 | 0.4 | 0.625 | 0.32 |
| 3.125 | 0.8 | 1.25 | 1.6 |

Figure it out⁴⁸

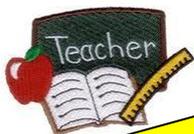
Complete the following magic squares. Here is a clue 1.5 is the sum of the numbers in each of the three rows, columns and two diagonals

| | | |
|--|-----|--|
| | | |
| | 0.5 | |
| | 0.1 | |

| | | |
|--|---------------|---------------|
| | | $\frac{1}{5}$ |
| | $\frac{1}{2}$ | |
| | | |

Fractions for decimals⁴⁹

| | |
|------|------|
| 0.44 | 0.66 |
| 0.22 | 0.88 |



What is the smallest denominator that could be the same for all four fractions equivalent to these decimals?
 What is the smallest numerator that could be the same for all four fractions equivalent to these decimals?

Name that decimal⁵⁰

⁴⁷ Millington (1999)

⁴⁸ Millington (1999)

⁴⁹ Millington (1999)

⁵⁰ Millington (1999)

| | | |
|-----|-----|-----|
| 567 | 189 | |
| 756 | 252 | 378 |



Make the following fractions and record their equivalent decimal value. Use one number as denominator and one as numerator:

1. A fraction equivalent to half
2. A fraction equivalent to one thirds
3. A fraction equivalent to two thirds
4. A fraction equivalent to three quarters

Missing the point ⁵¹

The decimal points have been left out but in each case the answer is a square. Every number lies between 1 and 100.

$$159+209+251= A^2$$

$$293+453+217= B^2$$

$$326+704+936= C^2$$

$$455+885+965= D^2$$

Adding decimals path-o-metre

Make a path through each number matrix so that the sum of the decimals is equal to the given answer.

Use only horizontal and vertical trails. You may not have to use all the numbers, and are not allowed to use a number more than once. This activity will also work well for multiplication of decimals.

| | | | |
|--------------|-----|-----|---------------|
| Start | 0.5 | 5.3 | 8.2 |
| | 7.2 | 4.1 | 0.7 |
| | | | 5.3 |
| | | | Answer |

| | | | |
|--------------|-----|-----|---------------|
| Start | 2.4 | 3.9 | 7.4 |
| | 0.8 | 0.6 | 5.5 |
| | | | 10.2 |
| | | | Answer |

⁵¹ Millington (1999)

How Many?

How many different equations can you make using the following numbers, 1.2, 0.7, 0.5, 0.6, 0.2, 0.4 and 1.0? Before starting, discuss with a partner what strategy if any you might use for this task.

Following completion, ask pupils to share their strategies with others.

Decimals Board Game

Ask pupils to work in pairs or small groups to design and make a board game that involves the use of decimal numbers, decimal fractions, decimal problems etc. Have pupils pitch their idea to another group of pupils or to the whole class before beginning to make it.

Decimals Video

In a small group plan and record an informative video on decimals for the pupils who will be in your class next year. Include in it: what decimal numbers are, what are they used for, where will you find them, any tips or hints you think might help next year's pupils etc.



Decimals Trail

Ask pupils to take a photograph of a decimal number found in their home for example, on a shopping receipt, a number in a newspaper, a measure in a recipe.

Forgot the numbers

This problem has the potential to address the relationship between division and decimal fractions. The richness of the activity lies in the various ways it can be solved as well as the discussion associated with each. You might spend a couple of lessons on this activity. Pose the following problem,

On my calculator I divided one whole number by another whole number and got the answer 3.125. I know that both numbers were less than 50, but can't remember what they were. Can you work out what they were?

Ask pupils to work in pairs to discuss possible strategies for attempting to solve this problem. Allow them to begin. When pupils have finished or if they become stuck, show them the following approaches of three pupils and ask them to continue each.

- a) Gemma and Flo: $47 \div 20 = 2.35$, $47 \div 15 = 3.1333$, $47 \div 17 = 2.704205$,
 $46 \div 17 = 2.70588$, $45 \div 17 = 2.64705$, $42 \div 17 = 2.470588$, $42 \div 15 = 2.8$
- b) Richard and Emily: $3.125 \times 1 =$, $3.125 \times 2 =$, $3.125 \times 3 =$, $3.125 \times 4 =$, etc.
- c) Chloe and Mustafa: “First we looked at the number 0.125 and worked out what fraction of 1 it is. It turned out that it was an eighth.”

The richness of this activity will come from the discussion that ensues. To extend the activity, ask the pupils to apply what they have learnt to a similar problem, where the answer is 3.375, 4.5, 4.1? Can they make up a similar question and solve it? The above problem was taken from www.nrich.maths.org which is a very useful website for sourcing rich mathematical problems.

Make 4.253

Gill, was playing with her name and with numbers. She let all her consonants equal 1.3 and all her vowels equal 0.5. So the value of Gill’s name is $1.3 + 0.5 + 1.3 + 1.3 = 4.4$. What is the value of your name? Change the rules so that the value of your name is 4.253. Initially, tell the pupils Gill’s story and let them find the value of an easy word for example, ‘room’. Then ask them to find the values of their names. Get their partner in their group to check that they have found the right value for their name. Then ask the pupils to put themselves in order by their value, around the side of the class. In groups, the students could then find some way of ending up with the same number, 4.253. They will most likely have to make up different rules to get this value. They could be asked to find more than one way to get this answer. Ask groups to report on what they have done. A possible extension to the problem is: Using Gill’s original substitution, what is the biggest and smallest value that you can find using names in your class? The above problem was taken from www.nzmaths.co.nz which is a very useful website for sourcing rich mathematical problems.

Problems involving money and measures

Growing Vegetables⁵²

Find the cost of growing your own vegetables. Devise a plant plan for your garden or for a standard allotment (9m x 30m). Consider: cost of seeds and plants / fertilisers / tools / labour / quantity consumed of each type of vegetable / amount of space needed. Now compare these costs to the cost of buying vegetables. Would it be cheaper to grow your own or to buy from the supermarket for a period of one week, one month, six months and a year?

Car parking

How many cars can reasonably be expected to park along a 100m stretch of road? Measurements will need to be made of the length of cars and of the distances between them for ease of parking. In the school parking lot, note how bays are marked. How much space needs to be allowed between rows for access? Would this differ for disabled driver bays? Survey a school playground and show how to mark it out as a car park.

ICT travel project

John must travel from Enniscorthy to Sligo on the 16th of June.

He wants to know if he should drive by car or take public transport. John's car travels 8.89km per litre and a full tank will take 40 litres. Find:

1. The quickest route for John and
2. The cheapest route for John

You may use the following websites:

<http://www.buseireann.ie> (Click on Journey Planner in Yellow)

<http://maps.google.com> OR http://www2.aaireland.ie/routes_beta/

<http://pumps.ie>



⁵² Adapted from Bolt & Hobbs (1989)

<http://www.irishrail.ie/home/>

Catalogue Shopping Trail

This example uses a LIDL brochure,

- You have €10.75 to spend. How will you spend it? (how near can you get to spending it all)
- Estimate the cost of buying one of everything on the first page (except the wine). Now calculate the actual cost.
- If 6 Muller Light Chocolate Yogurts cost €2.98 how much will one cost?
- Buy two packs of Lighter Mature Cheese - how much would you save from the original price?
- Buy a pack of Bacon Medallions, one bag of carrots, one Hovis loaf and two packets of Garlic Chicken Kiev's. How much would you spend?
- In question 2, you calculated the actual cost of everything on the first page. How much in total do you save in comparison with the original prices?

Level C.6

Add and subtract whole numbers and decimals up to 2 places

Teaching Notes

Estimation

Pupils should become adept at estimating decimal computations ever before they learn to compute using pen and paper⁵³.

Number Operations

The rules for adding and subtracting decimals are the same to those used for whole numbers, so most pupils who have grasped the concept of operations should find these procedures relatively straightforward. For pupils who have difficulty with these procedures, it may be necessary to re-visit the concrete and the pictorial phases of development. All pupils in the abstract phase should align the whole numbers with whole numbers, tenths with tenths and hundredths with hundredths. Getting pupils to start by adding one more will give the teacher an indication of the pupils' understanding of place value in relation to decimals. The following activity can be used as an assessment tool for this.

| | | | | | | | | | | | | | | | | | | | | | |
|--|---|---|---|----|----|---|---|---|---|---|---|---|--|---|---|----|---|---|--|--|--|
| <p>a.</p> <table border="1" style="margin-left: 20px; border-collapse: collapse; text-align: center;"> <tr> <td style="width: 20px;">O</td> <td style="width: 20px; background-color: yellow;"> </td> <td style="width: 20px;">t</td> <td style="width: 20px;">h</td> <td style="width: 20px;">th</td> </tr> <tr> <td>0</td> <td>.</td> <td>2</td> <td>8</td> <td>5</td> </tr> </table> <p>one tenth more than 0.285 _____</p> <p>one hundredth more than 0.285 _____</p> <p>one thousandth more than 0.285 _____</p> | O | | t | h | th | 0 | . | 2 | 8 | 5 | <p>a.</p> <table border="1" style="margin-left: 20px; border-collapse: collapse; text-align: center;"> <tr> <td style="width: 20px;">O</td> <td style="width: 20px; background-color: yellow;"> </td> <td style="width: 20px;">t</td> <td style="width: 20px;">h</td> <td style="width: 20px;">th</td> </tr> <tr> <td>0</td> <td>.</td> <td></td> <td></td> <td></td> </tr> </table> <p>one tenth more than 0.999 _____</p> <p>one hundredth more than 0.999 _____</p> <p>one thousandth more than 0.999 _____</p> | O | | t | h | th | 0 | . | | | |
| O | | t | h | th | | | | | | | | | | | | | | | | | |
| 0 | . | 2 | 8 | 5 | | | | | | | | | | | | | | | | | |
| O | | t | h | th | | | | | | | | | | | | | | | | | |
| 0 | . | | | | | | | | | | | | | | | | | | | | |
| <p>a.</p> <table border="1" style="margin-left: 20px; border-collapse: collapse; text-align: center;"> <tr> <td style="width: 20px;">O</td> <td style="width: 20px; background-color: yellow;"> </td> <td style="width: 20px;">t</td> <td style="width: 20px;">h</td> <td style="width: 20px;">th</td> </tr> <tr> <td></td> <td>.</td> <td></td> <td></td> <td></td> </tr> </table> <p>one tenth more than 0.345 _____</p> <p>one hundredth more than 0.345 _____</p> <p>one thousandth more than 0.345 _____</p> | O | | t | h | th | | . | | | | <p>d.</p> <table border="1" style="margin-left: 20px; border-collapse: collapse; text-align: center;"> <tr> <td style="width: 20px;">O</td> <td style="width: 20px; background-color: yellow;"> </td> <td style="width: 20px;">t</td> <td style="width: 20px;">h</td> <td style="width: 20px;">th</td> </tr> <tr> <td></td> <td>.</td> <td></td> <td></td> <td></td> </tr> </table> <p>one tenth more than 0.9 _____</p> <p>one hundredth more than 0.9 _____</p> <p>one thousandth more than 0.9 _____</p> | O | | t | h | th | | . | | | |
| O | | t | h | th | | | | | | | | | | | | | | | | | |
| | . | | | | | | | | | | | | | | | | | | | | |
| O | | t | h | th | | | | | | | | | | | | | | | | | |
| | . | | | | | | | | | | | | | | | | | | | | |

⁵³ Van de Walle (2007)

Sample Learning Experiences

Mental Maths



Before you begin to work out the answer make an estimate to help with your solution.

What is your estimate?

How did you arrive at that estimate?

Discuss your approach.

Reflect on your estimate.

How accurate was it?

Over or Under

Prepare a number of addition and subtraction algorithms. Pupils decide are they over or under a chosen number.

i.e.

$0.3 + 0.8$ over / under 1.0

$1.5 - 0.3$ over / under 1.0

$0.48 + 0.42$ over / under 1.0

$24.3 - 13.8$ over / under 10.0

$36.2 + 15.86$ over / under 50.0

$2.47 - 1.64$ over / under 0.55

Chance It

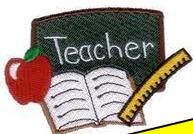
Teacher writes a selection of addition and subtraction algorithms on the board and asks the pupils to estimate the answer. (During the feedback on estimation strategies used by the pupils, highlight the different estimation strategies i.e. Front-end, rounding and clustering / special numbers).

$1.25 + 215.35 + 30.52$

$3.86 + 124.43 + 51.23$

$25.87 - 13.65$

$431.57 - 21.45$



Explain the strategy you used.

Did anyone else use a different strategy?

Which strategy was more effective?

Locate the Point

Teacher writes various algorithms on the board leaving the decimal point out of the solution. Pupils decide where the decimal point should be placed.

$$3.5 + 3.7 = 72$$

$$1.4 + 0.8 = 22$$

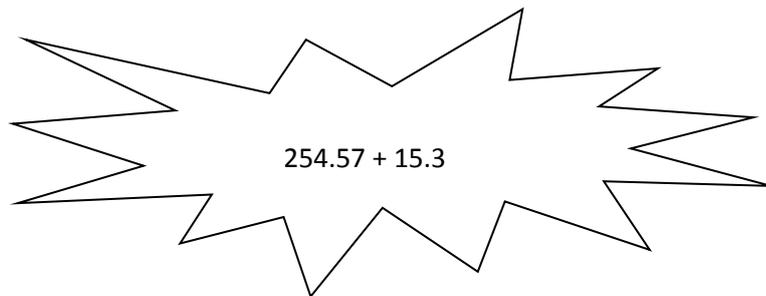
$$1.79 + 0.21 = 200$$

$$4.13 + 2.07 + 5.91 = 1211$$

High or Low Estimation Activity⁵⁴

Display a computation and three more computations that will help create an estimate.

The pupils decide if the estimations are higher or lower than the exact result and explain why they think so.


$$254.57 + 15.3$$

$$237.65 + 36.87$$

$$213.76 + 52.8$$

$$225.98 + 42.1$$

⁵⁴ Van de Walle (2007, p. 248)

Level C.7

Multiply and divide decimals up to 2 places by a single-digit whole number

Teaching Notes

Multiplication and Division

There is no need to develop new rules for multiplication and division of decimals because the same digits will be in the answer regardless of the decimal point. The computation can be performed as with whole numbers and the decimal can be placed in the correct position after the procedure is completed.

Level D.2

Express tenths, hundredths and thousandths as fractions and decimals

Teaching Notes

Materials

Various materials can be used for expressing tenths, hundredths and thousandths as fractions and decimals. The materials include:

- hundredth or decimal wheel
- 10×10 grids or dienes blocks
- measuring devices

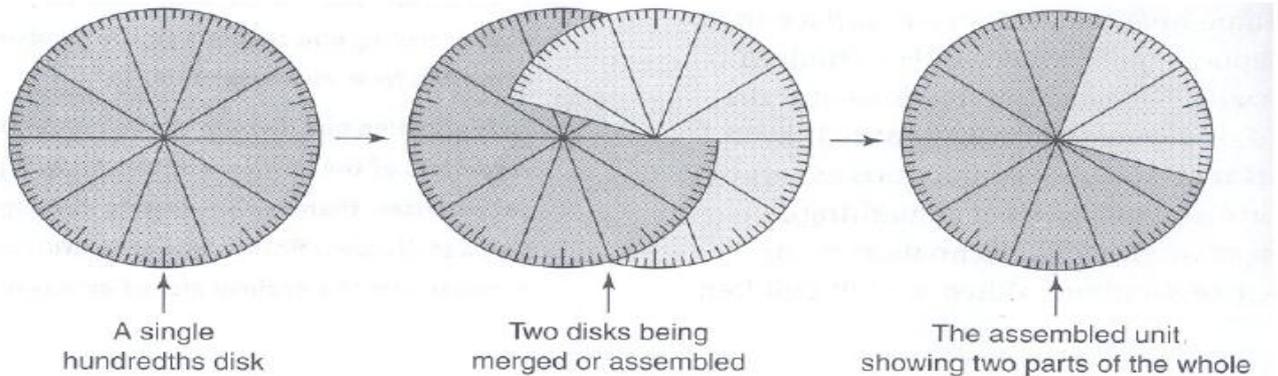
The use of these materials is illuminated in the sample learning experiences below.

Sample Learning Experiences

The Hundredth or Decimal Wheel

Provide pupils with two different coloured hundredths wheels (or decimal wheels) as shown below.

This is a good example of the area model for tenths and hundredths. A hundredths wheel is divided into 10 sections, each divided further into 10 equal intervals. When a slit is cut along one radius and two wheels of different colours are placed together, the model can be used to show decimals and fractions of less than one. This model will be familiar to pupils, as many have seen “pies” divided into thirds, fourths, tenths, and so on. Ignoring the smaller gradations, the hundredths wheel is simply a tenths wheel.



(Source: Van de Walle, *Teaching Student Centered Mathematics Grades 3-5*. Page 182)

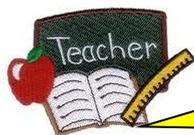
Begin by asking pupils to use their decimal wheels to show one half, then one fourth, then three fourths. Continue by having pupil's model different decimal numbers example 3 tenths, 6 tenths, 60 hundredths, 67 hundredths, 6 hundredths. Extend this activity by asking pupils: See Appendix C for a template for the hundredth or decimal wheel.



Is the decimal number shown on your wheel more or less than $\frac{1}{2}$?
Is the decimal number closer to $\frac{1}{2}$, $\frac{1}{4}$ or $\frac{3}{4}$?
How far from one whole is the decimal number?

Number equivalences

1. Pupils use their decimal wheels to model fractions and decimal numbers that you have recorded on the board, for example, $\frac{7}{10}$, $\frac{23}{100}$, $\frac{3}{10}$, $\frac{3}{100}$



Can you show and record the decimal equivalent for the fractions on the board using your decimal wheel?
Explain how you got your answer.

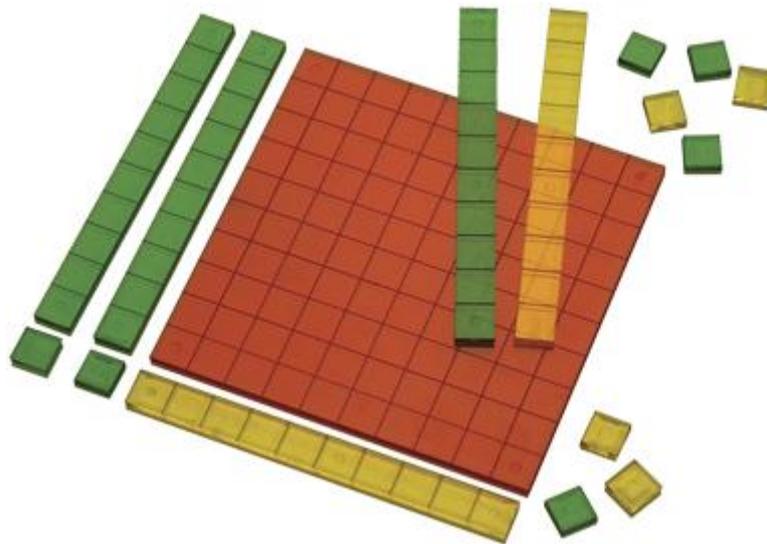
2. Pupils can discuss the following ideas by referring to their decimal wheels:

- Fraction-decimal number equivalencies (e.g., $\frac{7}{10} = 0.7$);

- The composition of fractions with denominators of 100 as tenths and as hundredths (e.g., $\frac{23}{100} = \frac{2}{10} + \frac{3}{100}$);
- The composition of decimal numbers as tenths and hundredths (e.g., $0.37 = 0.3 + 0.07$).

10x10 Grids or Dienes Blocks

10x10 grids (for example dienes blocks or paper 10 x 10 grids) represent one unit, strips of ten representing tenths and small squares representing hundredths



Start by asking pupils to represent half, fifths, fourths, eighths with the grids, strips and small squares. Represent the fractions using grids, strips and small squares. Write the fraction and the decimal equivalent. Give mixed numbers also, for example: $3 \frac{36}{100}$



*Write and read your representation as a decimal number.
Compare your method with your partners.
Record and explain your answer in your learning log.*

Now show me $\frac{1}{4}$



*How did you cover $\frac{1}{4}$ with strips and small squares?
Did anyone do it a different way?
Can you now represent $\frac{3}{8}$ on your grid and tell me what decimal it represents?*

Measuring Devices

Metre Stick or Measuring Jug

The metre stick is an excellent model for thousandths when it is marked with millimetre increments. The length of the whole does not change, and students can see that each interval can be further subdivided (decimetres into centimetres, centimetres into millimetres) while the whole always stays the same. Measuring jugs are useful for similar reasons.

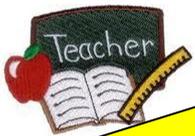
Figure these out with numbers *less than one*:

1. given the fraction, write the decimal equivalent;
2. given the decimal, write the fraction equivalent;
3. given a fraction and decimal, determine if they are equivalent representations.
4. Include fractional amounts greater than one for example write the decimal equivalent for

$$2 \frac{56}{100}$$

Various Values⁵⁵

| Kilometre | Hectometre | Decametre | Metre | Decimetre | Centimetre | millimetre |
|-----------|------------|-----------|-------|-----------|------------|------------|
| | | | 3 | 8 | 5 | |



How many ways can you represent the above information in different unit lengths using the decimal place?

Explain and justify your answer.

What is the function of the decimal place?

What, if anything, does the decimal place tell us about numbers sitting to the right/left of it?

Possible answers might be

1. 3.85 meters
2. 3850 millimetres
3. .00385 kilometres
4. 385 centimetres

Consolidation Activities

⁵⁵ Van de Walle (2013)

Number Lines - Fractions/Decimals/Percentages⁵⁶



⁵⁶ ICT link at <http://www.ictgames.com/equivalence.html>

Teaching Notes

Link with Fractions and Decimals

The conceptual connections between percentages, fractions and decimals are very strong. Once pupils have a good grasp of fractions and decimals, it is appropriate to start discussing percentages.

Hundredths as Percent

Once pupils have a good grasp of hundredths (in fractions or decimals), for example, $\frac{3}{4}$ as $\frac{75}{100}$ or 0.75, then the word 'percent' can sometimes be substituted for the word 'hundredth'. Percent is merely new terminology, not a new concept⁵⁷. Therefore, when the decimal point identifies hundredths as being the 'unit' (see notes for decimals) then 27.8 hundredths can also be referred to as 27.8 percent.

Language Benchmarks

It may be helpful for pupils to have language benchmarks for different percentages which will help with their understanding, for example, 1% as 'very little'; 98% as 'almost all'; 50% as 'exactly half'; 48% as 'nearly half'; etc. These can then be used for estimation and for checking answers.

Base-10 Materials

Pupils should use base-10 materials in much the same way as for decimals.

100-square

Visual representations can help children to understand portions within a 100, for example, a blank 100 square can be used to shade various portions in different colours.

Visual Representation

Representing decimals on a number line or a 'percentage bar' will help pupils to understand the relative size of decimals. This has advantages over using a 100 square where the relative positions of the decimals are unclear⁵⁸. Furthermore, a percentage bar can provide benchmarks for pupils. An example of a percentage bar might include:

⁵⁷ Van de Walle (2007)

⁵⁸ Anghileri (2007)

| | | | | | | | | | |
|------|------|------|------|------|------|------|------|------|------|
| | | | | | | | | | |
| 10% | 10% | 10% | 10% | 10% | 10% | 10% | 10% | 10% | 10% |
| 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| 1/10 | 1/10 | 1/10 | 1/10 | 1/10 | 1/10 | 1/10 | 1/10 | 1/10 | 1/10 |

This visual representation of decimals with percentages is one way of emphasising that, similar to fractions, decimals are parts of a whole unit; and that this whole unit does not necessarily have to be the number 1. 100% can represent any number and so 1/10 of that number could be any number (not necessarily a number smaller than 1).

Percentage Bar

A percentage bar can be used to visually represent 0% to 100% of a number⁵⁹; this will help pupils' understanding of the percentage scale in addition to providing pupils with a reference point when calculating.



Sample Learning Experiences

As for Learning Experiences in Level B.5

Adapt learning experiences in Level B.5 to include fractions and percentages. Refer also to Appendix A where counting activities are outlined.

Consolidation Activities

Loop Game

Each pupil has one card. The pupil with the start card reads his / her question, the pupil with the solution calls it out and reads the question on the same card. The game continues until the last card is read.

⁵⁹ Anghileri (2007)

| | |
|--|---|
| <p>I have 3</p> <p>What is $\frac{1}{2}$ of 4?</p> | <p>I have 2</p> <p>Which is bigger $\frac{1}{4}$ or $\frac{1}{8}$?</p> |
| <p>I have $\frac{1}{4}$</p> <p>$\frac{4}{4}$ the same as 1 whole True or false?</p> | <p>I have true</p> <p>A fraction that equals $\frac{1}{2}$ with a denominator of 4</p> |
| <p>I have $\frac{2}{4}$</p> <p>I ate 50% of my chocolate bar. What fraction is left?</p> | <p>I have $\frac{1}{2}$</p> <p>Which is bigger $\frac{1}{4}$ or $\frac{1}{3}$?</p> |
| <p>I have $\frac{1}{3}$</p> <p>I have a cake with 16 slices. I eat $\frac{1}{8}$ of it.</p> <p>How many slices are left?</p> | <p>I have 14.</p> <p>Another fraction that equals $\frac{1}{2}$ with a denominator of 8</p> |
| <p>I have $\frac{4}{8}$</p> <p>0.5 of €5.00</p> | <p>I have €2.50</p> <p>10% of 360 km</p> |
| <p>I have 36km</p> <p>$\frac{1}{8}$ of 32</p> | <p>I have 4</p> <p>Which is smaller $\frac{2}{8}$ or $\frac{3}{8}$?</p> |
| <p>I have $\frac{2}{8}$</p> <p>What is 25% of 32cm?</p> | <p>I have 8cm</p> <p>What is 0.6 of 300?</p> |
| <p>I have 180</p> <p>A fraction that equals 1 whole with a denominator of 2</p> | <p>I have $\frac{2}{2}$</p> <p>If I have eaten $\frac{1}{4}$ of a pizza. What decimal fraction is left?</p> |
| <p>I have 0.75</p> <p>Which is smaller $\frac{1}{8}$ or $\frac{1}{16}$?</p> | <p>I have $\frac{1}{16}$</p> <p>0.7 the same as $\frac{3}{5}$, true or false?</p> |
| <p>I have false</p> <p>What is 0.25 of 24</p> | <p>I have 6</p> <p>What is 40% of €2.00</p> |
| <p>I have 80c</p> | <p>I have 4 hundredths</p> |

| | |
|---|---|
| What does the 4 represent in 18.34? | Who has 4 tenths as a fraction? |
| I have $\frac{4}{10}$ Who has $0.07 + 0.02$? | I have 0.09 Who has 4 tenths add 2 tenths? |
| I have 6 tenths Who has 10% of 140km? | I have 14km Who has 0.1 of 90mls? |
| I have 9mls Which is bigger $\frac{2}{5}$ or $\frac{6}{9}$? | I have $\frac{6}{9}$ Who has $2.6 + 3$? |
| I have 5.6 Who has 20% of 250? | I have 50 Who has 10% of 30? |

Linear, Area and Set Models⁶⁰

Compare linear, area and set models for fractions, decimals and percentages.

ICT Opportunities

[Link: Linear, Area and Set Models](#)

⁶⁰ ICT link at <http://illuminations.nctm.org/ActivityDetail.aspx?ID=11>

Teaching Notes

Practical activities like those suggested below help pupils to relate the mathematical concepts of fractions, decimals and percentages to everyday life. This level provides opportunities for applying skills and conceptual understandings from previous levels, for example:

- compare, order and express fractions/ decimals / percentages as fractions / decimals / percentages; and
- addition, subtraction, multiplication and division of decimals,

This level brings together all previous work and therefore acts as a consolidation of skills applied to a specific area (money).



MONEY

Money is the context for the exploration and calculation of simple interest, profit, loss and VAT.

Sample Learning Experiences

Receipt Trail

A receipt trail is an excellent opportunity to reinforce the concepts that the pupils have learned to this point. The objective at this level is to use their understanding of fractions, decimals and percentages and apply it to a real situation (and the measure strand – money).

Each pupil has a copy of a receipt. Teacher calls out problems for the pupils to solve.



What was the total of my shopping?

What change did I get back?

Work out my new total if I got a 10% discount today.

What profit did the shopkeeper make if the cost price of the items was 75% of the total?

Today, scones are being sold on special offer for half price, what loss is the shopkeeper making on 12 scones?

How much VAT did I pay if my total included 20% VAT?

If I paid for my goods with my credit which charges 12.5% interest how much will my shopping cost me?



[Loan Calculator](#)

Please enter the 3 items in the table below to compute your monthly loan payments. For example:

200000 (amount) 7.625 (loan rate as a %) 30 (years)

| Loan Amount | Interest Rate | # of Years |
|----------------------|----------------------|----------------------|
| <input type="text"/> | <input type="text"/> | <input type="text"/> |

Monthly Payment: €

Sweet Treat Cafe⁶¹



⁶¹ ICT link at <http://www.mathplayground.com/MathApprentice/SweetTreat.html>

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Appendix A

Counting in Decimals

Counting activities should have:

- a lively pace
- enthusiastic participation
- two or three different short focussed activities (variety will maintain interest)
- physical activity
- a choral response
- an individual response

There are many ways of counting in decimals which include:

Counting Stick

- Count in different decimal decimals, for example, tenths, hundredths, thousandths.
- Start at different starting points, count forwards and backwards.
- Include whole numbers and decimal decimals
 - begin at one tenth (0.1), then count on by two tenths.
 - begin at 1.4 and count on in tenths (0.1)

Stamp and Tap

Pupils find a space facing the board. Count forwards in decimals stamping feet in time. Stop at required number and turn in opposite direction. Now count back tapping their shoulders in time. (Do this without pausing!)

Human Number Line

Each pupil is given a large card with a decimal on it. Pupils are asked to line up from the smallest decimal to the largest. Teacher/pupil then discuss the order of the decimals, for example, before/ after, more than /less than/ same as, between, first/second, etc.

The Sound of a Number Game (Counting Can)

Teacher shows/tell the pupils the decimal of a unit being dropped into a tin. The pupils count silently in their heads as the teacher drops the decimal pieces into the tin. When the teacher stops, the pupil can call out the answer, or show its place on a number line. Teacher completes step one but this times ask

pupils what decimal she/he would need to make 1 unit, 2 units etc. How many decimal pieces are in the tin, etc.

Fill the Bag

Place an even number of cubes on the desk, for example, 8. Tell the pupils you want them to place 0.5 of the cubes in the bag. Extend by asking questions, for example, how many cubes did you put in the bag? How many cubes are left? What decimal fraction does this represent? Etc. This can also be extended by using different numbers of cubes and different decimals.

Stand and Sit Game

Pupils stand and then sit while saying the decimal number sequence required, for example, Stand when our count is a whole unit. Pupils begin sitting and counting in tenths: 0.1, 0.2, 0.3.... (One tenth, two tenths, three tenths...) they stand and so on.

Count Around

Pupils stand in a circle and count around, each pupil saying the next number in the sequence. Start counting at 0.5. The pupil who says number 2 sits down. Keep going until only one pupil is standing.

This could be differentiated in a number of ways including:

- using different decimals and to more decimal places (tenths, hundredths, thousandths)
- using shorter/longer sequences
- using different starting/finishing points
- counting backwards

Counting Choir

Divide class into 3 groups. Give one group ones, one group tenths, one group hundredths. Teacher plays the role of conductor with a baton. Teacher begins to count and then points the baton at one group to continue to count in unison. Teacher then points to a different group and continues.

Hand Game

Teacher picks a starting point, for example 1.5. If teacher raises her/his hand up it means count 0.5 more, if the hand faces down it means 0.5 less. This activity can be extended to include hand to the right (count on in 0.1) and hand to the left (count back in 0.1).

Rope Activity

Stretch a skipping rope across the floor. Mark 0 at one end and 1 at the other end. Invite pupils to stand on or next to the rope to indicate positions of decimals, percentages and fractions, for example, 0.5,

0.33, 35%, 62%, $\frac{3}{8}$, $\frac{4}{9}$. Add an extra rope or two to extend the line to 2(3) so that the pupils can also represent larger decimal numbers.

Missing Number

Teacher counts forwards or backwards in a sequence, missing out one of the numbers, example 0.5, 0.4, 0.2, 0.1 Children discuss in pairs which number was missing.

Swap Shop

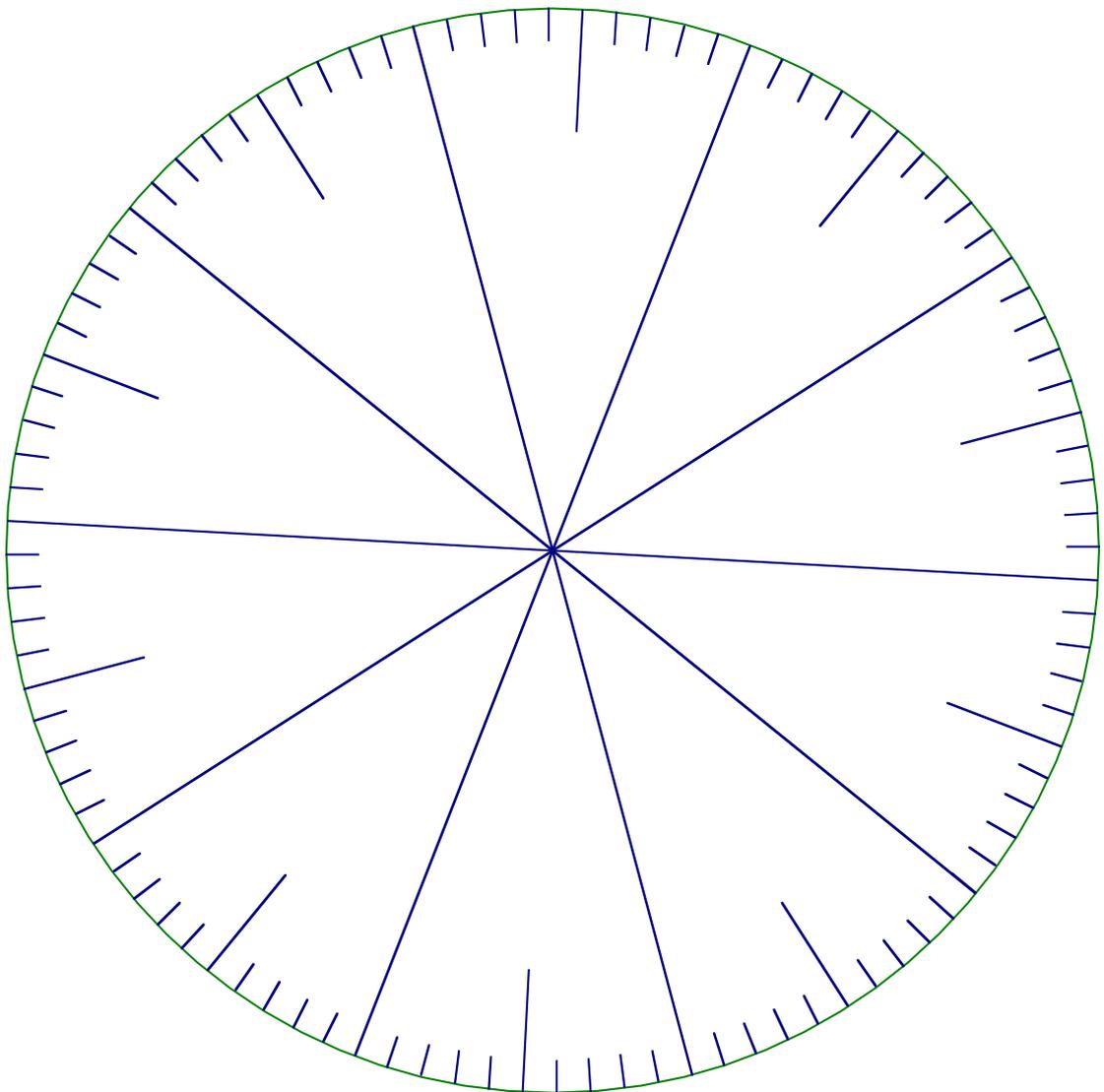
Pupils sit in a circle holding a number (include each number more than once). Teacher calls out a number and the children holding it have to swap places. Extend by saying different commands e.g. swap if your number is smaller than 4.5, swap if your number is bigger than 6.2, swap if your number is between 0.8 and 1.4

Near or Far

Select a number to two decimal places. Children work in pairs with digit cards / number fans. Teacher says a statement such as “close to 3.00”, “less than 4.00”, “close to 3 and 50 hundredths”. Children then have to rearrange the digits and show their answer. Discuss who is closest and why.

Appendix C

Hundredths Disk



Appendix D

Samples of Teacher Questioning and Teacher Talk: for Eliciting, Supporting and Extending Pupil's Mathematical Thinking

Eliciting

You have 23 counters, how many ways can you group them?

Estimate how many straws are in this box.

Did anyone do it another way?

Did anyone represent it in another way?

Seán can you tell me how you built your number?

Can anyone think of another name for forty?

Show me all the numbers you can build with the digits 456.

Show me 732 with arrow cards.

Here is 525. Discover how it is made up.

Did anyone else notice anything different?

What do you notice about the number to the left/right/above/below?

What do you notice about the numbers on the diagonal?

How many numbers can you find with a zero place holder?

Add or subtract two numbers to get a number with 3 in the hundreds place.

How many numbers can you find with an even number in the hundreds place?

Is there any other way to do it?

Did someone do it in a different way to Mary?

Using the equipment that we have (dienes blocks), how can we represent a one, a tenth and a hundredth?

Did anyone else use a different strategy?

Supporting

Discuss with your partner ways in which you can count the straws.

Represent this number in another way using any material you like.

Mary revoice for me what Seán said.

John, can you explain what Mary said in your own words?

Use concrete materials to support your answer.

Before you begin to work out the answer make an estimate to help with your solution.

What is your estimate?

How did you arrive at that estimate?

Discuss your approach.

Reflect on your estimate.

How accurate was it?

Compare your method with your partners.

Extending

Refining

Is there an easier way than counting by ones?

What is the fastest way to count these straws?

Is there one way that is better than the other?

Which strategy was more effective?

Explaining and Justifying

Explain why.

Explain why even though both numbers have the same digits they are not the same.

Explain how you know.

Justify your answer.

Explain how you got this answer.

Explain how you did it.

Explain why this is.

Explain how you got your answer.

Explain the strategy you used.

Explain and justify your answer.

Recording and Reasoning

Record your findings in your learning log.

Record what you notice.

Record and explain your thinking in your learning log.

Hypothesising

Explore if this will work for other numbers.

Did anyone notice anything else?

What do you notice when you compare all three.

Is there ever a smallest piece?

What have you and your partner noticed about when you round up?/round down?

Did anyone else notice anything different?

Share something that you notice about these decimals.

Does anyone have a conjecture about the relationship between any of these decimals?

How could we prove this conjecture?

Appendix E

Assessment

There are many forms of assessment which can be used effectively in mathematics lessons. The samples provided here are just a few. Please see the Assessment in the Primary School Curriculum: Guidelines for Schools (NCCA, 2007) for more information and guidance in relation to assessment - available at <http://www.ncca.ie/uploadedfiles/publications/assess%20%20guide.pdf> .

Two assessment checklists for place value, decimals and percentages are provided here – one for whole class assessment and one for individual pupil assessment. The individual pupil assessment checklist can be used to ‘track’ a number of pupils in the class over the course of a year. Similarly, it can be used to ‘track’ pupils from 1st to 6th class. It enables a dual-approach to assessment – assessment of the concepts of place value, decimals and percentages in addition to assessment of the developmental mathematical experiences (concrete, pictorial, abstract).

Individual Pupil Assessment: Place Value Learning Trajectory Level A

| | | | |
|--|--|---|--|
| Individual Pupil Name: | <p style="text-align: center;">Developmental Experiences</p>  | | |
| Concepts |  Concrete |  Pictorial |  Abstract |
| Level A.1 Explore, identify and record place value 0-99 | | | |
| Level A.2 Explore, identify and record place value 0-199 | | | |

Individual Pupil Assessment: Place Value Learning Trajectory Level B

| Individual Pupil Name: | Developmental Experiences  | | |
|---|---|--|---|
| Concepts |  Concrete |  Pictorial |  Abstract |
| Level B.1 Explore, identify and record place value 0-999 | | | |
| Level B.2 Read, write and order 3 digit numbers | | | |
| Level B.3 Round whole numbers to the nearest ten or hundred | | | |
| Level B.4 Explore, express and identify place value in decimal numbers to one place (tenths) | | | |
| Level B.5 Count, compare and order decimals | | | |
| Level B.6 Solve problems involving decimals | | | |

Individual Pupil Assessment: Place Value Learning Trajectory Level C

| Individual Pupil Name: | Developmental Experiences  | | |
|---|---|--|---|
| Concepts |  Concrete |  Pictorial |  Abstract |
| Level C.1 Explore, identify and record place value 0-9999 | | | |
| Level C.2 Read, write and order 4 digit numbers | | | |
| Level C.3 Round whole numbers to the nearest thousand | | | |
| Level C.4 Explore, express and identify place value in decimal numbers to two places (tenths and hundredths) | | | |
| Level C.5 Make, order, compare and count decimals | | | |
| Level C.6 Add and subtract whole numbers and decimals to 2 places | | | |

Individual Pupil Assessment: Place Value Learning Trajectory Level D

| Individual Pupil Name: | Developmental Experiences  | | |
|---|---|--|---|
| Concepts |  Concrete |  Pictorial |  Abstract |
| Level D.1 Round whole numbers and round decimals to nearest whole number (to one, two or three decimal places) | | | |
| Level D.2 Express tenths, hundredths and thousandths as fractions and decimals | | | |
| Level D.3 Compare and order fractions, percentages and decimals | | | |
| Level D.4 Explore and calculate simple interest, profit, loss, VAT | | | |

Teacher Reflection

Placing yourself on a continuum is a simple way for teachers to reflect on their own practice. The following reflection continuum (Table G.1) is adapted from work completed by Hufferd-Ackels, Fuson & Sherin (2004) when they devised a developmental learning trajectory based on classroom mathematical discourse and thinking.

Table F1 Reflection Continuum for Teachers

| Questioning | Explaining Mathematical Thinking | Source of Mathematical Ideas | Responsibility for Learning |
|---|--|---|--|
| Shift from teacher as questioner to student and teacher as questioners | Pupils increasingly explain and articulate their mathematical ideas to the teacher and to others | Shift from teacher as the source of ideas to pupils' ideas also influencing the direction of lesson | Pupils increasingly take responsibility for learning and evaluation of others and self. Maths sense becomes the criterion for evaluation |
| <i>Highlight or circle the most appropriate word in each section. Include the date and revisit after a particular instructional period.</i> | | | |
| Always Sometimes Never | Always Sometimes Never | Always Sometimes Never | Always Sometimes Never |