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Data and Chance Handbook for Teachers

A GUIDE TO TEACHING AND LEARNING IN IRISH PRIMARY SCHOOLS

DATA INTRODUCTION

This manual has been designed by members of the Professional Development Service for Teachers. Its sole purpose is to enhance teaching and learning of the strand of Data and strand unit Chance in Irish primary schools and will be mediated to practising teachers in the professional development setting. Thereafter it will be available as a free downloadable resource on www.pdst.ie for use in the classroom. This resource is strictly the intellectual property of PDST and it is not intended that it be made commercially available through publishers. All ideas, suggestions and activities remain the intellectual property of the PDST (all ideas and activities that were sourced elsewhere and are not those of the authors are acknowledged throughout the manual).

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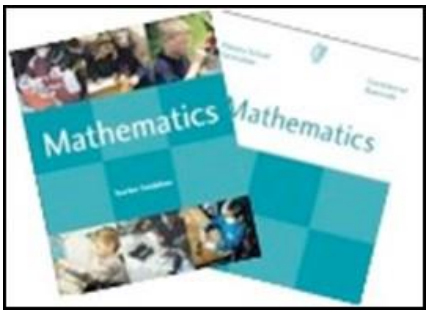
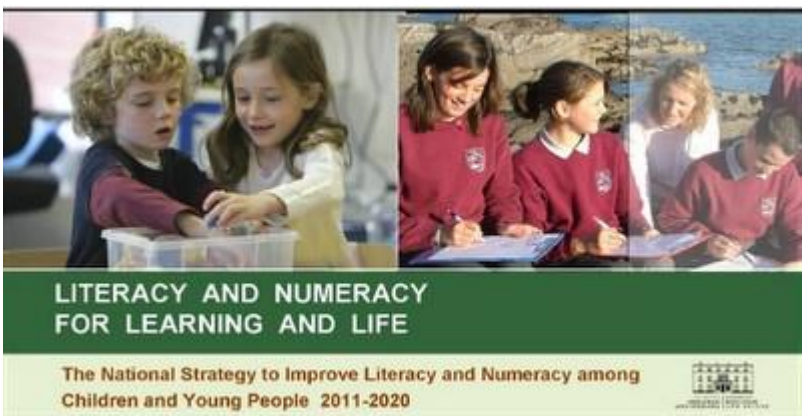
Contents

AIM

The aim of this Teacher Handbook is to assist teachers in teaching the strand of Data (infants to 6th class) and strand unit Chance (3rd to 6th class). This resource is intended to complement and support the implementation of the Primary School Mathematics Curriculum (PSMC) rather than replace it. By providing additional guidance in the teaching and learning of Data, this resource attempts to illuminate an instructional framework for enhancing mathematical thinking. This instructional framework advocates methods of eliciting, supporting and extending higher-order mathematics skills such as reasoning; communicating and expressing; integrating and connecting; and applying and problem solving. Although this resource highlights the Data strand, this instructional framework can be used for all strands and strand units of the PSMC.

To that end a problem-solving approach focusing on pupils' conceptual development of Data through engagement in practical hands-activities, is recommended throughout this resource. Enabling pupils to reason and communicate a variety of different approaches and strategies to solve problems, develops true mathematical understanding, rather than merely applying facts which have been learned 'by rote'. Engaging pupils in this way enhances pupils' learning experiences, enables multiple access points to the same tasks and encourages reflection and opportunities for self-assessment and self-directed learning.





INSTRUCTIONAL FRAMEWORK

INSTRUCTIONAL STRATEGIES

Table 1.1 illustrates a framework for advancing mathematical thinking. Although it does not explicitly refer to concrete materials or manipulatives, the use of these is often a prerequisite for developing mathematical thinking and can be used as a stimulus for this type of classroom discourse.

Table 1.1 Strategies for Supporting and Developing Mathematical Thinking[1]

Eliciting	Supporting	Extending
Facilitates pupils responding Elicits many solution methods for one problem from the entire class *e.g. "Who did it another way?; did anyone do it differently?; did someone do it in a different way to X?; is there another way of doing it?"* Waits for pupils' descriptions of solution methods and encourages elaboration Creates a safe environment for mathematical thinking *e.g. all efforts are valued and errors are used as learning points* Promotes collaborative problem solving *Orchestrates classroom discussions* Uses pupils explanations for lesson's content Identifies ideas and methods that need to be shared publicly *e.g. "John could you share your method with all of us; Mary has an interesting idea which I think would be useful for us to hear."*	*Supports describer's thinking* Reminds pupils of conceptually similar problem situations Directs group help for an individual student through collective group responsibility Assists individual pupils in clarifying their own solution methods *Supports listeners' thinking* Provides teacher-led instant replays *e.g. "Harry suggests that ...; So what you did was ...; So you think that ...".* Demonstrates teacher-selected solution methods without endorsing the adoption of a particular method *e.g. "I have an idea ...; How about ...?; Would it work if we ...?; Could we ...?".* *Supports describer's and listeners' thinking* Records representation of each solution method on the board Asks a different student to explain a peer's method *e.g. revoicing (see footnote on page 8)*	*Maintains high standards and expectations for all pupils* Asks all pupils to attempt to solve difficult problems and to try various solution methods *Encourages mathematical reflection* Facilitates development of mathematical skills as outlined in the PSMC for each class level *e.g. reasoning, hypothesising, justifying, etc.* Promotes use of learning logs by all pupils *Goes beyond initial solution methods* Pushes individual pupils to try alternative solution methods for one problem situation Encourages pupils to critically analyse and evaluate solution methods *e.g. by asking themselves "are there other ways of solving this?; which is the most efficient way?; which way is easiest to understand and why?".* Encourages pupils to articulate, justify and refine mathematical thinking *Revoicing can also be used here* Uses pupils' responses, questions, and problems as core lesson including student-generated problems *Cultivates love of challenge*

This is adapted from Fraivillig, Murphy and Fuson's (1999) Advancing Pupils' Mathematical Thinking (ACT) framework.

[1] Fraivillig, J.L., Murphy, L.A. & Fuson, K.C. (1999). Advancing pupils' mathematical thinking in everyday mathematics classrooms. Journal for Research in Mathematics Education, 30(2) 148-170.

DIFFERENTIATION

This manual is designed to support teachers in catering for individual pupil difference in each classroom. The learning trajectory outlines the developmental stages through which pupils progress, in order to become fluent in their mathematical understanding of concepts in Data and is based on the objectives for DATA in the Primary School Mathematics Curriculum. Supporting pupils through the concrete, pictorial, abstract stages, ensures each pupil has the opportunity to understand the concept being taught at their own pace. Some pupils will need to spend more time engaging in a variety of hands-on approaches in order to consolidate conceptual understanding. This approach is also advocated by Reys et al.[2] in describing methods to support children with SEN in mathematics.

"Research indicates that lessons using manipulative materials have a higher probability of producing greater mathematical achievement than do lessons without such materials. Handling the materials appears to help children construct mathematical ideas and retain them" (p.192).

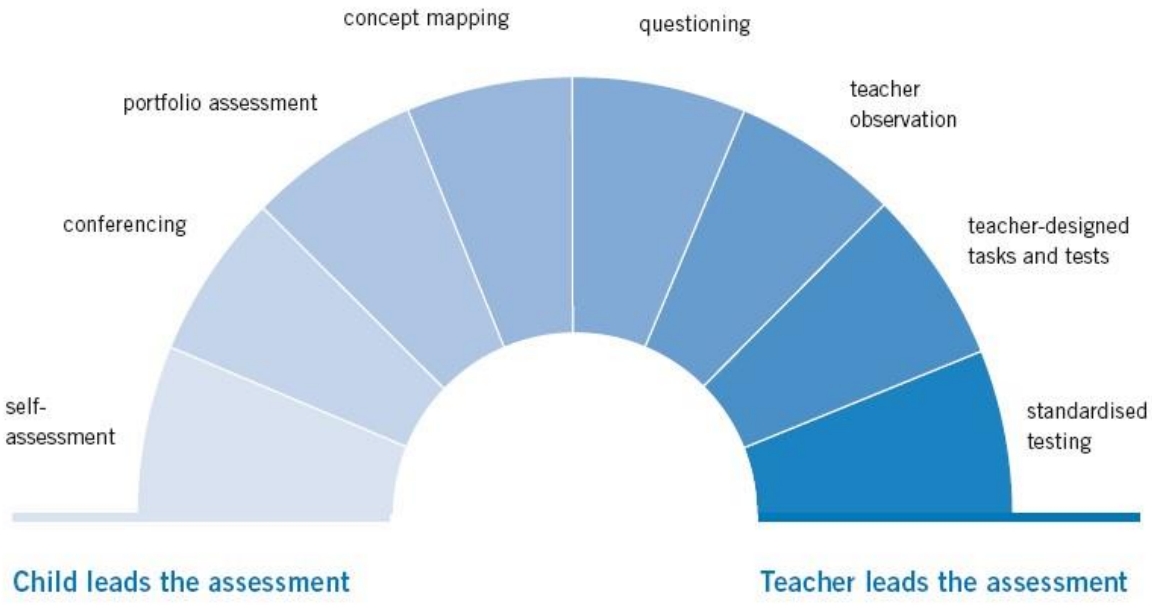
For most children, the content of each lesson on Data will remain similar, however, the support they receive can be adjusted. This can include more individualised support from the teacher, the use of a variety of materials to scaffold understanding, more opportunities to practise key vocabulary in context, use of appropriate ICT resources, greater links between special and mainstream teachers and home school links which espouse opportunities for practising real-life skills in Data. The way in which individual pupils demonstrate their learning can also vary, including, for example, the use of oral descriptions, pictures and concept maps, video, photographs and other forms of ICT to support assessment and learning. Many extension activities can arise from pupils' active engagement and self-reflection on learning, thereby enabling many pupils to ask further questions, create new problems, and develop more in-depth linkage with other aspects of mathematics and the wider curriculum. It is important to allow opportunities for all pupils to share their learning and strategies with peers and with the whole class, as appropriate. The way in which class groupings are organised, including individual, pair-work and group-work, will also enable differentiation of Data concepts, particularly in terms of opportunities for scaffolding pupils' learning.

[2] Westwood (2003).

ASSESSMENT OPPORTUNITIES IN DATA

A variety of assessment approaches is advocated in the Primary School Mathematics Curriculum (PSMC)[3] .

"Teaching and the gathering and analysis of assessment information should run concurrently, with the results of assessment feeding back into the teaching and learning process. Assessment should be a positive experience for the child, as this makes his/her learning more effective." [4] Multiple modes of assessment information garnered as pupils engage in the sample learning experiences described within this manual, will help to inform teaching and learning in Data. This assessment icon is used throughout the manual to indicate opportunities for teacher and pupil led assessment. These could include pupils taking photographs to document their work, the use of ICT to record and reflect on learning, a variety of samples of pupils' work, including strategies used to solve problems or demonstrate their learning. Individual and group outcomes could include projects combining other aspects of the curriculum, for example Geography, Literacy and Science.



A range of assessment tools, including pupil-led assessments, is recommended, in order to monitor pupil engagement and progress in learning. The use of journals in mathematics can be an effective pupil self-assessment tool, which can increase a pupil's awareness of how they learn and remember, as the art of writing is in itself a reflective process, so pupils have to think about the strategies and ideas used. Journals also serve to provide a record of thinking and learning and as a tool for supporting future conferencing with their teacher or indeed discussion with parents. Journals can also help pupils to identify challenges and set goals for future learning.

[3] NCCA (1999).
[4] NCCA (1999).

CLASSROOM CULTURE

Creating and maintaining the correct classroom culture is a pre-requisite for developing and enhancing mathematical thinking. This requires the teacher to:

- cultivate a 'have a go' attitude where all contributions are valued;
- emphasise the importance of the process and experimenting with various methods;
- facilitate collaborative learning through whole-class, pair and group work;
- praise effort;
- encourage pupils to share their ideas and solutions with others;
- recognise that he/she is not the sole validator of knowledge in the mathematics lesson;
- ask probing questions ;
- expect pupils to grapple with deep mathematical content;
- value understanding over 'quick-fix' answers; and
- use revoicing[6] (reformulation of ideas) as a tool for clarifying and extending thinking.

In this type of classroom pupils are expected to:

- share ideas and solutions but also be willing to listen to those of others; and
- take responsibility for their own understanding but also that of others.

[6] Revoicing is 'the reporting, repeating, expanding or reformulating a student's contribution so as to articulate presupposed information, emphasise particular aspects of the explanation, disambiguate terminology, align students with positions in an argument or attribute motivational states to students' (Forman & Larreamendy-Joerns, 1998, p. 106).

DATA - BACKGROUND KNOWLEDGE FOR TEACHERS

FUNDAMENTAL FACTS REGARDING DATA

What is Data?

'I'm 8 years old.', 'This bottle holds two litres of water.' 'She is wearing blue shoes.' 'This bag weighs half of a kilogramme'. Each of these statements contains descriptive information or data about a person or thing. **'Data'** is a plural noun; one datum is a single fact. Data are the facts or information that describe and differentiate people, objects or other entities. Data may be expressed as numbers, for example, 'she has four people in her family', or as other attributes 'her favourite flavour is chocolate'. Data can be defined as a collection of numbers or statistics in context. Collecting Data involves making detailed judgements on how to count, measure, describe and present. Data is a Strand on the Primary School Mathematics Curriculum and is first introduced to pupils at infant level. **Statistical literacy** is needed by all pupils to interpret the world and has been described as the 'cornerstone of present day numeracy'.

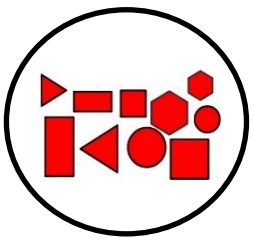
A **Set** is a well-defined collection of objects. Given any object it is possible to determine whether or not the object belongs to the set.

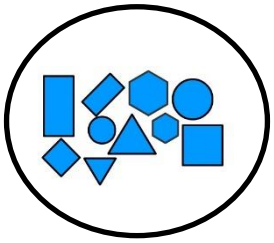
Universal Set

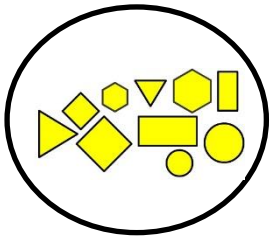
This is the set containing all items or members under consideration.

Variable

It is the variable that distinguishes between the members of the universal set. The variable can have a variety of **values**, thus enabling the sorting of the universal set into **sub-sets**. In this example, colour is the **variable**. It is used to distinguish between the members of the universal set. This variable has three values; red, blue and yellow. Three sub-sets can be created. The quantity of members in each subset is called the **frequency**. The frequency of red items is ten. The frequency of blue items is ten and the frequency of yellow items is ten. Using colour as the variable has led to the sorting of the set into three distinct separate subsets or '**discrete data'**.

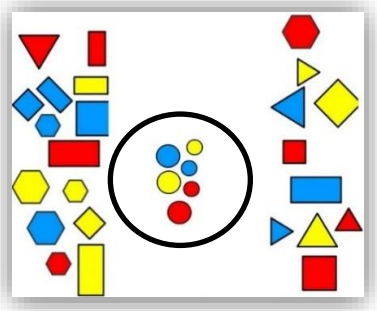






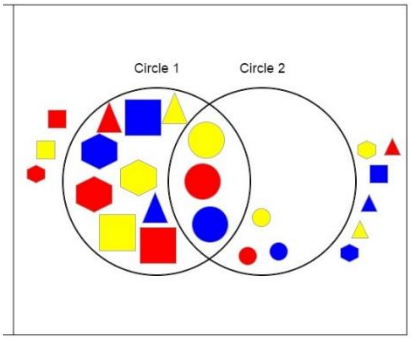
Set Diagrams

These are visual structures that aid children's understanding of the relationships between items in a set and the process of sorting a set into subsets. Pictured previously, is a set diagram known as a Venn diagram. This next example is another sort of Venn diagram. The variable this time is shape. Now there are two values; those that are circles, lying inside the

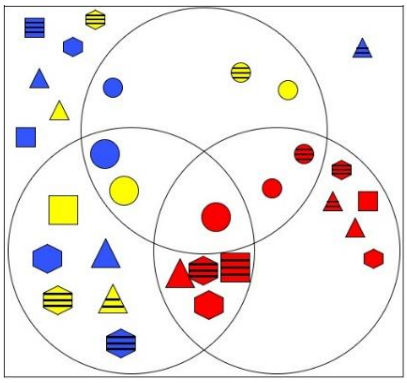


circle and those that are not circles lying outside the circle (also known as **the complement of the set**).

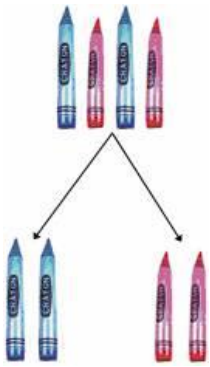
Use of Simultaneous Variables



An interesting example of sorting occurs when two variables are used simultaneously. In this Venn diagram the two variables chosen are Size and Shape. The values for size are big and small, while the values for shape are circle and not a circle. In the area where both sets overlap, called **the intersection,** the shapes that match the values of both variables are placed. In this example that is shapes that are big and circular. Outside the hoops are the complements of the set. Altogether the **universal set** when sorted this way has a total of three **subsets and the complement to the set;** items that are big, items that are circular, items that are big and circular, and items that are small but not circular (the complement to the set). Increasing the amount of simultaneous variables requires deeper levels of reasoning. In this diagram a **universal set** of shapes has been sorted simultaneously for the following variables; shape, colour and size. This has led to the creation of seven subsets and their complement.



Attribute and **property** are terms also used instead of **variable**. This set of crayons has been sorted according the **attribute** of colour. Two **subsets** have been created. A subset which share the attribute blue and a subset which share the **attribute** pink. There is now a **subset** of blue crayons and a **subset** of pink crayons.



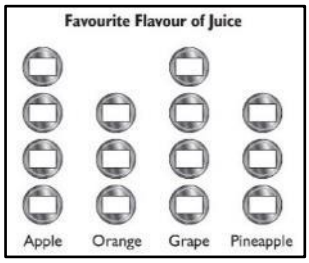
TEACHING NOTES

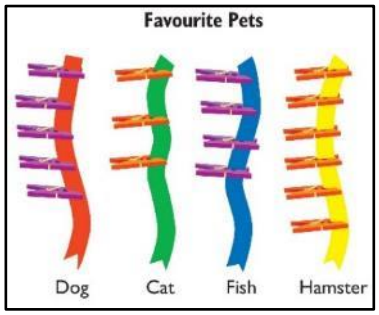
Communication is essential for learning. Having pupils work quietly and by themselves limits their learning opportunities. Interaction helps pupils clarify their ideas, get feedback on their thinking and hear other points of view
(Burns, pg. 16, 2004).

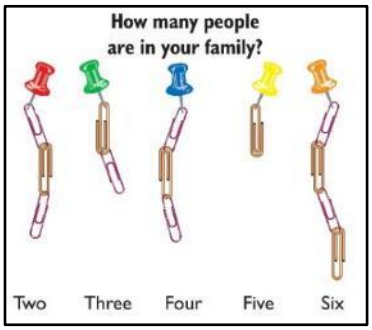
Sets that can be sorted according to one/two/three criteria	*Sets that can be sorted according to multiple criteria*
Attribute Bears Attribute Logic Blocks Attribute Beads Attribute Camels Unifix Cubes Coloured Counters Geometric Buttons Cotton Reels Coloured Lollipop Sticks Coloured Links	Collection of Clothes Collection of Socks Collection of Buttons Collection of Pencils Collection of Lids Found Collections Pupils Schools Bags Lunch Boxes

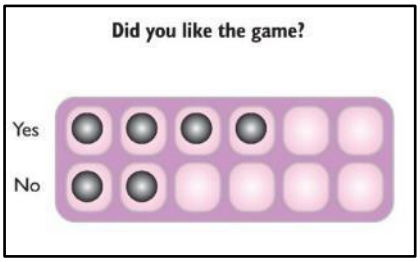
It is best to represent one set of Data in four ways, rather than four sets of Data in only one way. Questions asked should involve checking for understanding of each representation, comparisons and counting. If these questions are asked for each representation the same numbers turn up each time and this can be helpful in leading children towards conservation of number.[7]

There are many models that can be used to enable pupils to represent data. It is through exploring a variety of models that pupils will get to deepen their understanding of data. Various models display and communicate information in different ways. It is necessary to challenge pupils to consider the limitations of the models. Pupils should have a healthy scepticism about numerical information and this is a particularly useful disposition for pupils to have when interpreting data [8]'What does the model not show us?', 'What information could we show if we present the data in a different way?' [9]Questions such as these encourage pupils to consider the limitations of a set of Data. Every graph is only a partial representation of a topic or issue. The following are examples of various models that could be used to represent a set of data. Magnetised jar lids, an egg carton and a shower curtain with a grid drawn on it are some of the more inventive models of representing a set of data.





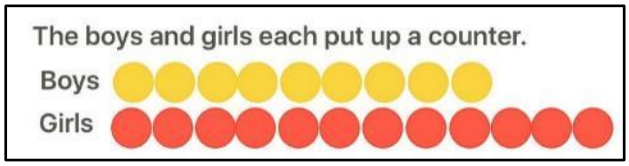


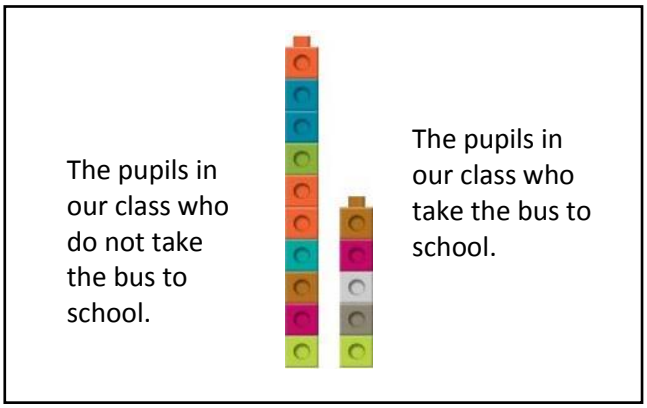


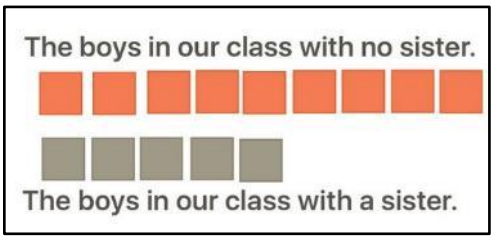
[7] (Deboys & Pitt, (1979, p.60).
[8] (Whitin, 2006).
[9] Whitin, (2006).

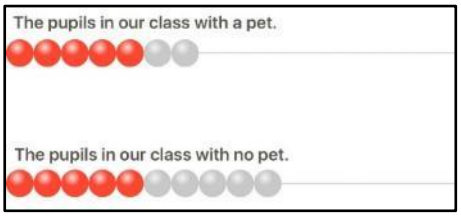
Concrete Experiences to Pictorial Representation

In the following pictures, counters, blocks, beads and tiles are used. This leads to a concrete representation of the set. The next step in each instance is to represent the ideas in pictorial form. The following are classroom comparisons which involve sorting a set to create subsets of two values and then representing that information pictorially. Each of these examples can and should be represented using a variety of models, with the pupils themselves creating the representations. Interactive tools can be effectively used to represent and record concrete representations pictorially—see interactive link for an example[10]. The arrangement and limited set of values allows for much discussion on 'rows' and or 'columns'.



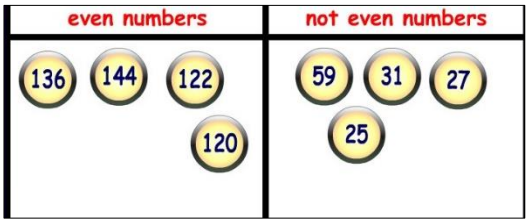






The activities suggested in the concrete representations can also be explored using Tree diagrams, Venn diagrams and Carroll diagrams.

A Carroll Diagram

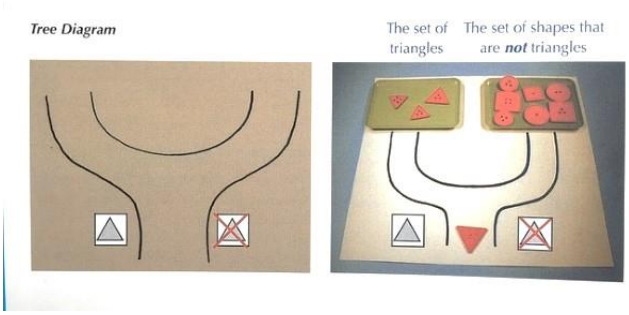


A Carroll diagram is a diagram that can be used to sort a set of items into subsets. It is a visual structure that provides another way to develop pupils' conceptual understanding of sorting a set, presenting a sorted set and analysing the relationship between its subsets. Carroll Diagrams use the principle of rows and columns to illustrate data that fit a criterion and data that does not fit a criterion. Deconstruction of Carroll diagrams is an important teaching strategy. An unlabelled Carroll diagram with the data already sorted can encourage pupils to think

[10] http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html

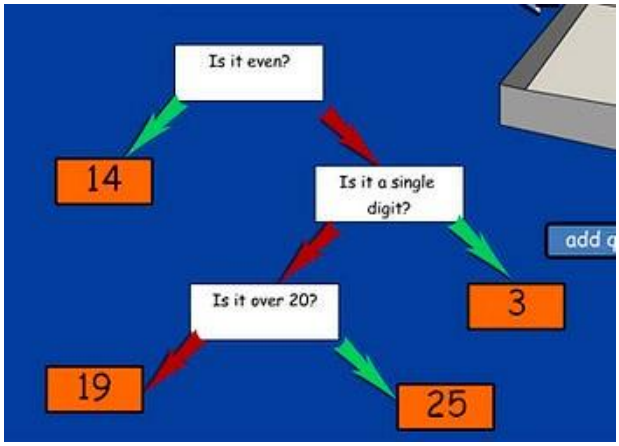
about the given attributes [11] In this image, the Carroll Diagram is used to sort a set according to one criteria, whether or not a number is even.

Tree Diagram[12]



A Tree diagram is a model which can be used to sort and represent a set of items. When using a tree diagram, work from the bottom of the tree up to a pair of branches. At the intersection of the branches, an item is moved to the left or the right, depending on whether or not it has the attribute under consideration. This method of sorting leads to a corresponding complement, for example, yellow and not yellow, or big and not big. Pictorial representation to define the attribute being considered is helpful. Tree diagrams are frequently used when sorting a set of shapes.

Decision Tree[13] A decision tree is used when two or more attributes of a diverse set are being considered. Decision trees are often used in Science to classify animals and plants and are called keys. The resulting subsets from the sorting involved in a decision tree, should lead to a means of defining a specific, limited, subset often of only one member. In a decision tree, the process of sorting a group of related items moves from the top down. Starting at the top, the collection of items is considered in relation to a closed question, continuing until small definitive subsets are created.



Types of Data

There are three main types of data covered in primary school. Analysis and presentation of the data hinges on the nature of the data collected.

Categorical Data

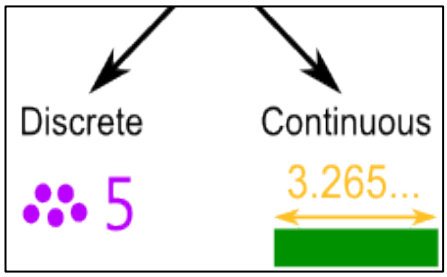
This is data grouped into subsets, with the number in each subset counted, for example, the colour of the children's hair, or, the way they come to school. In the junior and middle classes much of the data that interests pupils is categorical data, data that is based on counting the members of certain related categories.

Discrete Data

Only certain quantities can be obtained, usually whole numbers. Some examples of discrete data include age, house number, number of siblings. The subsets are discrete if there can never be a value intermediate between

[11] Turner and McCulloch, (2004).
[12] Deboys and Pitt, (1979).
[13] Suggate, J., Davis, A., Goulding, M. and Carroll, D. (2010).

them. For example, in the first Carroll diagram the data was sorted into discrete subsets, which were odd and even. In the second Carroll diagram there were intermediate values between the subsets '7 was odd and it was more than 6'

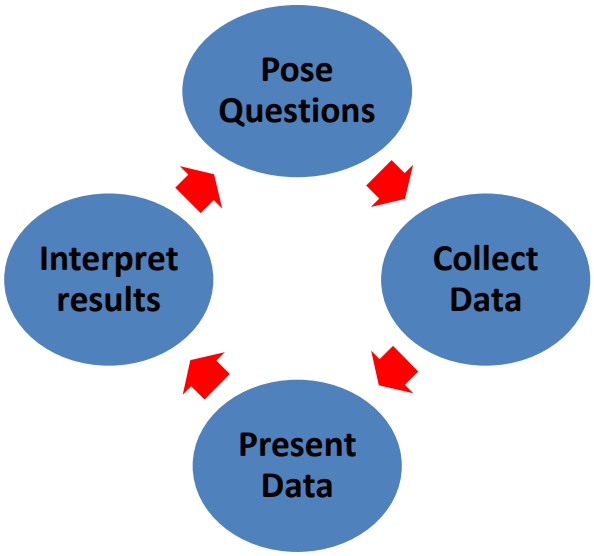


Continuous Data

Continuous **data** are not restricted to **defined** separate values, but can occupy any value over a continuous range. This type of data commonly involves measurement. Examples of continuous data include, a person's height which could be any value (within the range of human heights) not just certain fixed heights, the times in a race which could be measured to fractions of a second, or degrees of temperature.

The Data Handling Process

Many models of Data Handling are presented in mathematical literature.[14] [15] [16] Common to all is the premise that pupils should experience all stages of handling data and generally involves the stages of posing questions, collecting data, and presenting and analysing results from the data. This is a model which could help pupils understand this process.



Posing Questions:

The first stage in the data collection process is deciding on the point of inquiry, as in to formulate questions. Data collection should be 'for a purpose, to answer a question in the real world.'[17] Questions will often arise from the work pupils are engaged in or topics relating to them, for example; How many children in this class? When pupils formulate the questions, the data they gather becomes more meaningful.

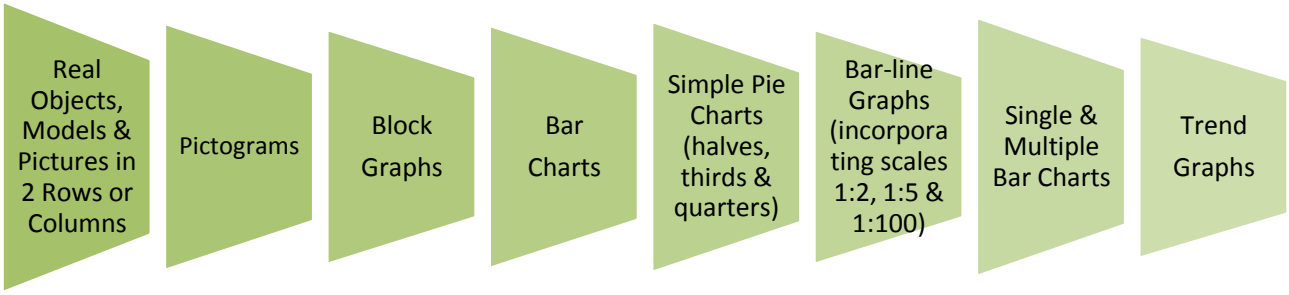
[14] Suggate, J., Davis, A., Goulding, M. and Carroll, D. (2010).
[15] Van De Walle, Karp and Bay Williams, (2013).
[16] Haylock, D., and Manning, R., (2014
[17] Van De Walle, Karp and Bay Williams, (2013).

Collecting Data

In the PSMC, collecting data is introduced to pupils from 3rd – 6th class. This stage of the data handling process is an excellent opportunity to discuss issues such as effective questions, accuracy, consistency and fair sampling, for example, discuss with pupils, is it better to ask 'What is your favourite food?', or 'Which of these foods (showing a list of about 6 or 8) is your favourite?' The latter is easier to analyse but does not yield a true picture of a pupil's favourite food, because their choice is constrained. Pupils can decide to either collect the data themselves or use pre-existing data.

Present Data

When presenting the results of the data collection process, pupils should be involved in deciding the mode of representation. The 1999 PSMC refers to the following modes of representation for presenting data:



Interpret Results

Once a graph is constructed, the most important activity is discussing what it tells the pupils who see it, especially those who were not involved in making the graph. Questioning that focuses on statistical thinking is a recommended way of assisting pupils in interpreting data gathered. The following is a list of some questions to stimulate meaningful discussions on interpreting data

What do the numbers (symbols) tell us about our class? If we asked another class, how would our data look? How do the numbers in Graph A compare to this graph? What does the graph not tell us? What might we infer? What new questions arise from these data?

At the end of the data handling process, a review of achievements, problems encountered and questions arising should be conducted. Questions about the reliability and validity of data gathered can also be discussed at this stage.

PUPIL MISCONCEPTIONS REGARDING DATA[18]

Pupils' conceptual ability to analyse data and draw conclusions and interpretations is often weak, so work is needed to emphasise this higher order skill. Note should be taken of common misconceptions in constructing and interpreting bar charts and pie charts, explored in detail in Levels C1 and C3.

[18] Van De Walle, Karp and Bay Williams, (2013).

DATA - LEARNING TRAJECTORY

The learning trajectory is based on the objectives for DATA in the Primary School Mathematics Curriculum. In some instances, similar objectives at the same class level have been collapsed into one objective. Objectives that only refer to problem solving have not been included as discrete objectives because a problem solving approach is advocated throughout all of the teaching and learning experiences. Problem-solving is viewed in this manual as a fundamental, integral part of mathematics teaching and learning that pupils should experience every day. The same colour coding from the curriculum is used – infants (green); first and second (red); third and fourth (blue); fifth and sixth (orange).

DATA LEARNING TRAJECTORY LEVEL A[19]

Trajectory Levels	Concept	Developmental Experiences Concrete → Pictorial → Abstract		
	Level A.1 Sort, classify, compare, match and discuss sets of objects by one and two criteria and by equivalence			
	Level A.2 Represent, interpret and discuss a set of simple mathematical data using real objects			
	Level A.3 Represent, interpret and discuss a set of simple mathematical data using models, diagrams and pictures in rows or columns			

[19] This level is generally aligned with the objectives for Junior and Senior infants.

DATA LEARNING TRAJECTORY LEVEL B [20]

Trajectory Levels	Concept	Developmental Experiences Concrete → Pictorial → Abstract		
↓	**Level B.1** Sort and classify sets of objects up to three criteria			
	Level B.2 Represent and interpret a set of data using real objects, models and pictures			
	Level B.3 Represent, read and interpret data in two, three or four rows or columns, tables, pictograms and block graphs			

[20] This level is generally aligned with the objectives for First and Second class.

DATA LEARNING TRAJECTORY LEVEL C [21]

Trajectory Levels	Concept	Developmental Experiences		
		Concrete	Pictorial	Abstract
	Level C.1 Collect, organise, represent and interpret data using pictograms, block graphs, bar charts and tables			
	Level C.2 Collect, organise, represent and interpret data using pictograms, block graphs, and bar charts incorporating the scales 1:2, 1:5, 1:10 and 1:100			
	Level C.3 Read and interpret data using bar-line graphs and simple pie charts involving halves, thirds and quarters			

[21] This level is generally aligned with the objectives for Third and Fourth class.

DATA LEARNING TRAJECTORY LEVEL D[22]

Trajectory Levels	Concept	Developmental Experiences		
		Concrete	Pictorial	Abstract
	Level D.1 Compile and use simple data sets and calculate averages			
	Level D.2 Represent, read, discuss and interpret data using pictograms, simple pie charts and single bar charts			
	Level D.3 Represent, read, discuss and interpret data using multiple bar charts and trend graphs			
	Level D.4 Explore frequency analysis and calculate averages of simple data sets			

[22] This level is generally aligned with the objectives for Fifth and Sixth class.


SAMPLE
TEACHING AND LEARNING EXPERIENCES
ASSESSMENT OPPORTUNITIES
DATA LEVEL A

LEVEL A.1

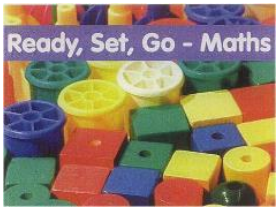
SORT, CLASSIFY, COMPARE, MATCH AND DISCUSS SETS OF OBJECTS BY ONE AND TWO CRITERIA AND BY EQUIVALENCE.

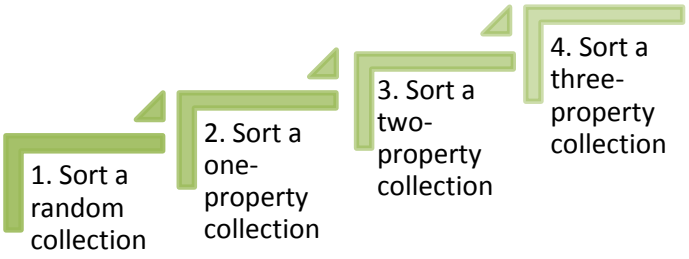
TEACHING NOTES

Young pupils need many opportunities to sort various types of sets. These sets should include items of interest to pupils, thus increasing motivation, encouraging insightful questions and answers and sustaining engagement. Describing, generalising, conjecturing and wondering are real-life mathematical practices that young pupils should engage with when exploring a data set. Such practice, with real-life items, will enable young pupils to develop descriptions and theories about aspects of the real world. The 1999 Primary School Mathematics Curriculum advocates use of the following criteria when sorting a set: colour, shape, size, texture, function and quantity. Therefore it is important to ensure pupils explore each property in depth, for example the following word-bank may be a useful reference when discussing 'texture' and similar word-banks could be developed with the pupils as 'living-charts' for the other attributes.

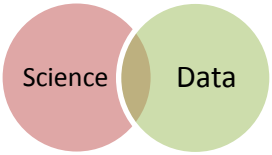
It is important also to use the pupils themselves for sorting, for example, get pupils to move to different corners of the classroom depending on their answers to questions like 'What age are you?', What class are you in?' (Multi-grade context), 'Have you a drink with you today?' Next do the same type of sorting by getting pupils to stand in circles drawn on the playground, or physically walking along chalk markings drawn to represent a tree diagram, before moving onto simple examples of set diagrams.

The Ready-Set-Go[23] Mathematics programme advocates an incremental approach to sorting experiences, following this line of development:





To support children's early experiences in sorting, use the word *set* frequently to help children build a foundation for future understanding and use of this concept. (Ritz, pg. 42, 2015).



SAMPLE LEARNING EXPERIENCES

Mother's Button Box[24]

This activity involves pupils sorting buttons using sectioned containers, for example, an egg carton.

1. Provide pairs with an egg carton, a group of 15 buttons and a magnifying glass.
2. Give pairs time to examine their collection of buttons and elicit from pupils ideas about which buttons are alike and what differences exist amongst their collection.
3. Invite pupils to use the magnifying glass to make more detailed observations of their buttons.
4. Elicit ideas from pupils on how the set could be sorted.
5. Challenge pupils to sort the set by placing buttons into sections in the carton, for example, sorting according to size, shape, number of holes. Elicit criteria from pupils for sorting. Facilitate pupils in sorting according to their own criteria.
6. After each 'sort' is complete encourage the closing of the carton, shaking of the set of buttons, re-opening and placing the set of buttons onto the empty tray before sorting them according to the new criteria into compartments. This will give pupils practical experience in recombining a set. In this image a large collection of buttons has been sorted according to colour.

[23] Pitt, E. (2000).
[24] Ritz, W. (2015).

What did you notice about your set of buttons Jack? In the egg carton how did you arrange the set? Tell me about the subsets you made. When you sorted for size Marie, did you create many subsets? What noise do you hear when you close the carton and recombine the set of buttons?

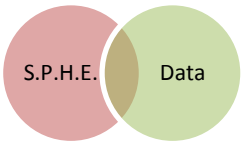
Texture Match-Ups

Different square swatches of materials are required for this activity; corduroy, velveteen, faux fur, wool, leather, felt etc. Fabric shops or second-hand shops may be a useful source for materials.

1. Display materials for pupils to explore and touch and encourage pupils to create ways to sort the fabrics.
2. Elicit from pupils different ways of sorting the fabrics.
3. Invite pupils to investigate their sorting method. Once a pupil has sorted the swatches in a certain way, encourage pupils to create questions based on that arrangement.
4. Pupils could also order the fabrics, for example, most soft to least soft, rough to smooth, largest to smallest etc.[25]

Sorting Pupils into Sets

Pupils can be sorted into sets using various criteria for example;



- The set of children who are five
- The set of children who had bread for lunch
- The set of children who came to school in a car

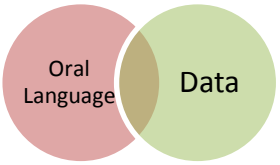
A mat, a large hoola-hoop or a ring of brightly coloured chalk all provide ways to mark out a set.

Click the video icon to view senior infants in an Irish Primary School where eye colour is used as the criteria for sorting pupils. Note the language that is used and the multiple ways in which the pupils record the sets. In each case pupils possessing the required characteristic stand within the defined area and are consequently part of that set. [26]

When sorting pupils into sets, it is important to also draw attention to pupils that are the complement of the set i.e. those that belong in the universal set, but not in the subset because they are not five, or, they did not have bread for lunch, or, they did not come to school in the car.

[25] Burns, M. (2004).
[26] http://pdst.ie/Good%20Practice%20Videos

Guess My Rule[27]



This is a classification guessing game in which players try to figure out the criteria for which a set of items has been sorted. Pupils involved are sorted into a set which is defined by a hoola-hoop or chalk circle on the floor.

1. Explain to all pupils that you have a secret rule in mind that some pupils in the class follow.
2. Write or draw on a small concealed whiteboard the rule pupils must follow in order to be part of the set, for example, 'Wears runners'.
3. Get pupils to stand near the hoop, one by one, invite pupils to place their foot into the hoop. The teacher then confirms whether or not the pupil can stand into the hoop based on the rule that the teacher has concealed.
4. When enough evidence has been gathered, elicit guesses on what the rule might be and reasons to support the guess.

Where should the pupils that are not part of the set stand? How could this help us discover the secret rule? Which has more, the subset of pupils that follow the rule or the subset of pupils that don't follow the rule?

LEVEL A.2

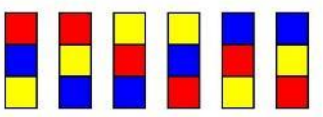
REPRESENT, INTERPRET AND DISCUSS A SET OF SIMPLE MATHEMATICAL DATA USING REAL OBJECTS

SAMPLE LEARNING EXPERIENCES

Three Block Tower[28]

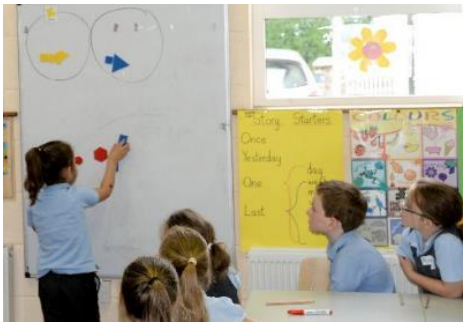
In this activity pupils use three coloured cubes to make as many different towers as they can. The key question is, "how do you keep track of the different towers?" Pupils could use actual coloured cubes, or draw them, or imagine them. Regardless of how the different coloured cubes are represented, what is helpful is some way to account systematically for all possibilities. This is a key concept for tasks that involve counting. It is helpful to let pupils realise for themselves the need for organising their data, and then letting them figure out a strategy.

1. Distribute three different coloured blocks to each pair of pupils.
2. Pupils investigate how many different 'three block tower' they can make.
3. Encourage the pupils to talk about the differences in each set of towers with their group.
4. Elicit why this is difficult (pupils may find it difficult to recall each representation, therefore elicit how to overcome this challenge, for example recording by drawing or uisng ICT as they proceed.



5. Elicit from pupils whether they think they have discovered all possible combinations; "Have we got them all? How can we be sure?

Sorting Logic Blocks[29]



1. Provide each pupil in the group with a logic block and allow small groups to explore and talk about their blocks.
2. Elicit from pupils their observations about the set of logic blocks in their group.
3. Pupils explore ways they can sort the blocks.
4. Pupils record ways in which they sorted the logic blocks.

[28] https://www.youcubed.org/task/3-block-towers/
[29] http://nrich.mathematics.org/6032

The Gate Game

1. Assign two 'Gatekeepers' who select an attribute block, and decide on which property to focus on for example 'thick' which they then conceal behind the 'gate'. This selected property of the attribute block defines the subset of logic blocks allowed pass through the gate.
2. Provide pupils in the class with a logic block which is their 'ticket' to get through the gate. Explain that the gatekeepers (two pupils holding the gate) have a rule to allow you through the gate or not. Two subsets will be created as pupils try to pass through with their ticket, those that share the characteristic of the hidden attribute block and those that do not.
3. After a few turns elicit ideas from pupils as to what possible property the gatekeepers have on the concealed attribute block this time.

Extension Activity

To further extend pupils' learning, attribute cards with more than one property could be used, for example, 'thick and red', or three properties, such as 'thin, yellow circle' etc.

CONSOLIDATION ACTIVITIES

Logic Block Collections[30]

This is a link to a collection of blocks. The blocks can be dragged around the screen and pupils can engage in different ways of sorting the collection. Pupils could also play a 'Guess my Rule' style game using the collection of blocks.

ICT Link
Logic Collection

[30] http://nrich.mathematics.org/6032

LEVEL A.3

REPRESENT, INTERPRET AND DISCUSS A SET OF SIMPLE MATHEMATICAL DATA USING MODELS, DIAGRAMS AND PICTURES IN ROWS OR COLUMNS.

TEACHING NOTES

Pictorial representation provides an interesting and attractive means of developing and reinforcing the following mathematical concepts:

- one-to-one correspondence
- comparisons of the language of inequalities
- conservation of number
- language of sets
- general computation based on particular data.

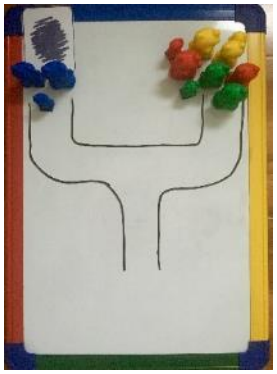
One-to-One Correspondence provides for the simplest form of pictorial representation. Initial comparisons should only involve two rows or columns and if possible all pupils in the class. The real value of such comparisons will be determined by the resulting discussion. When making such comparisons, provide pupils with objects to represent their position in the set.

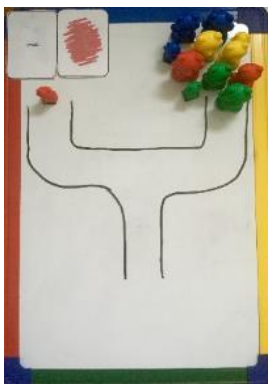
SAMPLE LEARNING EXPERIENCES

Sorting Attribute Bears using a Tree Diagram

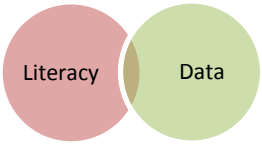
Attribute bears can engage young pupils in deep levels of sorting. The following images demonstrate how attribute cards can be used in conjunction with a set of bears and a sorting diagram; the Tree Diagram. An 'X' can also be used to denote where the object does not meet the criteria.

1. In the first image below, bears are sorted according to whether or not the bears match the property (colour) on the attribute card (purple).
2. In the second image below, bears are sorted according to whether or not the bears match the two properties (colour and size) displayed on the attribute cards (small and red).
3. In the third image below, bears are sorted according to whether or not the bears meet the criteria (yellow).

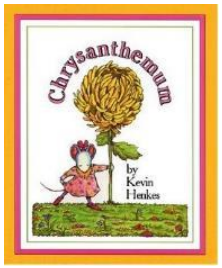




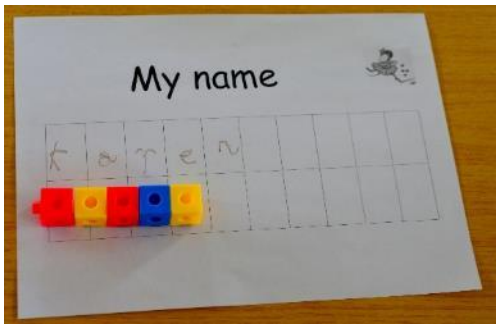
Chrysanthemum and the Name Trains



This activity can be used to help pupils investigate 'real data', for example the letters in their names.



1. After reading the book to pupils, discuss the name of the main character 'Chrysanthemum' and the letters in her name,
2. Elicit what pupils notice about the letters. Displaying the alphabet, mark off the letters in the name Chrysanthemum.
3. Create a name train for Chrysanthemum using blocks.
4. Challenge pupils to show the quantity of letters in their own name using cubes; their own name train.
5. Teacher models comparing his/her name with Chrysanthemum.
6. Elicit pupil responses on comparisons between their names and Chrysanthemums.
7. In pairs, pupils can find out from their partner how many extra letters their partner would need to make a name that matches Chrysanthemum.

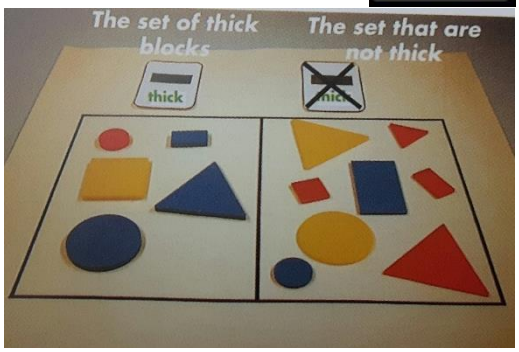


Who has the longest name in your group? Who has the shortest name in your group? Who in your group has one extra cube than you? Why do they have an extra cube?

This video clip focuses on the above learning experience with a mixed junior and senior infant class, in an Irish school.[31].

Sorting Logic Blocks[32]

The use of Carroll Diagrams to represent a set of mathematical data can add a further dimension to pupils' sorting experience. Logic Blocks are a more abstract form of representation and will challenge pupils to carefully consider the properties of the blocks in questions.



1. In this image, Logic Blocks are sorted according to whether or not the blocks match the property (thickness) on the attribute card (thick).

[31] http://pdst.ie/Good%20Practice%20Videos
[32] Pitt, E., (2000).

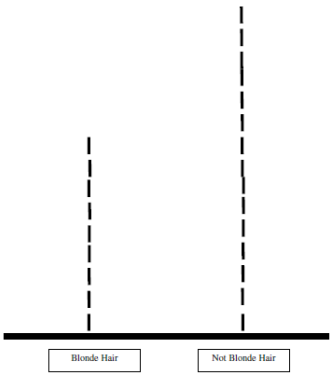
Further Development

Pupils can progress to drawing pictorial representations of the above sorting activity once ample experience of sorting with concrete representations has been provided.

Where Should I Put My Lollipop Stick?[33]

This activity provides pupils with an opportunity to represent data in columns and rows using real objects and pictures.

1. Clear a large floor area and put a metre stick on the floor to make the base line for a chart. (The pupils will later use their lollipop sticks to build columns on this base line.)
2. Provide each pupil with a lollipop stick.
3. Place the 2 labels which describe the sorting criteria, underneath the base line, to mark out where the 2 columns will be i.e. the label 'Blonde Hair' marks out the first column and the label 'Not Blonde Hair' marks out the second column.
4. Each pupil must decide which category he/she belongs and then place his/her lollipop stick above the appropriate label, building up a column of single lollipop sticks.



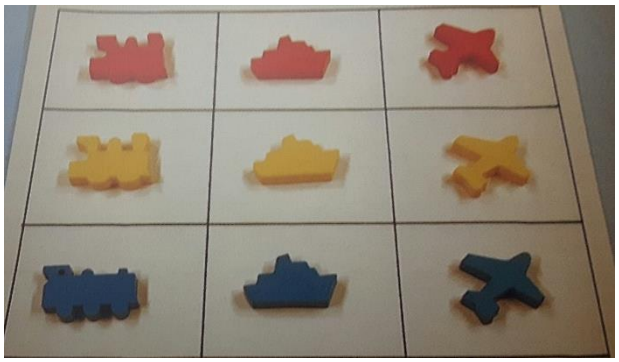
5. Once all pupils have recorded their hair colour, elicit the key features of the graph from pupils.
6. The activity can then be repeated to include a focus on **rows** instead of columns.
7. Rearrange the metre stick so that it lies in a vertical line on the floor. This will form the base line for the new chart.
8. Using the same criteria, pupils use their lollipop stick to represent their hair colour.
9. Elicit from pupils the difference between the two modes of representation.

CONSOLIDATION ACTIVITY

The Missing Piece [34]

This activity provides pupils with an opportunity to interpret concrete data arranged in rows and columns.

1. Provide pairs of pupils with a simple structured two – property collection on a grid as illustrated in the image below. In the image below the rows are determined by colour and the columns are determined by type of vehicle for example row 1 contains a red train, a red boat and a red plane.



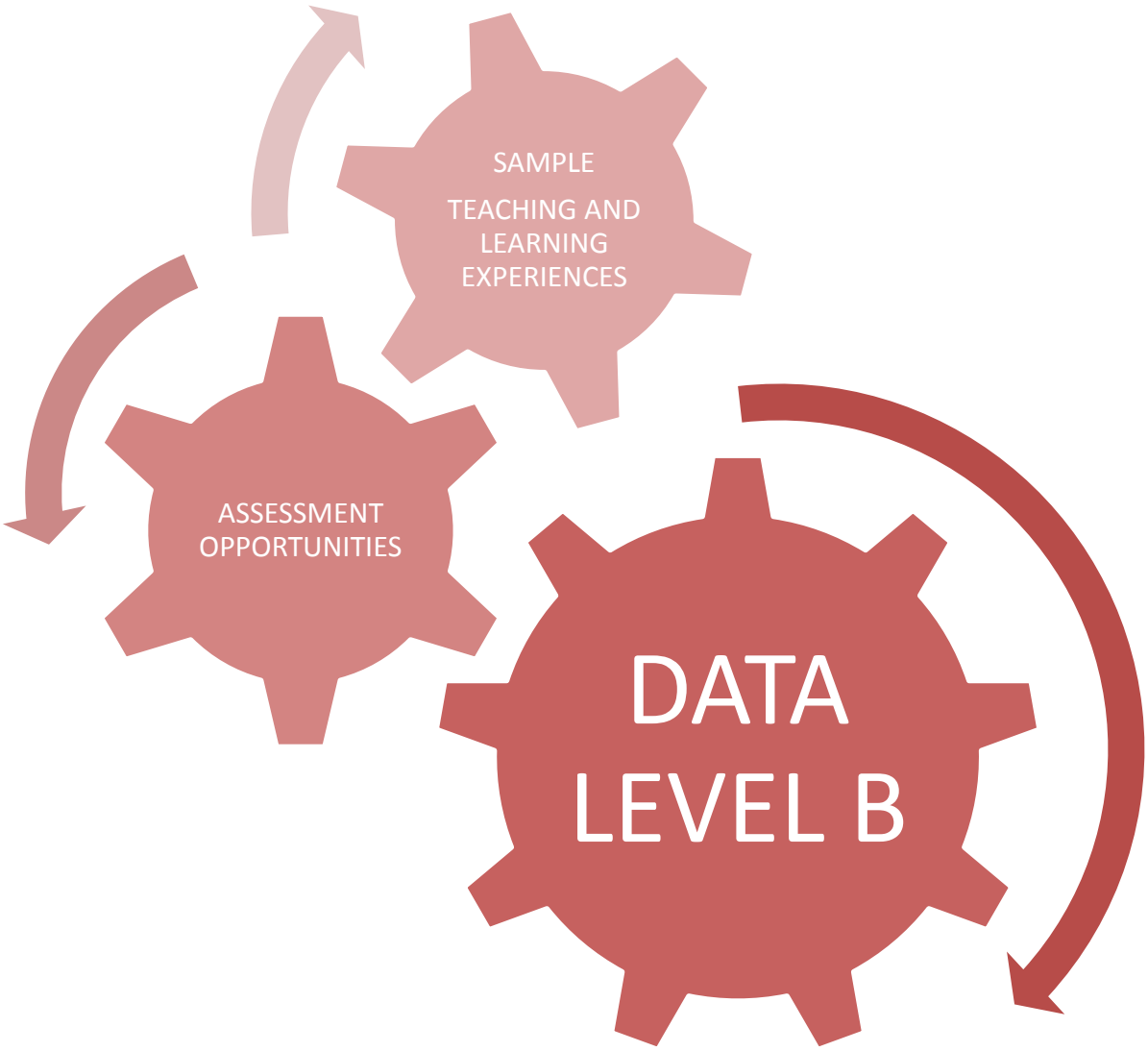
[33] https://www.scoilnet.ie/uploads/resources/13141/12778.pdf
[34] Pitt, E. (2000, p.32).

2. Pupils take turns to close their eyes while the other pupil removes one piece. The first pupil then opens his eyes and describes the missing piece. The missing piece is then replaced on the grid.
3. Encourage pupils to use the language of rows and columns as they describe where the missing piece should be located.

Mary, what do you notice about the collection? Is there anything missing? Let's examine the rows and columns. Where do you think the missing piece should go?

Further Development:

The size of the grid can be increased to provide pupils with a greater level of challenge for example extend grid to a 3 x 4 arrangement.


SAMPLE
TEACHING AND LEARNING EXPERIENCES
ASSESSMENT OPPORTUNITIES
DATA LEVEL B

LEVEL B.1

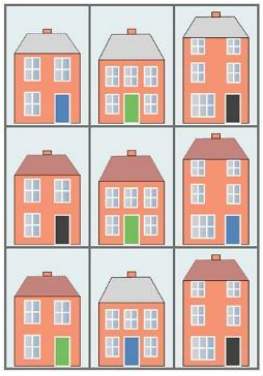
SORT AND CLASSIFY SETS OF OBJECTS UP TO THREE CRITERIA

SAMPLE LEARNING EXPERIENCES

Sorting Logic Blocks

This activity is outlined in Level A2 and at Level B1, can be extended to sort and classify logic blocks, up to three criteria.

Sort The Street[35]



In this activity pupils sort the houses on the street according to a variety of criteria.

1. Show pupils the picture of the houses. Pupils share what they notice with their partner.
2. Elicit from pupils, their partner's observations. Record these alongside the pupil's name on the board.
3. Explain that you want to find out the maximum number of ways these houses can be sorted.
4. Elicit from pupils ways in which they can be sorted and how the data can be recorded.
5. Provide a set of houses for pairs of pupils (These houses can be printed from the ICT link). Pupils begin to explore and sort the houses into groups. As pupils work in pairs or small groups there are opportunities to assess pupil understanding through teacher observation and questioning.

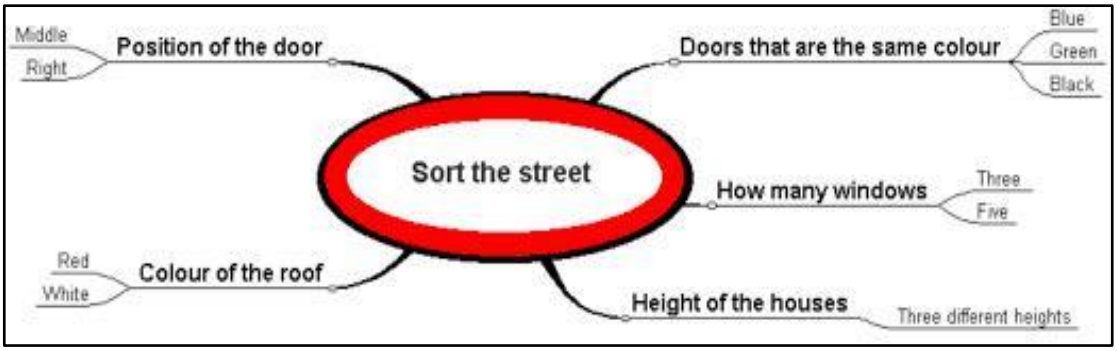
ICT Link
Sort the Street

6. Pupils record their different groupings in some way, for example, by making a drawing on their whiteboard. The mind-map image below shows a variety of criteria pupils may choose to use when sorting the houses.
7. Once pairs feel they have created a definitive list of criteria for sorting the houses, invite pairs to present and explain their ways of sorting the houses.

   There is an opportunity here to revisit models used earlier for sorting the houses, for example, the tree diagram, the Carroll diagram and the Venn diagram. See Level A.2 for more details

What is the same about these houses? Are there any others like that? Why have you grouped the houses in this way? What attribute do all houses in this subset share? Which subset has the greatest number of houses?

[35] https://nrich.mathematics.org/5157



Exploring and Sorting Pattern Blocks [36]

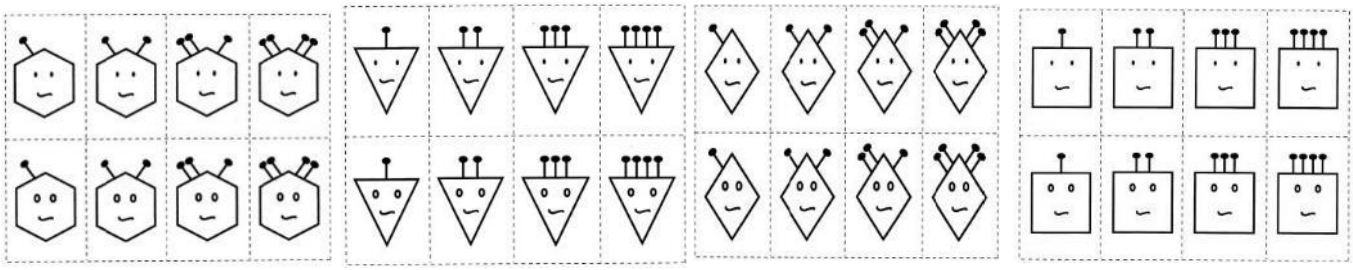
The learning experience above, can be repeated using pattern blocks instead of houses to sort and classify sets of objects up to three criteria. Interactive pattern blocks are available at the following link.

The PDST Shape and Space resource also explores the use of Pattern Blocks. Click the image to open this manual[37].

Yektees Investigation[38]

ICT Link
Interactive Pattern Blocks

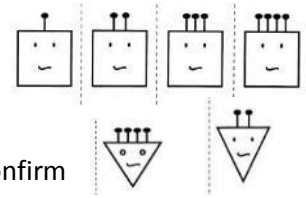
This investigation takes a trip into fantasy with a look at fantastical creatures called Yektees. These can be sorted according to three different criteria; head shape, eye type and antenna quantity. The 32 Yektee cards are essentially an attribute set, that is, a set of things that can be sorted and classified according to the characteristics of the set. (Appendix level B.2)



1. Read the story of 'Liam, Ann and the Yektees'. Then using the set of large Yektee cards
2. Show pupils a picture of one Yektee.
3. Elicit what pupils notice about this Yektee.

[36] Pattern blocks contain six different geometric shapes which are: hexagons (yellow), squares (orange), trapezoid (red), triangles reen), parallelogram (blue), rhombi (beige).
[37] http://www.pdst.ie/Shape-and-Space
[38] Russell, S.J. and Corwin, R.B., (1990).

4. Gradually reveal more Yektee cards and as the pupils develop their descriptions, encourage discussion on commonalities and differences amongst the set.
5. Challenge pupils to figure out the special features of these creatures. Through questioning pupils gradually infer the properties of the Yektees: the four head-shapes, the four numbers of antenna and the two types of eyes. (Pupils may have difficulty thinking of all the attributes at once. Therefore revealing Yektee cards that share the same head shape first, for example, reveal four square-headed Yektee cards followed by two triangular-headed Yektee cards).
6. After pupils have seen quite a few of the Yektees, challenge pupils to describe a Yektee that you may have in your set. Eventually pupils may specify completely all three attributes of the Yektee they want to see.



7. Encourage pupils to think about a way of sorting the Yektees so we can identify exactly which Yektees are missing.
8. Provide pupils with their own set of Yektee cards to explore. These cards will also confirm for pupils whether the Yektees they thought were missing, actually exist or not. They will also provide opportunities for pupils to analyse the Yektees further and notice their subtle and varied attributes.

9. In pairs pupils could summarise and record attributes that are the same and attributes that are different for all Yektees on their mini-whiteboards.

Exploration and discussion of Yektees and their 'fantastical' nature may lead to interesting opportunities for creative writing about Yektees. Here are samples of 'Yektee descriptions written by pupils aged eight.[39] At Level B.2 Yektees are used to further explore Venn Diagrams and Tree Diagrams.

[39] Russell, S.J. and Corwin, R.B. (1990).

Liam, Ann & the Yektees

Liam and Ann were 8-year-old twins. One day on their way to school they discovered some strange creatures near their home. These creatures were living in a burrow next to a path that the twins used as a shortcut on their way to school. Liam and Ann started studying these creatures and visited them every chance they had. Because these creatures never came all the way out of their burrow in the ground, Liam and Ann could see only their heads. The creatures looked as though they might have come from another planet. The creatures made a funny sound when they came up out of their burrows 'Yek'. The twins decided to call them 'Yektees'. Ann made sketches of the heads of each of the Yektees she had seen. She noticed that a lot of them were similar to each other, but that ***no two were exactly alike****. They discovered* ***32 Yektees*** *lived in the burrow.*

My Yektee's name is Mimi. It lives on a different planet that is far away. She only eats salad. She is a princess. On her planet they play nintendo for fun. They get electricity from their antennae. She has four antennae. All Yektees are peaceful.

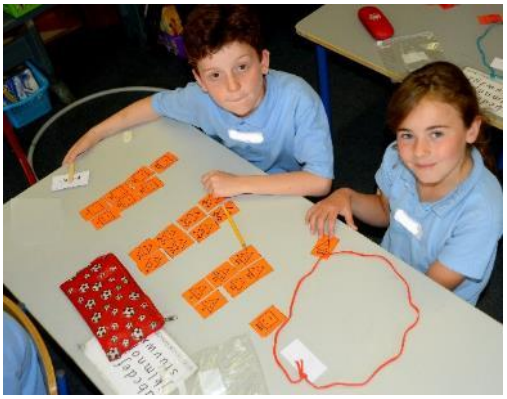
Yektees eat a strange kind of food. It is called mingilems. It is different colours. They live on a place cold and damp. No scientists know what it is called but I do. It is called Otulp. My Yektee's name is Rocky. The End.

LEVEL B.2

REPRESENT AND INTERPRET A SET OF DATA USING REAL OBJECTS, MODELS AND PICTURES

SAMPLE LEARNING EXPERIENCES

Venn Diagram: Guess My Rule With Yektee Cards

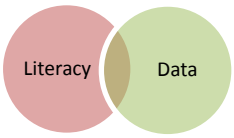


In this activity each group plays 'Guess the Rule' with Yektee Cards.

1. Give each group a set of attribute classification cards, a set of Yektee cards and a loop, for example a tied shoelace (Appendix B.2). The loop will help them define an area for the subset.
2. A pupil in the group selects an attribute card from the deck of attribute cards, reads it, and turns it face down inside the loop.
3. Pupils take turns asking that pupil with the attribute card (the 'Ruler'), does this Yektee with two antennae fit your rule? Or can this Yektee be part of your subset? If it does, the card is placed into the subset, if not, it stays outside the subset. The object of the game is to guess the rule.

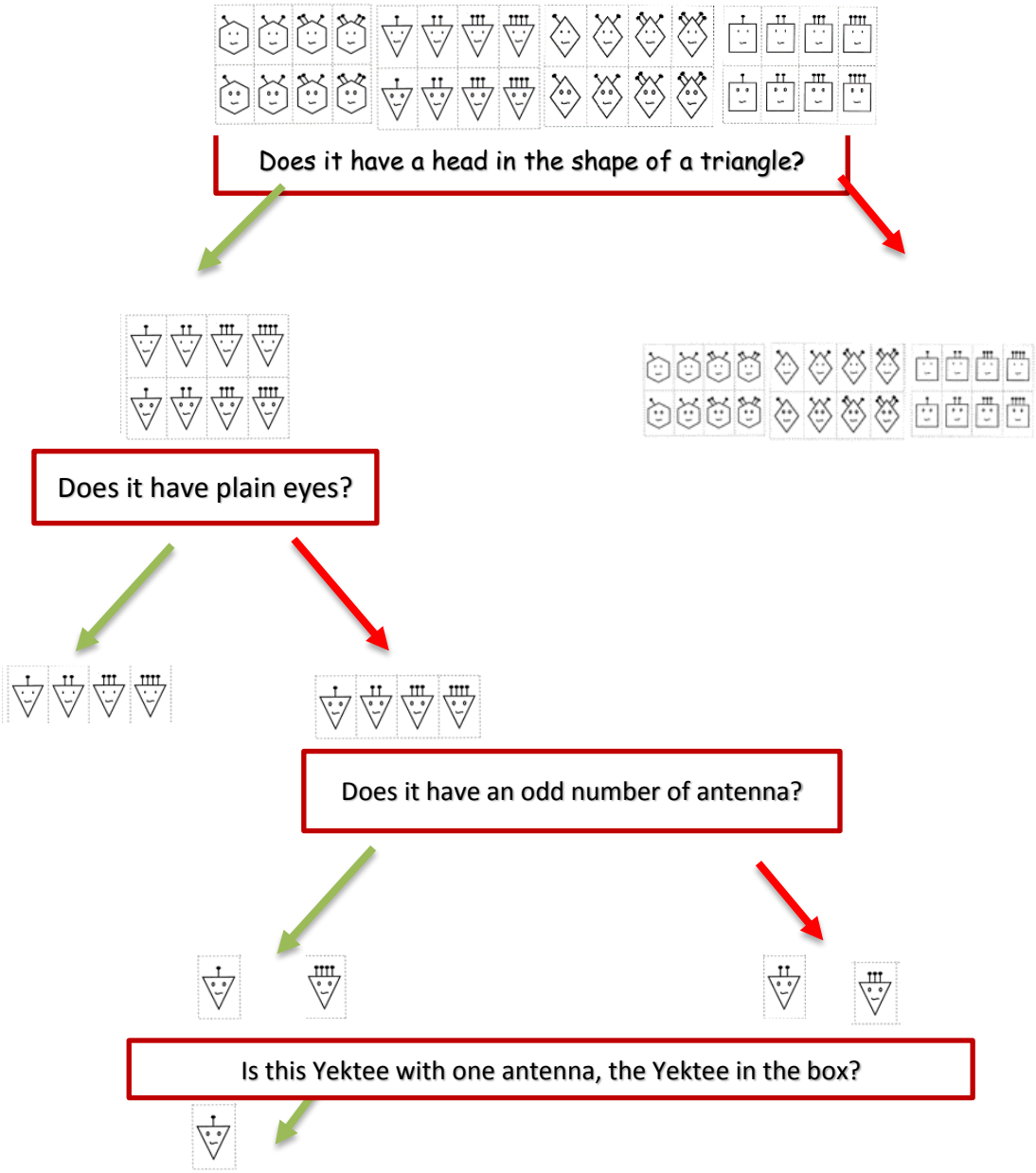
Remember the cards that are the complement to the set, i.e. not part of the subset, provide important information too. Sometimes pupils that chose the rule do not realise that all the evidence needs to remain visible to the guessers. Help pupils establish a place to put the 'does not fit' cards so everyone in the group can see them.

Tree Diagram: Guess The Yektee In The Box



1. Explain to pupils that you have placed a Yektee card in a box and you want them to guess which Yektee is in the box, using a decision tree. This tree could be generated on the board as pupils pose questions to guess what Yektee you have selected.
2. As questions are posed, record these on the tree and give the 'yes' or 'no' response to the question.
3. Invite pupils to come up and rearrange the Yektees, placing the Yektees under consideration to a more suitable location on the tree and also eliminating some options.

The following is an example of a decision tree which was used to identify the Yektee in the box. On this occasion it was a triangle-headed Yektee with ringed eyes and 1 antenna that was in the box.



Sorting with Carroll Diagrams[40]

This activity provides pupils with an opportunity to sort data according to a particular property and to also consider more than one attribute at a time. Click on the interactive link to access this problem and resources to support its exploration.

ICT Link
Carroll Diagrams

	Between 10 and 20	Not between 10 and 20
Even tens digit		
Odd tens digit		

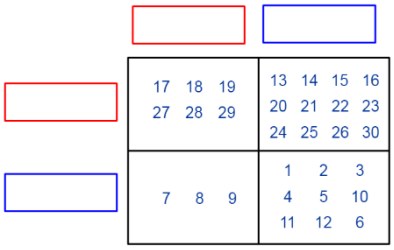
Put the numbers from one to thirty into the right boxes

[40] https://nrich.maths.org/5729

Further Development

ICT Link
Missing Labels

In this interactive activity, the Carroll Diagram has numbers in place, but no labels for the cells. Elicit from pupils what the categories could be. Encourage pupils to focus on how they might work out the labels for each row and column.



What do all the numbers in the top left box have in common? How are they different to the numbers in the bottom left box? What do you know about the numbers in the first box?

Four-Block Tower

This activity builds on the 'Three-Block Tower' at level A2 and the same process can be followed here. This exploration involves extra combinations and also another context in which to refine skills for organising and recording data. As the data set increases it becomes more difficult to represent ideas concretely. Therefore pupils are pushed to notice patterns, make conjectures and generalisations based on what they discover.

Sarah was asked to design towers for Tower Block City. Sarah was delighted to hear that there was an endless supply of blocks available for the project. Two of the rules for the project were that the towers all needed to contain the same four colours, using only four blocks but no tower could be identical to another tower. Sarah was disappointed to hear she could only use four colours and thought to herself that she could therefore only make four towers. What do you think? Will Sarah be able to create more than four towers? How can you convince Sarah? Investigate.

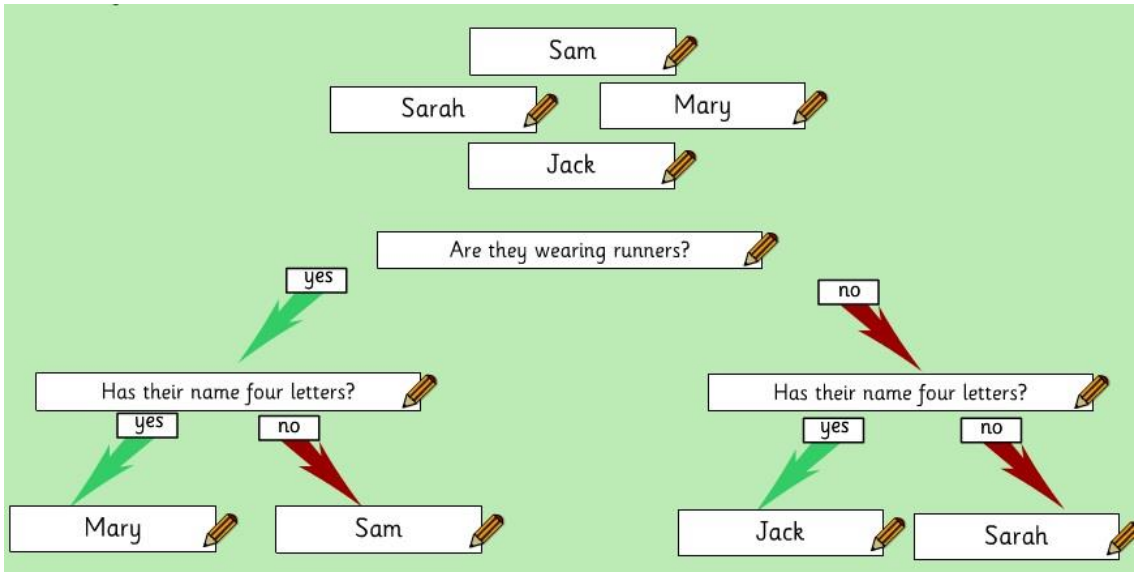
Guess Who Activity[41]

The purpose of this activity is to provide opportunities for pupils to create closed questions as they filter a set into subsets, using a decision tree.

1. Invite four pupils to the front to play 'Guess Who?' Explain that you are secretly thinking of one of the four pupils standing at the front and you have written that person's name secretly. The class needs to guess who you are thinking of, by posing yes/no questions to find clues that will enable the pupils to guess who you are thinking of.

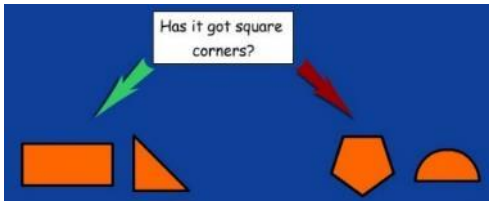
[41] http://mikeaskew.net/page3/page2/files/In%20good%20shape.pdf

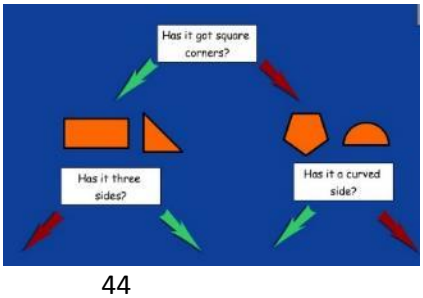
2. Elicit from pupils ideas for good questions, looking for those that separate the set of pupils into subsets and eventually individuals. For example, what would be an effective question 'are they in 1st class?' or 'are they wearing runners?'
3. Using appropriate questions, as expressed by the pupils, draw the decision tree diagram on the board. It will emerge that following a particular branch or line of inquiry will lead to a fast and efficient discovery of the mystery person. It is useful however to go back and complete the tree diagram to get a bigger picture of the data available. There are many opportunities to advance pupil reasoning skills by doing so.
4. Reflecting on the activity, elicit from pupils the extent to which questions made it easy or difficult to guess the mystery person.
5. Pupils take turns secretly thinking of one of the four children, while the class devise questions directed to them to find out who is the mystery person.

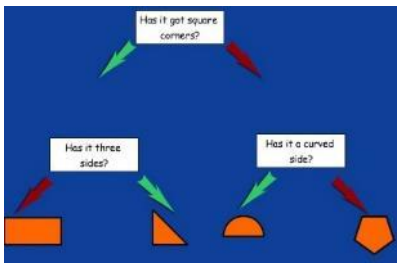


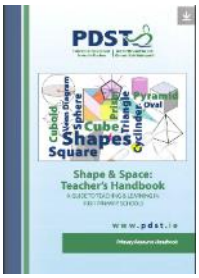
Extension Activity

Working in pairs, provide pupils with a small collection of four shapes to create their own tree sorting diagrams. Pupils can draw their tree diagrams and take it in turns to secretly think of a shape and investigate if the tree diagram helps them find that shape. Pupils can verify their sorting diagram works by swapping their diagram and shapes with another pair. The other pair can see if the tree diagram works. Begin by using a small quantity of shapes initially which will enable pupils to become familiar with the shapes, their properties and the process of classifying them.

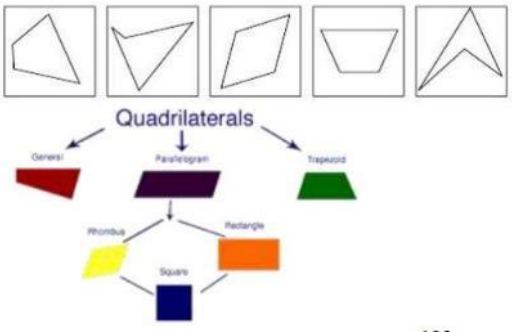








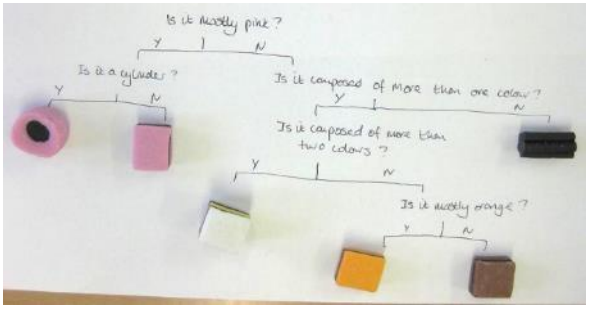
The PDST Shape and Space Resource has further quadrilateral investigations under Level C.4 on page 108.



CONSOLIDATION ACTIVITIES

Liquorice Sorting

Pupils can consolidate their understanding of the use of tree diagrams to sort the various types of liquorice according to different criteria.



LEVEL B.3

REPRESENT, READ AND INTERPRET DATA IN TWO, THREE OR FOUR ROWS OR COLUMNS, TABLES, PICTOGRAMS AND BLOCK GRAPHS.

TEACHING NOTES

Pictures, tables, sketches and tallies are working tools that can be used to organise data. Pupils should develop a repertoire of simple ways to record and display data quickly. Pupils may invent their own representations having experienced concrete and pictorial representations during previous activities. By doing so, pupils have the opportunity to see each other represent the same data in many different ways. Pupils gain new ideas about picturing data and begin to learn how different representations can communicate different information about the data. This is reflective of the Concrete, Pictorial, Abstract approach to mathematics where pupils have experienced concrete and pictorial representations of Data and are now moving to symbolic representations of Data. A useful technique here is that of tallying, based on counting in fives.[42]

Some children have problems with tallies. This usually concerns the cross stroke. Traditionally the fifth count is drawn as a stroke across the previous four so making a bundle of five altogether. Some children make upright strokes first, then draw a line across them. It is not clear whether this makes them a bundle of five or six. (Suggate, p.268).

SAMPLE LEARNING EXPERIENCES

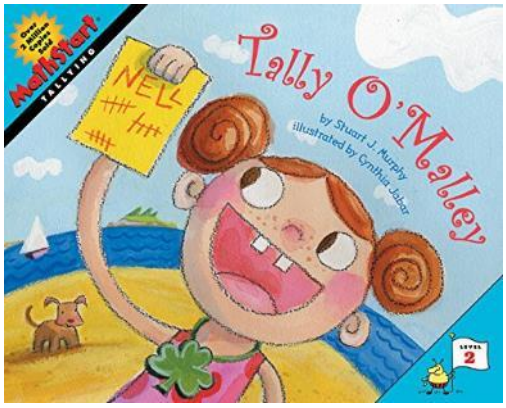
Fruit Chart

The purpose of this activity is to provide pupils with an opportunity to represent a set of data using rows and columns.

1. Elicit from pupils the type of fruit in their lunch box for example banana, apple, orange etc.
2. List the different types of fruit on the board and discuss with pupils a possible key to represent the different fruits e.g. yellow represent banana.
3. In groups, pupils discuss the types of fruits in their lunchboxes.
4. Provide each group with a selection of unfix cubes. These can be used to represent the different fruits in that group.
5. Elicit from pupils the difference ways of representing the results for example in rows or columns.
6. Encourage pupils to represent their results using both formats.
7. As a group, pupils devise questions based on their data for other groups.

[42] Haylock, D., and Manning, R., (2014).

Tally O' Malley[43]



This activity is based on the story Tally O' Malley[44]. The story gives pupils the opportunity to see and experience tallying being used in everyday life. The storyline evolves as the children get bored during the car journey, a scenario with which many pupils may be able to identify. The O' Malley family are on their way to the beach and during the long journey they decide to pass the time with short tallying competitions-- first with cars, then t-shirts, and finally train cars. The author Stuart Murphy provides the following suggested activities to do with pupils in conjunction with reading this book:

- Reread the story and have your pupils keep track of the data with their own tally marks.
- Make a chart with each of the characters' names, the colours they choose, and their tallies.
- Stop to see how the pupils tally marks compare with the marks in the book.
- Say a number between 10 -25 and ask pupils to make tally marks to represent that number.

Click the video icon to open a video of Tally O' Malley[45].

To practise the use of tallies, pupils could devise their own simple survey to conduct their family friends or classmates, for example a Pizza Survey: "What kind of pizza do you like best?" Pupils work together to tally the responses. Pupils can consider a number of questions, based on the data, for example, What kind of pizza is most popular? What kind of pizza is least popular?

If some pupils show signs of confusion with tallies, e.g. drawing 5 vertical lines and then a 6th line placed diagonally. The following rhyme may be useful for pupils:

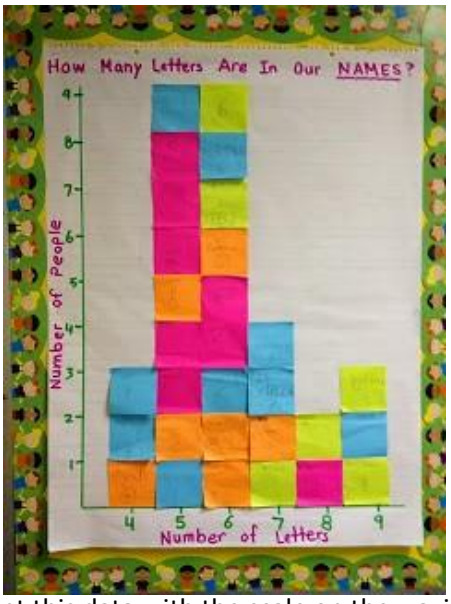
One, two, three, four,

Number five shut the door!

Equivalence is one of the most important concepts for children to learn in elementary school (NCTM 2000), graphing can provide a meaningful context for young children to use this concept for a real purpose.
(Dacey & Eston, 1999).

[43] http://www.mathematicstart.net/books.html
[44] Murphy, S.J. and Jabar, C. (2004).
[45] https://www.youtube.com/watch?v=-_ODQk0DquM&list=RD-_ODQk0DquM

Class Name Block Graph[46]

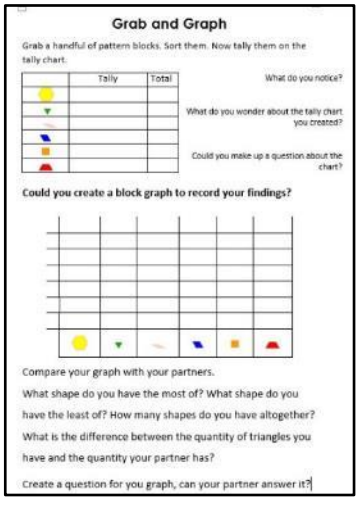


1. Elicit from pupils how we could discover which length of name (for example six letter names) is most common in the class.
2. Encourage pupils to record their name length using concrete materials, for example cubes or links.
3. Compare their names with other pupils and sort into appropriate sets.
4. Elicit from pupils how this information can be recorded (for example, a photo, pictures, concrete representations etc.)
5. Discuss the need to represent this information in a concise way that enables comparisons to be made. List suggestions from pupils.
6. Choose one of these suggestions, for example a block graph to represent this data with the scale on the y axis and the values on the x axis.
7. Pupils then write their name on a sticky note and place it above the corresponding value on the block graph.

The concept of developing class block graphs can be further reinforced using, other relevant topics, for example, preferred fruit, pets, number of family members etc.

If children are collecting data, use a variable with a limited, agreed amount of values. If you want to use data about a favourite topic, for example, books, then first agree with the class a menu of about six possibilities to choose from.
(Haylock, 2007, p.347).

Grab and Graph



In this activity each group is provided with a box of pattern blocks.

1. Group members put their hand into the box and grab a fistful of blocks. Once all members have a fistful of blocks, the box is closed.
2. Pupils talk about what they got from the box.
3. Pupils tally their fist of blocks and record their findings on the 'Grab and Graph' template (Appendix B.3).
4. This gives pupils the opportunity to represent their data in two complementary ways.

[46] http://anajessicakim.blogspot.ie/2013/09/name-bar-graph.html

5. First they tally the data, then they graph their findings using a block graph.

Pictograms

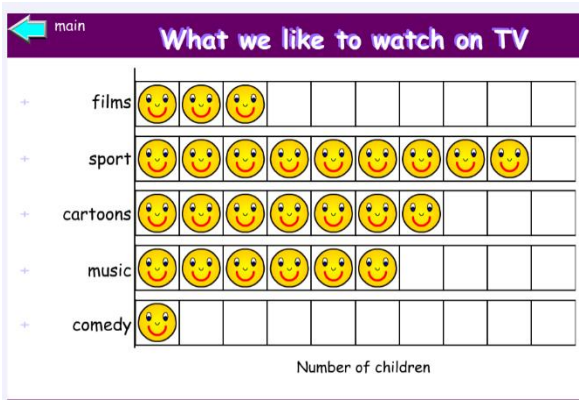
The purpose of this activity is represent and interpret data using simple pictograms.

1. Discuss with pupils what is the most popular way to come to school each day? Pupils may have ideas that most children come to school in a car while others may think most children get the bus.
2. Using a pictogram, pupils will represent and interpret data to find out what is the most popular way to come to school.
3. With the pupils, decide on a symbol/picture to represent a mode of transport, for example, a pair of shoes for walking.
4. Each pupil will represent their mode of transport and assemble on a class pictogram on the whiteboard.

What is the most popular way to come to this school? Why do you think this is more popular than cycling? What is the least popular way of coming to school? How many more pupils walk than cycle? Would we get the same results if we asked 6th class about how they travel to school? Why do you think this is so? What else do you notice?

Further Development:

The following interactive[47] is useful for developing pupils' understanding of pictograms and there are many options to choose from. Pictograms can be represented in horizontal or vertical format.



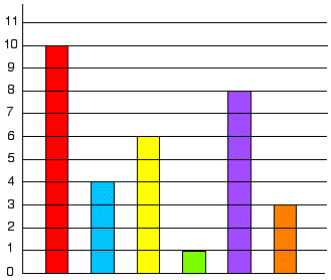
ICT
Pictograms

[47] http://www.topmarks.co.uk/Flash.aspx?f=pictograms

The Pet Graph[48]

This problem provides pupils with an opportunity to interpret a selection of clues in order to label the different columns in the block graph. Click on the interactive link to access this problem and resources to support its development.

ICT
The Pet Graph



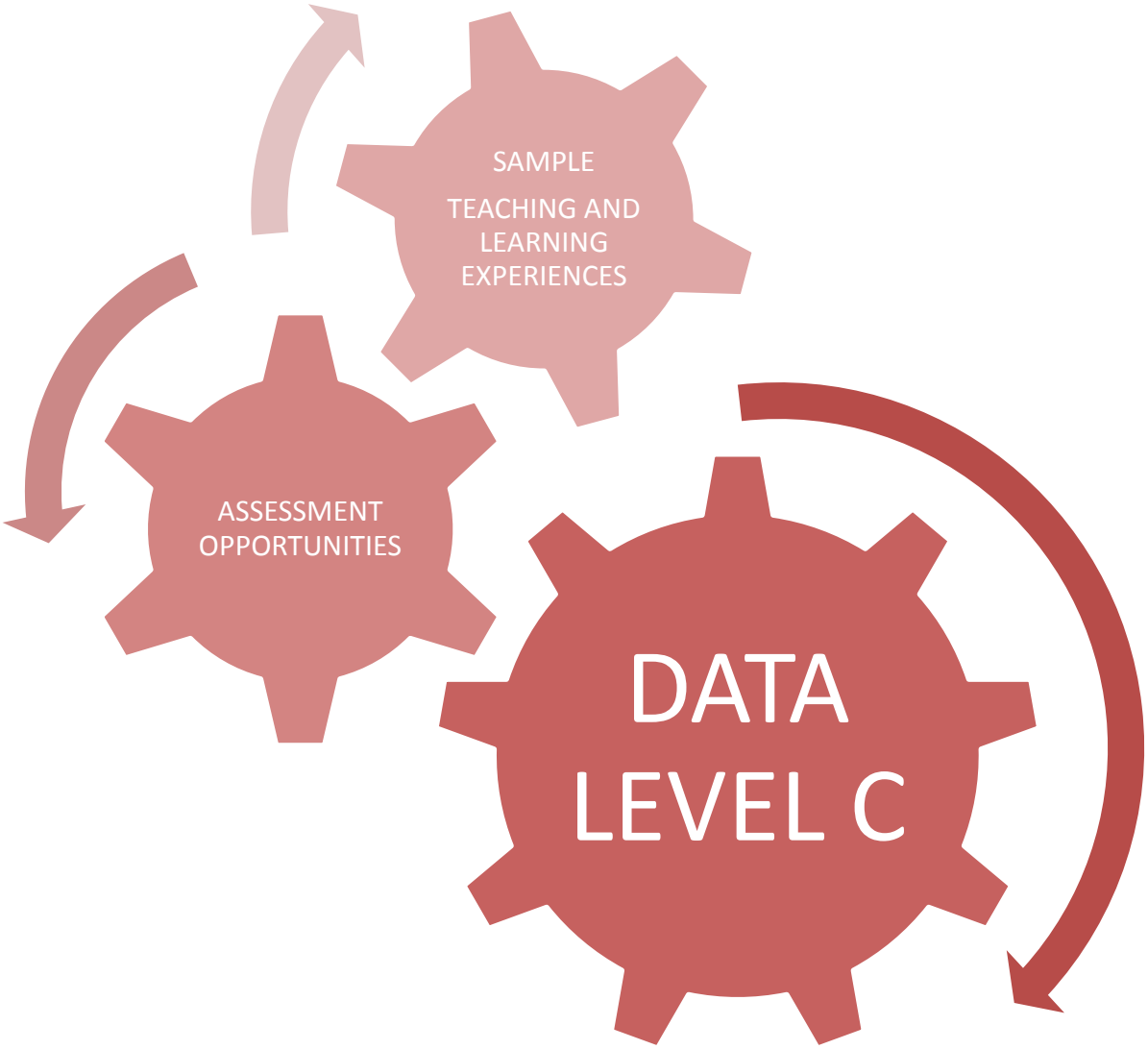
CONSOLIDATION ACTIVITIES

Sticky Data[49]

This activity invites pupils to work together, in a hands-on way, to display data about their own class. The use of sticky notes, which are all the same size, makes it a great introduction to block graphs.

ICT Link
Sticky Notes

[48] http://nrich.maths.org/247
[49] https://nrich.mathematics.org/7687

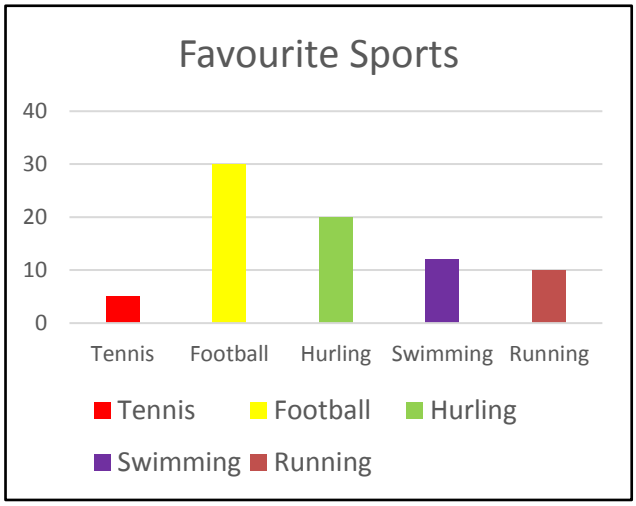

SAMPLE
TEACHING AND LEARNING EXPERIENCES
ASSESSMENT OPPORTUNITIES
DATA LEVEL C

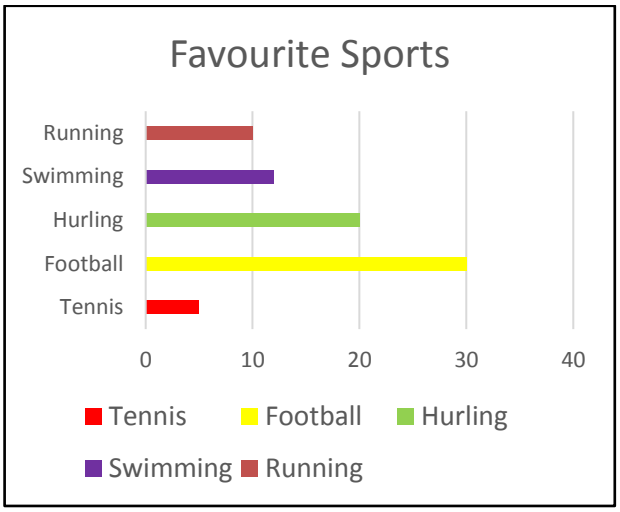
LEVEL C.1

COLLECT, ORGANISE, REPRESENT AND INTERPRET DATA USING PICTOGRAMS, BLOCK GRAPHS, BAR CHARTS AND TABLES

TEACHING NOTES

At this level, the introduction of bar charts requires pupils to move from colouring or counting squares one at a time to using vertical scales from frequency data[50]. This development leads to pupils seeing data as a collection of information about a group rather than individual pieces of data[51]. In contrast to block graphs it is the length or height of the column which gives the information required[52]. Bar charts can be represented in vertical and horizontal forms. It is important that pupils are familiar with both forms.



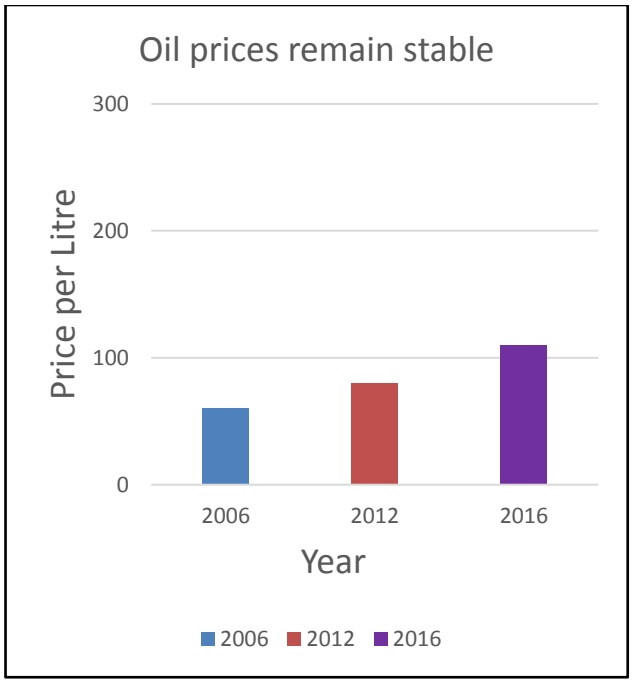


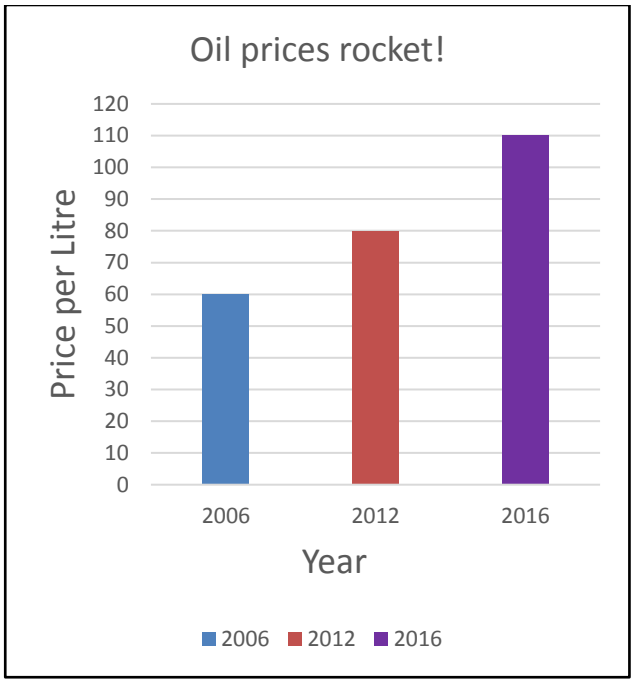
When constructing a bar chart, the scale must be labelled **on** each division not between the divisions (Crown, 2009, p.14).

When representing discrete or continuous data, it is important that the columns in a block graph or bar chart are of equal width and equally spaced apart in order to portray an accurate picture of the distribution of the data (Haylock, 2014, p.407).

[50] Willis, S., Hardman, C. and Sue, W., (2013, p.141).
[51] Willis, S., Hardman, C. and Sue, W., (2013, p.141).
[52] Haylock, D., and Manning, R., (2014, p.406).

A useful exercise is to show pupils two bar graphs of the same data but using different scales. The bar charts below highlight how data can be distorted to give the desired effect.





Once a graph is constructed, focus should be placed on the issue of analysis and communication. Encourage pupils to discuss the story of the graph and the key information it illustrates (Van de Walle et al, 2010, p.440).

Pupils should have regular opportunities for creating and analysing data using ICT. For example, pupils can create and recreate charts using programmes such as Microsoft Word and Excel. They can also explore how data can be represented in multiple ways using ICT, for example bar-line, bar chart, pie chart etc. This provides an opportunity for rich discussion about which representation is most suitable for interpreting the data in question.

The surveys and frequency tables which pupils experience at this level provide an ideal opportunity to consolidate earlier work in Level B on the use of tallies, lists and simple one–way frequency tables. Two-way frequency tables are a more complex concept as they require pupils to co-ordinate and understand more than one piece of data at a time[53]. Also at this stage, the simple factual and general questions explored in previous levels can be extended to include more inquiry based questions and those that are more statistical in nature for example the question "How much time did John spend on homework last night?" is not a statistical question because it has a pre-determined answer and is not answered by collecting data that varies. On the other hand, the question, "How much time do pupils in our class spend on homework each week?" is a statistical question because it is expected that not all pupils spend the same amount of time on homework.

A statistical question is one that anticipates variability in the data related to the question and accounts for it in its answer. Pupils need lots of opportunities to identify, explore and write statistical questions (CCSSO, 2010).

SAMPLE LEARNING EXPERIENCES

Birthday Pictograms

The purpose of this activity is to represent, read, discuss and interpret data from pictograms. Pupils will collect, represent and interpret data based on their birth months to create a class pictogram.

1. Elicit from pupils ideas about collecting and organising information about their birthday months. Pupils may suggest using tallies or post-it notes.
2. With collected data, create a class pictogram, but allow pupils the choice of how to represent their birthday month. For example, pupils born in March may decide to represent their birthday month using a shamrock or pupils born in December may choose to use a Christmas image.
3. Facilitate a discussion based on the data collected and assembled by the pupils. Encourage pupils to discuss the advantages of using a pictogram and also the limitations.

What do you notice when you look at the pictogram? What does this information tell you? Looking at the data about the pupils born in July, what can we say? What does this data not tell us about the children born in July?

[53] Willis, S., Hardman, C. and Sue, W., (2013, p.174).

Using Surveys[54]

The purpose of this activity is to provide pupils with an opportunity to create and investigate a statistical question.

1. Discuss with pupils the differences between simple factual questions and a statistical questions.
2. Explore with pupils an area of interest which could form the basis of a survey, for example what are the favourite hobbies of 5th and 6th class in our school?
3. In pairs, pupils predict the possible answers to the survey in advance.
4. Elicit from pupils the extent to which questions can lead to biased responses and the benefits of limiting the choice of possible responses.
5. In groups, pupils select a class grouping to administer the survey.
6. On completion of the survey pupils can design a simple table to illustrate the results of their selected class.

Extension Activity

This activity can be extended to introduce the concept of 'Fair Sampling' with pupils. Present pupils with a situation where we cannot collect data on everyone. Elicit from pupils the concept of using a representative sample.

We can't find out what hobby everyone in the county likes best. What could we do instead? Ann has an idea…could we ask a sample of pupils from every class? Would that be a fair sample? Why do you think so?

Two way tables can be a difficult concept for pupils as they require two constraints to be interpreted at one time. Pupils need to understand how frequencies in rows and columns may be summarised to show totals, often without the word 'total' being highlighted explicitly or the column or row being labelled (Willis, Hardman and Sue, 2013, p.174).

[54] Willis, S., Hardman, C. and Sue, W., (2013, p.174).

Exploring Frequency Tables[55]

The data gathered in the above survey can be used to form the basis of this activity. Elicit from pupils the breakdown of pupils who liked each subject in the 'favourite subject' survey for example how many boys/girls chose Art. Elicit from pupils how these results can be represented in a two way table.

Let's examine our results again. What does the data in the table not tell us? Could we reorganise the table to include more details? Will our new tables have more or less columns? How do you know? Prove it!

As an extension pupils can gather samples of frequency tables from various print media and compose their own questions based on the tables for use by other pupils.

	Art	Music	Total
Boys	37	8	45
Girls	26	14	40
Total	63	22	85

Class Bar Chart[56]

This activity provides pupils with an opportunity to gather data directly relating to their lives and represent their findings using a bar chart.

1. Pupils identify the duration of their journey to school by recording what time they leave home and when they arrive in school on an empty number line.

[55] http://www.cpalms.org/Public/PreviewResourceLesson/Preview/65965
[56] http://webarchive.nationalarchives.gov.uk/20110202093118/http://nationalstrategies.standards.dcsf.gov.uk/node/20332

2. Pupils cut a strip of paper to show the duration of their journeys, for example a 10cm strip to represent a 10 minute journey. These strips are stuck on a class bar chart.
3. Elicit from pupils a title for the graph and how each axis will be labelled. This activity can be extended by replacing the class names on the x axis with common time intervals for example 5 minutes, 10 minutes etc.
4. Pupils can also investigate how the data could be represented in a different type of bar graph for example horizontal format or using a different scale.

Magic Squares[57]

15	10	3	6
4	5	16	9
14	11	2	7
1	8	13	12

The "Magic Square" puzzle works on a similar principle to that of a two way table in that each row and column (and diagonal) must have a sum of a certain 'magic' number for example 15. In this interactive activity pupils are provided with a completed magic square. The 'Magic Constant' must be deciphered based on the data in the Square. Questions to support this activity can be accessed through the ICT Link.

ICT Link
Magic Constants

Who Is Absent From Ms. Marble's Class?[58]

The purpose of this activity is to present pupils with a large data set, provide pupils with opportunities to interpret limited representations of this data set and finally encourage pupils to enhance these representations in order to deduce 'Who is absent from Ms. Marble's Class?'. Click on the interactive link for further resources to support the development of this problem.

1. One day when 34 children were in class, Mrs Marble, their teacher, said they were going to make some block graphs and other things using their first names.

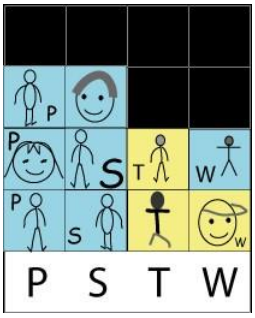
Girls in Class 5	Boys in Class 5
Hetty	David
Annie	Nelson
Tessa	Ali
Debbie	Jake
Willow	Harry P
Jess	William
Abby	Ben
Sindy	Tom
Penny	Dai
Bel	Arlo
Sara	Andrew
Pippa	Harry W
Selma	Tim
Becky	Joe
Mel	Alan
Pauline	James
Netty	Jeff
	Mohammed

[57] http://nrich.maths.org/87
[58] http://nrich.mathematics.org/7522

2. She put the class lists, which included those absent, onto the white board.
3. As the children looked at the lists, they did not include the pupils that were absent.
4. First, the class made tally charts of the initial letters of their names. They worked in pairs.

ICT Link
Class 5

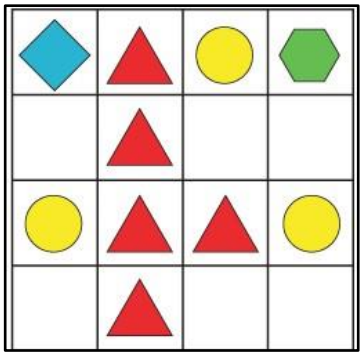
5. Next they decided which letters of the alphabet were needed and which were not needed to make a block graph of their class names.
6. The boys took yellow squares and the girls took pale blue squares, drew a picture of themselves and put the initial of their first name on the square and stuck it onto paper to make a pictogram graph, such as the one in this sample.



From this information could we find out who was missing from Ms. Marble's Class? Can you make a full tally chart using the class names? What does the tally chart tell us? What does it not tell us? Is there another way we could represent this information? Would it be possible to use numbers instead of tally marks?

CONSOLIDATION ACTIVITY

Shapes In A Grid[59]



In this interactive problem, pupils need to identify which shapes to put into the grid to make the totals at the end of each row and the bottom of each column. The shapes stand for four different numbers less than ten.

ICT Link
Shapes in a grid

[59] http://nrich.maths.org/7047

LEVEL C.2.

COLLECT, ORGANISE, REPRESENT AND INTERPRET DATA USING PICTOGRAMS, BLOCK GRAPHS, AND BAR CHARTS INCORPORATING THE SCALES 1:2, 1:5, 1:10 AND 1:100

TEACHING NOTES

At this level, the learning experiences in Level C.1. can be extended to incorporate scales.

SAMPLE LEARNING EXPERIENCES

Reading Scales

Reading scales is an important skill when interpreting bar charts[60]. A counting stick can be used to develop this concept with pupils. Hold the counting stick either horizontally or vertically to correspond with the axis in question. When reading bar charts, it is a specific point that represents a specific number. Therefore use a finger to point to the divisions, not the whole hand[61] . Deciding on what scale to use requires students to apply their knowledge of multiplication. Therefore, it is very helpful for pupils to have a good knowledge of these facts. The PDST Mental Maths Resource contains many activities to consolidate multiplication facts.

Birthday Pictograms

This activity is an extension of an earlier activity in Level C.1.

1. Facilitate a discussion based on the data collected and assembled by the pupils. Encourage pupils to discuss the advantages of using a pictogram and also the limitations.
2. Draw pupils' attention to the large number of birthdays in July. Elicit from pupils ideas on how this information could be represented quicker e.g. using half a symbol to represent one pupil or using one full symbol to represent a pair of pupils.
3. Pupils can then recreate the birthday pictogram using the selected scale for example 1:2

What are the advantages of using a pictogram to represent this data? Seán said there were a lot of pictures to do for July as there are 12 July birthdays. I wonder could we think of a way to create a pictogram which would need less drawings but show the same information.

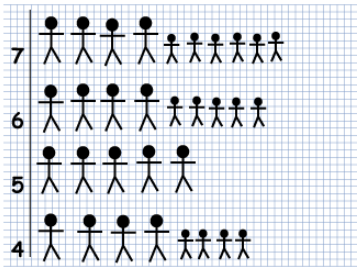
Further Development:

1. Pupils can identify pictograms from media sources which incorporate larger scales such as 1:00.

[60] Crown, (2009), p.16.
[61] Crown, (2009), p.16.

Class 5 Chart[62]

This problem provides pupils with an opportunity to interpret more than one piece of data while also consolidating their understanding of the use of scales. Click on the interactive link to access the problem and resources to support its development.



ICT Link
How big are classes 5, 6 and 7

Class Bar Chart[63]

This activity is an extension of an earlier activity in Level C.1.

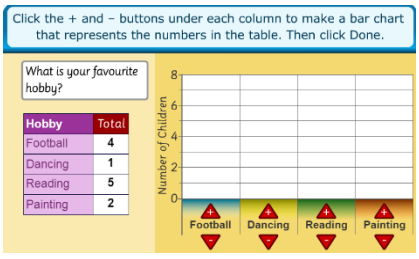
1. Facilitate a discussion based on the data collected by the pupils for the class bar chart.
2. Elicit from pupils the advantages of adding a more complex scale to the chart. Encourage pupils to predict how this could change the appearance of the graph.
3. Pupils construct a new chart using an agreed scale for example 1:10. Compare and contrast the original graph with the newly constructed graph.

How would the data look if we changed the scale? Sean did you notice any common multiples in the data that would help us decide on a scale to use?

CONSOLIDATION ACTIVITY

Graph Creator[64]

This interactive activity provides pupils with an opportunity to interpret and record data in a variety of different ways.



ICT Link
Class Graph

[62] https://nrich.maths.org/2399
[63] http://webarchive.nationalarchives.gov.uk/20110202093118/http://nationalstrategies.standards.dcsf.gov.uk/node/20332
[64] http://www.bbc.co.uk/staticarchive/88d1dc241__5404d3fbe483d29b5c81b57651d5d22.swf

LEVEL C.3

READ AND INTERPRET DATA USING BAR-LINE GRAPHS AND SIMPLE PIE CHARTS INVOLVING HALVES, THIRDS AND QUARTERS.

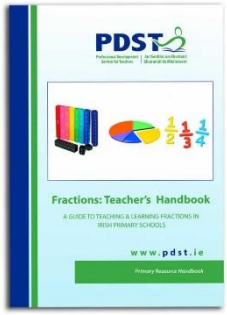
TEACHING NOTES

Pupils will encounter bar-line charts at this level. This type of chart is very similar to a bar chart, but the data is represented by lines rather than by bars. The Learning experiences in Level C.1. and Level C.2 can be used to develop an understanding of this mode of representation. Also at this level, pupils are introduced to Pie Charts which are a more sophisticated form of data representation and are used to represent discrete data[65]. In Pie Charts, it is the angle of each slice of the pie that represents the proportion of the population in each subset[66]. The larger the angle at the centre of the circle, the larger the frequency. At this level, more emphasis is given to interpreting pie charts than constructing them.

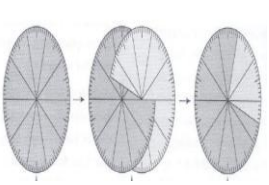
An important principle of the pie chart is that the whole pie must represent the whole population. Pie charts are really only suitable for discrete data with a small number of subsets for example six or fewer (Haylock, 2014, p.411).

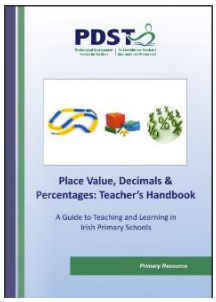
SAMPLE LEARNING EXPERIENCES

Hundredth Wheel[67]



Being able to estimate, measures and construct angles is a valuable skill when introducing pie charts. Level D.11 of the PDST Resource for Shape and Space outlines learning experiences for pupils working with angles. Using the hundredth wheel model encourage pupils to illustrate different fractions. Extend this activity to include questions that have a more numerical focus. The activities in Level D of the PDST Resource on *Place Value, Decimals and Percentages* contains further ideas on the use of the hundredth wheel. The Hundredth Wheel can also be used to develop the concept of fractions. More support is available to teachers in the PDST Fractions Resource.

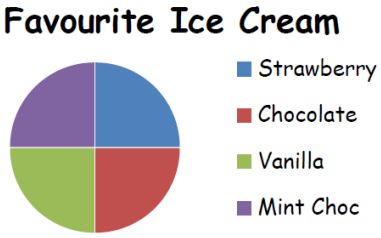




Show me halves, thirds, quarters on your hundredth wheel. If the whole pie represents 100 people, show me half. What would this number be? How far away from one whole unit is this fraction?

[65] Haylock, D., and Manning, R., (2014 p.411).
[66] Haylock, D., and Manning, R., (2014 p.411).
[67] http://www.pdst.ie/Mathematics

What Does The Pie Chart Tell Us?[68]



This problem solving task requires pupils to piece together different pieces of information in order to interpret the pie chart.

1. If 4 children were asked this question how many children does one segment represent?
2. If 8 Children were asked this question what does each segment represent?
3. If the total survey was of 20 children how many children's favourite ice cream was strawberry?
4. If the total survey was of 200 children how many children's favourite ice cream was strawberry?
5. If the total survey was of 200 children, how many children all together liked Vanilla and strawberry?
6. Can you explain why this pie chart cannot show 10 children's favourite ice cream?

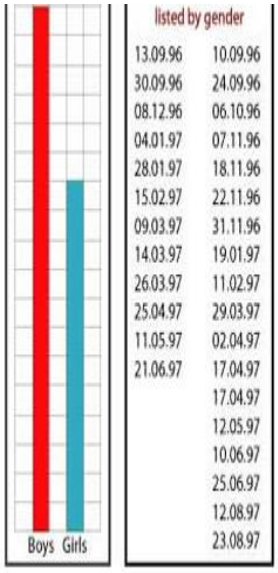
Further Development:

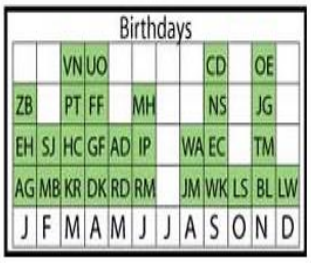
1. Using the pie chart above, pupils create their own story of what the pie chart represents and compose questions for other pupils to solve.

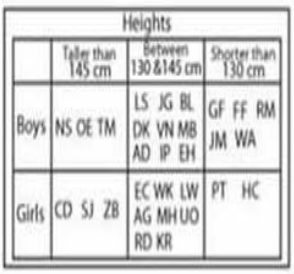
Presenting The Project[69]

This problem requires pupils to interpret different forms of data to identify on what day the pupils in Class 5 presented their work. In order to answer the questions, more than one piece of data needs to be interpreted at a time. The problem along with resources to support the exploration of this problem can be accessed through the interactive ICT Link.

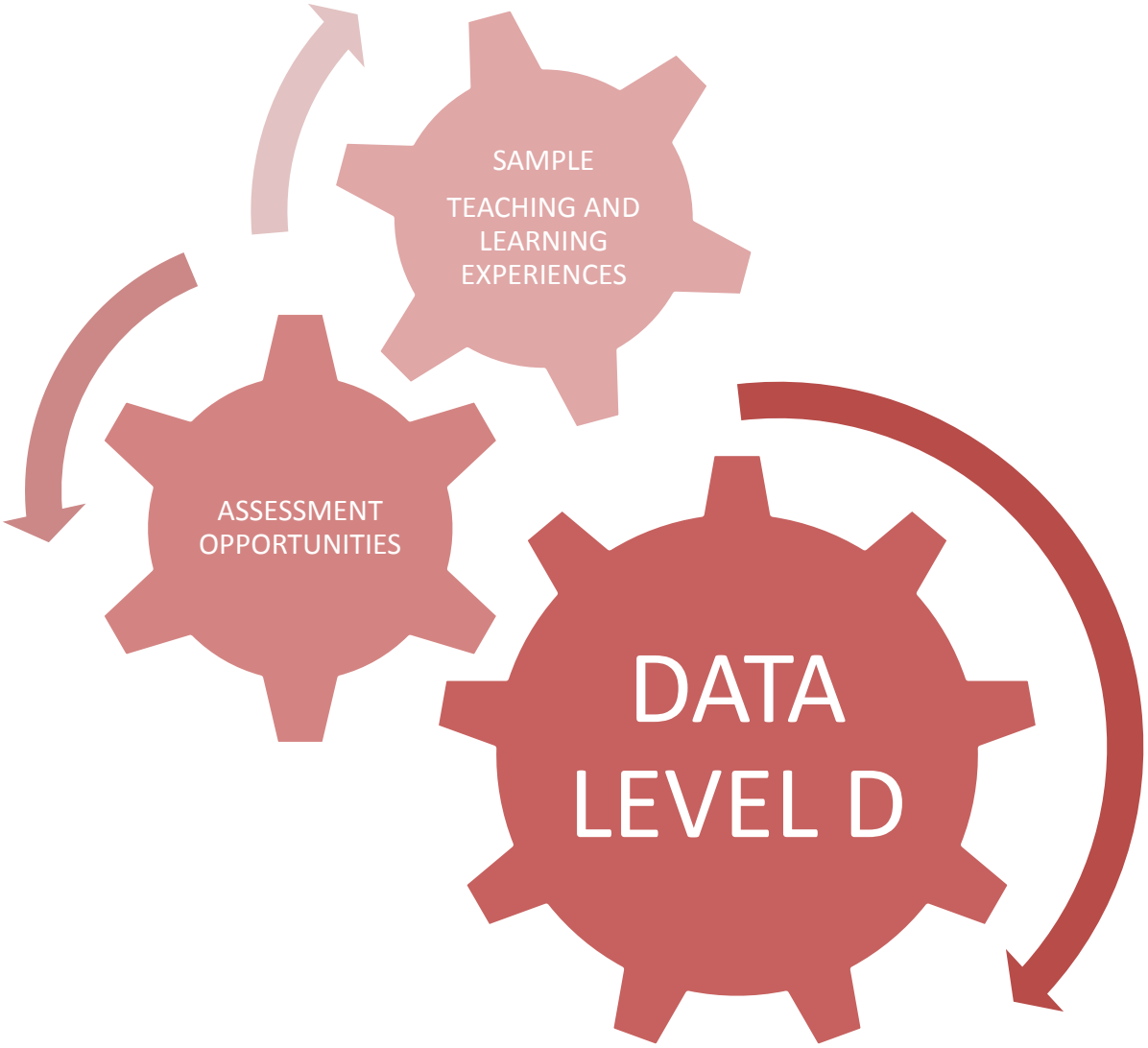
ICT Link
Presenting the Project







[68] file:///C:/Users/marymcmahon/Desktop/PDST/Data/Jan%202017/New%20Pie%20Chart%20Activity.pdf
[69] http://nrich.maths.org/4922


SAMPLE
TEACHING AND LEARNING EXPERIENCES
ASSESSMENT OPPORTUNITIES
DATA LEVEL D

LEVEL D.1

COMPILE AND USE SIMPLE DATA SETS AND CALCULATE AVERAGES

TEACHING NOTES

At this level, pupils should be encouraged to compile lists of statistics from their own experiences for example height, age, hair colour, sports results, preferences. This data can then be used as a source for representation, interpretation and setting problems. As pupils engage in data collection, opportunities will also arise to for pupils to differentiate between first hand and second hand data. Also at this level, further opportunities are provided for pupils to consolidate their understanding of two way frequency tables and to use this model as a means of representing data. The use of frequency charts and tables is a dual objective in both the Data strand and the Chance strand of the Primary Mathematics Curriculum. The activities at this level may also be explored as part of the Chance strand. At this level pupils calculate averages of simple data sets.

It is important that pupils have concrete opportunities to explore finding the 'mean' of a data set before teaching the traditional algorithm. This will help pupils develop a deeper understanding of these values and their relationship to each other
(Van De Walle, 2010, p.446).

Haylock[70] uses the image of pooling resources and then distributing them out equally to illustrate the concept of averages, for example, to find the average amount of money that a group have in their possession, all of the group put their money on the table and then share it out again equally between the members of the group.

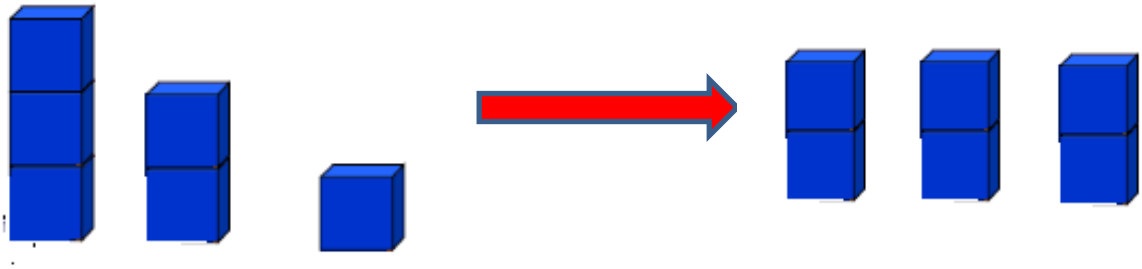
SAMPLE LEARNING EXPERIENCES

Levelling the Bars[71]

1. As a class, pupils gather data on the number of family members each pupil in the class has.
2. Pupils then represent this data on a class bar graph using connecting cubes (one cube per value).
3. Elicit from pupils what the 'typical' family size would be if all of the families were the same size.
4. Encourage pupils to use various techniques to rearrange the cubes to "level" the family size of each pupil. Pupils record the total as the 'average' family size in the class.

Let's examine our data. Who has the largest family? Who has the smallest family? Is there a way we could rearrange the cubes so everyone's family is the same size? What will we do if we have some left over cubes that cannot be distributed evenly?

[70] Haylock, D and Manning, R., (2014).
[71] Van De Walle, Karp and Bay Williams, (2013).

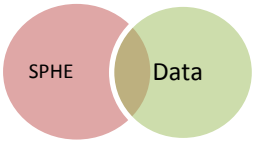


Averages and Frequency

Pupils need to understand the purpose of calculating average amounts and the most freuqently occuring item in a data set. Sample data sets that pupils could collect and analyse for this purpose include:

- Height
- Distance from school
- Duration of homework
- Number of family members

Food Diary[72]



1. Introduce the Food Pyramid to pupils and discuss the different food groups which make up the pyramid.
2. In groups, pupils devise a simple questionnaire and gather data on the different food groups each pupil in the group consumes during a typical day.
3. Once the data is collected, provide opportunities for pupils to compose questions for other groups based on their data set. Similarities and differences between each group's data 'set' can then be discussed.
4. Pupils identify the most frequently occurring food group in the food data set gathered above.

Further Development

As an extension, introduce the concept of fair sampling and a representative sample. Present pupils with a situation whereby the school principal wants to gather data on the food intake of the whole school population. Elicit ideas from pupils on how a representative sample could be obtained, for example using three pupils per class.

Would the results be similar or different if we gathered data on the food intake of the whole school? Would we have to ask every pupil? How can we make sure the results would be representative of the whole school?

[72] https://www.eathappyproject.com/resources/activity-sheets/food-diary/

1. Pupils gather data on the food intake of a representative sample of the whole school population using a questionnaire.
2. This data can then be compared to various second hand data available in the media on food intake of primary school children and pupils can hypothesise about any differences in the data trends.
3. Elicit from the pupils the advantages and disadvantages of using second hand data. Provide each group of pupils with a piece of second hand data on daily food intake. Pupils interpret the data and compose a summary report for the other groups in the class.

ICT Link
Medal Muddle

Medal Muddle[73]

In this problem-solving task pupils must piece together different pieces of information to complete the medal table of thirteen nations which completed in a sports tournament. The resources for this problem and further extension activities can be accessed through the interactive ICT Link.

1. Turkey and Mexico both finished above Italy and New Zealand.
2. Portugal finished above Venezuela, Mexico, Spain and Romania.
3. Romania finished below Algeria, Greece, Spain and Serbia.
4. Serbia finished above Turkey and Portugal, both of whom finished below Algeria and Russia.
5. Russia finished above France and Algeria.
6. Algeria finished below France but above Serbia and Spain.
7. Italy finished below Greece and Venezuela, but above New Zealand.
8. Venezuela finished above New Zealand but below Greece.
9. Greece finished below Turkey, who finished below France.
10. Portugal finished below Greece and France.
11. France finished above Serbia, who finished above Mexico.
12. Venezuela finished below Mexico, and New Zealand finished above Spain.

[73] http://nrich.mathematics.org/661

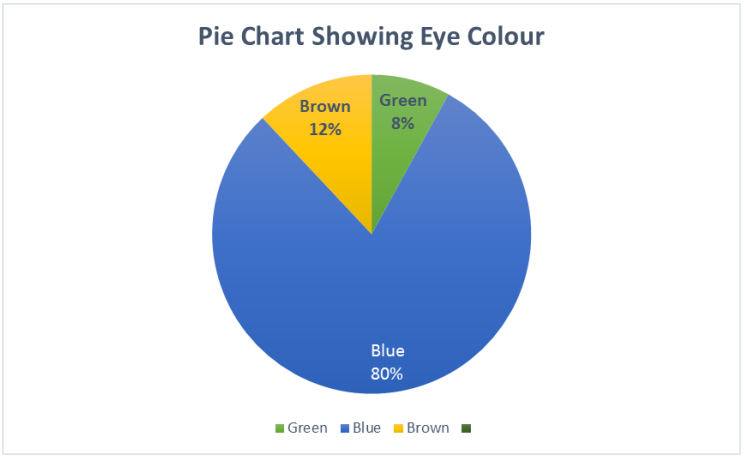
LEVEL D.2

REPRESENT, READ, DISCUSS AND INTERPRET DATA USING PICTOGRAMS, SIMPLE PIE CHARTS AND SINGLE BAR CHARTS.

TEACHING NOTES

The activities explored in Level C.2. can be revisited at this stage to reinforce pupils' understanding of pictograms and single bar charts. Earlier work on interpreting pie charts can be extended to include constructing pie charts. The mathematics involved in constructing a pie chart can be difficult for pupils. However as this can all be done by a computer, greater emphasis should be placed on the interpretation on the results rather than on the skill of constructing a pie chart.[74]

Pupils need to be aware that categories or classes with no representatives are not shown in a pie chart for example if the data were about pets and no pupil in the class has a tortoise then this category would not be represented in the pie chart. Pie charts should only be used when the total represented by the whole circle (or pie) has meaning. For example, using a pie chart for eye colour, the whole circle represents the total number of children in the sample.
(Suggate et al, 2010, p.273).



[74] Haylock, D. and Manning, R., (2014, p.412).

SAMPLE LEARNING EXPERIENCES

Constructing a Pie Chart[75]

For this activity, pupils can be used to create a human pie chart.

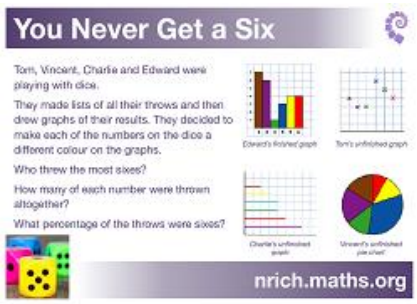
1. Provide pupils with a menu of different but limited food choices.
2. Pupils group together based on their food choice.
3. Pupils move into a large circle.
4. Elicit suggestions from pupils as to how to estimate the angles to represent their choices.
5. The end points of four long strips of paper or string are taped to the centre of the circle. Each strip can be extended to a different point in the circle where a group's food preference has changed.

Further Development

This activity can be extended by placing a cut-out of a rational number wheel (See PDST Place Value Resource, p.99) on the centre of the circle (Appendix, Level D.2). The strips of paper can then be used to show approximate percentages and fractions for each part of the pie chart. On completion of this activity, pupils can compose three statements about the pie chart and record in their Mathematics Journal. Record by drawing and then record more formally using Microsoft Word or Excel

It is important for pupils to represent data on pie charts using a variety of data sets, for example, personal data, sports results etc. These representations can be developed by moving from the concrete use of pupils and their ribbon strings, to recording using pictures, to the use of ICT.

You Never Get a Six[76]



This problem will challenge pupils to interpret data carefully and provides an opportunity to for pupils to contrast different ways of representing similar data. Click on the interactive link to access the problem and resources to support its exploration.

ICT Link
You never get a six

[75] Van De Walle, Karp and Bay Williams, (2013, p. 442).
[76] http://nrich.maths.org/2400

Scratch Pie Chart Creator[77]

Scratch is a programming language which allows participants to create their own interactive animations. Pupils can use this interactive tool to create a dynamic pie chart based on given information.

ICT Link
Scratch

CONSOLIDATION ACTIVITIY

The Pie Chart Game[78]

In pairs or groups, pupils compete against each other to gain the largest percentage covered on their pie chart.

1. To commence this activity, provide each pupil with a pie chart template (Appendix, Level D.2). The intervals within segments on the pie chart equal 5%. A six sided dice with faces marked 5%, 10%, 5% 30%, 5%, 20% will also be needed for this activity.
2. Each pupil starts with 50% of their pie chart.
3. Pupils take turns to roll the dice and slide their side of the pie chart to add on the percentages shown on the dice.
4. The game ends when one pupil reaches 100% of the pie chart or if their combined total exceeds 100%.

[77] https://scratch.mit.edu/projects/53651228/
[78] Lowe, E., (2014, p.3).

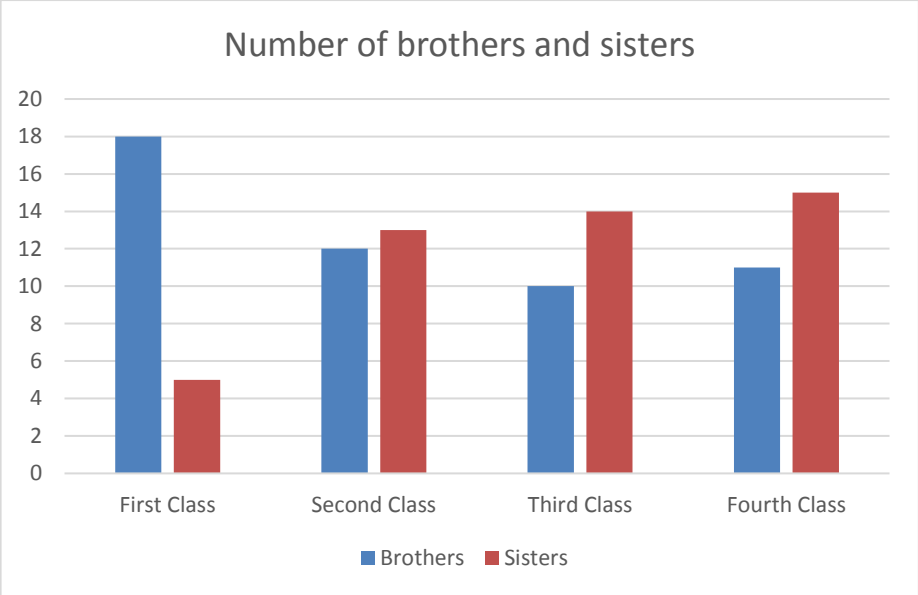
LEVEL D.3

REPRESENT, READ, DISCUSS AND INTERPRET DATA USING MULTIPLE BAR CHARTS AND TREND GRAPHS.

TEACHING NOTES

Multiple bar charts provide an opportunity to display two or more pieces of data side by side using the same scale for example, census data often shows male and female data separately for different years. This is usually done using a double bar graph. A legend is used to help the reader interpret a double bar graph.[79]

The example below highlights not only how many brothers and sisters the participants in each class in the survey have but also the number of brothers versus the number of sisters.



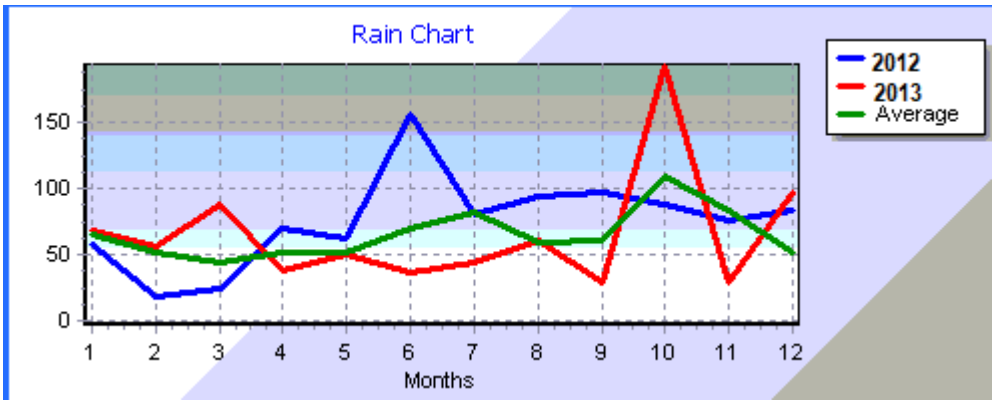
When constructing multiple bar graphs, the incremental numbers should be placed on each division and not in between the divisions. The pairs or multiples of bars on such a graph should be separated (Department of Education and Early Childhood Development, Canada, 2010)

Trend graphs (also known as Line graphs[80]) are used to illustrate two or more related pieces of continuous data[81]. A trend graph is drawn by plotting points related to these pieces of information, for example, time of day versus temperature in degrees. These points are then joined to create a line.

The example below shows the total amount of rainfall in County Carlow from 2012 - 2013. In a trend graph the movement of the line up and down gives a picture of how things are changing over time[82] for example

[79] Department of Education and Early Childhood Development, (2010, p. 111).
[80] Van De Walle, Karp and Bay Williams, (2013, p. 444).
[81] Haylock, D., and Manning, R., (2014, p.413).
[82] Van De Walle, Karp and Bay Williams, (2013, p. 442).

comparisons between the two years on the trend graph below shows a considerable increase in the amount of rainfall from September to November in 2013



Trend graphs are most suitable for variables involving time and are inappropriate for representing discrete data such as favourite pets as only plotted points have real meaning. The line only shows the pattern that may have occurred between the plotted points
(Haylock, 2014, p.413).

SAMPLE LEARNING EXPERIENCES

Salary Chart[83]

1. Elicit from pupils what types of data are appropriate to display in a multiple bar chart.
2. In groups, pupils gather data based on the salaries of their favourite professional sports man or woman. Pupils compare their salaries from their current team and previous team.
3. Pupils represent this data using a double bar graph. Elicit from pupils how they will distinguish between their sports star's current team and previous team on the graph.
4. On completion of the graph, pupils, in pairs make two statements about what the graph shows. Pupils then ask their partner to agree or disagree with the statement.
5. Encourage each group to represent the same data in a horizontal and vertical format and to estimate the values between the calibrations on the graph.

What do the two different coloured bars tell you? What do you notice about the salary scale of your favourite player? How would the data look if we used a larger scale?

[83] Department of Education and Early Childhood Development, (2010, p. 113).

What's the Weather Like?[84]

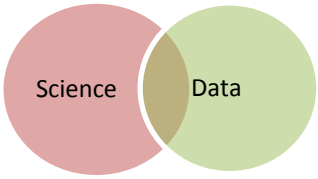
ICT Link
Met Eireann

ICT Link
Chart Wizard

1. In groups, pupils gather data on the average monthly rainfall recorded in the different weather stations in Ireland over a one-year period.
2. The data is entered into an excel spread sheet. Pupils use the functions of chart wizard to create a multiple bar chart.
3. Pupils record three conclusions about the bar chart in their Mathematics Journal.

What do you notice about the bar chart? Which province had the highest amount of rainfall? Which province had the least amount of rainfall? Why do you think that? What does the data not tell us?

Potato Planting[85]



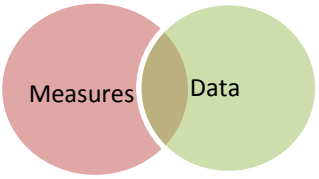
This activity provides opportunities for developing a trend graph.

1. Provide each group of pupils with a sample of potato seeds to plant.
2. Each group creates a prediction chart based on estimated heights of their potato plants.
3. Over the course of two months pupils monitor the increasing height of their group's potato plant in their Mathematics journal.
4. Each data entry can then be plotted on a trend graph.
5. Elicit from pupils a suitable scale for their trend graph.
6. Once the potatoes reach full height comparisons can be made between each group's trend graph.
7. Comparisons can be made between the original prediction chart and the actual height chart.
8. Questions can also be composed about the graph by pupils for other groups.

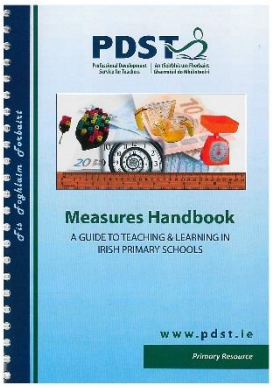
Louise, can you predict the final height of your group's potato plant? Why do you think this? How quickly do you think your plant will grow? Michael, do you agree or disagree with Louise's prediction?

[84] http://www.met.ie/climate/monthly-data.asp?Num=3904
[85] http://growyourownpotatoes.org.uk/pc70/Teacher-resources/Lesson-Content/Growing-potatoes

CONSOLIDATION ACTIVITIES



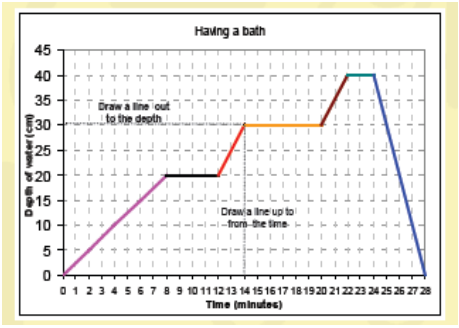
Archimedes Bath Tub[86]



In this interactive activity pupils can observe a trend graph being constructed based on Archimedes sitting in the bath tub. The variables include the power of the water flowing from the tap, whether the plug is in or out and the presence of Archimedes in the tub. Click the interactive link to explore the variables further. As teacher you could also generate a graph and allow pupils to investigate what the conditions were to lead to such a graph being created.

ICT
Archimedes Bathtub Activity

My Bath Story- Introducing The Mathematics World[87]



'Mathematics Eyes', a resource for developing pupils' mathematical skills also have a 'Bath Story' task on page 52 which can be used to consolidate pupils understanding of trend graphs. Click the interactive to access the Mathematics Eyes Handbook.

[88]

ICT
Have you got Maths Eyes

[86] http://www.colmanweb.co.uk/Assets/SWF/Archimedes.swf
[87] http://www.haveyougotmathematicseyes.com/wp-content/uploads/resources/mathematics_eyes_2012_resource_pack.pdf
[88] http://www.haveyougotmathematicseyes.com/wp-content/uploads/resources/mathematicseyes_resource_pack.pdf

LEVEL D.4

EXPLORE FREQUENCY ANALYSIS AND CALCULATE AVERAGES OF SIMPLE DATA SETS.

TEACHING NOTES

At this level pupils are introduced to the more specific kinds of averages known as the mean and the mode. This development allows pupils to progress from noticing individual features of the data to describing an overall feature of the data[89]. Also at this level pupils could be introduced to line plots. A line plot is a simple quick sketch showing the values of the data collected along a horizontal axis with X's used to mark the frequency of those values in the data set. As this is only an informal sketch it does not need to include a title, labels or a vertical axis[90]. Line plots are a useful tool which allows pupils to see data accumulate 'before their eyes' and to observe the changing shape of data as well as emerging patterns in the data. This is not as easily observed when constructing a bar graph.[91]

The **mode** is the value that occurs most frequently in a data set.

The **median** is the middle value in a set of ordered numbers.

The **mean** is a type of average or measure of central tendency.

To find the mean value of a set of numbers three steps are involved[92]:

1. Find the sum of all numbers in the set.
2. Divide by the number of numbers in the set.
3. Round the answer appropriately if necessary.

SAMPLE LEARNING EXPERIENCES

Using Statistical Language[93]

This activity provides pupils with an opportunity to construct line plots while also consolidating their use of statistical language.

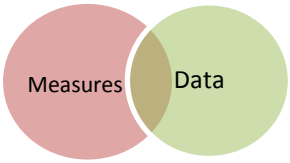
1. Working in groups, pupils record the results of tossing a dice 30 times on individual line plots.
2. Pupils arrange all their line plots in the centre of their group's table.
3. Pupils then take turns to describe their line plot without revealing to the other pupils in the group which line plot it is.

[89] Haylock, D., and Manning, R., (2014, p.421).
[90] Russell, S.J. and Corwin, R.B., (1990, p.17).
[91] Willis, S., Hardman, C. and Sue, W., (2013, p.194).
[92] Haylock, D. and Manning, R., (2014) (2014, p.412).
[93] Willis, S., Hardman, C. and Sue, W., (2013, p.197).

4. Encourage pupils to use words which describe the 'shape' of the data whilst discussing the line plot for example range, outliers, clusters, gaps. The remainder of the group must identify which line plot is being described.
5. On completion of this activity, pupils can compose statements about their line plot in their Mathematics journal.

Lorraine noticed that her data clusters between 3 and 5. Kate, did you notice something similar or different in your line plot? What conclusions could you reach based on the data you collected?

The Mean Foot[94]



1. Elicit predictions from pupils on the average length of our feet in centimetres.
2. In groups, pupils cut a length of till roll that matches the length of his or her foot. Pupils record their names and the length of their feet in centimetres on the strips. Pupils share and discuss the different lengths.
3. Using the data gathered, each group comes up with one value which is representative of the length of a typical foot in their group. Each group records and justifies their chosen value.
4. In order to test the results, the foot strips of the group are then taped together end to end. To evenly distribute the centimetres for each pupil's foot among the members of the group, the strip can be folded into equal parts so that there are as many sections as pupils in the group for example into four pieces of equal length if there are four pupils in the group.
5. The length of any one strip is measured to find the 'average' foot length of each group. Pupils can then compare their individual foot length to the mean value and discuss why some foot lengths are above and below the mean.

[94] Van De Walle, Karp and Bay Williams, (2013, p. 447).

Introducing the Mean Algorithm[95]

The above activity can be extended by finding the mean foot length for the whole class.

1. Pupils predict whether the mean foot length for each group is equal to the mean foot length for the entire class. Elicit from pupils how the mean foot length for the entire class could be obtained and the value of the paper strip method.
2. In order to link the mean algorithm to the activity encourage pupils to imagine a long paper strip representative of all the pupils' foot length in the class. The total length would be the sum of the lengths of each individual's foot. Imagine dividing the strip into 28 equal pieces. To find the length of one section after the strip was divided into 28 equal sections, pupils can divide the total sum by 28.

Mean Balance[96]

1. This problem-solving task provides a good opportunity for pupils to use their understanding of the term 'mean' to identify the missing values in the equations.
2. In order to extend this activity pupils can work in pairs and create their own equations with missing numbers based on the 'mean'.

> The mean of the set of numbers 8, 4, 5, 4, ●, ■ and 6 is 6.
>
> If another ■ is added to the set, then the mean is still 6.
>
> What numbers do ● and ■ represent?

Mean Versus Mode[97]

1. The data gathered in the above activity can form a starting point for this investigation
2. Introduce the line plot to pupils as a useful quick graph to record data. Revisit the shoe sizes of pupils in the class. Pupils identify the range of data values in the class. These are noted in order and form the 'x' axis of the line plot.
3. Pupils mark their individual shoe size on the graph by placing an X above the particular number which represents their shoe size.
4. Draw pupils' attention to the shape of the data. Highlight features such as outliers, gaps and clusters in the data.
5. Highlight the 'bump' in the data as this can be used as formal introduction to the **mode** of the data set (most popular/common value).

---

[95] Van De Walle, Karp and Bay Williams, (2013, p. 448).
[96] http://nrich.mathematics.org/12810
[97] Adapted from:
https://www.into.ie/ROI/Publications/InTouch/FullLengthArticles/FullLengthArticles2012/Mathematics_May2012.pdf

6. Elicit from pupils the advantages of this value in comparison to the **mean**. Working in groups, pupils record three conclusions about the graph. Allow each group to report on their findings. Pupils compare their original predictions with their findings.

What do you think a line plot for 3rd class shoes would look like? Anna can you revoice Sean's idea? What would happen if a value of 35cm was added to the graph? Would the mode or mean change? How do you know? Can you prove it?

Searching for Mean(ing)[98]

This problem solving task requires pupils to calculate the average (mean) of sets of whole numbers. Simply click the interactive to access this task and the possible strategies and solutions linked to the task.

ICT Link
Searching for Mean(ing)

CONSOLIDATION ACTIVITY

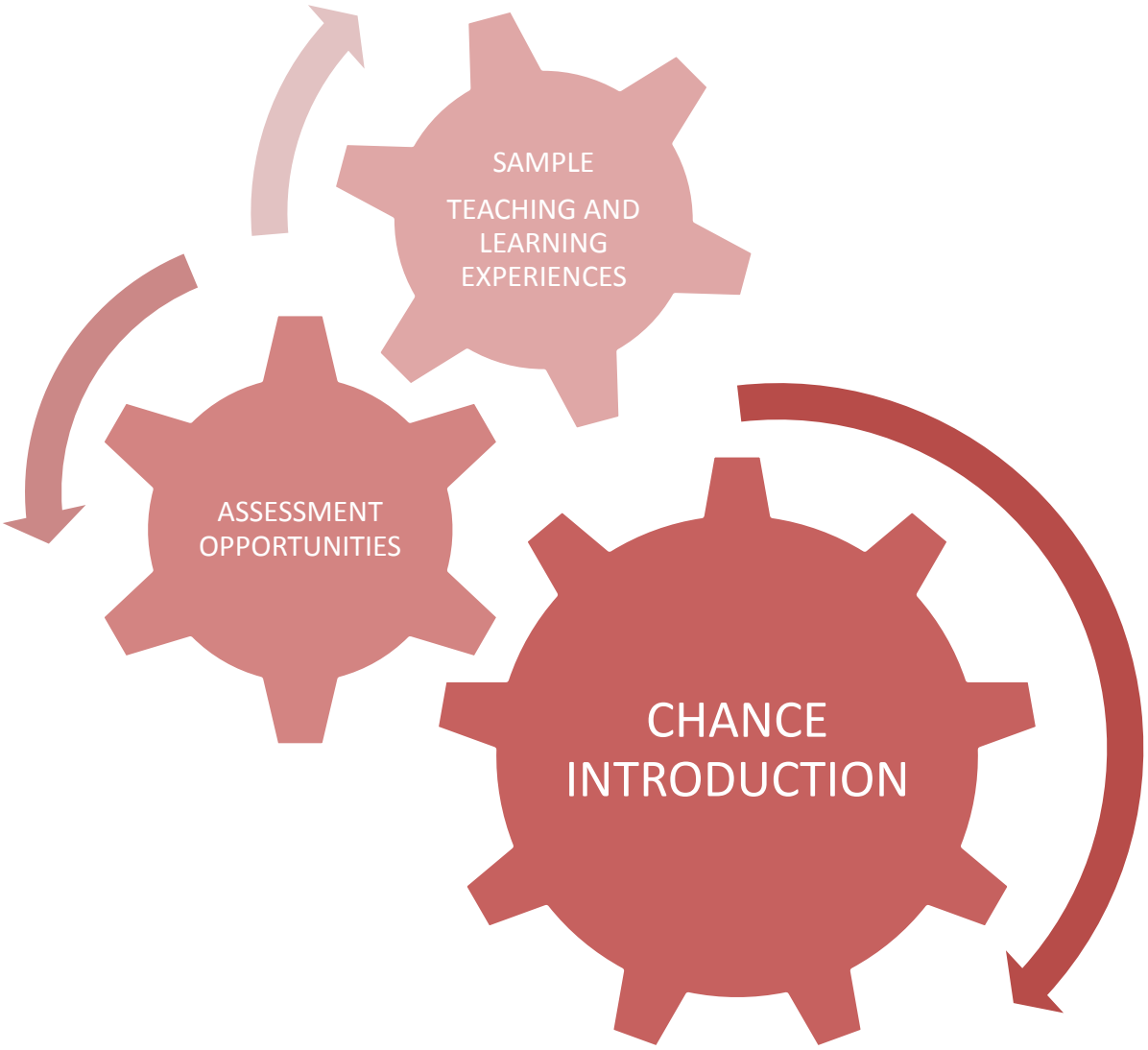
Average Facts[99]

In this activity pupils must use their knowledge and understanding of 'averages' to determine whether or not their fact is true or false for example *the average hen lays 228 eggs per year* (Appendix, Level D.4). On completion of this activity pupils can work in pairs and compile their own true/false average facts based on a topic of interest.

Finding Averages[100]

This card game can be played in groups and can be used to consolidate understanding of the **mean**, the **mode** and the **range** in a data set (Appendix, Level D.4). A dealer and score keeper are identified in each group. The game consists of three rounds with each round focusing on a different average – **mean, mode and range.** At the end of the three rounds pupils add up their total score. The winner is the person with the highest score.

[98] https://nrich.mathematics.org/7204
[99] http://www.did-you-knows.com/did-you-know-facts/average.php
[100] https://www.tes.com/teaching-resource/averages-game-playing-cards-ks3-group-activity-6124913


SAMPLE
TEACHING AND LEARNING EXPERIENCES
ASSESSMENT OPPORTUNITIES
CHANCE INTRODUCTION

CHANCE

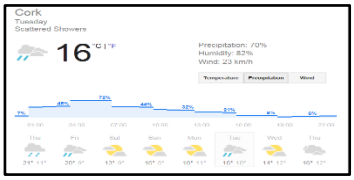
BACKGROUND KNOWLEDGE FOR TEACHERS

Chance and Probability

- Chance is the likelihood that a particular outcome will occur. There is a good chance it will rain tomorrow.
- Probability is the chance that a particular outcome will occur, **measured as a ratio of the total of possible outcomes**. There is 50% Probability of rain tomorrow.

The concept of Chance is introduced and developed from 3rd – 6th class. At post-primary level, students advance in their knowledge of Chance to deal more formally with Probability.[101] With the introduction of Project Mathematics at post-primary level, greater emphasis was placed on the area of Statistics & Probability. Probability is an important branch in mathematics as it provides opportunities for pupils to engage in purposeful and meaningful learning activities and enhances mathematical thinking and reasoning skills[102].

The Primary Mathematics Teacher Guidelines (1999) notes that while references to Chance are all around us, many children and adults have poor or inadequate ideas of Chance and likelihood. Van de Walle et al. suggest that '*realistic concepts of chance require considerable development before students are ready to construct formal ideas about the probability of an event*'[103]. It is crucial that pupils actively participate in experiments related to Chance, rather than discuss hypothetical situations from textbooks and worksheets. *'Concepts in Probability can be more readily understood if pupils are first exposed to Probability via experiment'.*[104] Pupils should engage in discussion-based activities while considering the outcomes of a variety of situations. These discussions provide a foundation on which more formal ideas can be developed at post-primary level.



References to Chance and Probability exist widely in our every-day lives, for example, the weather. Young children will have informal experiences of these concepts from playing board games and sports. Probability theory is linked inextricably to insurance premiums, lottery games, investments and many more areas of daily life. Weather forecasters often predict a percentage chance of precipitation or rain. This refers to the probability that at least some minimum quantity of precipitation will occur within a specified

[101] http://www.ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Mathematics/Syllabuses_and_Assessment/JC_Mathematics_English_2013.pdf
[102] Tsakirdou and Vavyla, (2015).
[103] Van De Walle, Karp and Bay Williams, (2013).
[104] Tsakirdou and Vavyla, (2015).

forecast period and location. Conversely, a report published by the NCCA[105] (2005) found that teachers believed the strand unit Chance to be the least useful in the Mathematics curriculum from 3rd – 6th class.

FUNDAMENTAL FACTS

The Law of Large Numbers was proven by the Swiss mathematician Jakob Bernoulli in 1713. This law states that as the number of trials of a random process increases, the percentage difference between the expected and actual values goes to zero. Therefore, the greater the number of trials, the closer the results will be to the expected probability[106] . While there is no reference to the Bernoulli's Law in the 1999 Mathematics Curriculum, (DES, 1999), research suggests the Law of Large Numbers is undoubtedly a fundamental rule to be learned early on by pupils[107].

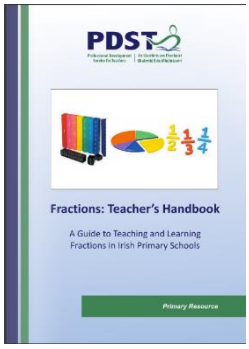
POSSIBLE PUPIL MISCONCEPTIONS

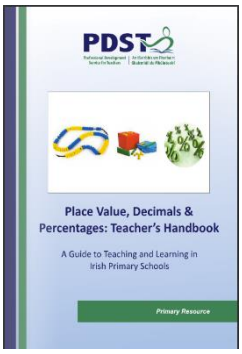
Conducting experiments and examining the outcomes is important in helping pupils address common misconceptions and in developing deeper understanding for why certain things are more likely than others[108].

- As adults, we often hold preconceived ideas about chance. For example, when a person fills out a lotto ticket, they generally spread out their numbers, believing this to give them a better chance of winning the prize. However, the numbers have an equal chance of winning and their order makes no difference to the chance of winning. Similarly, people often use 'lucky numbers' believing they can influence random events.

- At Level D (5th & 6th class), pupils begin to express probability of events in numerical terms, using decimals, percentages and fractions. Weak understanding of fractions may impose limitations on pupils' ability to make probabilistic judgements. PDST have produced comprehensive resources for teaching fractions, decimals and percentages in Irish schools. These two manuals are available to download free from www.pdst.ie and focus on developing pupils' conceptual understanding.

[105] http://www.ncca.ie/uploadedfiles/Publications/PrimaryCurriculumReview.pdf
[106] Leavy and Hourigan, (2015).
[107] Falk and Lann, (2015).
[108] Van De Walle, Karp and Bay Williams, (2013).





TEACHING NOTES

Pupils will need opportunities to become familiar with the language of Chance. It may be useful to begin with a focus on possible and not possible[109]. Later, more vocabulary such as certain and impossible can be introduced. It should be noted that pupils' understanding of straightforward language such as certain, possible and impossible should not be taken for granted. Vocabulary can be woven into discussions in the classroom across various subject areas. The following is a list of words which may be appropriate to use when discussing Chance at primary school level.

Never	Occasionally	Sometimes	Always	Unlikely	Probably
Rarely	Usually	Certain	Often	Maybe	Seldom
Mostly	Possibly	Unsure	Perhaps	Hardly	Likely

The following is a list of resources which may be useful in developing pupils' concepts of Chance.

Dice	Marbles
Playing Cards	Bag
Numeral Cards	Counters
Stickers	Spinners
Coins	Cubes

[109] Van De Walle, Karp and Bay Williams, (2013).

CHANCE LEARNING TRAJECTORY LEVEL C110

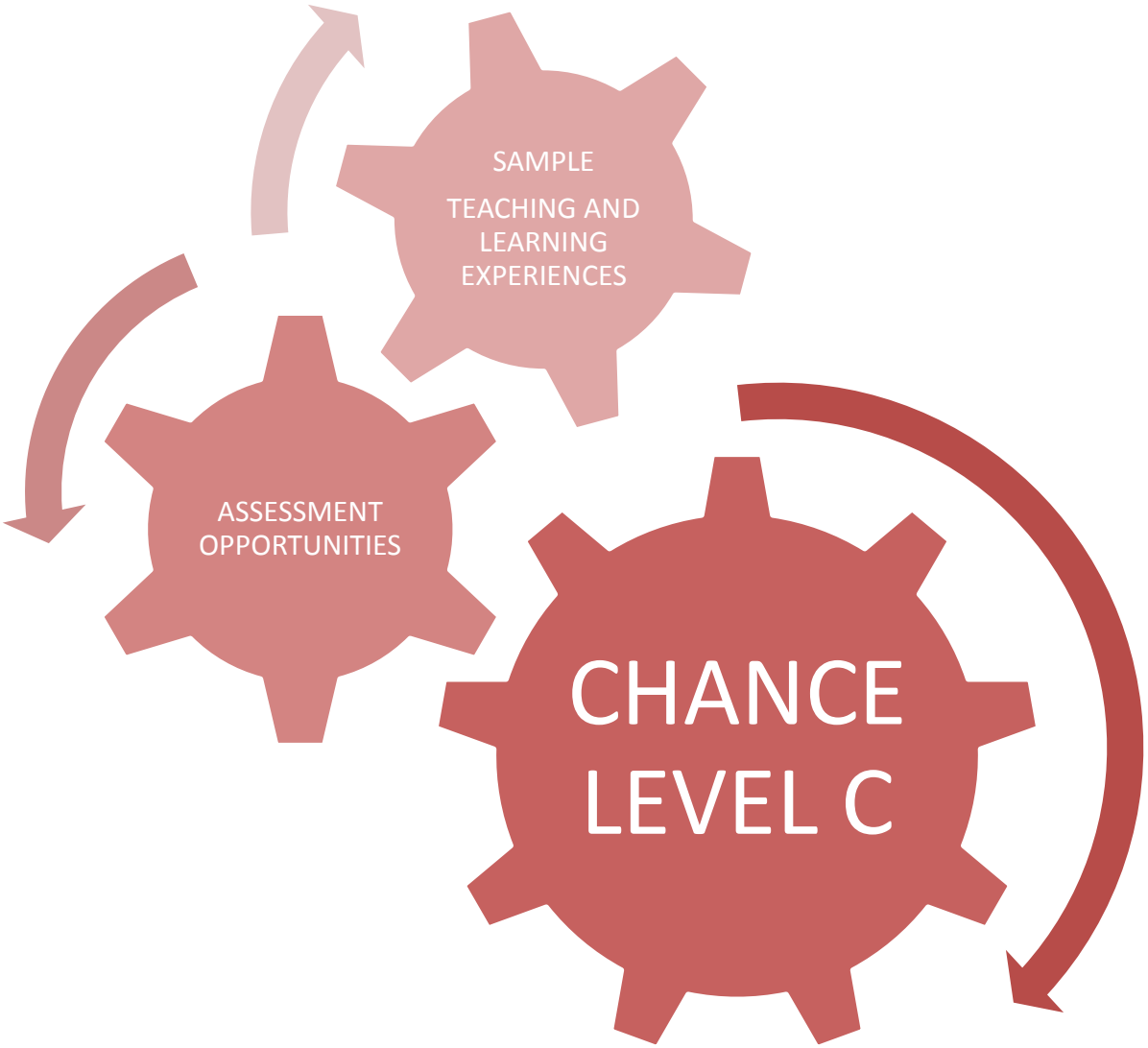
Trajectory Levels	Concept	Developmental Experiences		
		Concrete	Pictorial	Abstract
	Level C.1 Use vocabulary of uncertainty and chance.			
	Level C.2 Order events in terms of likelihood of occurrence.			
	Level C.3 Identify and record outcomes of simple random processes.			

[110] This level is generally aligned with the objectives for Third and Fourth class.

CHANCE LEARNING TRAJECTORY LEVEL D[111]

Trajectory Levels	Concept	Developmental Experiences → Concrete	Pictorial	Abstract
↓	**Level D.1** Consolidate vocabulary of uncertainty and chance			
	Level D.2 Estimate the likelihood of occurrence of events and order on a scale from 0 to 100%, 0 to 1			
	Level D.3 Identify and list all possible outcomes of simple random processes			
	Level D.4 Construct and use frequency charts and tables			

[111] This level is generally aligned with the objectives for Fifth & Sixth Class.


SAMPLE
TEACHING AND LEARNING EXPERIENCES
ASSESSMENT OPPORTUNITIES
CHANCE LEVEL C

LEVEL C.1

USE VOCABULARY OF UNCERTAINTY AND CHANCE

TEACHING NOTES

In 3rd Class, pupils are introduced to the Strand Unit of Chance. However, young pupils will have experiences of early concepts of Chance through playing games and every day experiences. Young pupils' concepts of the likeliness of a future event can be surprising[112]. For example, pupils playing Snakes and Ladders may believe it is harder to roll a 6 than a 1. The language of Chance is the most commonly used mathematical language and is often used in a colloquial or informal manner, for example, tomorrow will probably be fine, the team has no chance of winning the match. According to First Steps Mathematics (2010), pupils need to hear terms such as 'possible', 'impossible', 'unlikely', 'likely', 'certain', 'probable' and 'improbable' modelled and have the opportunities to use this vocabulary in an appropriate context. Since chance is about how likely an event is, a good place to begin is with a focus on possible or not possible[113]. In preparation for the activity, discuss the meaning of impossible and certain. These discussions can be woven into ideas in other curricular areas for example, Literacy.

SAMPLE LEARNING EXPERIENCES

Newspaper Sorting[114]

Pupils, over the course of a few days, collect pictures and headlines from magazines or newspapers which depict a wide range of events from possible to impossible. Under two headings on a notice board or chart, pupils can create a collage of events which are 'possible' and 'not possible/impossible'. Discuss the pictures with the pupils.

Is it really possible for a pig to jump over a moon like that or is it impossible? Why do you think that? Sarah's picture shows a tiger on a busy street. Is that possible or impossible? What about if the animal was a dinosaur?

[112] Van De Walle, Karp and Bay Williams, (2013).
[113] Van De Walle, Karp and Bay Williams, (2013).
[114] Willis, Hardman and Sue, (2013).

Pupils will need to engage in discussion to be exposed to the real meaning of impossibility as referring to things that cannot happen. Through discussion and questioning, challenge pupils' misconceptions and ideas. These images and headlines could later be used to extend pupils' thinking about likely/unlikely.

Considerable life experience is involved in distinguishing those things that are subject to chance variation from those that are not. Events that appear unpredictable may simply be those we do not know enough about, and those we regard as quite predictable can surprise us.
(Willis, S., Hardman, C. and Sue, W., First Steps in Mathematics: Chance and Data, 2013).

Possible or Impossible

The purpose of this activity is to enable pupils to distinguish between possible and impossible scenarios and justify their reasoning. Pairs of pupils or small groups are given a set of 'Chance Cards' (Appendix, Level C.1). Pupils decide which scenarios are possible or impossible.

Why do you think this scenario is impossible? How can you be sure? Compare your set of your 'possible' scenarios to another pair. What surprises you about their set?

Pupils could discuss the differences in their ideas. Allow pupils time to justify and defend their reasons for believing something is possible or impossible. Through discussion elicit events that are unlikely or still possible.
Variation: The same set of Chance Cards could be used to give pupils opportunities to discuss and sort the scenarios into the following categories:

- **Certain/Uncertain**
- **Likely/Unlikely**

Alternatively, pupils could use index cards to create their own set of Chance Cards for use in discussion.

Pass the Parcel

The purpose of this activity is to enable pupils to discuss a variety of scenarios in terms of Chance. A shoe box of probability statements is passed around as music plays. When the music stops, the pupil holding the box draws out a statement and reads it to the group. The pupils must then decide where to place the statement on the continuum that is drawn on the whiteboard.

Least likely

Most likely

Sentences could be about events in school or about random events. Possible examples of Chance statements include:

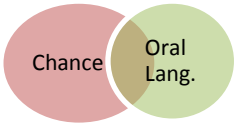
1. The principal will be the next person to walk into the classroom.
2. There will be a dog in the yard at lunch time.
3. The teacher will not be in school tomorrow.
4. An elephant will visit our school.
5. We won't get to play outside tomorrow.
6. The school team will win the match.
7. We will get extra homework on Friday.
8. The principal will decide to close the school tomorrow.
9. We will eat our lunches at lunchtime.
10. I will roll a 7 if I throw a dice.
11. The school bus will be late today.

Record pupils' language as they discuss the chances of events happening. Compare event likelihood with other events that have been placed on the continuum.

Seán says there is no chance of an elephant arriving into school today. Can anybody suggest another word for 'no chance'? Does anybody think there is a chance of an elephant coming? Explain your thinking.

Allow the pupils time to discuss their reasoning and have an opportunity to use the language of chance.

CONSOLIDATION ACTIVITY



Walking Debate

A walking debate is a useful methodology to develop pupils' communication and reasoning skills. Place labels indicating 'possible' and 'impossible' at either side of the classroom. Call out a statement/scenario (see examples above), then pupils must decide whether to walk to the possible or impossible side of the room. Allow pupils time to state why they chose that side of the room and provide each side a chance to persuade pupils to join their side. As pupils listen to each other, they may indicate that they would like to change their mind and move to the other side of the room. Provide an opportunity for pupils to do so. Vary the labels to include more vocabulary including likely/unlikely, certain/not certain etc. You may also like to include an 'unsure' station.

LEVEL C.2

ORDER EVENTS IN TERMS OF LIKELIHOOD OF OCCURRENCE

TEACHING NOTES

At this level, pupils should discuss and order events by how likely they are to happen. Being able to say that two events are more, less or equally likely does not require pupils to have any idea of the numerical probability[115]. In fact, focusing on numerical probability too early is likely to be unhelpful and prevent some pupils from developing the concept of Chance. Teachers can enable pupils to understand what it means to say events are equally likely by thinking about familiar events which are more or less likely. Pupils should begin to use expressions such as very likely, fairly likely, equally likely, fairly unlikely and very unlikely. It is important pupils are provided with opportunities to explain their reasoning. Pupils' understanding of the concept of 'how likely?' will be best developed if initially the events being ordered are obviously very different in likelihood[116]. As pupils grow in confidence, they may be asked to compare and order events which are closer in likelihood.

SAMPLE LEARNING EXPERIENCES

Bags of Bears

For this activity, pupils will need access to attribute bears. However cubes, lollipop sticks or any other coloured resource available will work and could be used to extend and consolidate this activity. Pupils can work in pairs to create the following 'bags of bears'. Combinations in the bags could include opportunities for the following possibilities to occur:

- A bag of red and yellow bears where you are very likely to pull out a red bear
- A bag of two different coloured bears where you have no chance of pulling out a red bear
- A bag of bears where you are certain you will pull out a yellow bear
- A bag of bears where you are unlikely to pull out a blue bear
- A bag of bears where you are equally likely to pull out a red bear or a green bear.

The criteria can vary depending on pupils' abilities. Pupils should discuss their reasoning when showing their choices. Pupils should discuss the likelihood of pulling bears from each other's bags of bears. Encourage pupils to extend their thinking about their bear combinations.

What else can you tell me about your bag of bears? What do you notice about Sarah & Tom's bag?

[115] Willis, Hardman and Sue, (2013).
[116] Willis, Hardman and Sue, (2013).

Pupils could record their findings in a learning log or use a digital portfolio to document and share their learning. The Seesaw app enables pupils to take pictures or videos and talk about their learning. [117]

Equal Chances[118]

Using a black cube and a white cube in a feely bag, pupils discuss the chances a particular colour will be pulled out. Elicit that that both cubes have an equal chance of being picked so the chances are equally likely.

Am I more likely or less likely to pick the black cube? How do you know? Could I pull out a red cube? Could we put more blocks in the bag but still have equal chances of getting a white or a black? What numbers would not give us an equal chance? If we were added a third colour, would black and white still have equal chances? Convince me.

This activity could provide opportunities to informally explore a common misconception known as Gambler's Fallacy.

The gambler's fallacy is when a person believes that the onset of a certain random event is less likely to happen following an event or a series of events. This line of thinking is incorrect because past events do not change the probability that certain events will occur in the future. In a simple experiment, a coin was tossed 5 times and landed on heads 5 times in a row. If a person believes it is less likely to land on tails a 6th time, this would be incorrect. The chances of the coin landing on tails, remains 50-50.

Sarah has pulled out the red cube four times. Does this help us predict what colour come next? Is she more likely or less likely to pull out red again? Explain your thinking.

[117] http://web.seesaw.me/
[118] Willis, Hardman and Sue, (2013).

How Can It Change?[119]

The purpose of this activity is to raise awareness of the impact of various factors or variables on Chance. After introducing a possible scenario or event, pupils discuss factors which might affect the chance of that event happening.

What could you do to make it less likely that you will fall in the yard at play times? What would make it likely that you will fall in the yard?

Repeat this activity with various scenarios which the pupils can relate to, extending pupils' understanding of more likely/less likely, emphasising factors or variables each time.

Let's have a look at the weather for the coming week. How likely that we will be able to have the sports day on Thursday? Why do you think that?

Through the primary years, students should begin to take chance variation into account in sensible rather than technical ways. They should learn not to assume that what happens in a sample will exactly predict what happens in the whole population, and to develop an everyday sense of what is normal variation and what is unusual

(Willis, S., Hardman, C. and Sue, W. First Steps in Mathematics: Chance and Data, 2013).

[119] Willis, Hardman and Sue, (2013).

LEVEL C.3

IDENTIFY AND RECORD OUTCOMES OF SIMPLE RANDOM PROCESSES

TEACHING NOTES

Before pupils are able to make numerical statements, they need to be able to list all possible outcomes for an event and make decisions whether or not the outcomes are equally likely[120]. Often young pupils will focus on a particular outcome of relevance to them or the outcome they believe most likely. Encourage pupils to consider all possibilities of an event. Pupils will need to develop conceptual understanding of Chance through experimentation and discussion while ordering events non-numerically.

Early experiences should emphasise 'What are all the possibilities?' rather than 'what is the chance of one possibility happening?'
(Willis, S., Hardman, C. and Sue, W. First Steps in Mathematics: Chance and Data, 2013.)

SAMPLE LEARNING EXPERIENCES

Ice-Cream

The purpose of this activity is to find possible outcomes of an event using a problem solving strategy. An ice-cream parlour has four flavours of ice-cream. Challenge the pupils to discover how many two-scoop combinations can be made from the four flavours. Pose questions that focus the pupils on counting the outcomes systematically.

How many different combinations have you found?
Have you found them all? How do you know? Share your strategy for working out the combinations.
How could you convince others that you have found all the ice-creams? How could you record your findings?

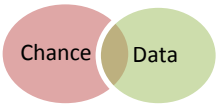
Pupils will discuss and share the different ways that have been used to find all the outcomes.

Further Development: The following ICT link[121] is an interactive where pupils must calculate how many two scoop cones can be created from six different flavours. Create a poster that shows your group's thinking.

ICT
Ice- Cream

[120] Willis, Hardman and Sue, (2013).
[121] http://www.transum.org/Software/SW/Starter_of_the_day/starter_November12.ASP

What Colour?[122]



In the following three activities, pupils will use small boxes of smarties to find and record all possible outcomes of random events. Showing pupils a small box of Smarties™, pose the following questions.

If you close your eyes and chose a colour, what colour might it be? Is it possible you might get another colour? Which colours might you get? Can you get a black smartie™? How do you know?

1. Provide each pair/small group with a small box of Smarties ™ and a paper cup/bag. Pupils should now list the colour contained within their box and record how many of each colour. Pupils might construct a graph to record the colours within. At this stage pupils could compare and discuss each other's graphs. What was the most common colour/least common etc.?

Looking at the data, what colour would you most likely chose if you picked a smartie™ from a cup at random? Why do you think this? What colour would you least likely chose? Explain your thinking? How does your data help you make a prediction?

2. Pupils could experiment and test their ideas by randomly selecting a Smartie™ and recording their choice. **N.B. Remind pupils to replace the Smartie™ before testing again**. Extend pupils' mathematical thinking by posing the following question:

Can you use numbers to say how likely you are to get a red? What numbers would you need to know?

3. Pupils can test their ideas by exploring and recording the results of 20 tests. Pupils could use a tally chart to record their results. Pupils should be given the opportunities to discuss their experiment and findings.

[122] Willis, Hardman and Sue, (2013).

Roll These Dice[123]

The purpose of this activity is to enable pupils to find and record all outcomes of a simple process.

1. Pupils work in pairs or small groups. Each pair/group should have 2 red dice and 1 green dice. (Colours can vary as long as pupils have 2 identical dice and 1 different dice).
2. Pupils roll the dice and add up the numbers on the two RED dice and then subtract the number on the GREEN. For example, if one RED is 4 and the other RED is 5 and the GREEN is 3, we should add together 4 and 5 to make 9 and then subtract the 3 so that gives us a final answer of 6.
3. Challenge pupils to consider all possible outcomes through questioning.
4. Extend pupils' mathematical thinking by asking them to look at their results and write down any questions they have.

What are the final answers by doing the addition and subtraction each time? What are all the different possible numbers? Is there an efficient way of making sure you find them all? Can you prove it? How will you record what you've found out?

CONSOLIDATION ACTIVITY

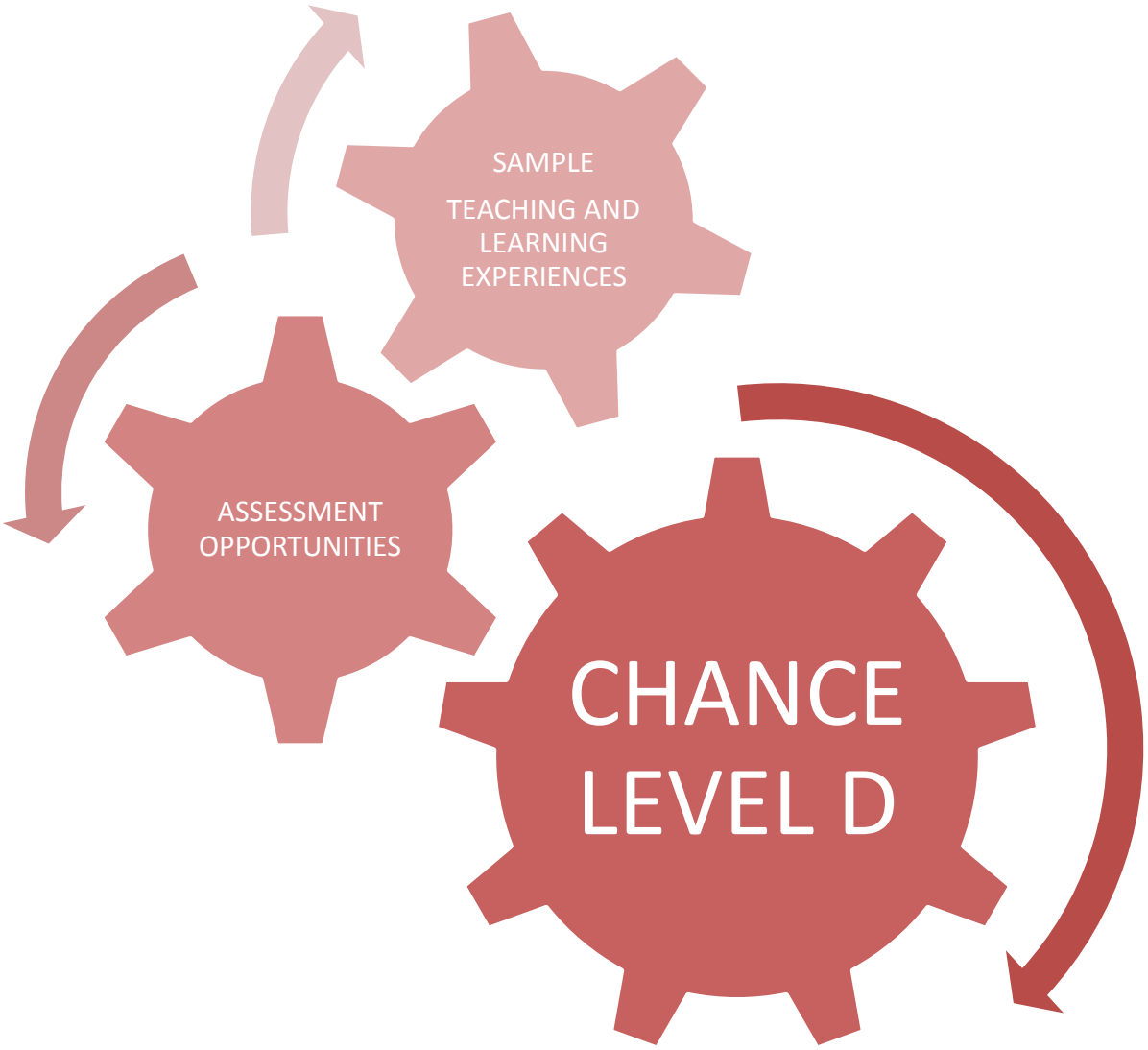
Same or Different[124]

The purpose of this activity from the NRICH website is to enable pupils to consider all outcomes of a simple game and consider the fairness of the game. Pairs of pupils play the following game several times, record the outcomes and decide upon the fairness of the game. Pupils put one purple cube and two yellow cubes in a bag. Each pupil takes a turn pulling a cube without looking. If the two cubes are the same colour, Pupil A wins but if the cubes are different, Pupil B wins the game.

Is this a fair game? Does everybody have an equal chance of winning? Explain your answer. How can you prove this is/is not a fair game?

Further Development: Challenge the pupils to test out some ideas they may have for creating a fair game using various combinations of cubes and colours

[123] https://nrich.mathematics.org/53
[124] https://nrich.mathematics.org/1176


SAMPLE
TEACHING AND LEARNING EXPERIENCES
ASSESSMENT OPPORTUNITIES
CHANCE LEVEL D

LEVEL D.1

CONSOLIDATE VOCABULARY OF UNCERTAINTY AND CHANCE.

TEACHING NOTES

At this level, pupils should be familiar with the language associated with Chance, such as, possible, impossible, certain, likely, unlikely etc. Activities from Level C.1 may be used to revise this vocabulary. This level contains learning experiences to consolidate pupils' vocabulary of uncertainty and chance. All pupils should be able to discuss probable outcomes of events. As referenced in recent Irish research, an understanding of the language of probability is crucial as pupils will become critical consumers of the data presented in newspapers, surveys and consumer reports[125]. This vocabulary is the most commonly used mathematical language. Pupils should begin to compare and order events based on likelihood. This can be followed by introducing probability scales which uses everyday language such as:

Impossible – almost impossible – unlikely – even chance – likely – almost certain –certain – impossible before introducing numerical scales.

SAMPLE LEARNING EXPERIENCES

What are the Chances?

The purpose of this activity is to enable pupils to consider the everyday nature of the language of chance. Working in small groups or pairs, pupils brainstorm phrases used to describe the probability or chance of an event occurring on a scale from absolute certainty to no chance at all. Pupils can discuss the meaning of the terms and phrases used in everyday situations. Some examples may be:

- Fat chance
- A dead cert
- No hope at all
- Never in a month of Sundays
- Unlikely
- Certain
- Fairly unlikely
- Impossible
- Even chance
- 50/50

Pupils can create statements to match each word/phrase. This activity should lead to rich discussion as to the meanings of the words and statements.

[125] Leavy and Hourigan, (2015).

Likelihood Line

The purpose of this activity to enable pupils to compare and order events in terms of likelihood. Pupils at this level may like to discuss topics in current affairs, popular culture or sport.

1. Encourage pupils to compare and order their ideas in terms of likelihood. For example, Tipperary are more likely to win the All–Ireland hurling final this year than Wexford. Kilkenny are fairly likely to win but not certain to.
2. Pupils could design a Likelihood Line based on their ideas in small groups or pairs based on a topic of interest, using posters or ICT. Alternatively, pupils could use a rope or string, for example a skipping rope and label the rope from Impossible to Certain.
3. Pupils can order statements along the line, offering reasoning and proof for the placement of their statements.

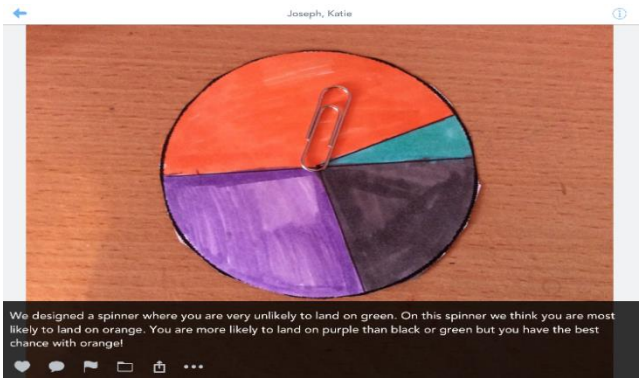
CONSOLIDATION ACTIVITY

Design A Spinner[126]

For this activity, pupils will need cardboard or card to design spinners. A template for spinners can be found through the link below.[127] A paper clip on a pencil can be used to spin on the coloured wheel (See image below). Pupils work in pairs to design spinners to meet certain criteria such as below. Pupils swap their spinners and test their designs.

- Design a 4 colour spinner with an even chance of landing on each colour.
- Draw a 4 colour spinner such that landing on green is fairly unlikely.
- Draw a 4 colour-colour spinner such that landing on red is most likely.

Pupils should compare and discuss their spinners, for example, estimating and measuring the angles created on the different spinners. Pupils should test their ideas by spinning 20 times and recording the results of their experiments, for example with a tally chart . Pupils can record their designs and findings using a learning log. Here is an image from a free app 'Seesaw'[128] where pupils have recorded their work and ideas using a digital portfolio. There are many opportunties to elicit and extend pupils' understanding of angles in the PDST Shape and Space Teaching Resource.



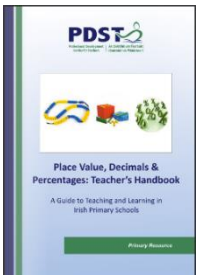
[126] http://www2.math.umd.edu/~tjp/214%2001.0%20Beckmann%20Activity%20Manual%20Chap%2016.pdf
[127] http://nzmathematics.co.nz/sites/default/files/SpinningBlankSpinners.pdf
[128] http://web.seesaw.me/

LEVEL D.2

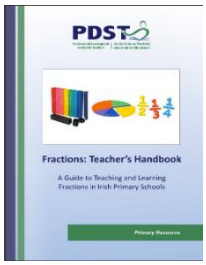
ESTIMATE THE LIKELIHOOD OF OCCURRENCE OF EVENTS AND ORDER ON A SCALE FROM 0 TO 100%, 0 TO 1.

TEACHING NOTES

At this level, pupils have been developing their understanding and use of the language of Chance by comparing and ordering ideas. While comparison and ordering are the first stages of the development of any aspect of measurement, the next stage is to introduce a probability scale[129]. Level D.1 introduces a probability scale based on the language of probability and at this level pupils will progress to explore the idea of measuring probability on a numerical scale informally. Haylock recommends encouraging pupils to assign points out of 100 (percentage scores) to various events which might happen, with 0 points for impossible and 100 points for certain. Pupils can then convert these percentages to decimals ranging from 0 (impossible) to 0.5 (evens) to 1 (certain).



At this level, pupils should have a solid conceptual understanding of fractions, decimals and percentages. PDST have produced two resources to assist teachers in teaching place value, decimals and percentages and fractions. Teachers may find the activities contained within these resources useful at this level. The resources are available free to download from www.pdst.ie.



The process of estimating probabilities is very much a personal matter, and sometimes uncovers superstitious beliefs. It is a sensitive area, so care should be taken not to ridicule children's ideas. (Suggate et al, p.284).

SAMPLE LEARNING EXPERIENCES

What are the Chances (2)

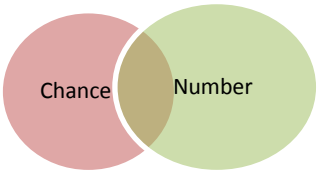
Pupils will revisit and consider some of the phrases used in the previous activity in level D.1.

With your partner, think of a scenario that is certain to happen tomorrow. Out of 100, what mark would you give the chance of this event happening? What is the percentage score of this happening? Now think of a scenario that has a 0% chance of happening tomorrow? What can we say about the chances of this event happening?

[129] Haylock, D., and Manning, R., (2014).

Through discussion, pupils can assign percentage scores to the phrases used in the earlier activity. Pupils' percentage scores may vary and it is important to allow time for discussion around pupils' ideas.

Likelihood Line (2)



This builds on the likelihood line activity in Level D.1. Using skipping ropes or pieces of string, pupils invent statements and place them along a line with 0% at one end, 50% in the middle and 100% at the other end. Alternatively a likelihood line may be created on the playground yard using chalk. Pupils may decide to include other percentages along the line. It is important that pupils discuss their reasons. If pupils disagrees with the placement of a statement, they should defend their reasoning.

Claire, can you explain why you think you have 90% chance of having no homework next Friday? David do you agree or disagree with Claire? What are the chances of getting homework off on Tuesday? Using our knowledge of percentages, are there any other ways we could represent the chances of these events using numbers?

Elicit how percentages could be converted to decimals or fractions. Pupils can add labels with decimals to the likelihood line using decimals to highlight the standard scale used for measuring probability.

Andrew (2009) stresses the importance of pupils gaining concrete experience as concepts in probability can be more readily understood if the pupils are first exposed to probability via experiment.

I'm Spinning[130]

The purpose of this activity is to determine the probability of simple events using percentages, fractions and decimals. Pupils will need templates for spinners in thirds and quarters. These templates can be downloaded and printed from http://nzmathematics.co.nz/resource/im-spinning .

1. Pupils look at a spinner, which is equally divided into two colours, for example, red and green. Pupils predict which colour the spinner will land on. Spin and repeat a few times allowing pupils to change their guess each time.

What can you tell me about this game? Is it fair? Why or why not? Did everyone have an equal chance of winning? What fraction of the spinner is red? How else could we represent this using numbers? If we spun the spinner 30 times, how many times do you think it would land on red? Record your estimate.

[130] http://nzmathematics.co.nz/resource/im-spinning

2. Spin the spinner 30 times and record the results on a tally chart. Pupils could also record their results using a frequency table. (See Level D.4 for more information). Compare the pupils' estimates. Repeat the game with a spinner that is ¾ red and ¼ green and discuss.

Which colour did you choose? Why? Did the spinner always land on that colour? Why not? Is this a fair spinner? Why not?
Would this be a good spinner to use in a game? Why or why not?
What else can you tell me about the spinner? What fractions are the colours on the spinner? Why do you think that the size of the segments (colours) is important?
What do you think would happen if I spun the spinner 20 times?

3. Pupils work in pairs to design a spinner using three colours that they think would be fair. Pupils can test their designs to see if they seem fair. Swap and share spinners.

Further Development: Using a blank circle, pupils can make a spinner for free choice activities. Pupils must include at least one activity they don't want to complete. Pupils can test out their spinner, keeping track of their results using a tally chart. Pupils could also record their results using a frequency table. (See Level D.4 for more information). Pupils predict the number of times the spinner will land on their favourite activity. Conclude the activity by sharing spinners and tally charts. Encourage pupils to use numerical probability when discussing the spinners.

Did your spinner work the way you thought it would? Why or why not? Was your prediction close? Which spinner would you choose to use? Why?

Emphasise the idea that probability does not tell you anything about what will happen next, but predicts what will happen in the long run.
(Haylock, D. 2015 p.441).

What are My Chances[131]

The purpose of this activity is to foster pupils' curiousity regarding probability while engaging in experiments. Through discussion and experimentation, pupils informally discover that the number of trials impacts the relationship between theoretical and experimental probability. Pupils conduct 5 simple games of chance. If there are enough cards, coins and dice for each group, pupils can work at their desks. Otherwise, pupils can

[131] https://illuminations.nctm.org/Lesson.aspx?id=2895

work at 5 stations and rotate. Pupils could record their results using tally charts or frequency tables (See Level D.4 for more information). Depending on class size, further stations could be added.

- **Stations**
- Flipping a coin
- Rolling a dice
- Picking a card colour
- Picking a card suit
- Picking an exact card

1. Begin by showing the pupils the coin. Ask the pupils 'what are the chances of flipping the coin and landing on tails'? Pupils may answer with even chance, equal etc. but elicit numerical values from the children, for example. 1/2, 50% or 0.5. Continue this with the other games of chance eliciting the probabilities of each game.
2. Record the pupils' responses on the board to refer to later.
3. Pupils rotate in groups to the stations, playing each game once and recording the results.

In which games of chance were your predictions most accurate? Why you think this is? Which game was the hardest to win? Why do you this is so? What surprised you about each game? Compare the probability we recorded to your result? What questions do you now have about the chances of winning each game?

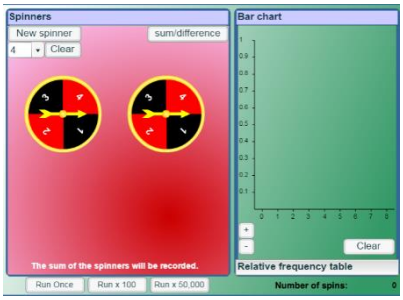
4. Each group could now revisit a station and play the game 20 times, recording the result each time. Through discussion, children should come to the realisation that it is sometimes difficult to predict the outcome of an event using an isolated or small number of trials.

After playing the game 20 times, what did you notice? How did the result compare to the result the first time you played?

Interactive Spinner[132]

ICT
Nrich Spinner

Using ICT, teachers can develop pupils' understanding of the Law of Large Numbers, which states the greater the number of trials, the closer the experimental results will be to the theoretical probability[133]. The following is an online resource which may be useful in developing understanding of the Law of Large Numbers.



1. Set the spinner to the required number of segments, e.g. 4, 8, 12. Pupils should discuss the chances of landing on a chosen number.

What are the chances of landing on 7? Seán thinks there is a 12.5% chance. Could we represent this answer using decimals? How else could we represent the chance of landing on 7?

2. Spin the spinner once and then five times and discuss the results. How do the actual results compare the pupils' predictions and expectations?
3. Use the interactive buttons to spin the spinner 100 times and discuss the chances of landing on 7. Follow with discussion. Follow this with 50,000 times.

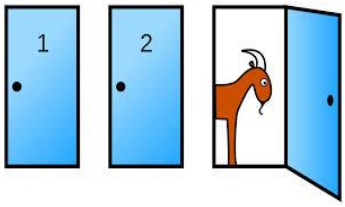
What did you notice as we increased the number of times we spun the spinner? What idea does this give us about the chance of landing on a 7? How could we test our ideas?

Stick or Switch

The purpose of this activity is to develop pupils' reasoning skills and consider the chance of winning a competition. This activity is based on a classic game show scenario and is often called 'The Monty Hall Problem'. A contestant picks one of three doors in the hopes of winning a car. The host, who knows the location of the prize, first opens one of the remaining doors revealing there to be a gag prize there (usually a goat!). The contestant at this point can either stick with their original choice or switch to the unopened door. Many people will stick with their original choice.

ICT
Stick or Switch

[132] https://nrich.mathematics.org/content/id/6033/experimenter.swf
[133] Leavy and Hourigan, (2015).



However, mathematicians have proven you have a greater chance of winning the prize if you switch. At the beginning, you have a 33% chance of winning the prize. After the host reveals a gag prize, there are now two doors left. The chances of winning remain at 33% if you stick to your first choice. However, if you switch, your chance has increased to 50% as you are choosing between two doors. Further explanations of this problem are can be found at the link below.[134]

This problem could be introduced to pupils using an interactive game[135].

1. Discuss strategies for the game with the pupils. Pupils should investigate whether sticking or switching gives you the best chance of winning.
2. Pupils design an experiment in groups and use the online simulator to further investigate this. It is important that pupils run a large amount of trials and discuss their findings. Groups could present their findings.

Let's discuss the contestant's options. Which strategy would you choose and why? Would you stick with your original choice or change your door? Which choice gives you the best chance to win? Convince me. How could we test our ideas? Is this a fair test?

Share your findings with us. Did anything surprise you during the experiment? Did you notice any patterns? After running the experiment, how did your ideas about the strategies change? What questions do you now have?

[134] http://www.stayorswitch.com/explanation.php
[135] http://www.stayorswitch.com/

LEVEL D.3

IDENTIFY AND LIST ALL OUTCOMES OF SIMPLE RANDOM PROCESSES

TEACHING NOTES

At this level, pupils play games, record results and use these results to make predictions. They find out that with probability they can never know exactly what will happen next, but they get an idea about what to expect. Probability can only help us make our best guess about what might happen next. However, pupils should become aware that sometimes what we least expect actually happens.

Fair or Unfair[136]

The purpose of the activity is to enable pupils to consider all outcomes of a simple game.

1. In groups of three, pupils will play a game of tossing a coin twice. Pupils must record the points based on the following: Player 1 gets a point if the coin toss results in 'two heads', player 2 gets a point if the coin toss results in 'two tails' and player 3 gets a point if the coin toss results in 'one head, one tail'.
2. Pupils will play and record the results 20 times and the winner is the player with the most points.
3. Pupils can play the game a number of times before a class discussion is held.

What did you notice after playing the game? Is this game fair? Why or why not? What are chances of getting two tails? Does every player have an even chance? With your group, discuss every outcome of the coin tosses.

Challenge pupils to reason why the game is fair/unfair. Pupils should come to realise that there are four possible outcomes of the game:

1. Heads, heads
2. Tails, tails
3. Heads, tails
4. Tails, heads

Therefore, the Pupil C has a 50% chance of winning, whereas A & B have a 25% chance each. It is important that pupils can justify and explain their reasoning about the fairness of this task.

In a research study with a similar task, more than half of the pupils correctly answered about the probability of the result, but many of these pupils had used faulty or incomplete reasoning. These pupils had failed to recognise that there were four possible outcomes of the experiment. Encouraging pupils to examine all outcomes helps pupils connect with the fairness of the game.
(Van de Walle et al., 2011, p.460).

[136] Van De Walle, Karp and Bay Williams, (2013).

Bags of Probability 1[137]

1. Put four cubes in a bag (3 red & 1 blue). Tell pupils there are 4 cubes in the bag and they are either red or blue but don't tell them how many of each.
2. After shaking the bag, a pupil selects one and shows the class. Record the colour on the board and the pupil replaces the cube in the bag (Note: Each time a pupils takes a cube it must be returned before the next pupil draws a cube. Otherwise, the probabilities will change).
3. Another pupil draws a cube. If it is the same colour pose the question: 'Is *this the same cube as last time? Explain.'*
4. A third pupil draws a cube but this time, the class might predict what the cube might be. Repeat for a fourth time.

Do you think that we have seen all the cubes?
Do we know what the 4 cubes are? Why or why not?
Would we find out more if we had more turns?

5. Four more pupils repeat the process and record these on the board. Each time ask the pupil to predict the colour of the cube. Pupils will work in pairs to consider all combinations for the 4 cubes.

Possible combinations for 4 cubes

Red	Blue
1	3
2	2
3	1

6. Pupils should decide what they believe the combination of cubes are and offer reasoning for their answer.

Bags of Probability 2[138]

The purpose of this activity is to provide opportunities for pupils to design and perform simple experiments to determine all outcomes of simple random processes.

1. Give each pair of pupils 10 cubes each of 2 colours. Explain to pupils they must put 10 cubes in their bag using any combination of the 2 colours.
2. Pupils will swap bags with another pair and predict how many cubes there are of each colour in the bag by taking turns drawing cubes from the bag. Remind pupils to put the cube back in the bag after each draw.
3. Pupils can make as many draws as they like before taking their guess. It is important that pupils make a recording of their draws and also they may write about why they made the prediction.

[137] http://nzmathematics.co.nz/resource/whats-bag
[138] http://nzmathematics.co.nz/resource/whats-bag

4. At the end of each session pairs of pupils share their predictions and then open the bags.

What did you predict? Why?
Were you surprised when you looked in the bag? Why?
How many times did you draw from the bag?
What helped you with your prediction?

Further Development

This activity could be extended by varying the number of cubes but limiting the choices to 2 colours.

CONSOLIDATION ACTIVITY

ICT
Hi-Low

Hi- Low[139]

This interactive game challenges pupils to predict whether the next playing card to be turned over will have a higher or lower value. Pupils should consider all outcomes before making a decision. This interactive version also asks pupil to numerically predict the chance of getting a higher/lower card. Alternatively pupils could play this game using suits of cards in pairs or small groups.

Problems with Probability[140]

While tackling these problems, it is important that pupils record chance associated with each of the problems and also justify their reasoning through discussion.

Sam's Sandwich

Every morning Sam makes his own sandwich. He can use brown or white bread and can choose between honey, cheese, jam, peanut butter or chocolate spread. How many different sandwiches can Sam make with one filling? If Sam makes each possible sandwich equally often, what are the chances that he made a white sandwich with honey for lunch today?

Pete's Pizza

At Pete's Pizza there is a special on. You can choose two toppings on your pizza. One choice has to be made from the ham, mushroom, pepperoni and bacon bin, while the other choice has to come from the pineapple, tomato and extra cheese bin. How many different pizzas are possible with 2 toppings?

Coin Tossing

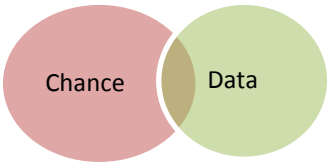
Áine is captain of her camogie team and always selects heads at the start of the game. What is the probability that she will win the toss for the next four games?

[139] http://www.transum.org/Software/Online_Exercise/Probability/PlayYourCardsRight/Default.asp
[140] http://nzmathematics.co.nz/resource/counting-probability

LEVEL D.4

CONSTRUCT AND USE FREQUENCY CHARTS AND TABLES

TEACHING NOTES



Level D.4 is inextricably linked with Level D.1 and D.4 of the Data Trajectory. Activities from Level D.1 and D.4 enable pupils to explore, analyse and compile statistics and frequency analysis. Pupils should have prior experience of using tallies from Level B.2 of the Data Trajectory. The use of frequency tables is a progression from tallying. A useful technique when collecting and organising data is that of tallying. If some pupils show signs of confusion with tallies, for example, drawing 5 vertical lines and then a 6th line placed diagonally. The following rhyme may be useful for pupils:

One, two, three, four,

Number five shut the door!

Data should then be organised in a frequency table. Level C and D of the Data Trajectory presents opportunities for pupils to develop their skills at constructing and using simple frequency charts and tables. Further experiences can be found below.

SAMPLE LEARNING EXPERIENCES

The following activity uses M & Ms™ which may contain traces of nuts. Therefore these sweets could pose a danger to children with a nut allergy. If in doubt, the sweets used could be easily substituted for a nut free alternative.

M & M-aths

The purpose of this activity is to develop pupils' understanding of frequency tables and the use of probability. M & Ms™ are popular sweets manufactured by Mars™. They are produced in six different colours ; yellow, orange, green, red, blue, brown. In this activity, pupils estimate, record and calculate the most common colour of M&Ms™ using an individual bag and multiple bags.

1. Pairs or small groups of pupils are given a bag of plain M&Ms. Discuss with pupils the chances of removing a particular colour from the bag without looking. Pupils discuss, predict and record (without opening the bag!) the following:
    - The number of sweets in the bag
    - The most popular colour in the bag
    - The least popular colour in the bag

2. Pupils perform the following experiment. After opening the bag, pupils remove one sweet from the bag, record its colour and return the sweet.
3. Pupils repeat the procedure the same number of times as their intial prediction of the total sweets , recording their findings using a tally chart. Pupils could use the following chart.

Predicted Data			
	Tally	**Out of Total**	**Percentage**
Yellow			
Orange			
Green			
Red			
Blue			
Brown			

For example, if pupils predict there are 30 sweets in total and 6 yellow sweets, their predicted percentage would be 20%. Discuss predictions with pupils and allow them time to justify their reasoning. Pupils may believe there is an even distribution of colours or some may believe one colour to be more popular than another.

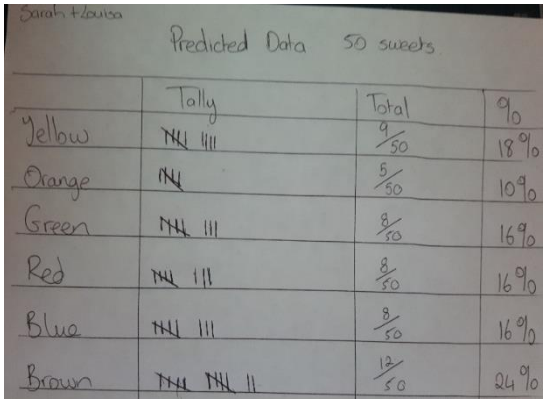
4. Pupils remove the M&Ms and calculate the actual data. Stress the importance of not eating the data (yet!).

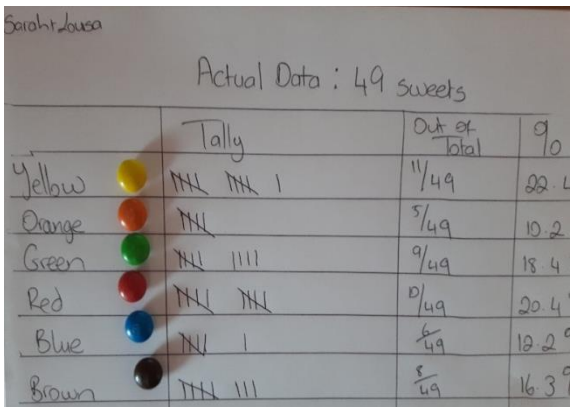
Actual Data			
	Tally	**Out of Total**	**Percentage**
Yellow			
Orange			
Green			
Red			
Blue			
Brown			

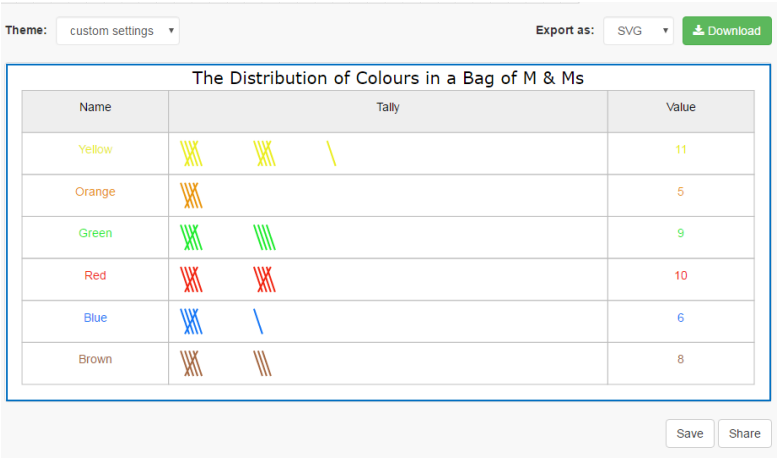
Compare your predictions with the actual data. What did you notice? Can you explain any differences? Which is the most popular colour of your sample? Which is the least popular colour in your sample? How could we check using a larger sample? Are the results from the class sample the same the smaller samples? Can you think of a reason for this? What questions do you now have about using samples?

5. As a class, construct a tally chart using the entire class' data set. Pupils could show the results of both data sets using frequency tables.
6. Further activities with sweets could include:
    - Calculating the average number of M & M's in a bag/ family bag
    - Using different types of data charts to show the distribution of colours e.g. pie chart, bar charts and multiple bar charts.

Pupils could also use ICT to create tally charts & frequency table. The tally chart here was created using the website www.meta-chart.com[141]







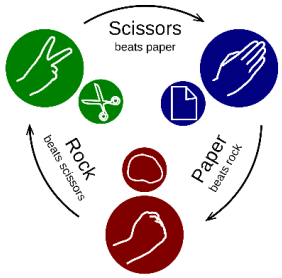
ICT
Meta-Chart

Rock Paper Scissors

Although Rock-Paper-Scissors is a random game, much research has been done on strategies to win. Although the game is random, people are not usually random and can be analysed. More information on strategies and running Rock Paper Scissors tournaments can be found on http://worldrps.com/advanced-rps/sample-page/

[141] www.meta-chart.com

Pupils will need to be familiar with the simple game of Rock-Paper-Scissors. Ask a pair of pupils to play the game 2/3 times in front of the class before posing the question.

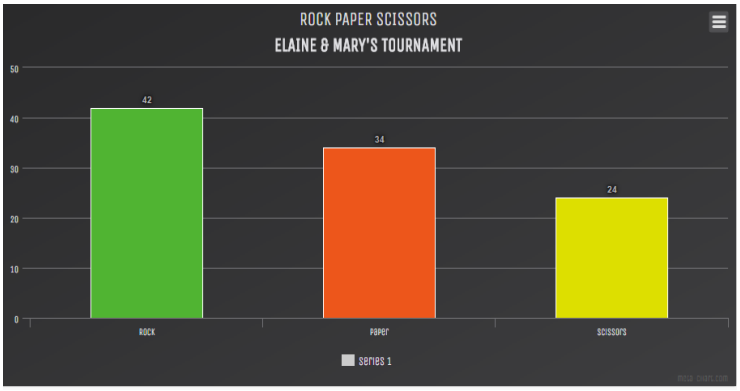


Is Rock-Paper-Scissors a fair game? Why/why not? What are the chances of winning?

Pupils in pairs play the game 100 times, recording what gesture each used and identifying the winner each time. Pupils will now consider all possible outcomes of playing the game. How many different possible outcomes can they find? Pupils could use a frequency table to record the outcomes of their game before transferring this data to a graph for analysis and discussion.

What was the most common gesture you used? What was the most common gesture you used? Which combination seemed to 'win' most often?

142



[142] This graph was created using the website https://www.meta-chart.com

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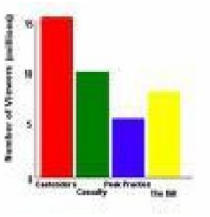
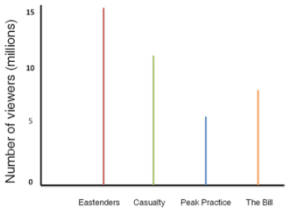
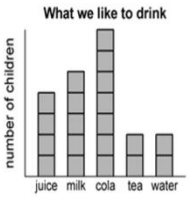
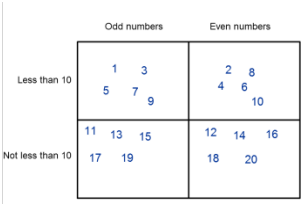
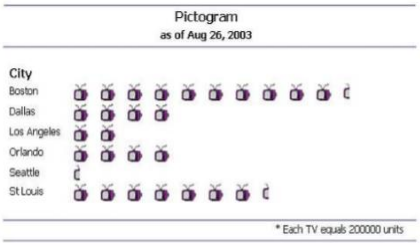
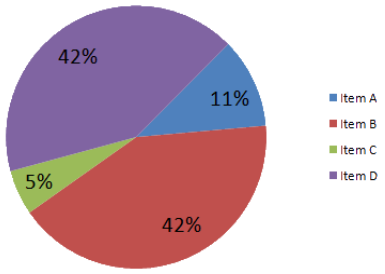
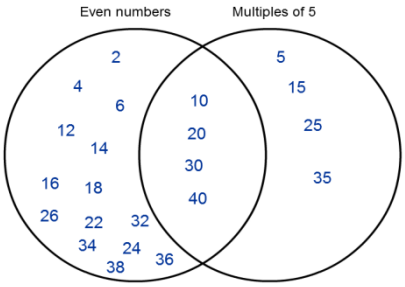
GLOSSARY OF DATA TERMS

TERM	DEFINITION
Average	The calculated 'central' value of a set of numbers. Also known as the 'mean'. To calculate, add up all the number, then divide by how many numbers there were.
Bar Chart	This is a diagram used to display data in rectangular bars. It is used to summarise and display information in a diagram. In general there should be spaces between the columns. 
Bar-line Graph	This is a way to show and compare data by using horizontal or vertical lines. The bars in a bar chart are simply replaced by straight lines. 
Block Graph	This is an introductory way of representing discrete (separate) data, in which each member of the population is represented by an individual square. In general there should be spaces between the columns. 
Carroll Diagram	A sorting diagram named after Lewis Carroll mathematician and author of "Alice in Wonderland". A Carroll diagram is used to sort items according to the attributes of two or more categories. The numbers or objects are categorised as belonging or not belonging to the set[143]. 
Chance	This is measurement that applies to events. Chance does not tell you what will happen next but predicts what will happen in the long run.
Data	This is information. Data handling involves practice in questioning, collecting information, analysing and recording or representing data visually using some form of a graph or table.

[143] www.nrich.mathematics.org/5728

GLOSSARY

<table>
<tr><td>Experiment</td><td>This is an activity which allows information/data to be collected and recorded (often called the results of the experiment).</td></tr>
<tr><td>Frequency</td><td>This is the number of times an event occurs in an experiment. Frequencies are often summarised in a table or a histogram. Example: in nine soccer matches played on a school pitch during a tournament the number of goals scored was recorded as 0, 1, 1, 0, 2, 2, 0, 2, 0. This information could be summarised in a frequency table.
<table>
<tr><td>Number of goals</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>Frequency</td><td>4</td><td>2</td><td>3</td></tr>
</table></td></tr>
<tr><td>Mean</td><td>This is the simple average of a given set of data.
The mean of 8,7,12,0, 3 = 8 +7 +12 +0 +3 = 30 ÷5 = 6</td></tr>
<tr><td>Median</td><td>This is the middle value (or two values) of a set of data arranged in order. Example: 18, 3, 7, 8, 16, 2, 3 becomes 2, 3, 3, 7, 8, 16, 18 and 7 is the median.</td></tr>
<tr><td>Mode</td><td>This is the most commonly occurring value in a set of data. Example:12, 34 , 25,17, 34, 56,12, 67, 43, 68, 93, 34, 33, 21, 25 the mode is 34</td></tr>
<tr><td>Outcome</td><td>This is the result of an experiment. Example: Roll a die as an experiment and the outcome is a number between 1 and 6.
  </td></tr>
<tr><td>Pictogram</td><td>This is a way of representing discrete (separate) data, in which each member of the population is represented by an individual picture or icon arranged in rows or columns.
 
</td></tr>
</table>

Pie Chart	This is a diagram in the shape of a circle or disc that is used to represent data. The 360° of the disc is divided in ratio into pieces of the pie. 
Probability	This is the study of chance; its value varies between 0 and 1. Example: The probability of a fair coin landing on heads = 0.5. 
Survey	This is a method of collecting data often by asking questions of a population or a sample of a population.
Tally	This is made by recording a series of single strokes. Usually every fifth stroke is a bar to the other four for easy counting. 
Venn Diagram	A diagram using circles or other shapes, to show the relationship between sets. [144] 

www.amathematicsdictionaryforkids.com provides highly visual, interactive definitions on over 650 common mathematical terms.

[144] www.nrich.mathematics.org/6290

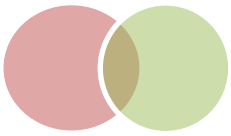
GLOSSARY

Click the image to access the draft primary mathematical glossary designed by the NCCA to provide clarity of mathematical terms used in the Mathematics Curriculum. It is not a mathematical dictionary, but is written in 'plain English'

GUIDE TO DATA HEADINGS AND SYMBOLS

Symbol	Explanation
TEACHING NOTES	These notes provide essential pedagogical knowledge on conceptual development of the specific objective. Information on the mathematical content of the objective may also be contained in these notes. These notes precede the *Sample Learning Experiences.*
SAMPLE LEARNING EXPERIENCES	These are child-centred activities designed to develop conceptual understanding. A selection of sample learning experiences are outlined for each objective in the learning trajectory. This is not intended to be an exhaustive list of learning experiences. These learning experiences are colour coded according to the levels: **Level A** **Level B** **Level C** **Level D**
CONSOLIDATION ACTIVITIES	These consolidation activities follow on from the sample learning experiences. There are no new concepts introduced during these activities, rather, their purpose is to support and consolidate pupil understanding.
	This symbol signals important points to be aware of when teaching a particular concept.
	This symbol denotes that a learning experience can be extended to challenge and deepen pupils' problem solving, understanding and reasoning.
	The speech bubble contains possible language that can be used by the teacher when teaching a particular concept. This language is intended to activate and extend pupils' mathematical thinking. For more information on Teacher Questioning please see the 'Instructional Framework' in this manual. The *NCCA Primary Assessment Guidelines* also have a section on Teacher Questioning.
	This is an interactive activity which is hyperlinked and so can be accessed online by clicking on the icon. The website address for each interactive activity is provided as a footnote, so that teachers using the hard copies of this Measures manual, can also access these activities.

GLOSSARY

	This icon highlights the opportunity for linkage or integration with another strand/strand unit or subject.
	This icon highlights opportunities for assessing pupil understanding. For more information on assessment, please see the *NCCA Primary Assessment Guidelines*.

APPENDICES

Helping Your Child Understand Data

Parents play a strong role in supporting their children's learning. The following are a list of ideas and activities that may help develop your child's understanding of Data and Chance in the world around them. The following suggestions are based on ideas from The National Council of Teachers of Mathematics (NCTM)[145] and The National Council for Curriculum and Assessment (NCCA)[146]

Data:

1. Practise sorting and classifying objects by separating toys into sets e.g. cars, blocks and action figures. Ask questions such as 'How could we sort these toys? Which one doesn't belong in this set? Why do you think so?'

2. Ask your child to help sort the laundry. Children might decide to sort by type of clothing or by family member or by gender. Encourage your child to discuss their reasons for sorting.

3. Encourage your child to keep organise information and keep track of data using charts and graphs. For example, if your child is saving for a new toy or bike, they could keep track of their money saved using a pie chart or graph.

4. Statistics and data feature every prominently in sports. Encourage your child to keep track of their favourite sportsperson or team and associated statistics. Children may collect data from the newspaper or internet and use it to keep track of points or goals scored over a season.

5. Discuss charts, graphs and tables from newspapers and magazines with your child. Encourage your child to analyse the data by asking, 'What does this data tell us?' and 'What does this data not tell us?'

6. Encourage children to collect, organise and record personal data such as how much time they spend watching television during the week, how many servings of vegetables they ate etc. At the end of the week, encourage your child to discuss what findings they can draw from data they collected and represented.

[145] http://illuminations.nctm.org/uploadedfiles/activities_home/familyguide_fulltext.pdf
[146] www.ncca.ie

Chance:

1. What words are useful in describing how likely something is to occur? Ask your child to draw pictures of things your family does often, things you do sometimes, and things you never do. Discuss why you never do some things, for example, swim outside in January. Ask your child if it is likely to rain today.

2. When playing games e.g. Snakes and Ladders, ask your child are they more or less likely to get certain numbers e.g. 1 or 6 on a dice. Many children have preconceived ideas about how likely or unlikely they are to get a certain number, whereas the chances of getting a 6 are equal to getting a 1.

3. When a coin is flipped, which is more likely, heads or tails? Ask your child to predict whether a coin will show a head or a tail when it is flipped. Together, record your results over 10 flips, and compare the results with the prediction. Then flip the coin 10 more times. Are the results the same? Would they be the same if you flipped the coins 100 more times?[147]

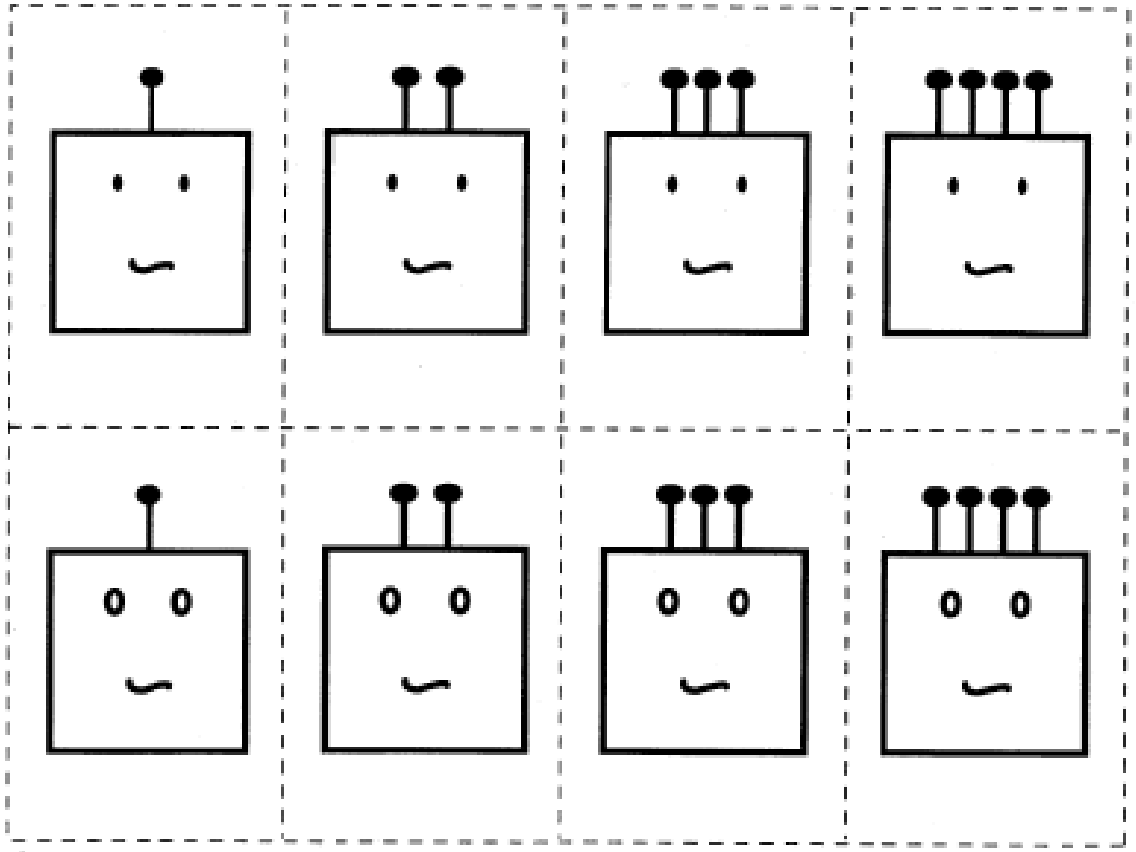
4. Hold a discussion with your child about how they make predictions every day. How are predictions useful? How might people be able to forecast the weather? What clues do they use to make their predictions? Give your child the opportunity to make a prediction and test it.

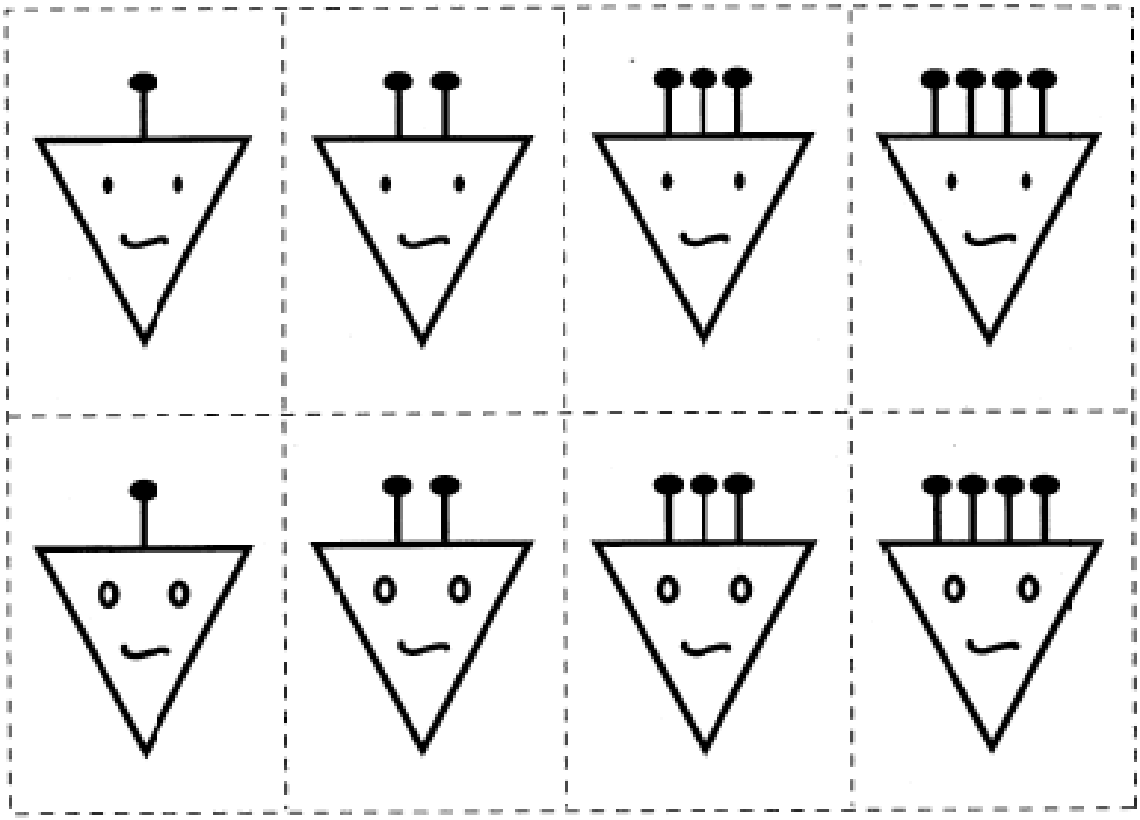
Appendix Level B.2

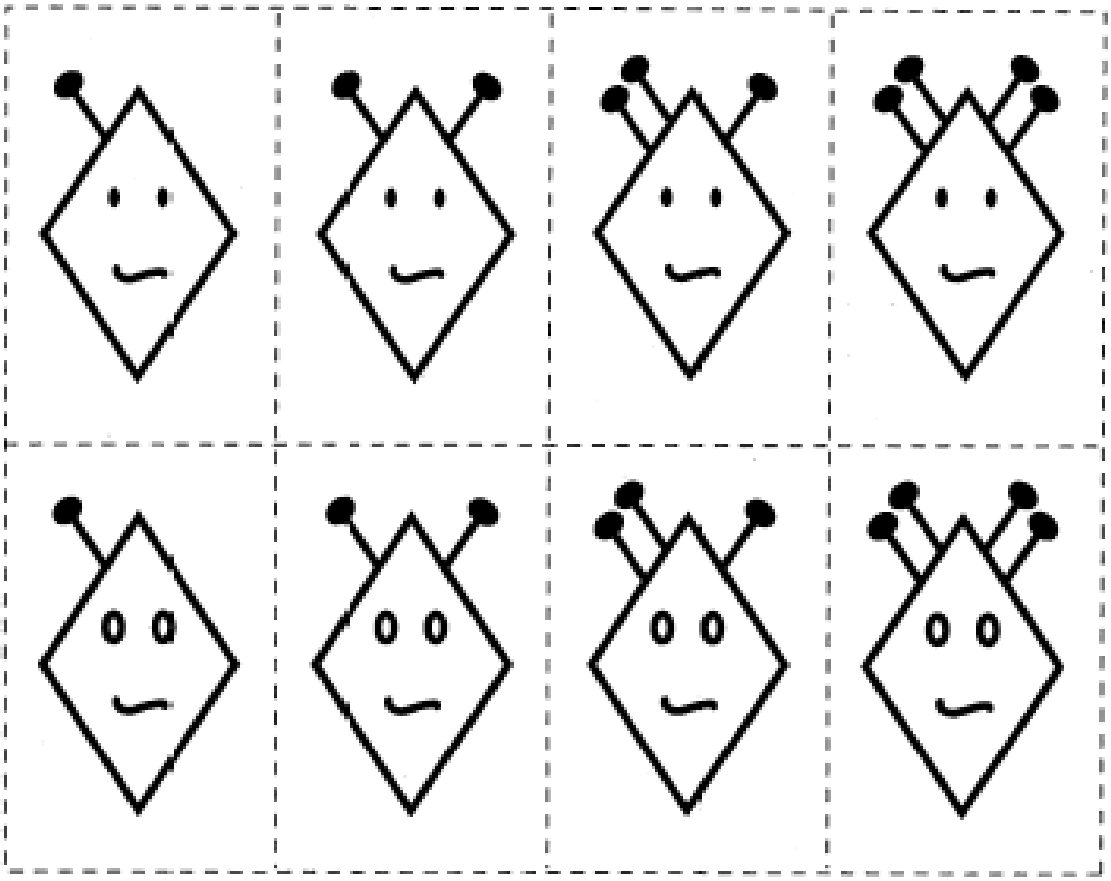
Yektee Attribute Cards

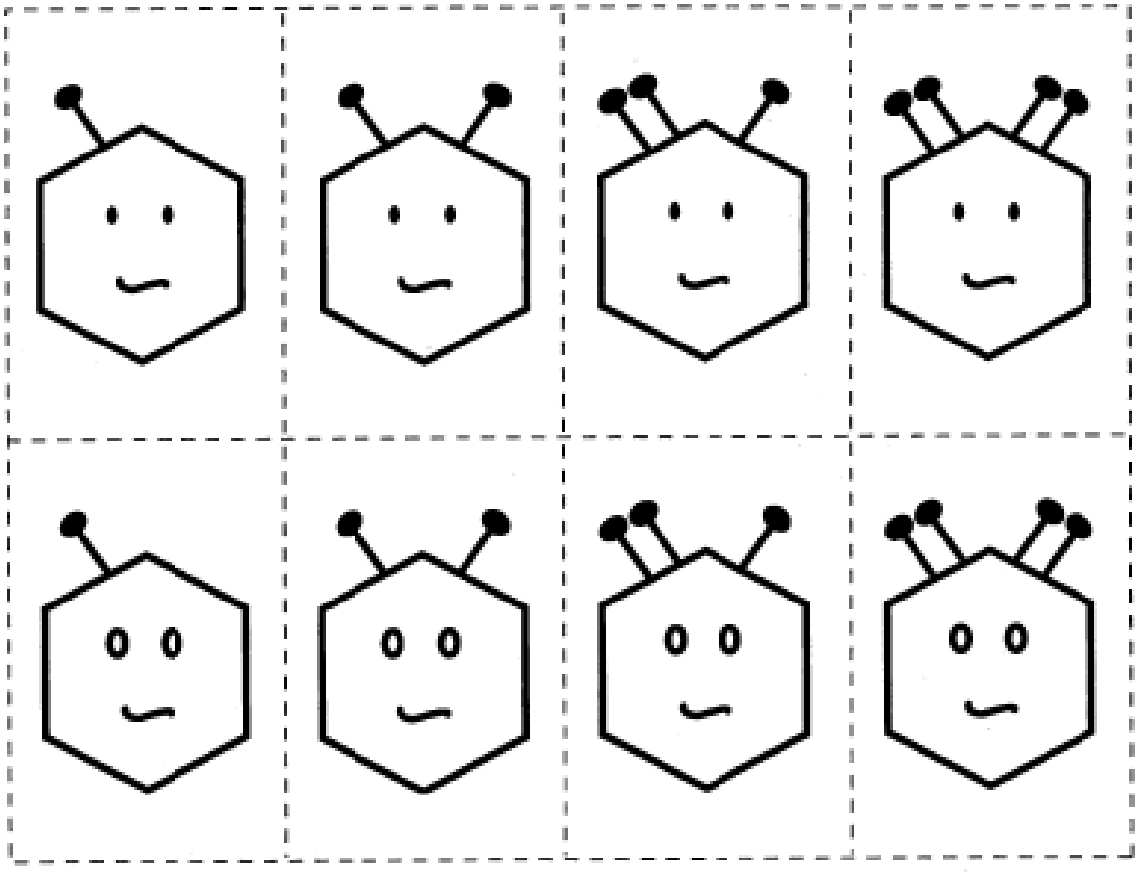
square	triangle
rhombus	hexagon
plain eyes	ringed eyes
1 antenna	2 antenna
3 antenna	4 antenna

Small Yektee Cards

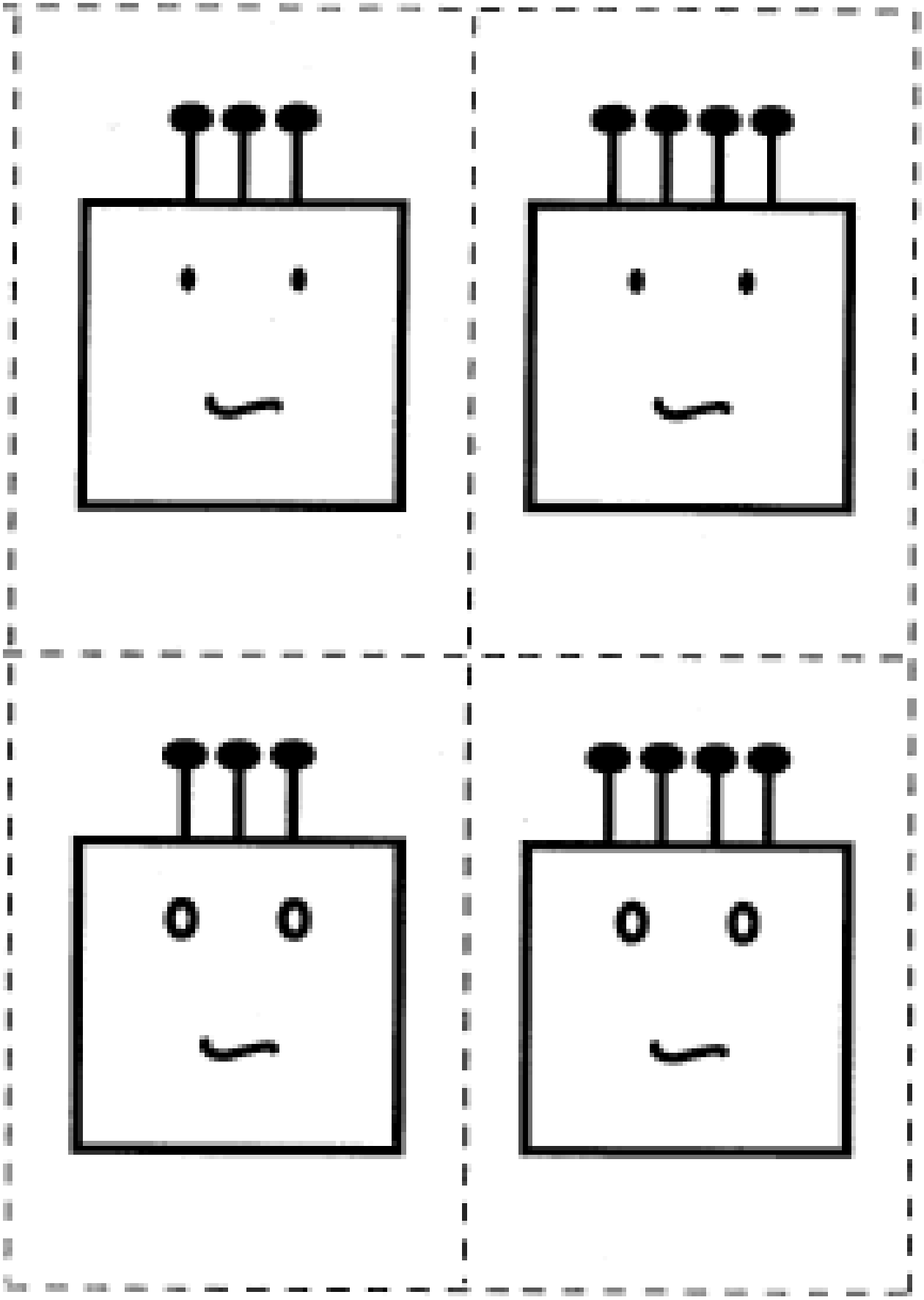


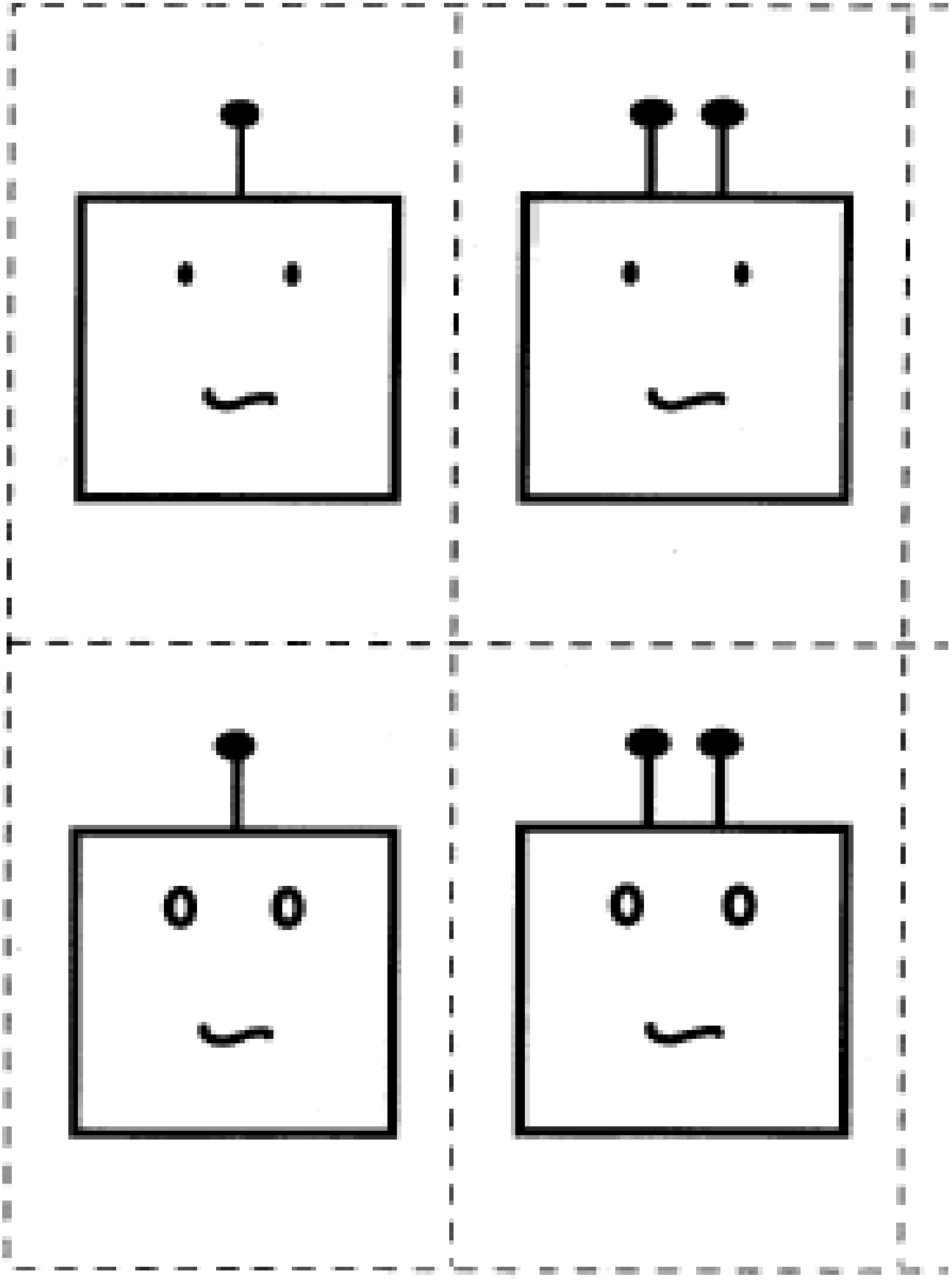


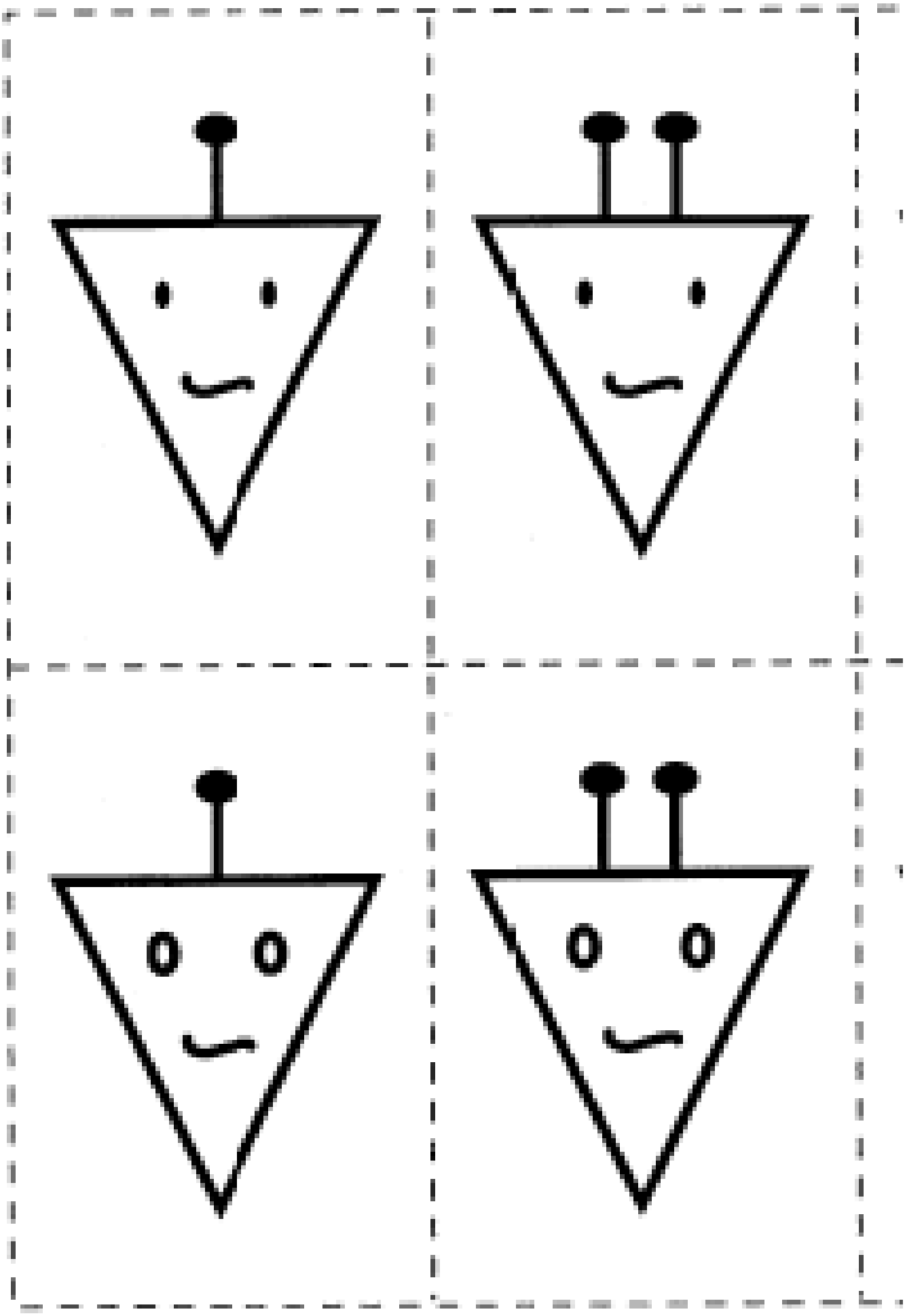


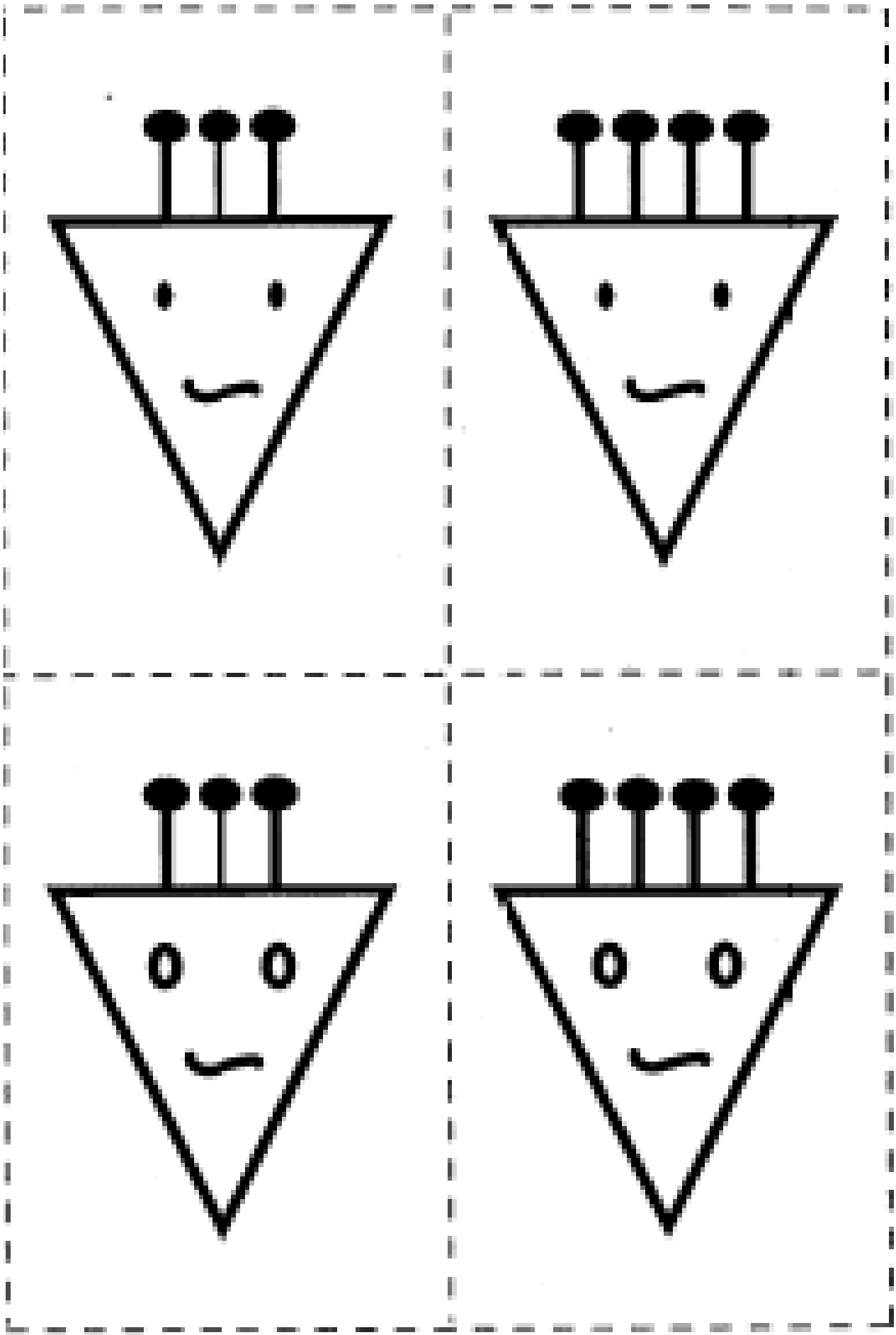


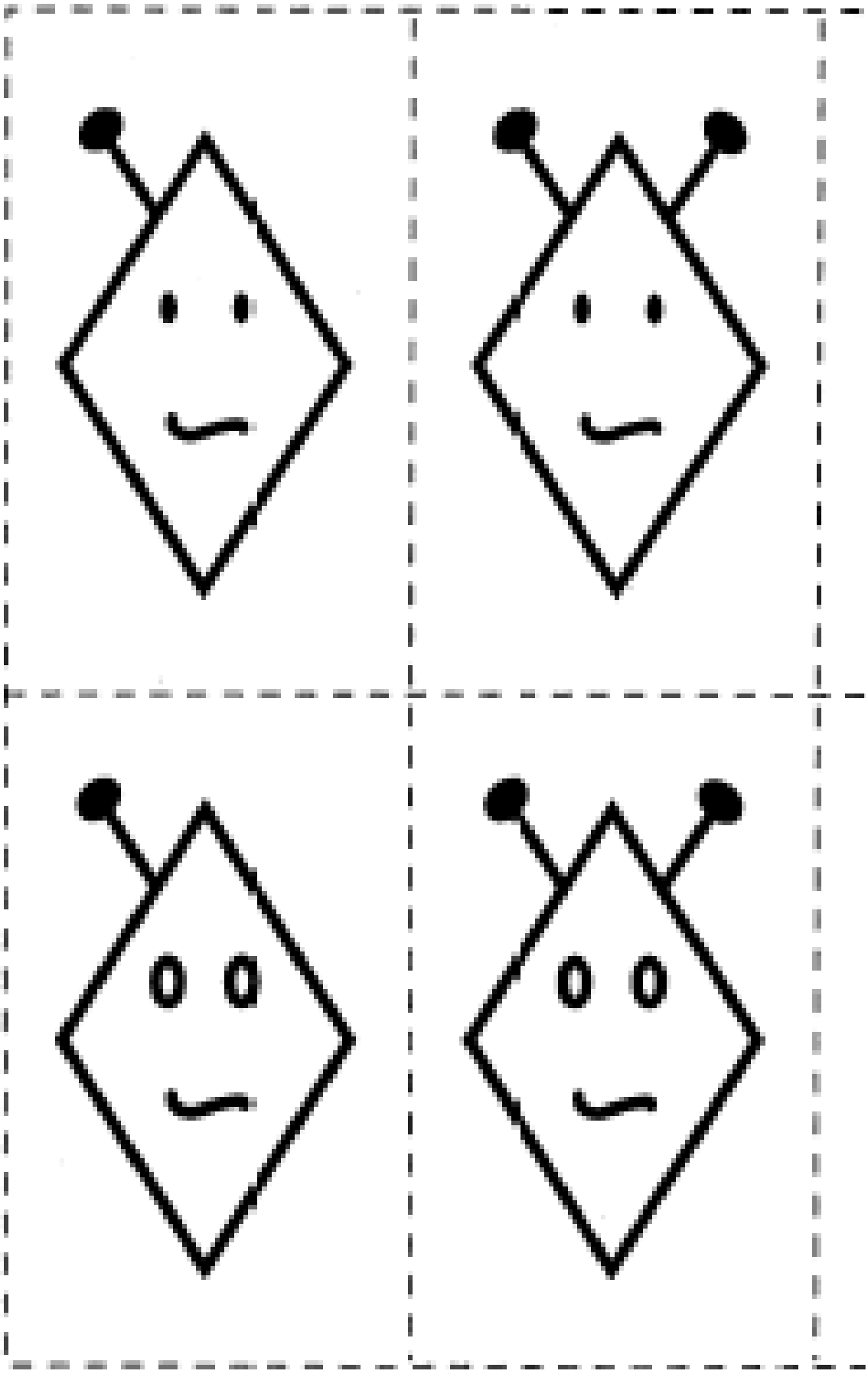
Large Yektee Cards

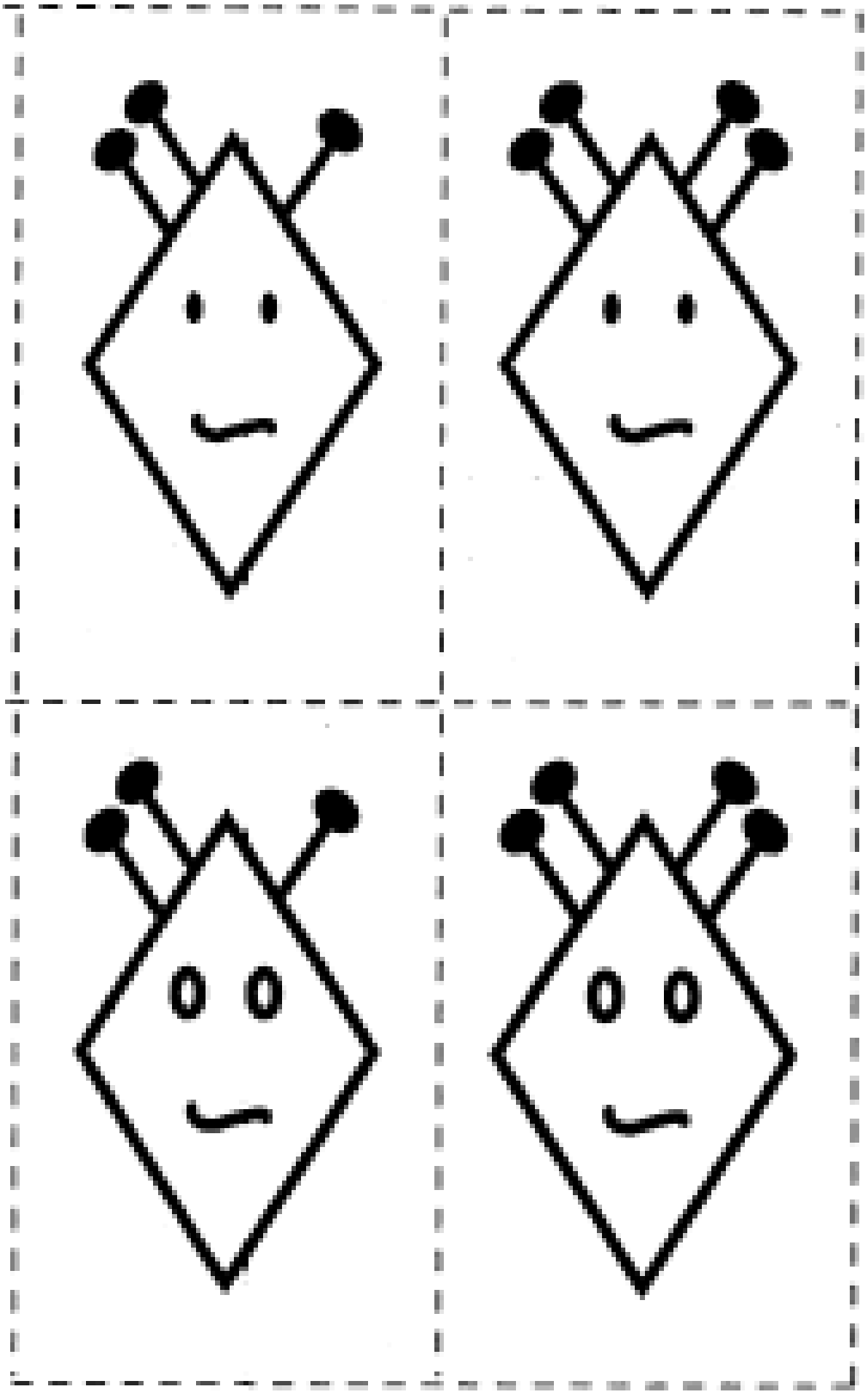


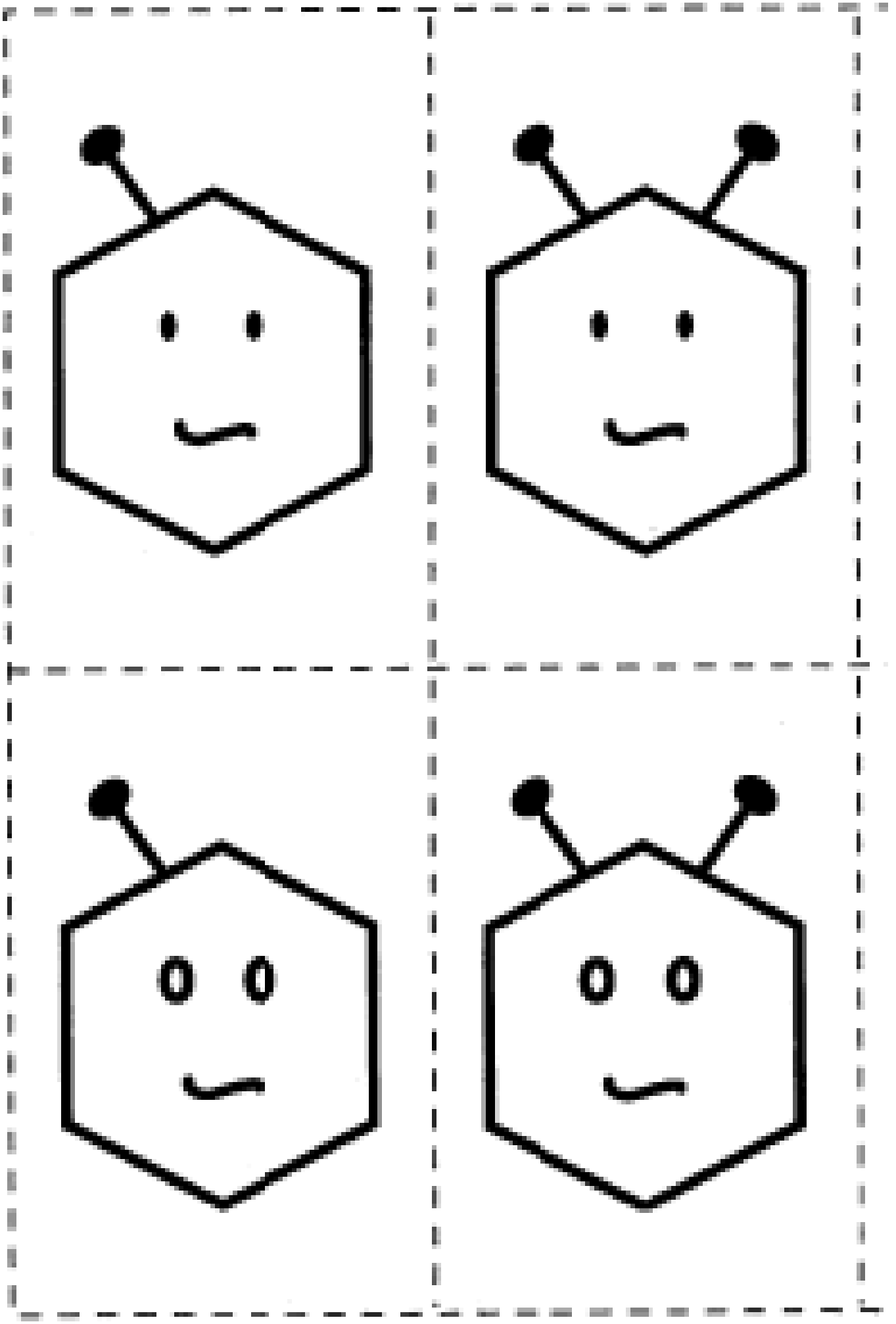


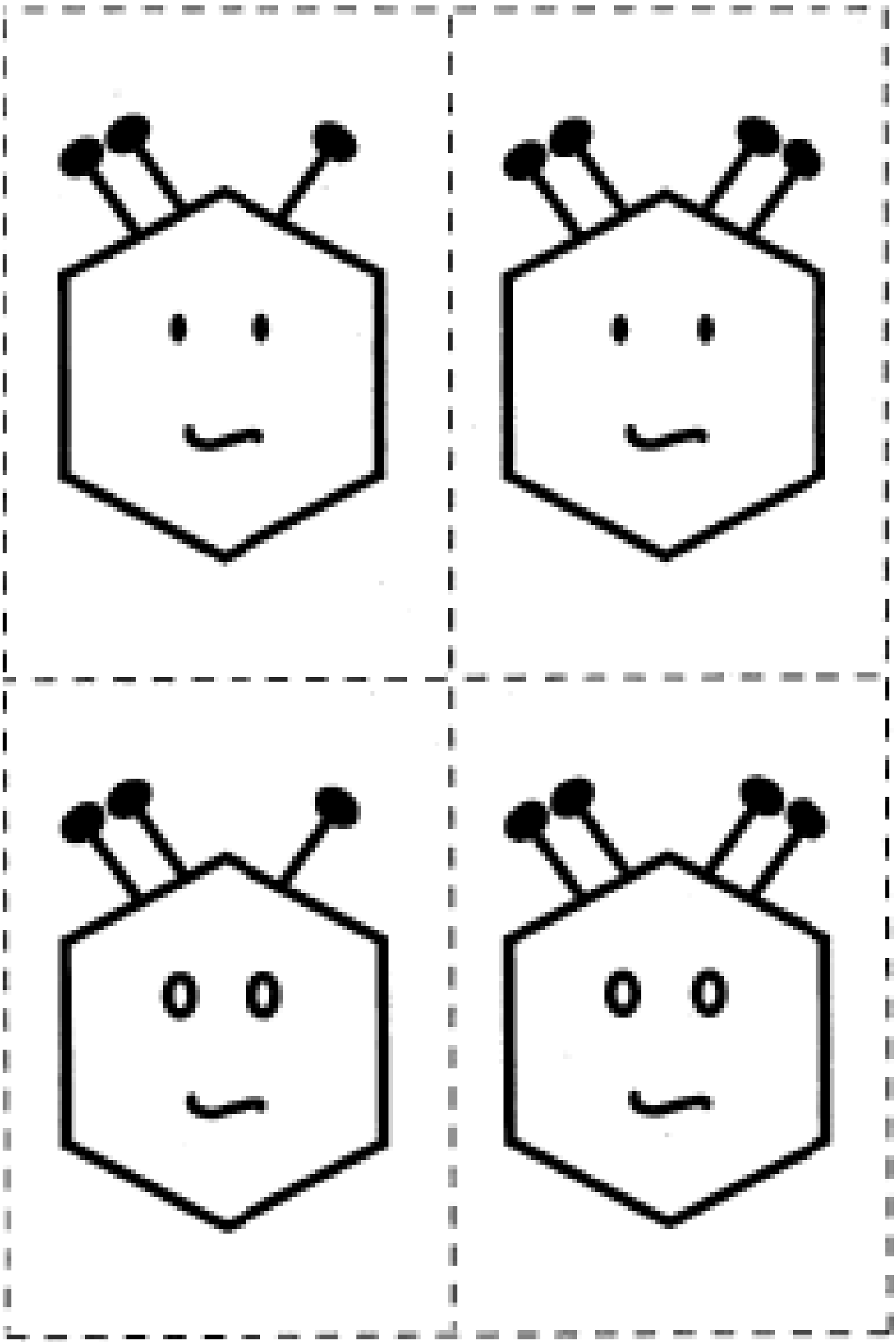












Appendix Level B.3 Grab and Graph

Grab a handful of pattern blocks. Sort them. Now tally them on the tally chart.

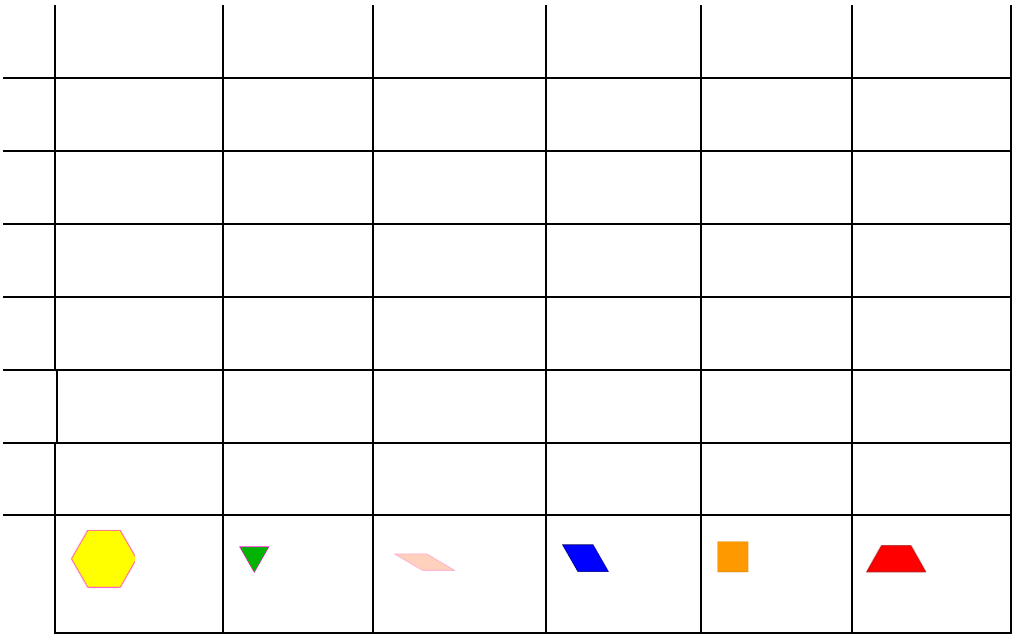
	Tally	Total
		
		
		
		
		

What do you notice?

What do you wonder about the tally chart you created?

Could you make up a question about the chart?

Could you create a block graph to record your findings?

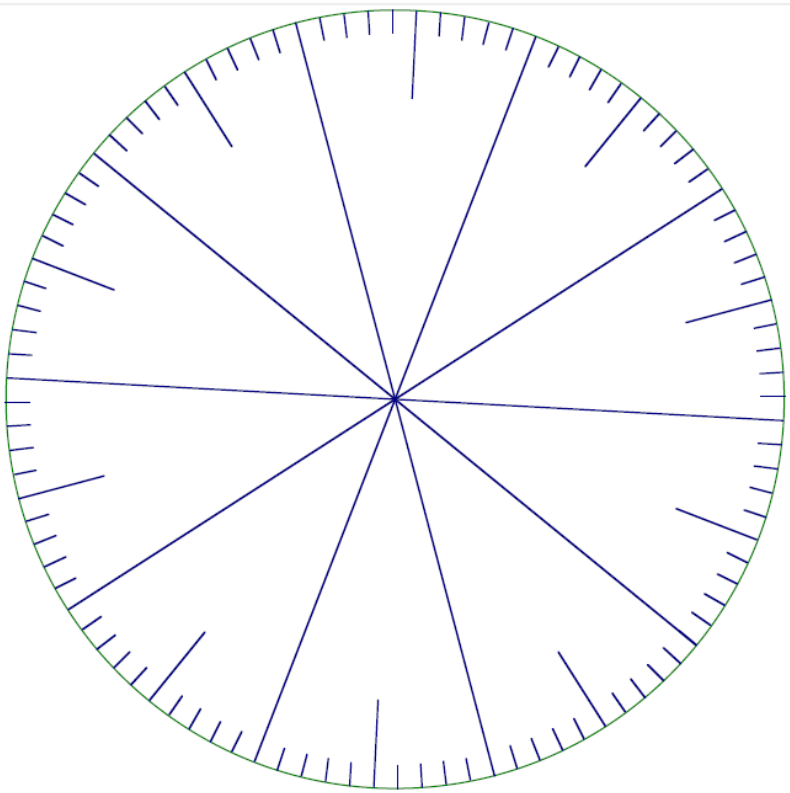


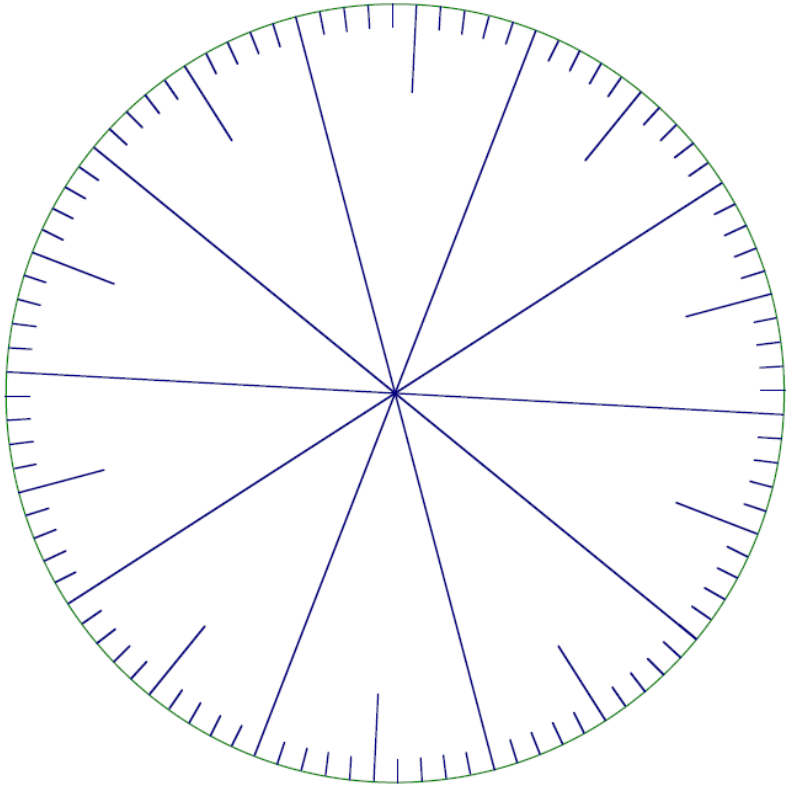
Compare your graph with your partner's graph.

What shape do you have the most of? What shape do you have the least of? How many shapes do you have altogether? What is the difference between the quantity of triangles you have and the quantity your partner has?

Create a question for you graph, can your partner answer it?

Appendix Level D.2 Rational Wheel





Appendix Level D.4 Average Facts

Average Facts

The average person falls asleep in 7 minutes.

The average human brain contains around 5% water.

An average person will spend 25 years asleep.

The average hen lays 100 eggs a year.

The average person goes to the toilet 6 times a day.

The average porcupine has 10,000 spikes.

The average lifespan of a squirrel is 9 years.

The average person eats 5 tons of a food in a lifetime.

Average Facts

An average human heart will have beat approximately 1.5. Billion times when it reaches the age of 66 years.

On average cats sleep 16 hours a day.

On average pigs live for about 15 years.

The average cow produces 5 glasses of milk each day.

15: The average number of litres an elephant's trunk can hold.

100: The average number of hairs in a person's eyebrow.

The average human head weights about 8 pounds.

The average person will drink 70,000 cups of coffee in a lifetime.

The average person falls asleep in 7 minutes: True	The average human brain contains around 5% water: False: 78%	An average person will spend 25 years asleep: True	The average hen lays 100 eggs a year: False: 228
The average person goes to the toilet 6 times a day: True	The average porcupine has 10,000 spikes: False: 30,000	The average lifetime of a squirrel is 9 years: True	The average person eats 5 tons of food in a lifetime: False: 35 tonne
An average human heart will have beat approximately 1.5. billion times when it reaches the age of 66 years: False: 2.5 Billion Times	On average cats sleep 16 hours a day. True	The average cow produces 5 glasses of milk each day: False: 15	15: The average number of litres an elephant's trunk can hold: False: 9
100: The average number of hairs in a person's eyebrow: False: 450	The average human head weights about 8 pounds: True	The average person will drink 70,000 cups of coffee in a lifetime: True	On average pigs live for about 15 years: True

http://www.free-for-kids.com/True-or-False-Quiz.pdf

http://www.did-you-knows.com/did-you-know-facts/average.php

http://www.express.co.uk/news/weird/437344/Average-human-grows-590-miles-of-hair-and-eats-35-tons-of-food-AMAZING-human-stats

Appendix Level D.4 Finding Averages

Finding Averages

- Agree on a dealer and a score keeper
- The dealer should remove all the Jacks, Queens, Kings & Jokers from the pack and then shuffle
- The score keeper should create a score board

Round 1:

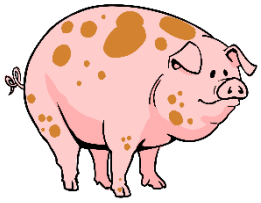
- The dealer deals 7 cards to each player. Each player works out the Mode for their 7 cards
- Each player gets the score of their mode. If a player has no mode, they get 0 points. Repeat, after collecting in the cards & shuffling

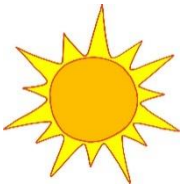
Round 2:

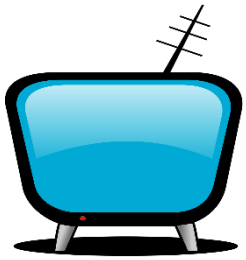
- The dealer deals **5** cards to each player. Each player works out the **Mean** for their **5** cards
- Each player gets the **score** of their **Mean**. Repeat, after collecting in the cards & shuffling

Round 3:

- The dealer deals **7** cards to each player. Each player works out the **Range** for their **7** cards
- Each player gets the **score** of their **Range**. Repeat, after collecting in the cards & shuffling

You throw a dice once and it lands 	***You will receive a present next Christmas.*** 	***School will be closed tomorrow.*** 
We will see a pig fly in the sky 	***You will get a goldfish this weekend.*** 	***You will be a millionaire when you grow up.*** 
You will be in 1st class next year 	***You will go swimming during the holidays.*** 	***There will be an ice cream truck at the school tomorrow.*** 

It will snow in the Sahara  ***Desert.***	***Fish will rain from the sky*** 	***You will eat your dinner at home this evening.*** 
If you roll a dice, you will get a six. 	***You will go to London this weekend.*** 	***You will get a new book for your birthday.*** 
A pride of lions will wander down the street. 	***You will see the sun in the sky at midnight.*** 	***An alien will visit our***  ***classroom.***

You will watch television after school. 	***You will eat pizza for dinner.*** 	***You will walk to school***  ***tomorrow.***
You will see a live tiger after school. 	***You will do all your homework this evening.*** 	***The President of Ireland will visit this week.***